Is ‘Knowing that P’ Identical with ‘Knowing that “P” Is True’?

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Received: 2 May 2019 / Revised: 17 July 2019 / Accepted: 22 July 2019 /

Published online: 9 August 2019

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Abstract

It is epistemological orthodoxy that the object of propositional knowledge is the truth of propositions. This traditional view is based on what I call the ‘KT-schema’, viz, ‘S knows that p , *iff*, S knows that “ p ” is true’. The purpose of this paper is to reject the *KT-schema*. By showing the paradoxical upshot of the *KT-schema* and providing counter-examples to the *KT-schema*, this paper argues that ‘knowing that p ’ is more than ‘knowing that “ p ” is true’. Consequently, we shall rethink the object problem of propositional knowledge – if knowing that p is not merely knowing that “ p ” is true, then what is indeed the object of propositional knowledge? I will also attempt to solve this problem by proposing a complementary answer: knowing that p requires at least knowing the truth of p , plus, understanding the content of p .

Keywords T-schema · The object of propositional knowledge · Knower paradox · Understanding

Knowledge has its object. The object of a piece of knowledge-who can be a person (or propositions regarding the person). The object of a piece of knowledge-where can be a location (or propositions regarding the location). But what is the object of propositional knowledge? To be more specific, when we claim to know a proposition p , what is known?

A traditional answer that I take to be platitudinous is: the truth of p is known. The object of an item of propositional knowledge is the truth of that proposition. This traditional view can be motivated by the widely-accepted T-schema (see Tarski 1944, 1983; Dummett 1959):

[T-schema] p , *iff*, ‘ p ’ is true.

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Given the T-schema, to say that ‘snow is white’ is equivalent to saying that “‘snow is white’ is true”. Accordingly, it is natural to infer that ‘know that p ’ is equivalent to ‘know that “ p ” is true’ by simply adding the ‘K’ predicate. Hence we have:

[KT-schema] S knows p , *iff*, S knows that ‘ p ’ is true.

The purpose of this paper is to disprove the KT-schema. The paradoxical upshot of the KT-schema will be revealed. Moreover, counterexamples to the KT-schema will be provided. A conclusion that I will draw is: knowing that p is more than knowing that ‘ p ’ is true. Consequently, we shall rethink the object problem of propositional knowledge – if knowing that p is not merely knowing that ‘ p ’ is true, then what is indeed the object of propositional knowledge? I will try to carve out a potential complementary answer – knowing that p requires (at least) knowing the truth of p , plus, understanding the content of p .

This paper will proceed as follows. In Section 1, I will introduce how the KT-schema is deeply entrenched in standard epistemological thinking and why it is intuitively appealing. Section 2 will construct a dilemma, which shows that the KT-schema will give rise to a contradiction to the effect that every unknown true proposition would be both known and unknown. The failure of the KT-schema would be further exposed by some counterexamples in Section 3. Section 4 will address some possible objections. The effect of rejecting the KT-schema will be discussed in Section 5.

1 KT-Schema

Albeit highly intuitively plausible, the KT-schema has rarely been systematically proved by epistemologists – perhaps because the KT-schema is taken to be too uncontroversial. Nonetheless, the deflationary understanding of propositional knowledge behind the KT-schema is widely reflected in epistemologists’ interchangeable uses of ‘know that p ’ and ‘know that p is true’. For example, Dretske (2014) uses ‘knows Q ’ and ‘knows that Q is true’ interchangeably when defining the closure principle (pp. 27–28). Neta (2002) also deemed ‘knows the truth of some proposition p ’ and ‘knows that p ’ as identical (p. 663). Likewise, JC Beall interprets ‘ k is known’ and ‘ k is known to be true’ convertibly when characterising the knower paradox (see Beall 2000, p. 243). Plenty of examples can be found elsewhere.¹

Also, the KT-schema is embodied in some influential epistemological theories. Take epistemic absolutism (the view that propositional knowledge is not gradable) as an example. Hetherington argues that the ‘strongest form of defence’ for epistemic absolutism is based on the idea that ‘to know that p is to know the truth of the proposition p ’ (Hetherington 2005, p. 148). Accordingly, knowledge-that seems to be absolute, in the sense that the truth of a proposition cannot be known more or less – it can only be known as an absolute whole. Thus, epistemic absolutism can arguably be built on the basis of the KT-schema. Another example is the truth-tracking theory of knowledge (see Nozick 1981; Dretske 1971; Goldman 1967; etc.), which claims that to

¹ I cannot exhaust all relevant instances here. For more data, readers are invited to refer to Moser (1987:91), Moffett (2003:82), Hetherington (2005:148), Jespersen (2008:125), etc.

know that p is to track the truth of p . With the KT-schema in play, it would be easy to comprehend tracking theorists' emphasis on the truth, as tracking the truth is tracking the object of knowledge. Notice that I do not mean to say those theories *have to be* based on the KT-schema – instead, my purpose is just to show how the KT-schema can serve as the underlying intuition that helps to motivate those epistemological theories.

So why is the KT-schema intuitive? I believe that one main reason is the appeal of the T-schema. Apart from the naïve 'adding K-predicate' approach stated above, another way to demonstrate the KT-schema is to combine the T-schema with the closure principle. The naïve unrestricted version of closure predicts that if a subject S knows that p , and that p entails that q , then S knows that q . Thus we can have:

- (i) If S knows that p and that p implies q , then S knows that q .
- (ii) S knows that [$p \leftrightarrow$ ' p ' is true].
- (iii) [S knows that p] $\leftrightarrow$ [S knows that ' p ' is true].

We will return to this *prima facie* plausible argument for the KT-schema in Section 4. Now we have seen how and why the KT-schema is apparently appealing. As a biconditional proposition, the KT-schema can be falsified by rejecting either direction of entailment. Here, I would grant that [knowing that p] implies that [knowing that ' p ' is true]. Instead, this paper will argue that [knowing that ' p ' is true] does not always suffice to imply that [knowing that p]. To achieve this, I will take two steps: (i) showing the unacceptable upshot that the KT-schema will result in; (ii) providing counterexamples where the subject knows a proposition to be true, whilst fails to know that proposition. I will take the first step by constructing a dilemma for the KT-schema.

2 Dilemma for the KT-Schema

To begin with, let me state some background assumptions of my argument. First, I take it to be relatively uncontroversial that there are *unknown* truths.² Second, I assume that our knowledge is consistent. To be more specific, one cannot know both p and the negation of p – only true proposition can be known. Also, a proposition cannot be both known and unknown to a subject – a proposition is either known or unknown. Readers who disagree with my assumptions are invited to deem the conclusion of this paper as a conditional: *if* knowledge does not accept that there are true contradictions, *then* the KT-schema is false.

Let us turn to the main argument. Consider the following proposition Φ :

(Φ) Σ is an unknown truth.

Let Σ represent any unknown true proposition so that Φ is true. Given our aforementioned assumption that there are unknown truths, it is possible that there exists at least a

² Please note that I am not advocating an anti-verificationist view and assuming that there are *unknowable* truths. Rather, I just assume that there are true propositions that are not yet *known* by us.

proposition Σ that can make Φ true. *Qua* a proposition, Φ is either known or unknown. By supposing that Φ is known and unknown respectively, we can see how the KT-schema will encounter a dilemma as follows:

2.1 First Horn

Suppose (for *reductio*) that Φ is known (by at least one subject). If one knows that Φ , then one knows that Σ is an unknown truth. That means two things: (i) one knows that the proposition denoted by Σ is not known; (ii) one knows that *the proposition denoted by Σ is true* – in short, one *knows that Σ is true*. By the KT-schema, knowing that a proposition is true means knowing that proposition. Consequently, by (ii), one *knows that Σ* . However, if one *knows that Σ* , then Φ would turn out to be a false proposition – since according to Φ , Σ is unknown. No false proposition can be known. Hence one cannot know that Φ , which contradicts our assumption. The KT-schema thus leads to inconsistency. Precisely put, the inconsistency occurs because:

1. A subject S knows that Φ , viz, S knows that Σ is an unknown truth. [Assume for *reductio*]
2. 2. S knows that: 1) the proposition denoted by Σ is unknown; 2) the proposition denoted by Σ is true. [From 1]
3. The KT-schema.
4. S knows the proposition denoted by Σ – in short, Σ is known. [From 2.2 and 3]
5. 5. Φ is false. [From 4]
6. No false proposition can be known.
7. S does not know that Φ .

The contradiction between 1 and 7 is obvious. However, one might worry whether the statement that ‘one knows that Σ is an unknown truth’ is consistent. If this statement is inconsistent, then the primary assumption of my argument is untenable. However, if this concern is of any intuitive appeal, then what can the inconsistency be? Is it inconsistent to claim that we know a proposition to be of a certain property, while not knowing the proposition *per se*? No. I take the following statements to be consistent:

- a. ‘We all know that the proposition expressed by the longest sentence in the world has its truth value, but we just do not know what the proposition is.’
- b. ‘I know that the philosophical proposition you were talking about is famous, but sorry I just don’t know that proposition as I am not familiar with the relevant literature.’
- c. ‘I know that the last proposition of this book is written by you, but I do not know that proposition as I haven’t read it yet.’

All of these statements involve ‘knowing a proposition to be of a certain property’ and ‘not knowing that proposition *per se*’. None of them is self-contradictory. ‘Having a truth value’, ‘being famous’, ‘being written by someone’ are all

properties of a proposition. Those properties of a proposition p can be known without knowing that p .

Likewise, ‘one knows that Σ is an unknown truth’ means that one knows that the proposition denoted by Σ has two properties – first, it is true; second, it is unknown. So, are these two properties so special that they cannot be known without possessing knowledge-that- Σ per se? Let us discuss them respectively.

Is the property ‘being true’ so special, such that one cannot know Σ to be true without knowing that Σ per se? No. As a matter of fact, this specificity of ‘being true’ is exactly what the KT-schema implies – the KT-schema insists that knowing a proposition to be true requires knowing the proposition per se. One cannot defend the KT-schema by using the KT-schema – this is begging the question. The latter part of this paper will provide cases where one knows a proposition *to be true* without knowing that proposition. At this stage, let us move forward to the property of ‘being unknown’.

Is the property ‘being unknown’ so special, such that ‘we know that Σ is an unknown truth’ is inconsistent? Precisely put, can we know the fact that a *specific* proposition is not known by *anyone*? If it is unknown to anyone, how can that proposition be a *specific*³ one? There seems to be a kind of prima facie inconsistency. Nevertheless, this inconsistency is also illusory. There are many ways to make a proposition a *specific* one. Consider the proposition expressed by the longest statement in the world. No one knows what the longest statement in the world is. Nevertheless, the proposition expressed by such a statement is a *specific* proposition. When talking about it, we are talking about *that particular* proposition rather than any other random proposition in general. There is nothing inconsistent in the claim that ‘we know that the longest proposition is not known’. There are many other specific propositions unknown to anyone (yet): the last proposition that will be uttered in the history of the human; the second proposition in *The Guardian* printed on July 1st, 2028; etc. We know, consistently, that they are unknown.⁴

In conclusion, there is no inconsistency in our primary assumption. We can thus plausibly conclude that when assuming Φ to be known, the KT-schema would derive that Φ cannot be known. The KT-schema causes contradiction on the first horn of the dilemma.

2.2 Second Horn

Assume (for *reductio*) that Φ is not known (by anyone). That is, we do not know that Σ is an unknown truth. Remember that we use Σ to refer to an unknown truth. Hence it is hard to deny that we thereby *know that Φ is true*, now that *we know that Σ* denotes an unknown true proposition. However, by the KT-schema, if *we know that Φ is true*, then *we know that Φ* , which contradicts our assumption that Φ is unknown. To be specific, the contradiction occurs, because:

³ Notice that, by ‘specific’, I do not mean ‘making its content transparent’ or ‘spell the proposition out word by word’ – that is not how ‘specific’ is understood when one takes Σ to be a *specific* proposition. Instead, ‘specific’ can be simply understood as the opposite of ‘general’.

⁴ See Carrara et al. (2019) for a recent discussion on unknown truth, known unknowns, and levels of ignorance.

- 1*. Φ is unknown to anyone. [Assume for *reductio*]
- 2*. We use Σ to denote an unknown true proposition.
- 3*. We know that Σ denotes an unknown true proposition. [From 2*]
- 4*. We know that: 1) The proposition denoted by Σ is true; 2) The proposition denoted by Σ is unknown.
- 5*. In short, we know that Σ is an unknown truth.
- 6*. Φ is known by us. [From 5*]

Clearly, 1* and 6* are incompatible with each other. Hence, again, the KT-schema gives rise to the inconsistency.

One might question the derivation from 2* to 3*. It might be argued that, we just *assume* that Σ is true, rather than *know* that Σ is true. This objection neglects the fact that propositions can be known *by definition*. We define Σ as an unknown truth, and hence by definition, Σ is true – which can be known by us. Similarly, if we use T to denote a tautology, then we can thereby know *by definition* that T is true. Knowledge of propositions (like T and Σ) can be trivial and uninformative. But they are knowledge indeed.

Admittedly, the contradiction between 1* and 6* is not caused by the KT-schema directly. In fact, statement 1* just does not obtain. This means that, strictly speaking, the only considerable scenario is the scenario characterised by the first horn – it has been shown that it is an inconsistent scenario *due to* the KT-schema.

In summary, whether Φ is known or unknown, given the KT-schema, contradictions would occur regardless. If the KT-schema is true then Φ is both known and unknown. Being committed to the KT-schema means being committed to self-contradiction. Another unwelcomed corollary of the KT-schema is that we cannot know that there are unknown truths – if we know that there are true propositions unknown to us, then we know that those propositions are true, by the KT-schema, we thereby know those unknown propositions, which is incoherent. However, it is extremely counterintuitive to deny that we can know the existence of unknown truths. The cost of preserving the KT-schema can be enormous.

2.3 Discussions

If we discard the KT-schema, then the dilemma aforesaid can be avoided. On the one hand, suppose that Φ is known, then it is known that Σ is an unknown truth. Accordingly, we know that Σ is true. Nevertheless, this does not imply that we know that Σ , as the KT-schema is abandoned. That is, statement 4 in our original argument cannot be derived. The first horn of the dilemma is thus disarmed and the contradiction is avoided. On the other hand, the second horn of our dilemma assumes that Φ is unknown to anyone. Given our considerations stated in Section 2.2, it can be concluded that this assumption is untenable, as we can know that Φ by definition. Therefore, the second horn is dissolved. The conclusion is, the putative dilemma disappears. We both know that Φ is true, and Φ per se; whereas, we only know that Σ is true, without knowing Σ per se.

So why does the KT-schema give rise to this dilemma? One explanation might suggest that it is because Σ seemingly only serves as an opaque symbol of propositions, rather than as an actual statement. The semantic content of Σ is undisclosed. If the

content of Σ is articulated, given that Σ is true, it is hard to conceive how Σ can remain unknown. However, since Σ is only a symbol rather than a statement, we do not know the proposition denoted by Σ .

Call this explanation ‘the symbol account’. This explanation is of partial explanatory force, in the sense that it can indeed explain examples as below:

Math Textbook Suppose that your math teacher told you that ‘every proposition in your textbook is true’. Your textbook is classic and well-proofread. Your teacher is reliable. You believe her testimony firmly as there is no proper reason to doubt it. Consequently, your belief constitutes knowledge in that case, *viz*, you know that every proposition in your textbook is true. Accordingly, you also know that the last proposition of your textbook is true. However, you have not read the last proposition of the textbook, so what the proposition expresses is completely unknown to you. And as a matter of fact, that proposition involves a formula that you are completely ignorant of.

That is a case where you know the truth of a proposition without knowing the content of the proposition. In that sense, the symbol account can conclude that the KT-schema overlooks the gap between knowing the truth of p and knowing the content of p when the former is known merely via a symbol of p . However, the symbol account is not a complete explanation for the KT-schema’s failure as it only focuses on cases where the symbol of propositions is involved. In what follows, I will show that even if the truth of p is known via a spelt-out statement (rather than merely a symbol of p), one might still fail to know that p . Hence, the crux does not lie in whether the content of Σ is *disclosed*, but whether it is *understood*. I will argue that [knowing that p] requires understanding the meaning of p at least to a certain extent, while [knowing that ‘ p ’ is true] does not.

3 Counterexamples to the KT-Schema

Counterexamples in this section turn on the idea that knowing that p requires understanding the meaning of p (to a certain extent), while a subject who is (completely) ignorant of the content of a proposition p can still know that ‘ p ’ is true. In other words, one can *know* a proposition, which he does not understand at all, *to be true*; but one cannot *know* a proposition that he completely does not understand. This asymmetry also occurs in cases where the relevant proposition is uttered as a statement rather than just a symbol. Consider the following case:

Foreign Language There is a true Chinese sentence ‘爱丁堡是苏格兰首都’ (which means ‘Edinburgh is the capital of Scotland’) printed in a pub quiz book. A non-Chinese speaker, Jonny, read this sentence but failed to understand it. So he asked his Chinese friend Chen, a reliable man who lived in Scotland, for help. Chen told Jonny that this sentence is true. Jonny believed Chen’s testimony. Then Jonny read the next question in the pub quiz book, which asks whether Glasgow is the capital of Scotland. This time, Jonny understood the

question as it is printed in English. Jonny spoke to Chen: ‘Yeah, I know this one. The answer is YES!’

Arguably, by virtue of Chen’s testimony, Jonny can be recognised as knowing that the proposition expressed by the Chinese sentence ‘爱丁堡是苏格兰首都’ *is true*. This is a typical case of testimonial knowledge. However, it is also obvious that Jonny *does not* know that Edinburgh, rather than Glasgow, is the capital of Scotland. Therefore, it is highly implausible to claim that he knows the proposition delivered by the Chinese sentence ‘爱丁堡是苏格兰首都’. The KT-schema is thus violated in this case.

The example above involves multilingual expressions, which might not look straightforward enough. Consider a modified case:

Foreign Language 2 There is a true English sentence ‘Edinburgh is the capital of Scotland’ printed in a pub quiz book. A non-English speaker, Chen, read this English sentence but failed to understand it. So he asked his Scottish friend Jonny, a reliable man who could also speak Chinese, for help. Jonny told Chen that this sentence is true. Chen believed Jonny’s testimony, but he still did not understand that English sentence. In fact, Chen just did not know which city is the capital of Scotland – even if the question was asked in his native language (say, Chinese). He was just ignorant of the existence of Edinburgh.

Here, Chen can also be recognised as knowing the proposition expressed by the English sentence ‘Edinburgh is the capital of Scotland’ *to be true* by virtue of Jonny’s testimony. However, Chen does not know that Edinburgh is the capital of Scotland. Moreover, he is completely ignorant of the relevant fact that the proposition supposes to deliver. The content of the sentence is not understandable to him. Chen does not know which city the capital of Scotland is, nor that there is a city named Edinburgh. It is incoherent to say ‘I know that Edinburgh is the capital of Scotland’, but I do not know which city is the capital of Scotland’. If our analysis of this case is right, then the KT-schema is disproved. In both cases above, the proposition in question (‘爱丁堡是苏格兰首都’/‘Edinburgh is the capital of Scotland’) is just like a string of mysterious codes for our protagonist. The semantic content of the string of codes does not influence the protagonist’s judgment of the proposition’s truth value. Our protagonists know those propositions to be true, merely on the basis of true reliable testimonies.

We have seen that, if the subject does not understand the meaning of the words constituting the proposition in question, then it is possible that the KT-schema would be violated. Besides, lacking an understanding of the proposition’s logical structure might also lead to the failure of the KT-schema. Consider another example:

Logic Exam There is a multiple-choice question in a logic exam asking candidates to read a reading material and choose the only true proposition from three options. A student, Maria, knew clearly that option A and option B were both false, while she did not understand what option C ‘P is contraposed to Q’ means. That is because Maria did not remember what ‘contraposition’ means. Nevertheless, Maria still decided to choose C confidently, as she knew that A and B could not be the correct answer, and thus option C must be true. Maria was right. C is

the correct answer indeed. However, without understanding the contraposition relationship, after the exam, when asked whether ‘P and not Q’ is true, Maria answered, ‘I think so’.

In this case, it is reasonable to argue that Maria knows the sentence ‘P is contraposed to Q’ to be true. She came to know this by the method of exclusion, i.e., she inferred that option C is true from the premises: (i) neither A nor B is true; and (ii) one among the three options has to be true (presume that there was no sign indicating that the test question was misprinted). Her reasoning is correct. Hence Maria can be recognised as knowing that ‘P is contraposed to Q’ is true. On the contrary, it would be much more improper to claim that Maria knows that P is contraposed to Q. That is because she does not really understand the logical structure of the proposition, and thus fails to comprehend its semantic meaning. Moreover, even if the proposition expressed by option C was ‘P and not Q’ (if misprinted), it is still highly likely that Maria would choose option C regardless by the method of exclusion. The content of option C is not important for Maria’s choosing C. In addition, it would be extremely counterintuitive to claim that one can know that two propositions are contraposed to each other without knowing their contraposition relationship. It is incoherent to say ‘I know that P is contraposed to Q, and P is true while Q is not’. In contrast, the following sentence is coherent: ‘I know that the sentence ‘P is contraposed to Q’ is true, but I do not understand the meaning of the relevant proposition, therefore I do not know whether P is logically equivalent to Q or not’. The upshot is, we can hardly grant that Maria knows the proposition, even though she knows (*de dicto*) that the proposition is true. Therefore, the KT-schema is violated again in this case.

Let us now return to the math textbook case in Section 2.3. It concerns a situation where the content of the proposition is *completely unrevealed* to the subject and thus the KT-schema is violated. The two foreign language cases involve scenarios where the content of the proposition is *exposed* to the subject, nevertheless its semantic meaning is *completely mysterious* to the subject – and hence the KT-schema fails again. The logic exam case further constructs a scenario where the content of the proposition, of which the semantic meaning is *exposed to* and *partially understood* by the subject. However, the incomprehension of the proposition’s logical structure still prevents the subject from knowing the proposition – and thereby falsifies the KT-schema. Comparing the three types of counterexample, we can find that the extent to which the protagonist understands the content of the given proposition seemingly increases by degrees. Nevertheless, what those cases have in common is that the protagonist’s understanding is insufficient for obtaining knowledge. So, how much understanding is sufficient? It is an interesting yet knotty question that is beyond this paper’s purpose. The moral that I hope to draw from those counterexamples is: (a certain extent of) understanding of the meaning of a proposition *p* is necessary for knowing that *p*, while it is not necessary for knowing that ‘*p*’ is true. With this distinction in play, we can better understand why there is a gap between [knowing that ‘*p*’ is true] and [knowing that *p*], and why the KT-schema fails.

4 Objections and Rejoinders

Objection One Remember that in Section 1 we noted that the KT-schema could *prima facie* be entailed from the T-schema and the closure principle. Now that the KT-schema is falsified, a natural consequence is that we should re-examine the closure-based argument for the KT-schema. That argument trades on two essential premises, namely the T-schema and (the naïve) closure. If the widely accepted T-schema is unproblematic, does this mean that we shall *abandon* the closure principle?

Reply No. Some restrictions on closure will suffice. Notice that the naïve closure utilised in Section 1 is an unrestricted one. That is, it simply predicts that for any proposition p , if p is known and the subject also knows that p implies q , then q should be known. This unrestricted closure is oversimplified. In order to restrict and save the closure principle, there are at least two ways to choose from. The first way is to adopt the competent deduction closure (hereafter, CDC):

[Competent Deduction Closure] If a subject S knows that p and comes to believe that q by competently deducing q from p , then S knows that q .

It can be argued that *none* of the counterexamples stated before fit the bill of the CDC. To meet the competent deduction restriction, the subject should, first, know that a proposition p is true, second, form the belief that p by competently deducing p from “‘ p ’ is true”. Only when the two conditions are satisfied, would the CDC predict that the subject knows that p (per se). However, it is highly doubtful whether, in any counterexample discussed before, the protagonist comes to believe that p by competently deducing it from that ‘ p ’ is true. In particular, it is doubtful whether the subject forms the belief that p , given that the content of p is not understood.

‘Believes that’, as well as ‘knows that’, is a propositional attitude. It constitutes a subject’s cognitive relation to propositions. We usually expect this cognitive relation to be substantive. If a subject believes that p without understanding the meaning of p , then what is the content of that belief? The content of the belief seems to be empty (or at least mysterious). It is odd to grant that a propositional attitude like this can substantively serve as one’s cognitive relation to a proposition. This oddness would be salient when it comes to the math textbook case. In that case, you do not know what the last proposition of the math textbook is – so, what is the object of your propositional attitude? It is simply infelicitous to say ‘I believe that the last proposition of the textbook’ – this statement is incomplete. Again, at most what you believe is just that the last proposition of the textbook *is true*. Similar to the distinction between *knowing that p* and *knowing that ‘ p ’ is true*, there should be a distinction between *believing that p* and *believing that ‘ p ’ is true* as well. In light of this, it can be argued that one cannot virtually come to believe a proposition that she does not understand. At most, one can only come to believe a proposition that she does not understand *to be true*.

Therefore, it is justifiable to argue that the CDC does not apply to the counterexamples aforementioned. Moreover, the KT-schema cannot be derived from the CDC and the T-schema. Readers are invited to rework the argument given in Section 1 and replace the naïve closure with the CDC. What can be entailed, this time, is just ‘if S knows that p is true, and S comes to believe that p by competently deducing it from p is true, then S knows that p ’. This new thesis obviously differs from the KT-schema.

However, I am not completely confident of the new thesis derived from the CDC. That is because, we assume that the protagonists in the aforementioned counterexamples do not really form relevant propositional beliefs. This assumption relies on the claim that [believing that p requires understanding the meaning of p], which might be questionable. Perhaps an *obscure belief* is at least conceptually possible.⁵ I do not mean that the CDC is thus wrong – it is still a promising and plausible formulation of closure. I just suggest that it would be ideal if we could have a safer option to restrict closure. Here is an alternative:

[Content-Understanding Closure] If a subject S knows that p and comes to believe that q by competently deducing q from p , where q is a proposition the meaning of which is understood by S , then S knows that q .

The content-understanding closure (hereafter, CUC) is a weaker formulation of closure compared with the CDC, in the sense that it does not turn on a dubitable presumption that believing-that- p requires understanding. Again, the problematic the KT-schema cannot be derived from this form of closure. What can be derived is just a new thesis: ‘If a subject S knows that p is true and comes to believe that p is true implies that p , and S understands the content of p , then S knows that p .’ The new thesis is compatible with those counterexamples to the KT-schema. In those cases, our protagonists fail to meet the condition that ‘ S understands the meaning of p ’, and thus fail to know that p . Thus, given this content-understanding restriction, closure can be saved from the failure of the KT-schema. One does not need to endure the pain of losing closure in order to disprove the KT-schema. Closure might be rejected for some other reasons, but at least, not for the failure of the KT-schema.

Objection Two There can be a Fitch-style proof (see Fitch 1963) proving that your assumption that ‘ Φ is known’ is inconsistent. The proposition Φ claims that Σ is an unknown truth, which can be formulated as follows:

$$\Sigma \wedge \neg K\Sigma$$

The first horn of your alleged dilemma assumes that Φ is known, viz, ‘ $K(\Sigma \wedge \neg K\Sigma)$ ’. Since knowing a conjunction entails knowing every conjunct, we have:

$$K\Sigma \wedge K(\neg K\Sigma)$$

Since knowledge implies truth, ‘ $K(\neg K\Sigma)$ ’ entails ‘ $\neg K\Sigma$ ’. Hence, a contradiction occurs:

$$K\Sigma \wedge \neg K\Sigma$$

⁵ Here is a potential example for *obscure beliefs*. When Tim was a child, his mother told him that ‘You will find your soulmate’. Little Tim did not understand what ‘soulmate’ meant, but he trusted everything that his mother said. He kept her word in mind. ‘Yes, I will find my soulmate.’ When Tim was twenty, he fell in love with Mary. One day he accidentally read the definition of ‘soulmate’ on the Internet, then he spoke to Mary: ‘Yes, I’ve always believed that I would find my soulmate. Now I realise that it is you!’ Did Tim *believe* that he would find his soulmate – even though for a long time he did not understand the concept ‘soulmate’? I think one can plausibly argue that ‘yes’.

This means that the primary assumption of the first horn of your alleged dilemma is inconsistent. One cannot consistently know a proposition to be an unknown truth. If that is the case, then your argument against the KT-schema is blocked.

Reply This objection mistakenly formulates the proposition Φ . The faithful formulation of Φ should be $[\Sigma]^t \sim K\Sigma$, where the predicate $[p]^t$ reads “‘ p ’ is true”. That is because, Φ only asserts that Σ is an unknown true proposition. By assuming that Φ is known, we just assume that it is known that Σ is a true proposition, and, that Σ is an unknown proposition. To be more specific, what is assumed to be known is only that ‘ Σ is true’, rather than Σ per se. The content of the proposition Σ is not revealed. In contrast, ‘ $\Sigma \sim K\Sigma$ ’ straightforwardly deliver the content of Σ , and thus should be read as ‘ Σ and Σ is unknown’, which fundamentally differs from our original claim that ‘ Σ is an unknown truth’. In fact, this unfaithful formulation simply presumes that the KT-schema is true and identifies $K[\Sigma]^t$ with $K\Sigma$. Accordingly, the faithful formulation of our original assumption should be ‘ $K([\Sigma]^t \sim K\Sigma)$ ’ rather than ‘ $K(\Sigma \sim K\Sigma)$ ’. That is because the latter, *contra* what we originally assume, indicates that the proposition Σ per se is known. With this formulation in play, the aforementioned Fitch-style reasoning cannot run, as ‘ $K\Sigma$ ’ will not be entailed from our correctly formulated assumption. Thus there is a logical lacuna in the Fitch-style objection.

Can the logical lacuna be avoided by employing the T-schema? One may argue that according to the T-schema, “‘ p ’ is true” implies that p , hence $[\Sigma]^t$ implies Σ , so the Fitch-style reasoning can still proceed as follows:

- (1) $[\Sigma]^t \sim K\Sigma$ [Φ]
- (2) $[\Sigma]^t \rightarrow \Sigma$ [From T-schema]
- (3) Σ [From 1, 2]
- (4) $\Sigma \sim K\Sigma$ [From 1, 3]
- (5) $K(\Sigma \sim K\Sigma)$ [Assumption]
- (6) $K\Sigma \quad K\sim K\Sigma$ [From 5]
- (7) $K\Sigma$ [From 6]
- (8) $K\sim K\Sigma \rightarrow \sim K\Sigma$ [Knowledge entails truth]
- (9) $\sim K\Sigma$ [From 6, 8; contradicting 7]

This objection still involves an unfaithful formulation. Line 5 is not a correct formulation of our original assumption. As we have noted, the original assumption should be formulated as ‘ $K([\Sigma]^t \sim K\Sigma)$ ’, rather than ‘ $K(\Sigma \sim K\Sigma)$ ’. So what about another variation of Fitch-style objection employing the correct formulation? Consider the following argument:

- (1) $[\Sigma]^t \sim K\Sigma$ [Φ]
- (2) $K([\Sigma]^t \sim K\Sigma)$ [Assumption]
- (3) $K[\Sigma]^t \quad K\sim K\Sigma$ [From 2]
- (4) $[\Sigma]^t \rightarrow \Sigma$ [From T-schema]
- (5) $K([\Sigma]^t \rightarrow \Sigma)$ [T-schema is known]
- (6) $(K[\Sigma]^t \quad K([\Sigma]^t \rightarrow \Sigma)) \rightarrow K\Sigma$ [From closure]
- (7) $K\Sigma$ [From 3, 5, 6]
- (8) $K\sim K\Sigma \rightarrow \sim K\Sigma$ [Knowledge entails truth]
- (9) $\sim K\Sigma$ [From 3, 8; contradicting 7]

This argument seems to serve as a *reductio* to our original assumption. However, the problem with this argument lies in line 6. In fact, the closure principle employed here is still the problematic naïve version of closure. Here, neither the CDC nor the CUC is applicable, given that the content of Σ is not understood, so according to the CDC and the CUC, the consequent of line 6 (i.e., ' $K\Sigma$ ') cannot be entailed. Only the naïve closure will allow this entailment, as the naïve closure also applies to a proposition like Σ whose content is not understood. The defects of the naïve closure have been adequately discussed in contemporary literature (e.g., Dretske 1970, 2014; Nozick 1981). Counterexamples to the KT-schema previously mentioned can also be seen as cases against the naïve closure. Take the second foreign language case for an example. Chen knows that the proposition delivered by the English sentence 'Edinburgh is the capital of Scotland' is true. It can also be assumed that Chen knows the T-schema. However, he does not know the proposition, contra what the naïve closure would predict. Since Σ is an unknown true proposition, its content is not revealed to, much less be understood by us. One cannot actually form a propositional belief of Σ given that its propositional content is inaccessible, so line 6 is false as its consequent does not follow.

Furthermore, if the Fitch-style argument above is accepted, then an unpalatable consequence is that we should also endorse the following argument to the effect that there is no unknown truth:

- (1) $\exists p ([p]^t \sim Kp)$ [Assume for *reductio*]
- (2) $[\Sigma]^t \sim K\Sigma$ [1, existential instantiation]
- (3) $K([\Sigma]^t \sim K\Sigma)$ [Assumption]
- (4) $K[\Sigma]^t K\sim K\Sigma$ [From 3]
- (5) $K([\Sigma]^t \rightarrow \Sigma)$ [From T-schema]
- (6) $K[\Sigma]^t$ [From 4]
- (7) $(K([\Sigma]^t \rightarrow \Sigma) K[\Sigma]^t) \rightarrow K\Sigma$ [From the naïve closure]
- (8) $K\Sigma$ [From 5, 6, 7]
- (9) $K\sim K\Sigma \rightarrow \sim K\Sigma$ [Knowledge entails truth]
- (10) $\sim K\Sigma$ [From 4, 9; contradicting 8]

The first premise of this argument assumes that there exists at least an unknown truth. The second premise, by existential instantiation, assumes Σ to be an unknown truth. From these plausible assumptions, the aforementioned Fitch-style reasoning will entail a contradiction (i.e., $K\Sigma$ and $\sim K\Sigma$). Therefore, if we preserve the naïve closure and apply it to unknown true propositions, then the Fitch-style argument would also conclude the uncontroversial statement that 'there are unknown truths' to be inconsistent. The Fitch-style proof will not only become a *reductio* to my dilemma, but also a *reductio* to the existence of unknown truths.

Objection Three Is your argument a variation of the knower paradox? The knower paradox argues that a proposition G that 'G is not known to be true' can lead to incoherence, because if G is known to be true, then G is false and thus G is not known to be true; while if G is not known to be true then G can be demonstrated and thus known to be true. Can solutions to the knower paradox solve your dilemma as well?

Reply First, the dilemma for the KT-schema significantly differs from the knower paradox. There appears to be little consensus as to the *strongest* form of the knower

paradox in the literature (see Kaplan and Montague 1960; Maitzen 1998; Cross 2001; etc.), nor as to its *best* resolution. Nevertheless, almost all orthodox forms of knower paradox involve self-reference, *contra* the dilemma for the KT-schema. Unlike ‘G’ in the knower paradox, ‘ Φ ’ in my dilemma is not self-referential – instead, it describes another proposition Σ . Thus any solution to the knower paradox trading on the undesirability of self-reference (e.g., the simplest way, banning self-references) can hardly help to solve our dilemma.

Admittedly, some solutions to the knower paradox in the literature do not rely on self-reference. For example, Maitzen (1998) proposes that to solve the knower paradox, we should reject epistemic closure. I will not comment on whether Maitzen’s solution *to the knower paradox* is successful (there are objections to Maitzen’s solution arguing that the knower paradox can be independent of the epistemic closure⁶). What I shall emphasise is that my argument *against* the KT-schema does not rely on the success of epistemic closure (in contrast, it is interesting that the argument *for* the KT-schema seemingly does hinge on closure). Quite the opposite, my dilemma indicates that we should at least restrict, if not abandon completely, the closure principle. There are also plenty of logical solutions to the knower paradox⁷ that I cannot detail here, but it is unclear how those solutions relate to our dilemma – especially if we do not prefer to base our epistemological system on dialetheism.

5 The Effect of Rejecting the KT-Schema

Presume that my argument against the KT-schema is correct, then there will be some noteworthy consequences of rejecting KT.

5.1 Restricting Closure

We have noted that, the failure of the KT-schema suggests that the naïve closure should be restricted. Suitable alternatives can be the CUC, or negotiably, the CDC.

5.2 Epistemic Gradualism?

We emphasise the importance of understanding when it comes to the object problem of propositional knowledge. However, ‘understanding’ is a somewhat ambiguous concept. To be specific, understanding is not an absolute concept. On the contrary, it is gradable. This inspires us to conceive a gradable account of knowledge-that, as opposed to the standard *epistemic absolutist* view that propositional knowledge is absolute and ungradable. Hetherington (2001, 2005) proposes *epistemic gradualism*, which argues that propositional knowledge is gradable. According to Hetherington, there is a close connection between ‘knowing’ and ‘understanding’, which coincides with our conclusion that knowing that p requires understanding the content of p . One can understand the content of a proposition better than another person does. If ‘understanding-that’ partially constitutes ‘knowing-that’, then it is reasonable to infer

⁶ See Cross (2001), (Cross 2004).

⁷ For example, see Anderson (1983); Solovay (1976); Égré (2005).

that knowing-that can also be gradable as understanding-that. That is, it will be defensible to argue that knowledge-that- p can be graded in terms of one's understanding of the semantic meaning of p – the better the meaning is understood, the better the proposition is known.

Note that the main point of this paper does not base on the gradualist account of knowledge-that. One can retain the absolutist stance while rejecting the KT-schema and recognising the importance of understanding. Also, I do not plan to conclude that epistemic gradualism is correct.⁸ Rather, my attitude is just that the denial of the KT-schema might shed more light on the underestimated gradualist stance. It would be a thought-provoking attempt to explore epistemic gradualism as it might help to resolve a significant problem: why is knowledge-that seen as ungradable, whilst knowledge-wh and knowledge-how are admitted to be gradable?

The traditional explanation of this putative asymmetry is that there are fundamental differences between the object of knowledge-that and that of knowledge-wh or knowledge-how. The object of an item of knowledge-that is the truth of a proposition, which – unlike a person, a location, or a way of doing something – can neither be known more or less, nor better or worse. This idea, according to Hetherington (2005), also underpins 'the strongest defence' for epistemic absolutism: knowledge-that is absolute in the sense that its object, to wit, the truth, is absolute.

While in accordance with our discussions above, the object of knowledge-that- p is *not only* the truth of p , hence the defence for absolutism is ill-grounded. Conversely, we have noted that the content of p should also constitute the object of knowledge-that- p . Epistemic gradualism suggests that the content of p can be known better or worse in terms of the degrees of relevant understanding. As a result, knowledge-that also involves epistemically gradable object as well as knowledge-how and knowledge-wh. We can thus have a unified gradable account of different types of knowledge.

Admittedly, it will be a non sequitur to conclude directly that knowledge-that is gradable from that its necessary conditions are gradable. Beliefs and justifications are also ordinarily taken to be necessary for knowledge, nevertheless, their gradability does not imply the gradability of knowledge-that per se. Thus the discussion regarding the gradability of understanding should better be understood as an internal criticism to epistemic absolutism. That is, if the ungradability of truth (*qua* the alleged object of knowledge-that) can be employed to defend epistemic absolutism, then the gradability of understanding should be able to undermine absolutism and support gradualism. That is because, we have noted that the semantic content of p , as well as the truth of p , also constitutes the object of knowledge-that- p . I take it to be justifiable that there is no substantial difference between 'knowing the semantic content of p ' and 'understanding the semantic content of p '. Admittedly, this claim needs to be borne out by more intensive analyses. But if this claim turns out to be true, then because understanding is gradable, knowing that p can also be deemed gradable. This inference does not resort to the gradability of the *necessary condition* of knowledge, but the gradability of the *object* of knowledge. If one finds it reasonable to see knowledge-who as gradable because a person (the object of knowledge-who) can be known more or less, then it should also be reasonable to see knowledge-that as gradable because the semantic content of a proposition can be known more or less, or, better or worse.

⁸ Therefore, there is a question mark in the title of this sub-section.

It is difficult indeed to determine how much understanding is *enough* for constituting knowledge, as it is almost unlikely to locate a non-arbitrary and clear cut-off point between ‘sufficient’ and ‘insufficient’ understanding (for constituting knowledge). It is as difficult as locating a non-arbitrary and clear cut-off point for ‘enough justification (for constituting knowledge)’. Notwithstanding, this sort of ‘boundary problem’⁹ should not hinder us from taking gradable terms such as ‘understanding’ into consideration when addressing the object problem. After all, an intrinsic trait of most gradable terms is the fuzziness of their boundaries. *Belief* requires confidence, but how confident is ‘confident enough’ for one’s forming a belief? *Gloom* requires sadness, but how sad is ‘sad enough’ for one’s being counted as gloomy? Likewise, clear-cut boundaries of many other gradable concepts – such as ‘enjoy’, ‘hatred’, ‘devoutness’, etc. – can hardly be determined in a non-arbitrary way. Hence, if knowledge-that is gradable, then we shall not even be perturbed by the boundary problem.

5.3 Rethink the Object Problem

Traditionally, the KT-schema is taken to be the standard answer to the object problem of propositional knowledge. Hence an immediate consequence of the KT-schema’s failure is that we should rethink the object problem. We have seen that ‘knowing the content of *p*’ seems to be a complement. That is to say, from what we have discussed, ‘to know that *p*’ seemingly means that ‘to know that “*p*” is true + to know the content of *p*’. To be specific, we reject:

[T Account] To know that *p* is to know that ‘*p*’ is true.

But advocate:

[T+C Account] To know that *p* is to know that ‘*p*’ is *true*, plus, to know *p*’s *content*, which requires understanding the meaning of *p*.

There are two points that I wish to clarify regarding the T + C account. First, this account purports to answer the object problem rather than the definition problem of propositional knowledge. The object problem *differs from*, albeit closely related to, the definition problem. Theories attempting to answer the definition problem include the historical ‘JTB’ template, the relatively recent ‘JTB + anti-Gettier factors’ template, and the AAA-model of virtue epistemology, etc. However, none of them is answering the object problem of propositional knowledge. Any attempt to solve the object problem should be careful about the subtle distinction between the two problems. Given this distinction, one should not worry that our ‘truth and content’ account will invite circular definition or infinite regress. That is because, we are not *defining* ‘knowing-that’ by ‘knowing the truth and the content of *p*’. We are just *analysing the object* of ‘knowing-that’ by the ‘truth and content’ account.

⁹ For example, when discussing knowledge’s justificatory boundary problem, Hetherington (2006) argues that, lacking the knowledge of *where* the boundary is would only *weaken* our knowledge that there is such a boundary, rather than *eliminate* the latter knowledge.

Second, the T + C account parallels ‘knowing the truth of p ’ with ‘knowing the content of p ’ as two factors constituting the object of knowledge-that- p . This does not mean that these two factors are completely independent of each other. They could be interdependent and even overlapped to some extent. We have seen how one can know the truth of p without knowing the content of p . It is also easy to imagine scenarios where one knows the content of p without knowing its truth – for example, comprehending a proposition while disbelieving it. The union of the two factors can constitute a *fuller* and thus better answer to the object problem (than T-account), which can at least avoid counterexamples that we discussed before. However, is this T + C alternative the *full* answer to the object problem of propositional knowledge? Is there any other factor constituting the object of propositional knowledge except for ‘truth and content’? Or, it may be questioned that, is ‘the content of p ’ a necessary component of the object of knowledge-that- p ? Furthermore, is ‘the truth of p ’ really necessary as it is orthodoxly taken to be? I will leave these questions open to my readers and I think these questions deserve further intensive explorations.

6 Concluding Remarks

I have disproved the KT-schema by constructing a dilemma where the KT-schema will perform result in contradictions, and providing counterexamples where the KT-schema does not obtain. The long-held view that ‘knowing that p ’ is equivalent to ‘knowing that “ p ” is true’ is thus falsified. ‘Knowing that “ p ” is true’ does not suffice to entail ‘knowing that p ’, hence the traditional T-account fails to address the object problem of propositional knowledge satisfactorily. It has been suggested that, apart from ‘knowing the truth of p ’, the object of an item of knowledge-that- p should also consist of ‘knowing the content of p ’, which requires a certain degree of understanding of p . Three upshots of the KT-schema’s failure have been discussed – first, the closure principle should be modified and restricted; second, a gradable account of propositional knowledge is promisingly justifiable; last but not least, we should rethink the object problem of propositional knowledge, which can benefit the contemporary epistemological discussions.

Acknowledgments I am grateful to Duncan Pritchard and Martin Smith for their patient guidance and helpful discussions about this article. Special thanks to Yijie Shen for valuable feedback. This work was funded by China Scholarship Council.

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