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SURVEY PAPERS

Emergence in complex networks of simple agents

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Abstract
Patterns and processes emerge unbidden in complex systems when many simple entities interact. This overview emphasizes the role of networks in emergence, notably network topology, modules, motifs, critical phase changes, networks of networks and dual-phase evolution. Several driving mechanisms are examined, including percolation, entrainment, and feedback. The account also outlines some of the modelling paradigms and methods used to study emergence, and presents cases to show how emergence occurs, and its implications in economics and other real-world contexts.

Keywords Complex networks · Self-organization · Network · Phase change · Agents based models · Emergence

Mathematics Subject Classification 68P30 · 37B15 · 91A40 · 91D30

JEL Classification B16 · C31 · C63 · D85

1 Introduction

Almost everywhere we look, we find examples of large-scale patterns and organisation that emerge in collections of simple entities. On the one hand, there are natural wonders, from growing crystals (Bisoyi and Kumar 2011) and flocks of birds to the beauty of growing plants (Reuter et al. 2010). On the other hand, there are devastating events, such as market crashes (Shi et al. 2022), bushfires (Zinck et al. 2011), epidemics (Jenkins et al. 2020) and accidents (Coze 2015). There are also unanticipated trends in society (Green 2014) and in the business world (Arthur 2021). Understanding emergent phenomena, and in some cases controlling them, poses a huge challenge. In every field of activity, society needs to understand how emergence occurs and what its implications are.

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The idea of emergence is intimately bound up with the study of complex systems. In the past few decades, research on complexity has exploded into a vast literature (Haynes and Alemna 2022), and has revealed many insights about ways in which complexity fosters emergence. In this perspective survey, I begin by defining emergence and briefly overview examples in economics and society. This includes a discussion of how intelligence emerges in groups of agents. I then survey common mechanisms that lead to emergence. Finally, I survey some modelling paradigms and tools that have been widely used to study emergence.

My account focuses on emergence in *networks* of simple agents. The reason for this is that many, if not most, cases of emergence can be explained in terms of interactions within networks. The *network model of complexity* is based on the observation that all complex systems can be represented as networks (Green 1994, 2000). This has two important implications. First, properties emerge out of interactions between the nodes of the network, rather than the properties of individual nodes. Second, patterns and processes in networks underlie many properties and behaviours that we see in real systems.

Although humans are certainly not "simple" agents in the general sense, simple rules often govern human behaviour, rather than considered decisions. I therefore include examples of emergence, such as crowds and markets, where networks of humans often interact and behave in simple ways.

This account puts a strong emphasis on emergent phenomena in economics and related fields. However, it also draws examples from across a broad spectrum of fields, as these often throw a different light on emergence. Markets, for instance, are often compared to ecosystems. For this reason, I include ecological examples that may hold useful lessons for economics. One of the issues impeding complexity theory as a field of research is that it has been studied in so many different disciplines, often with relatively little dialogue between them. By presenting a wide range of examples, my aim is to give readers a wider perspective for understanding emergence.

2 What is emergence?

Complex systems are collections of entities that are rich in interactions between them. Emergent properties are the outcome of these interactions. We can define emergence as the appearance of patterns, properties and behaviours within a system that are not evident in individual components. Some examples (discussed later) include the emergence of small world structures, modules, and hierarchies (patterns); the spread of rumours (processes); cost spirals, and panic (behaviour).

The above definition of complexity is captured by the popular expression: *the whole is greater than the sum of its parts*[1]. An economic example of this would be a group of companies that form the supply chain for manufacturing a product. Together they achieve a result that no one company could alone.

There is a close relationship between emergence and self-organization. Here I will use *self-organization* as a general term for processes (e.g. feedback) that create order

[1] Attributed to Aristotle

within a system, as distinct from order that is imposed from outside. Emergence usually arises by some form of self-organization, but some forms of self-organization can be said to emerge in a system. In complexity theory, self-organization and emergence are linked to the concept of *Complex Adaptive System* (CAS) (Holland 1992). These are complex networks of dynamic interactions in which the collective behaviour adapts, but is not predictable from the behaviour of its individual components. The term is often applied to systems where conventional models (e.g. differential equations) provide no insights into understanding the system's behaviour.

In some respects, emergence is in the "eye of the beholder". That is, its existence depends on the way you observe a system (Goldstone and Janssen 2005; Green et al. 2020b). This problem is often a matter of scale: "seeing the wood for the trees". Random, local variations often obscure patterns that become apparent only when observed at large scale (Hoel et al. 2013; Varley and Hoel 2022). A related issue is the distinction between strong and weak emergence. Bedau (2008) defined a macro-property P of a system S as *weakly emergent* if and only if P can be explained by the system's prior micro-facts, but without providing a way to compress the description of it.

The above distinctions can have huge practical implications (Tian et al. 2011). The tension between economic development and environmental conservation provides a good example (Schoelynck et al. 2011). Whether or not to preserve or develop patches of forest sometimes hinges on the way forests are classified. Developers are likely to claim that every patch is just an instance of the same kind of forest, so all but one or two representative patches could be cleared. On the one hand, environmentalists might claim that every patch is a unique type of community and should be preserved. Certainly the mix of species may differ from one patch to another, but the central question is whether each patch is really distinct? In the end, the issue hinges on what led to the particular assemblages of species found at each site. Are they merely random combinations? or are they communities that *emerged* from interactions between species over long periods of time?

3 Emergence in real world systems of simple agents

3.1 Emergence in economics and finance

In a sense, the idea of emergence is implicit in the foundations of economics. Smith (1776) argued that when a need or opportunity arises, a solution emerges in the market to meet it. Stock markets themselves have been characterized as emergent structures that arise in trading networks (Overholt 1991; Arthur 1999; Odell 2000; Stringham 2002; Padgett and Powell 2012; Saeedian et al. 2019).

Economics played a significant role in the development of complexity theory during the late Twentieth Century. This arose from a need to understand emergent aspects of market behaviour that could not be explained by traditional economics. Economists were quick to recognize that stock markets and economies are complex systems (Arrow 1969; Arthur et al. 1987; Krugman 1996) and that economics needs to apply ideas and methods from complexity theory (Beinhocker 2006). So-called "complexity economics” upturned conventional thinking (Arthur 2021). “*Complexity portrays the economy not as deterministic, predictable, and mechanistic, but as process dependent, organic, and always evolving.*” Arthur (1999)

In essence, conventional economics adopted a top-down approach. It assumes that agents are rational, have perfect information and adopt consistent, optimal, strategies. Moreover, large-scale market behaviour is just the sum of all these individual behaviours, and markets settle into static equilibria (Gatti et al. 2010). Complexity theory upturned these assumptions (Arthur 1999). In contrast to traditional economics, it assumes that many features of market behaviour emerge bottom-up, arising from interactions between individual agents.

Emergence takes many forms in economics. Perhaps the most widely studied are processes that affect market volatility. Many processes contribute to the emergence of stability in markets. They include: resilience (Stanley 2020), cooperation (Bargigli and Tedeschi 2013; Aydogmus et al. 2020), confidence in decision-making (Rolls 2019; Lorscheid and Meyer 2021), moral behaviour (Gaus 2019), and trust (Phuong et al. 2020; Kato and Sbicca 2021). In contrast, other processes can lead to the emergence of market instability (Cavalli et al. 2022), including: cycles (Donaghy 2022), crises (Kirsch and Rühmkorf 2017), criticality (Harré 2018), and financial bubbles (White 1990; Rappoport and White 1993; Seyrich 2015; Barbie and Hillebrand 2018). There are also other ways in which changes emerge in markets, notably new technologies (Jung 2019) and entrepreneurism (Yun et al. 2018).

In later sections, we will examine many processes that contribute to the above cases of emergence in markets and finance. See percolation (Sect. 4.2), networks (Sect. 4.3), feedback (Sect. 4.6), entrainment (Sect. 4.7), and modules and motifs (Sect. 4.8).

3.2 Human society

Although humans are intelligent, many phenomena in society emerge because people act as simple agents.

In an influential series of studies, Dunbar (1992) argued that cohesion in social groups emerges from interactions between individuals, and the nature of those interactions places an upper limit on group size. He argued that in apes, grooming leads to a natural group size of 30-60 individuals. However, in human social groups, speech provides a more efficient ‘social glue’, which makes it possible for larger natural groups (100-150 individuals) to emerge. The formation of cohesive groups can be linked to the emergence of consensus by the spread of a common belief (e.g. agreement that “I belong to this group”) (Jiménez-Martínez 2015). A number of agent-based models have supported Dunbar’s hypothesis (Stocker et al. 2002; Seeme et al. 2019; Seeme 2021). They represent human opinions as switches in Boolean network models (see Sect. 5.10), in which changes of state are determined by simple interactions between nodes.

Many kinds of dysfunctional behaviours emerge in social groups. They often arise from poor communication and the limits on human perception. For instance, *Confirmation Bias* occurs when prior beliefs cloud a person’s perception (Nickerson 1998). Suppose that an individual believes “I am always in the slow queue”, then confirming instances reinforce the bias, whereas negative instances are ignored. Confirmation bias can skew decision-making (e.g. risk aversion, poor judgement) (Nelson 2014) and contributes to emergent trends, such as movements in market prices (Cafferata and Tramontana 2019; Cipriano and Gruca 2014). Automation can also contribute to the emergence of bias. Online search engines, for instance, form *filter bubbles* that restrict what individuals see by feeding them information that is consistent with their past searches (Pariser 2011).

Another kind of bias is *Groupthink*, which arises when members of a group suppress their individual opinions, and base judgements on what they think the group's consensus would be. Groupthink leads to alternative ideas being dismissed or ignored, and results in the emergence of poor decision-making (Schafer and Crichlow 2010), and even collective delusions (Bénabou 2013). In contrast, Surowiecki (2005) argued for the "wisdom of the crowd." That is, good decision-making can emerge where a group is comprised of people with a variety of differing views.

Many other kinds of dysfunctional behaviour also emerge in social groups. They include: the impact of media on public opinion (Stocker et al. 2002), pluralistic ignorance (Seeme et al. 2019), the spiral of silence (Ross et al. 2019), and misinformation (Brumley et al. 2012).

As we shall see later (Sect. 4.3), many trends emerge in human society as unintended side effects of individual actions (Merton 1936). Interaction between different spheres (e.g. work-life conflict) can lead to entrainment (Sect. 4.7), resulting in cascading side-effects (Sect. 4.3). During economic booms, for instance, employees work more, spend less time sleeping, and less time with family, or on recreational activities; but in recessions, the opposite is true (Barnes et al. 2016).

3.3 Emergent intelligence

By *emergent intelligence*, I refer here to the ability of ensembles of simple agents to solve problems collectively. The concept takes in many problems and many fields. One of the first questions is how to recognize and measure intelligence. The problem has a long and chequered history, but the advent of Artificial Intelligence (AI) extended the question by asking how to measure intelligence, not only in humans, but also in animals, machines, and its emergence in collections of agents. Given the vast scope of the problem, a general solution needs to go beyond any specific domain. A promising approach has been to link it to the measurement of complexity (Sect. 5.1). That is, the more complex the problem, the greater the intelligence of an entity (of whatever kind) that can solve the problem (Hernández-Orallo and Dowe 2010). This approach raises the prospect of measuring (say) how much intelligence emerges when a team of agents is brought together (Chmait et al. 2016).

A broad, and diverse area of AI research is *Swarm Intelligence*. This concerns the way intelligent behaviour and problem-solving emerge within groups of simple agents (Dorigo and Birattari 2007). It has its roots in flocking and similar behaviours (see Sect. 5.6). In recent years, swarm intelligence has gained attention because of the need to coordinate the movements of fleets of drones, as well as flocks of online software agents.

Swarm Intelligence has gained many applications in business, especially as financial activities are conducted increasingly online. There is now a wide range of applications, such as sentiment analysis (Yildirim 2022), security (Mishra et al. 2021), competitive bidding (Bajpai et al. 2008), and predicting financial markets (Rosenberg et al. 2017) (see Sect. 5.6).

A simple example of swarm intelligence is *Stigmergy* (see Sect. 5.6). It has been adapted to provide an algorithm (ant sort) by which simple agents sort items (Fig. 7). Stigmergy is also an example of a broad class, known as *Nature-Inspired Algorithms* (NIAs), which are often used to solve problems of optimisation, searching and sorting, especially where complexity confounds traditional, analytic methods. Inspired by the observation that optimal solutions to problems often emerge in nature, NIAs represent a problem so that it imitates a natural process. Some NIAs are widely used, but many have been criticized for associated practical problems, especially "... *algorithmic convergence and stability, parameter tuning, mathematical framework, role of benchmarking and scalability*" (Yang 2020). In recent years, much ingenuity has been devoted to borrowing problem-solving methods from nature. Stripped of their finery, however, 'new' NIAs often turn out to be existing meta-heuristics in disguise (Tzanetos and Dounias 2021).

The best known NIAs are *Genetic Algorithms* (GAs), which mimic the way mutation and recombination help populations of organisms to adapt (Holland and Reitman 1978; Goldberg 1989). The GA method has advantages, especially its ability to deal with poorly specified problems. However, GA-derived solutions are likely to be only near-optimal. There have been many modifications, mostly aiming to deal with problems that bedevil GA applications. A well-known problem is local convergence. That is, an entire population converges on the same genotype, before reaching an optimum solution. Many approaches have been introduced to deal with this problem, such as imitating evolution in a landscape (Kirley 2002).

Nevertheless, GAs have proved useful in some economic applications. For example, a recent study demonstrated their usefulness for finding optimal trade-offs between energy, economics and environment in agricultural production (Mousavi-Avval et al. 2017). Other uses have included trading models for foreign exchange (Drake and Marks 2002).

Many algorithms deal with search problems. Optimisation, for example, can be treated as a search through a *fitness landscape*, where points in the landscape correspond to different combinations of parameter values and the "fitness" indicates how good the solution is. These searches typically combine two kinds of search: *exploration*, in which the agents seek to find the peak in the "landscape" where the best solution lies, and *exploitation*, in which agents takes advantage of a known peak (e.g. to search locally to find its highest point). Many kinds of search require a trade-off between these two extremes, both for animals and humans (Mehlhorn et al. 2015). In ant colonies, for instance, ants spread out and explore widely, then use pheromone trails to bring many ants together and exploit each food source they find.

Animal intelligence emerges out of complex interactions among neurons, which are nerve cells that excite each other electrically via synapses that link them. Consciousness and intelligence can be seen as phenomena that emerge out of these interactions. A popular model, based on the idea of clusters of neurons, is the Artificial Neural

Network (ANN), which had its origins in early experiments to replicate the way perception and thinking emerge in the brain (McCulloch and Pitts 1943; Rosenblatt 1958). A typical ANN consists of layers of cells ("neurons"), with outputs from cells in each layer becoming inputs to cells in the layer above. The bottom layer receives inputs (the problem), and the outputs (results) emerge from the top layer. In a simple feed-forward network, the neurons all process inputs in the same way. For instance, if the input is a number, then the neuron might apply a mathematical transform and output the result. The only differences between neurons in this case are the values of the parameters applied.

A common criticism of ANNs is that they act like a black box: you feed in a question and an answer pops out, but the reason for the answer is unknown. Another issue, especially with early ANNs, is that they differ from real brains in several fundamental ways. Attempts to improve performance involve overcoming some of these differences. First, *Deep Learning* has increased the sheer size of networks from a few hundred neurons, in a typical early ANN, to millions of neurons (LeCun et al. 2015). Second, Deep Learning builds modularity into its training and networks (Rotaru-Varga 1999). Early ANNs were typically trained ab initio to solve a single specific problem. Deep Learning involves training a system on many kinds of problems, and in particular on problems that contribute to solving increasingly complex problems.

However, real brains still differ from ANNs in some important ways. One is that ANNs are usually designed to converge on a particular behaviour, whereas real brains exhibit sensitivity to initial conditions (Freeman 1975, 1991; Skarda and Freeman 1987). That is, small differences in inputs produce vastly different excitation patterns. One advantage of this is that it allows brains to make fine discrimination between similar patterns.

Artificial Neural Networks have been applied to a wide range of problems, such as forecasting and pattern recognition. They are normally trained by being fed test cases where the desired result is known (*supervised learning*). The ANN adapts by altering settings within neurons to improve the results.

A recent review (Nosratabadi et al. 2020) highlighted the growing importance, in economics and finance, of ANNs, and other artificial intelligence tools. It showed that the applications of ANNs to economics has grown rapidly since 2016. The review also found that several main areas of application have emerged. The most widespread were in the stock market to forecast price movements from financial time series, and algorithmic trading. Marketing applications included automated sentiment analysis and evaluating impressions on social media. In e-commerce, they were used to improve the performance of web sites, analyze customer behaviour by categorizing items and by recommending products to customers. Other applications included cryptocurrency (forecasting the price movements of digital currency), and predicting corporate bankruptcies.

4 Mechanisms of emergence

The following is a brief survey of processes that promote or influence emergence in complex networks. In many cases, properties emerge from the combined effects of more than one of these mechanisms.

4.1 Graphs and networks

Fundamental to understanding emergence and complexity are the ideas of graphs and networks. Here we define a *graph* G to be a tuple $G = \langle V, E \rangle$, where V is a set of *vertices* (also termed 'nodes') and $E \subseteq V \times V$ is a set of *edges* linking them. A *network* is a graph in which the vertices and/or edges have associated attributes (e.g. names of the vertices, quantities for the edges). Edges can be directed (e.g. "A affects B") or undirected ("A and B are linked"). Directed edges are often called "arcs". The above terms are often used interchangeably, but in general, *graph* is used for discussing abstract structures and *network* for structures in the real world. Here I will use the term 'vertex' when discussing graphs and 'node' when discussing networks.

Many kinds of networks can be found in economics (Arthur 2021). Instances include trading networks (Saeedian et al. 2019), supply networks (Hearnshaw and Wilson 2013), and international agreements (Bartesaghi et al. 2020; Fisher 2022). Societies also contain many kinds of networks, including neighbourhoods, families, friends, co-workers, and organizations.

Graphs are important for understanding emergence because they are inherent in the structure and behaviour of all complex systems (Green 1994, 2000). So properties of graphs are fundamental for understanding complex networks in the real world. In particular, it means that features of graphs are involved in the emergence of many properties and behaviours (Gignoux et al. 2017).

Perhaps the most widely studied feature of graphs is their topology (patterns of connections). Different topologies, and the ways they emerge, have far-reaching consequences in networks. A *random graph* is formed by adding edges to pairs of vertices with probability p, which is the *edge density* of the resulting graph. This edge density is also the ratio of the number of edges to the maximum possible number $n(n-1)/2$ (for an undirected graph of n vertices). Random graphs have been widely studied and are often treated as a null hypothesis about the topology of observed networks. For instance, *Exponential Random Graph Models* (ERGMs) (Lusher et al. 2013) provide a tool for testing competing hypotheses about the way a network was formed.

Other topologies emerge when the formation rules change. If the graph is formed from k vertices by adding edges that connect vertex v_i to vertex v_{i+1} for $i \in [0, k-1]$ then a *path* of length k forms. If, at the end of that process, an edge is added between the first and last vertex in the path, then the result is a *cycle*.

A *tree* (hierarchy) can form from a partition (disjoint subsets) of the vertices. Suppose we have a vertex partition $V = V_0 \cup V_1 \cup \ldots V_k$ and $V_0 = \{r\}$, where r is denoted the root node. We create a tree by iteratively adding edges as follows: For each vertex v in V_{i+1}, we create a directed edge (arc) (u, v) where $u \in V_i$. The resulting tree is a hierarchy.

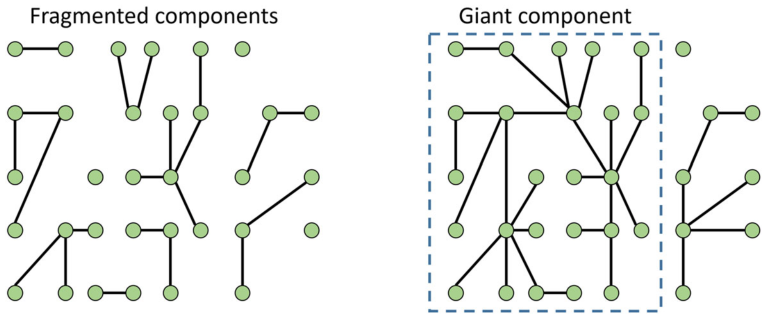

Fig. 1 The connectivity avalanche in a random graph. At left the graph is fragmented, and composed of small components. At right, the addition of more edges connects most of the fragments into a single giant component, indicated by the box. In dual-phase evolution (Sect. 4.4), a system flips back and forth between the local phase (left) and the global phase (right)

Two of the best known topologies are *small worlds* (Watts and Strogatz 1998) and *scale-free networks* (Barabási and Bonabeau 2003). Small worlds are typical of traditional societies (see Sect. 4.4). A small-world emerges when random edges are added to a cycle. The name "small" recognizes that this process creates shorter paths between vertices that were otherwise widely separated. A scale-free network emerges when a graph forms by preferential attachment. That is, the probability of a new vertex attaching to any existing vertex is a function of the degree of the vertex (the number of edges already incident to it). Both of these topologies emerge in real systems, especially social networks in which people create new contacts in a consistent fashion (Barabási and Bonabeau 2003; Watts and Strogatz 1998) (see Sect. 4.4).

A crucial feature of random graphs is the *connectivity avalanche* (Fig. 1). It occurs when edges are added to a set of vertices to form a random graph (Erdös and Rényi 1960). The addition of edges creates *components*. These are sub-graphs in which there are paths between every pair of vertices. The avalanche occurs at a critical point, when there are $n/2$ edges; that it, when the edge density is $1/(n-1)$ (Fig. 2). At this critical point, separate components are rapidly absorbed into a single "giant component" (Fig. 2a). Other changes also accompany the avalanche: the diameter of the largest component is greatest at the critical point (Fig. 2b), but decreases rapidly with increasing edge density. Also, the variation in the size of components reaches a maximum (Fig. 2c).

The connectivity avalanche manifests itself in the emergence of many kinds of phenomena (see Sect. 4.2). It usually occurs in combination with other processes discussed below. One result is that networks exhibit two distinct phases: *connected*, in which almost all vertices are linked into a single *giant component*; and *fragmented*, in which the vertices are either isolated, or form small components. The transition (i.e. the connectivity avalanche) between the two phases is marked by a critical region, within which the giant component emerges. Its size increases exponentially as larger and larger clumps of nodes become joined to it. We will discuss some of the implications in Sects. 4.2–4.4.

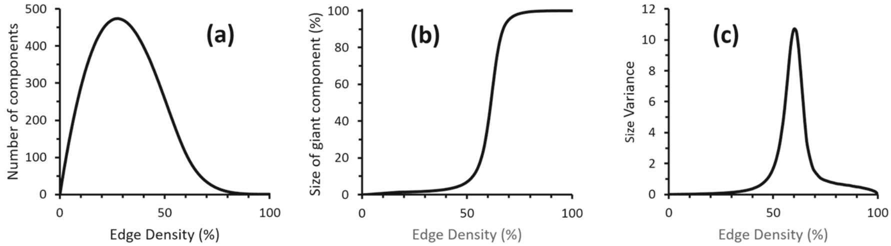

Fig. 2 The connectivity avalanche in a random graph as a function of the edge density (ratio of observed to maximum possible number of edges). In this example, the critical point lies where the edge density is ~59%. **a** The number of separate components; **b** The size of the giant component; **c** The variance in size of the components. See text for explanation. (Redrawn from Green et al. 2020a)

4.2 Percolation

In abstract terms, percolation is an emergent process, in which a change of state spreads from node to node through a network (Stauffer 1979). In physical terms, this change of state may be the flow of a substance from node to node, for instance water seeping through a porous medium. Alternatively, it may be a physical change brought on by contact with neighbouring nodes, such as spread of a fire through a fuel bed, or the spread of a rumour through a social network.

Originally studied to understand the movement of fluids through porous media, percolation also occurs in many other contexts. It can be understood by considering flow through a simple lattice, consisting of sites and bonds between them. There are two kinds of percolation (Li et al. 2021a): *bond percolation* refers to links that enable flow between neighbouring sites (nodes), and *site percolation* refers to flows in which sites either allow flow through them, or not.

The critical phase change in networks, which we saw earlier (Fig. 2), applies to both kinds of percolation (Yanuka and Englman 1990). Some phenomena can involve either bond, or site percolation. In fire spread, for instance, a fire cannot spread if the fuel is patchy and its density is below a critical level (site percolation). Similarly, in cold weather (say), fire cannot spread, even in dense fuel, if the heat of burning fuel is below the threshold needed to ignite neighbouring fuel (bond percolation). The same duality applies to the spread of an epidemic. The greater the degree of contact between potential hosts (site percolation), the less contagious the disease needs to be to spread. Conversely, the more contagious it is, the less contact is needed for it to spread (bond percolation).

We can interpret several economic processes as percolation. One is the spread of information in financial markets (Duffie and Manso 2007; Byachkova and Simonov 2015; Chiaradonna and Lanchier 2021), especially between traders (Asparouhova and Bossaerts 2017). This information can include the spread of rumours (Gaildraud et al. 2009), and, in extreme cases, can lead to herding behaviour and financial bubbles (Seyrich 2015). Another financial process is the spread of new market technologies. A good example is the Bitcoin Lightning Network. It is a “second-layer technology” that provides channels for making fast payments off the main blockchain. It has spread

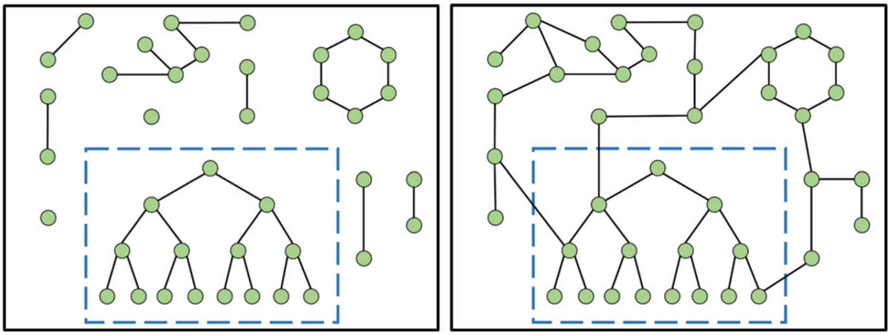

Fig. 3 Emergence of cascading failures. This is an example both of networks of networks, and of dual-phase evolution in a complex system. In normal conditions (left), a system behaves as closed (dashed box). In this local phase, the system has no links to outside networks. Changed conditions (right) create new connections with outside networks. This global phase allows unanticipated cascades of effects to spread through the system

through markets by bond percolation (Bartolucci et al. 2020), thus enabling blockchain applications to scale more easily.

4.3 Networks of networks

In recent years, there has been increasing recognition of the need to understand complex systems that are not a single network, but networks of networks (Karimi et al. 2021). As the name implies, these are comprised of different networks that are interlinked in some way (Gao et al. 2012, 2014). Infrastructure (roads, power, water, etc) provides good examples. A railway system, for example, consists of networks of rails, power supply, and communications. Failure in any one of these can cascade into failure of the entire system. Human society consists of many overlapping networks, including family, friends, and neighbours, as well as transport and communication networks that link people.

Several kinds of behaviours can emerge from interactions between different networks. Events in one network can trigger emergent events in another. A common problem is percolation leading to cascading failures (Buldyrev et al. 2010; Gao et al. 2013). A power grid, for example, compensates for failure in one local network by shifting power across from another network. However, if demand is high, that demand can overload the second network, setting off a series of failures across the entire grid. Widespread power blackouts, such as the infamous New York blackout of 1965, often emerge from this kind of cascade (Green 2014).

More generally, accidents and failures usually emerge as by-products of changes within interlinked networks (Fig. 3). For instance, the Tenerife disaster (the worst accident in aviation history) emerged from a sequence of forced changes, triggered initially by a bomb placed in an airport waste bin. No single change—switching flights to a secondary airport, fog on the runway, parking planes on the taxiway, garbled communications—was itself fatal, but they cascaded to cause the deaths of 583 people (Bruggink 2000; Green 2014).

Conceptually, we usually model working systems as closed boxes within a wider environment that includes many other networks (Fig. 3). In these models, we assume that interactions between a system and its external environment are negligible, and can be ignored. However, a change in some part of that environment can impinge significantly on the system, rendering the normal model invalid.

Perhaps the most widespread implication of the above process lies in the emergence of unintended consequences in decision-making (Merton 1936). The dashed box in Fig. 3 represents the range of issues considered in making a decision. The assumption is that outside factors neither influence the system, nor are influenced by it. However, if connections to the outside actually exist, or if changed conditions create connections (Fig. 3), then outside influences can create unforeseen consequences. In this way, decisions made in one context can have implications that are impossible to predict, in another, different context (Merton 1936; Kozlowski et al. 2013). Decision-making here can also take many forms. The introduction of new technology, for instance, can be seen as a decision to change the way things are done.

We can see a good example of new technology driving unintended social consequences in the case of labour-saving devices. In many countries, refrigerators, washing machines, and other devices were introduced into homes during the late Twentieth Century (Green 2014). In Australia, for instance, their introduction set off a cascade of unanticipated social changes (Green 2014). Labour saved in the home created greater free time and led to a large increase in the number of women in paid employment. That increase launched a cascade of other social changes. Women in the workforce had even less free time at home, which led to the rapid spread of child-minding centres, and growth of the fast food industry. Financial independence for women also led to increases in the number of single-parent families.

Similar cascades of changes have emerged following other technological innovations, including industrial automation and the communications revolution, marked by the introduction of the World Wide Web and mobile technology (Green 2014).

Supply networks can be vast and complex. Their diverse nature makes them highly susceptible to the emergence of unanticipated consequences (Matos et al. 2020). For instance, the global spread of supply chains has made them vulnerable to exploitation as commercial and military weapons (Farrell and Newman 2022). The quest for sustainability has also impinged on many supply networks. Subsidising sustainable and other socially beneficial behaviour can have the effect of inflating prices, thus harming consumers (Arya and Mittendorf 2015). Similarly, "... *lean logistics practices improve operational performance, [but] they may unintentionally increase environmental impacts*" (Ugarte et al. 2016).

In ecosystems, the introduction of new species can set off a "trophic cascade", leading to the emergence of instability and the loss of species (Walsh et al. 2016). Conversely, the removal of a "keystone species" can also trigger a trophic cascade. An infamous example was the removal of wolves from Yellowstone National Park. Reintroduction of wolves in the 1990 s largely restored the park's ecosystems (Ripple and Beschta 2012).

Arguably, one of the most important examples of interlinked networks lies in the interactions between human socio-economic networks and natural ecosystems. A simple model of traditional economics casts the flow of money as a closed system that forms a virtuous circle: *income–investment–production–growth*. This circle is a positive feedback loop, in which each step enables the next. The model casts economic activity as a closed box, ignoring interactions with outside influences, such as environmental constraints. A controversial demonstration of the danger of closed box models was the Club of Rome model (Meadows et al. 1972). By linking economic activity to human and environmental networks, it showed the potential for crashes to emerge in several vital systems. *Environmental Economics* embeds economic activity within larger environmental networks (Hanley et al. 2019). In particular, economic activities rely on a range of *ecosystem services* (Reid et al. 2005; Arrow et al. 2014). Failure of any one of these services could trigger a cascade of economic disasters (Mastny 2015).

4.4 Dual-phase evolution

As shown earlier (Figs. 1 and 2), the connectivity avalanche in random graphs means that networks normally exist in one of two phases: *connected* or *fragmented*. This property underlies the process of *Dual-Phase Evolution* (DPE), which occurs when a network exhibits three features (Green et al. 2014; Paperin et al. 2011):

- The network switches back and forth between the connected and fragmented phases;
- Different processes, *selection* and *variation*, predominate in each phase. In a fragmented phase, small component networks are separate from one another, allowing variations to appear. In a connected phase, links within a giant component dampen variations;
- The system has memory. That is, changes that occur in one phase carry over to the other phase, allowing order to accumulate.

An example is the formation of small worlds in human society (Watts and Strogatz 1998). Traditionally people are limited by time and distance to contact with people in their immediate neighbourhood (local phase). Sometimes, however, individuals visit distant places and meet people from elsewhere (global phase). These contacts create long-distance connections, which typify small-worlds (Paperin et al. 2011).

Studies have shown that dual-phases underlie many environmental processes. Oxbow lakes, for instance, emerge from alternation of flooding events and slow erosion (Paperin et al. 2011). Storms in central Australia flood dry water channels, creating migration pathways that allow widespread populations of water birds to emerge (Roshier et al. 2001). Another example can be seen in the post-glacial forest history of North America and Europe. Preserved pollen records show clear forest zones over time (Green 1994). These zones emerged because existing tree populations excluded northward migrating species, until intermittent wildfires cleared tracts of land, allowing new ecosystem assemblages to emerge.

In economics, dual-phase evolution has been linked to changes occurring during social (Xu et al. 2013) and market cycles (Goodman 2014). It has been suggested that prosperity is "*... a dual phase process of alternating highly prosperous, connected phases and non-prosperous, fragmented phases*." Cavaliere et al. (2012) More generally, dual-phases are often associated with processes in networks of networks. As noted earlier (Sect. 4.3), a change in the environment of a system can create new interactions between networks. In effect, the larger, surrounding network enters a more connected phase (Fig. 3). As we saw above, unintended economic consequences can emerge this way. The process can also trigger the emergence of permanent change in a market, such as the disrupting effect of a new technology.

Dual-Phase Evolution (DPE) should not be confused with the theory of *Self-Organized Criticality* (SOC) (Bak et al. 1988). Many cases of DPE were incorrectly interpreted as SOC (Green et al. 2014). SOC proposed that for many systems, their natural state lay in a critical region, to which they return after any disturbance. The chief identifying signature was presence of a power law (the so-called "$1/f$ noise") in the distribution of avalanche size. SOC has been invoked to explain many phenomena, but critiques have pointed out the theory's flaws (e.g. Paperin et al. 2011; Watkins et al. 2016). First, the theory offered no underlying causal mechanism. However, its signature $1/f$ noise can be explained by the exponential nature of the connectivity avalanche that occurs in random graphs. Also, $1/f$ noise is not a definitive indicator. Based on detection of power laws, many studies blindly claimed to find SOC in systems, even where the power law resulted from external factors (e.g. size of storms), not self-organization. Even the validity of SOC's exemplar sand-pile model (Bak et al. 1988) can be questioned: the sand pile's natural state need not be within the critical region; it has a stable configuration, and collapses only when it is driven into the critical region by falling sand.

4.5 Evolution: selection and adaptation

Species evolution, in particular the mechanism of natural selection, shapes organisms so that new phenotypic features emerge. Selection occurs when some aspect of an organism's world conveys an advantage (greater *fitness*) to individuals that possess a certain attribute. This selective pressure may be imposed by the physical environment (e.g. climate, soil), by interactions with other species (e.g. predation), or by competition within a population (e.g. sexual selection). Interactions between species can lead to co-evolution, in which both species evolve in relation to mutual selection, such as the need for predators to catch fast-running prey.

The mathematics of genetic recombination mean that mutations within an interbreeding population tend to be suppressed by crossover. However, in a small isolated population there is a much higher chance of a mutation persisting and even becoming fixed in the population. That is, every member of the population shares the mutation. Hence, the usual process of speciation is by isolation (*allopatry*) of small populations. However, other modes of speciation can occur. Both field and modelling studies show that speciation can occur within populations that share the same geographic area (*sympatry*). An example is where local variations and mating sort a species in ecotypes that minimise hybridisation (Sadedin et al. 2009).

Glacial cycles during the Pleistocene present examples of species emergence by allopatry in different parts of the world. For example, the repeated cycles of glaciation and thawing during the Pleistocene in Europe led to dual-phase evolution (Sect. 4.4). The cycles forced alternating retreats of species into refugia, followed by rapid expansion, accompanied by genetic variation (Hewitt 2000). In North America, the effect of repeated glacial cycles on the emergence of novel flora and fauna has been likened to a "speciation pump" (April et al. 2013).

Some theoretical models have characterised evolution as a search through a hypothetical *fitness landscape*. In these models, phenotypic attributes (e.g. size) of an organism define the dimensions of a landscape, and the organism's fitness defines the elevation of each point in the landscape. One drawback of the fitness landscape model is that it considers only selection within a fixed range of variables or attributes. For instance, placing organisms in a different environment could lead to a different "landscape".

A common issue in evolution is *genetic trade-off*. That is, selection for one attribute comes at the cost of another attribute. A classic example is seen in reproduction. Animals can either produce few offspring and nurture them to ensure they survive (e.g. birds, mammals), or they can produce many offspring, relying on chance to ensure that some survive (e.g. many insects). Likewise, different operational models emerge when businesses face similar kinds of trade-offs. For instance, restaurants usually invest resources into fostering a regular clientele, whereas fast-food outlets invest in efficiency and high throughput.

4.6 Feedback

Feedback occurs when the output of a process leads to an input. If a positive output returns as negative input, then the feedback is negative. If it returns as positive input, then the feedback is positive. Familiar examples are thermostats (negative feedback) and compound interest (positive feedback).

In dynamic systems, negative feedback tends to dampen changes. Hence it promotes stable equilibrium. More generally, by suppressing change, it acts to preserve the state of a system. However, cyclic behaviour can emerge if there are delays in a negative feedback loop. For example, in a growing population, if seasonal reproduction is too fast, then the population can exceed the carrying capacity of its environment. This causes the population size to fall and cycles of growth and collapse occur. Some systems avoid population cycles through dual feedback loops, where one loop eliminates oscillations that the other promotes (Nguyen 2012). In a survey of emergence in economics, Fromm (2005) argued that optimal "... *pricing of goods in an economy and free markets emerges from the interaction of agents obeying the local rules of commerce and the law of supply and demand. It is based on negative, stabilizing feedback.*"

In contrast, positive feedback accelerates change, hence destabilizing systems. In complex systems, it allows local variations to grow into global properties or behaviour. It is thus a mechanism that drives emergence in many natural systems. Numerous studies have reviewed the literature on its role in emergence (e.g. DeAngelis et al. 2012; Strogatz 2018). This research touches on too many aspects of science to review here, but they include areas such as: population ecology (Boone et al. 2011), physiology (Prochazka et al. 1997), metabolism (Chimenti et al. 2015), cellular control systems (Mitrophanov and Groisman 2008), reproduction (Harter et al. 2018) and develop-

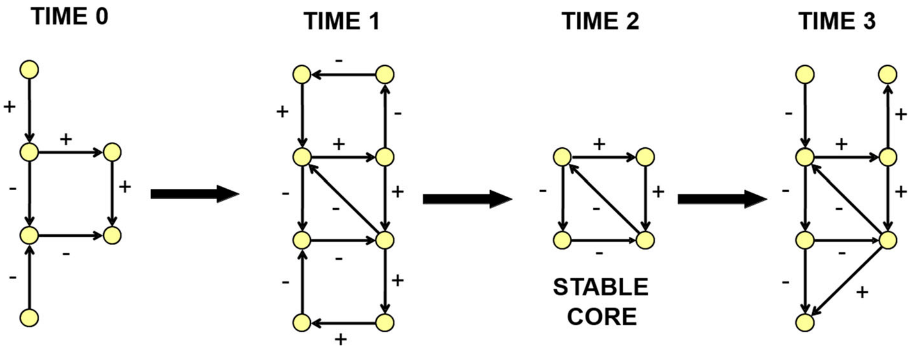

Fig. 4 Conceptual model of the formation of a stable core in a dynamic system. Addition of new nodes (Time=1) produces several feedback loops. The positive loops collapse (Time=2), leaving behind negative feedback loops (Time=2), around which a larger, stable network can emerge (Time=3)

ment (Cheng et al. 2003), neurodegenerative disease (Zilberter and Zilberter 2017), autoimmune disease (Shlomchik et al. 2001), and memory (Yang et al. 2011).

In most cases, positive feedback combines with other processes to produce emergent features or behaviour. The exact mechanisms depend on the nature of the system involved. However, a few examples can show that combinations of feedback and network processes are widespread and occur in many kinds of systems. In the onset of autism, for instance, "*...a positive feedback loop ... amplifies small functional variations in early-developing sensory-processing pathways into structural and functional imbalances in the global neuronal workspace*" (Fields and Glazebrook 2017). A biological example is stigmergy (Sect. 3.3). Positive feedback occurs because large clumps of eggs grow faster than small ones. This reinforces the results of sorting arising from insect behaviour and leads to the emergence of organisation in ant colonies (see Sect. 5.6).

In many instances, positive and negative feedback combine to produce emergent order in complex adaptive systems. In general terms, this occurs where positive feedback creates emergent features, and negative feedback operates to stabilize those features (e.g. by dual-phase evolution).

As a specific example, the above process operates in ecosystems, where it drives the emergence of stable species assemblages (Green et al. 2020a). We can view an ecosystem as a food web, in which populations of different species interact with one another. Repeated arrival of new species in such a system can create feedback loops. Positive loops lead to collapses, whereas negative loops create *stable cores* that enable component networks to persist, and grow (Neutel et al. 2002). This idea resolved an apparent paradox: complex ecosystems, such as coral reefs and rain forests, are known to be stable (they recover from disturbances). So it was assumed that complexity begat stability. However, in model ecosystems, the more complex the system, the more unstable it is May (1972). However, the stable core hypothesis resolved the paradox by showing that ecosystems can grow complex only if they are stable (Fig. 4).

Positive feedback plays a prominent role in the market economy (Arthur 1990; Dobusch and Schüßler 2013; Arthur 2021). A widespread practice is *positive feedback* *trading* (De Long et al. 1990; Antoniou et al. 2005; Koutmos 2014). Often a deliberate strategy, this occurs when trading success leads agents to continue and expand on the same activity. Recent studies show that it has had widespread influence, for instance on stock pricing in China (Liu and Wan 2022) and in the behaviour of Bitcoin markets (Wang et al. 2022).

Positive feedback also plays a significant role in the emergence of boom and bust cycles in markets (Arthur 1990, 2021). As early as 1841, Mackay (1980) provided vivid, detailed accounts of several market bubbles and the way they burst, including Tulip Mania in Holland (1634–36), the South Seas bubble in Britain (1711), and the Mississippi Company (1715–16) in France. There are many, recent examples, such as the collapse of the Dot-com bubble in 2000 (Wheale and Amin 2003),

Given the prominence of boom and bust cycles in economics, much effort has been devoted to understanding them. One problem is that positive feedback trading can lead to herd behaviour (Zhou and Lai 2008; van Roekel and Smit 2022), so a significant movement in stock prices can become a loop (Blomme 2012), which quickly turns into a virtual stampede. Loops can also arise between agents (Mercure et al. 2016) and if the resulting feedback is positive, it can drive debt cycles (Lojak 2018).

The introduction of automated agents for buying and selling shares had some unfortunate side-effects on market behaviour. The early agents used simple criteria for decision-making. Essentially, if the price of a stock went up by a set amount, they would buy; if it went down, they would sell. Single agents following such rules helped brokers react quickly to price movements. A problem emerged when large numbers of agents used the same principle for trading at speeds humans cannot match (Poirer 2012). Automated buying by many agents at the same time led to rapid price increases, which drove automated agents to buy more, and hence sent the price spiralling. Automated selling can also produce crashes: "*The stock market crash of '29 was a result of individual human agents–not a central controller. The crash of October '87 partly resulted from individual software agents that buy and sell securities according to programmed rules*" (Odell 2000). More sophisticated logic has reduced the problem, but automation ensures that rapid market movements remain common.

4.7 Entrainment

A widespread mechanism for emergence is *entrainment*. It is often associated with feed-forward (Takahashi et al. 2009), which occurs where node B passes on inputs from node A as inputs to other nodes. An example is retweeting messages on Twitter. In dynamic systems, entrainment can be interpreted as a process in which a system's state falls into an attractor region, from which it cannot escape. That is, it falls into a stable equilibrium, a cycle, or into a chaotic state (strange attractor). Entrainment can arise from negative feedback, if (say) one agent constrains and restricts the behaviour of another.

Entrainment is most famously associated with the mode-locking of oscillations in cyclic phenomena (Kuramoto 2003). It occurs when interactions between neighbouring agents leads to synchronization, so that small local vibrations can affect a global whole. The phenomenon has long been known to occur in the pendulums of mechanical clocks mounted on a wall (Nikhil et al. 2015). Other famous examples are spontaneous synchronised clapping in a crowd and the spread of laughter within groups of people (Lee et al. 2020). Physical examples of contexts where entrainment occurs include material rheology in debris and snow avalanches (Cuomo et al. 2014; Issler and Pérez 2011). In the firing of a laser, light emitted from atoms stimulates emissions of the same wavelength in other atoms. When the energy of synchronized atoms exceeds a critical level, the laser fires (Schawlow and Townes 1958). Entrainment also appears in biological contexts, such as the timing of circadian rhythms (Golombek and Rosenstein 2010; Brodsky 2006), the emergence of bipedal locomotion (Taga 1994), cognitive development (Wass et al. 2022), and shimmering, where bees flip abdomens up simultaneously in self-defence (Kastberger et al. 2010).

In economics and finance, entrainment arises in two ways: by the appearance of constraints that limit behaviour; or by reinforcement (Lussange et al. 2021), which can act as positive feedback, locking market players into particular strategies or behaviours (Arthur 1990; Forrester 1997; Anesi and De Donder 2013; Li et al. 2021b; Lussange et al. 2021). The emergence of trust can be interpreted as entrainment (Phuong et al. 2020; Kato and Sbicca 2021). Other consequence of entrainment include the formation of economic clusters (van Roekel and Smit 2022), the adoption of new technologies, and the emergence of economic niches (Cazzolla Gatti et al. 2020; Li et al. 2021b). In prehistoric times, for instance, the domestication of animal species produced a cycle of positive feedback, which "... *sparked ongoing intensification of agriculture production*" (Shelach-Lavi 2022). A present-day example is the emerging transition to renewables in energy markets (Chappin and Blomme 2022).

Entrainment can be seen as a mechanism by which the stockmarket adapts (Holland 2018). However, it can also lead to market volatility (Koulakiotis and Kiohos 2016; Song 2021). For instance, long-term cycles can emerge through locking of capital in sectors (Haxholdt et al. 1995). Entrainment, combined with positive feedback, also leads to herd behaviour, and the kinds of boom and bust cycles described earlier.

4.8 Modules and motifs

Two network structures—motifs and modules—often play roles in emergence. In network terms, a *motif* is a small set of nodes and edges, often repeated, that plays a well-defined role in a complex system (Milo et al. 2002) (Fig. 5). Feedback loops (see Sect. 4.6) provide a good example. In financial markets, modules can take several forms, such as economic clusters (van Roekel and Smit 2022) and niches (Cazzolla Gatti et al. 2020).

In general, both modules and motifs are structures that serve to fix emergent patterns in a network. Elsewhere, I present two examples of this. One is stigmergy (Sect. 3.3) and the way clumps (effectively modules) form and grow, under the influence of positive feedback. The other is the formation of a stable core in dynamic systems, where negative feedback stabilizes a set of nodes (Sect. 4.6). In general, complex systems can accumulate order by forming motifs that provide stabilizing structures. These can play roles in the emergence of network organisation (Benson et al. 2016). Motifs emerge in many settings, including protein interactions (Yeger-Lotem et al.

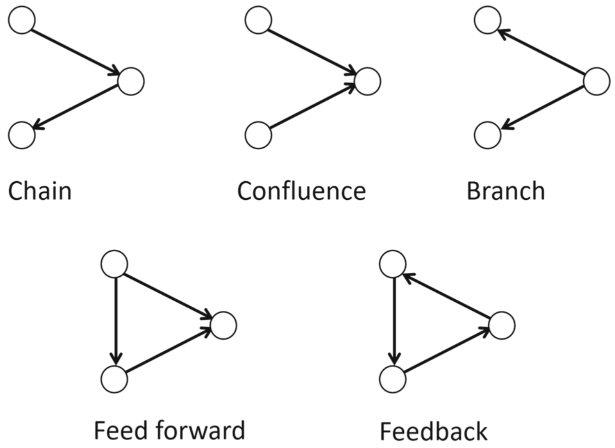

Fig. 5 Examples of common motifs of three vertices that can appear in directed graphs

2004), emergence of molecular functions (Aziz et al. 2016), genetic regulation (Burda et al. 2011), and social networks (Girvan and Newman 2002). Proteins, for instance, exhibit motifs comprised of short sequences of amino acids (Hofmann et al. 1999). Typically, the sequences form structural elements, such as bends, sheets and helices, that play a crucial role in the way a protein folds.

Some motifs, especially feedback loops, are common in finance (see Sect. 4.6). Motif patterns have also been shown to play a role in certain economic conditions, such as price spillover (Liu et al. 2019).

Modules take encapsulation of ordered elements a step further. We can define a *module* to be a network component that is well-connected internally, but minimally connected to the rest of the network. Tree growth, for instance, is highly modular, being dominated by the repetition of elements such as buds, branches, leaves and fruit. Genetic regulatory networks also exhibit modularity (Caetano-Anollés et al. 2019). In development, the genome includes modules that control the growth of anatomical features (Gu et al. 2015; Szenker-Ravi et al. 2022), such as the eye (Halder et al. 1995; Arnoult et al. 2013), neural development (Gu et al. 2015), and anatomical asymmetry (Szenker-Ravi et al. 2022). Simulation studies have accounted for the prevalence of modules in neurological and genetic regulation by showing that selection favours the emergence of modules because they reduce the cost of connections between nodes (Clune et al. 2013; Mengistu et al. 2016). Other studies show that relatively simple processes, such as gene relocation and selection, can account for the emergence of genetic modules (Karlebach and Shamir 2008; Newth and Green 2007; Mittenthal et al. 2012).

Modular organisation lends itself to the emergence of hierarchical structures, consisting of modules within modules. Such hierarchies convey the advantage of breaking big processes or structures into smaller, efficient, and easily managed ones. Human organisations exploit this property deliberately. A large company, for instance, can achieve efficiency by dividing its operations into separate divisions, each dealing with a different area of work (e.g. manufacturing, transport, marketing). Engineering and architecture adopt the same approach to design, which provides efficiencies for construction and maintenance. The downside of modularity is the emergence of failures arising in cases that do not fit neatly into the hierarchy. A business example would be a problem that goes unnoticed because it does not fit neatly under (say) manufacturing or marketing.

In networks, a module is a set of nodes that are connected internally, but with minimal edges joining them to the rest of the network. Attempts to measure modularity in these terms have not been completely satisfactory. Some metrics requires identification of a candidate module. However, this approach cannot work in a search to detect unknown modules. Newman introduced a general metric to assess the degree of modularity in an arbitrary network (Newman 2006). However, this metric suffers from being highly correlated with the network's edge density.

5 Modelling paradigms

Several approaches to modelling have been widely used in studies of emergence in complex systems. In many cases, they provide alternative ways to represent the same phenomena.

5.1 Measuring emergence

In many contexts, it is useful to know whether new features emerge in a system. To this end, metrics that test for emergence are appealing. By analogy with "seeing the wood in the trees", emergent features are often obscured by random details in observations made at micro-scale, and become obvious only in macro-scale, compressed data (Hoel et al. 2013; Varley and Hoel 2022). For this reason, metrics to detect emergence have usually been based on changes in complexity (Gershenson and Fernández 2012).

The problem of capturing complexity in a single number amounts to finding a simple solution to something that is, by definition, not simple. A complexity metric does away with all the richness. However, it can be useful when comparing the *relative* complexity of different systems, or changes in complexity over time or scale.

Most attempts to measure complexity have adopted one of two approaches: description or thermodynamics. The first approach is to consider how a system is described. In early studies, several authors argued that complexity can be measured as the length of the code or algorithm that generates a description of the system (Solomonoff 1964; Chaitin 1966; Kolmogorov 1968). However, these metrics depend on the encoding system employed, and different encoding systems may yield inconsistent results. A greater problem is that adopting this interpretation suffers from the drawback that a random system would be considered complex. This confuses complexity with complication. That is, these metrics fail to capture the richness of connections that make systems truly complex.

To deal with the above problem, a refinement is to regard complexity as a message that falls into two parts: *program* plus *data* (Wallace 2005). Similarly, Papentin (1980b) proposed two kinds of order: primary (orderly pattern) and secondary (random entropy). The ordered component can be taken as an indicator of emergence in the system.

The alternative approach is to apply thermodynamic concepts, which have motivated several measures of complexity, and hence emergence (Prokopenko et al. 2009), Mnif and Müller-Schloer (2011). The Shannon-Wiener index relates information to the entropy of a system. Thus, a quantitative measure H of complexity, based on Shannon's definition, is given by

$$H = -\sum_{i=0}^{n} p(i) \log(p(j))$$

for a system with n attributes, where $p(i)$ is the probability of attribute $a(i)$. This metric reaches its maximum value, $log(1/n)$ when all n states have equal probability, that is, when the system is random. The entropy in the system is higher when the system is less ordered. Emergent order can then be measured by the difference of the observed entropy from the maximum (e.g. Mnif and Müller-Schloer (2011)):

$$H_{\text{emergence}} = H_{max} - H_{observed},$$

where H_{max} is the entropy obtained from the uniform probability distribution.

Note that information measures can be misleading. For example, the same system can appear to be self-organizing, or disorganizing, when partitioned at different scales (Gershenson and Fernández 2012). Two further objections can be raised about the above direct entropy measure (Nowosad and Stepinski 2019). The first is that, like the previous approach, it does not discriminate between ordered and random systems. The second objection is that it focuses only on proportions, not on organization: it fails to consider interactions within a system.

To capture interactions within a system, an alternative is to use *mutual information* between parts of a system. We can measure this via joint entropy, which extends the Shannon-Wiener index by considering interactions (Szabo et al. 2014; Varley and Hoel 2022). For a collection of n entities, the (pairwise) joint entropy is given by

$$H_{joint} = -\sum_{i=0}^{n}\sum_{j=0}^{n} p(i,j) \log(p(i,j)),$$

where $p(i, j)$ is the probability of entities i and j occurring together.

A visual example highlights the difference between entropy and joint entropy. Consider a grid in which the entities are n colours of cells in a grid (Fig. 6). In this grid, colours are linked if they appear in adjacent cells. A simple entropy metric yields the same result for both random and organized patterns, whereas joint entropy identifies the difference clearly (Fig. 6).

Metrics that use variations on joint entropy, or related ideas, have been applied to the emergence of various group forms, such as flocks, crowds, and traffic congestion (Wright et al. 2000, 2001). Other criteria have also been applied as measures of emergence. For instance, fractal patterns often appear as by-products of emergence,

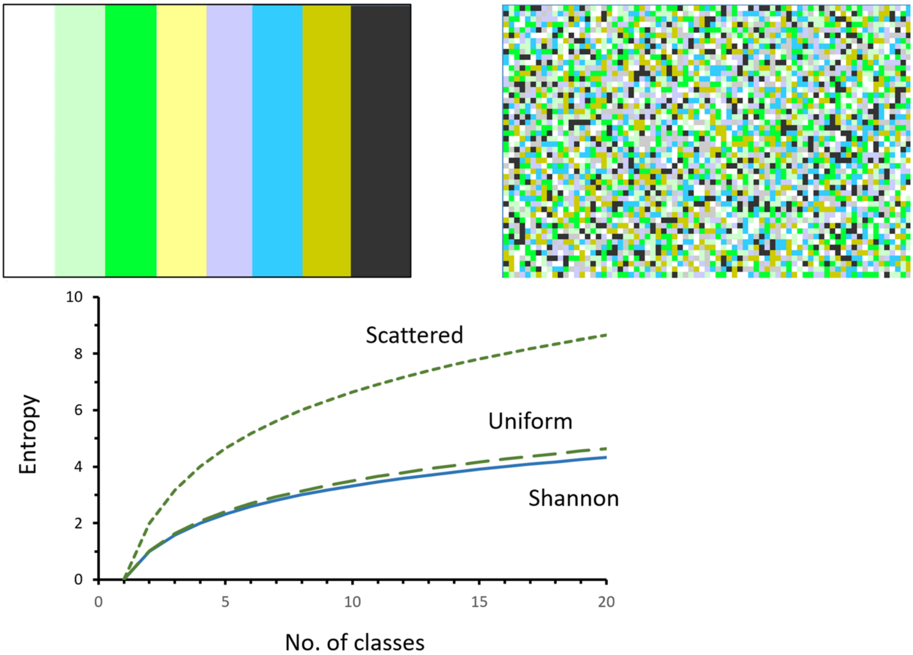

Fig. 6 Use of entropy measures to compare spatial patterns. At the top are uniform (left) and scattered (right) patterns. The plot below them shows entropy as a function of the number of colour classes present. The Shannon index (solid line), is based on the frequency of each class and gives the same values for both uniform and scattered patterns. The dashed lines show the joint entropy for the two patterns. (Redrawn from Green et al. 2020a)

so fractal dimension, and related measures, have sometimes been treated as metrics (see Sect. 5.7).

Most alternatives to the above metrics were developed to deal with specialized contexts, using suitable kinds of data. Examples include information discovery (Kerne et al. 2008); collective intelligence in groups (Chmait 2017), emergence of new technology (Li et al. 2021b), and entrepreneurial activity (Lichtenstein et al. 2006). As discussed in Sect. 4.1, the increasing availability of network data inspires other approaches.

5.2 Network modelling and analysis

The increasing use of network data raises the question of how to detect and test for emergence, based on network structure. The essential problem is that causal patterns can get lost amidst the noisy detail of individual nodes and edges (see Sect. 2). An obvious solution is to reduce the effect of local detail by taking a higher-level view of a network. One such approach is embodied in the idea of "causal emergence" (Hoel et al. 2013). This method calculates the difference in "effective information" between microscopic topology and one in which individual nodes are grouped into "macronodes" (Hoel et al. 2013; Klein and Hoel 2020). However, the method has so far proved difficult to apply, because it makes assumptions that real networks rarely satisfy (Rosas et al. 2020).

Network metrics are widely used to test for emergent features and other issues (Boccaletti et al. 2006; Newman et al. 2002; Oehlers and Fabian 2021). We met modularity earlier (see 4.8). Some other common metrics include the following:

- *Clustering* tests for the presence of tightly linked clusters, which are common in small worlds.
- *Centrality* tests for the presence of important nodes (Rodrigues 2019), such as influential people in a social network.
- *Assortativity* tests for the tendency of similar nodes to link together. Social applications usually base assortativity on node degree (the number of edges a node has), but any numerical property could be used (e.g. company size).
- *Path length* tests for the average length of paths within a network. In small worlds, for instance, the path length is relatively low.

A rapidly growing area of research concerns graph similarity (Emmert-Streib et al. 2016). These methods compare how alike two networks are, based on various criteria (Bause et al. 2021; Zager and Verghese 2008). There have been many applications, often using specialized methods. However, we can identify three broad areas of application:

- *Searching for known patterns in a network*. This kind of application has long been common in bioinformatics, notably in motif searching in proteins and genes.
- *Comparing different observed networks*. Comparison of networks is increasingly used to study social networks and markets. For instance, a recent study (Faizliev et al. 2019) constructed networks of links between companies, based on co-mentions in newspaper reports. Comparing similarity of the resulting networks at different times provided a test of market stability.
- *Testing models against observed networks*. Similarity has been widely used to test whether social and other networks conform to known models (Lusher et al. 2013).

5.3 Thermodynamics and self-organisation

Thermodynamics treats emergence and self-organization in a system at a macroscopic scale. This "top-down" approach contrasts with (and complements) network models, which provide a "bottom-up" approach to emergence. Non-linear interactions between objects produce emergent structure in many kinds of systems. Gravity, for instance, drives clouds of dust and gas to aggregate into stars, and the emergence of solar systems with orbiting planets. In water, if the heat energy of atoms falls below a critical threshold (freezing point), then hydrogen bonds form between the water molecules and ice crystals emerge.

Prigogine (Prigogine and Lefever 1968) used the term *dissipative* to describe open systems that are far from equilibrium and share energy with the outside. Near-equilibrium systems tend to dampen variations; non-equilibrium behaviour is very different. Local irregularities can expand (e.g. by positive feedback, entrainment), leading to the emergence of large-scale order. The thermodynamic idea of open systems corresponds to the model of a system as a network embedded within an environment of other networks (see Sect. 4.3). So, local irregularities in the system can arise by interactions with an outside network, setting off a cascade of changes, as we saw earlier (Sect. 4.3).

5.4 Dynamic systems

A dynamic system is a system whose state at any time is a function of its previous states. The term is usually applied to systems in which changes are represented by variable quantities that are associated with the system. A simple example is logistic growth.

$$\frac{\mathrm{d}p}{\mathrm{d}t} = rp\,(1 - p/L)\,,$$

where p is population size, t is time, r is growth rate, and L is the environment's carrying capacity. In this continuous form, it has the explicit solution

$$p(t) = \frac{L}{1 + e^{-rt}}.$$

In ecology, the logistic model describes the growth of a population whose size is limited by the carrying capacity of its environment. In economics, it leads to the *Diffusion of Innovation* model (Rogers et al. 2014; Meade and Islam 2006), which describes the sigmoid pattern of growth that typically follows the entry of a new product into the market.

The continuous form of the logistic model converges to L as a limit. However, in the discrete form

$$p' = rp(1 - p/L),$$

different kinds of behaviour emerge, depending on the value of r. For positive values of r, the system exhibits five main patterns of behaviour: converging asymptotically to zero ($r < 1$), converging to $(r - 1)/r$ ($1 < r < 2$), emerging cycles and period doubling according to Feigenbaum's ratio 4.66 ($2 < r < 3.57$), sensitivity to initial conditions (chaos) ($3.57 < r \leq 4$), and rapid population crash ($r > 4$).

In general, dynamic systems often exhibit *attractors* as emergent patterns. From its initial state, a system will undergo transient behaviour before falling into an attractor state. This state may be an equilibrium, a limit cycle, or a strange attractor. As described above, all of these behaviours can emerge in logistic growth. *Strange attractors* are associated with chaotic behaviour and exhibit several characteristic properties. First, they are sensitive to initial conditions. In the logistic, for instance, a small difference in initial population size soon leads to completely different behaviour by the population. This makes the future behaviour of a chaotic system essentially unpredictable. However, the strange attractor is usually confined to a finite region, so values will lie within a limited range.

For multi-species communities, we can express system dynamics using the generalized Lotka–Volterra equation,

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = r_i p_i \left(1 - \sum_{j=1}^{n} \alpha_{ij} p_j\right),$$

where p_i is the size of population i of n populations, r_i is the population growth rate for population i and α_{ij} is the interaction between populations i and j.

By interpreting populations as economic agents, and biomass as capital, we can draw parallels between ecosystems and financial systems. The Lotka–Volterra equations, originally introduced to describe the dynamics of food webs, can be applied to financial transactions. Many of these applications concern transactions in the stock market (Solomon et al. 2000) and trace the movement of wealth between traders (Samanidou et al. 2007). Other applications include models of competition for market share (Marasco et al. 2016; Ziegler et al. 2020) and the movement of capital between banks (Comes 2012).

The above, continuous model assumes that interactions within the system are immediate, and synchronous. However, the model can fail in several ways, causing different behaviours to emerge. First, a population consists of discrete entities, so when populations are small, individual interactions are likely to be asynchronous. Also, the model implicitly assumes a uniform spatial distribution of individuals at all times. Spatial heterogeneity can alter the dynamics completely. Both of these issues have raised the need for agent-based models to understand the patterns that emerge in non-uniform conditions (see Sect. 5.6).

Temporal heterogeneity can likewise cause different behaviour to emerge. For instance, seasonal breeding introduces delays into population growth. In a simple, predator–prey system, for example, these delays can be seen as feedback (see Sect. 4.6). Cyclic behaviour often emerges in the presence of negative feedback, with the cycling period being governed by the delays in the loop.

Although the emergence of chaos is easy to show in models, the combination of sampling and other practical issues makes it difficult to prove in real populations. This raised the question, does it actually occur in nature? (Turchin and Ellner 2000) Subsequent studies into the necessary field methods, as well as careful experiments, appear to show it does occur in some organisms (Becks et al. 2005).

5.5 Automata

Many processes that occur in discrete steps can be represented as *automata*. These are abstract entities that change state (behave) according to inbuilt programming, and to inputs (from their environment). Note that networks are inherent in the behaviour of automata. We can treat an automaton's state at any time as a node, and the transitions to the next state as a (directed) edge. Features of automata can contribute to emergent patterns and behaviour, which may arise out of combinations of the following:

- *Programming*, especially iteration. Examples include emergence of periodic or chaotic behaviour.
- *System memory.* Retention of states during iteration is essential for anything to emerge in most of these processes.
- *Interactions between different automata* ("agents", see Sect. 5.6). This is a rich source of emergent behaviour, especially promoting changes of state, e.g. percolation, entrainment.
- *Interactions between automata and their environment.* These might be inputs, constraints, or effects that the automaton (agent) has on its environment.
- *Other contributing processes.* Examples include feedback (output returning as input), phase changes in the connections between automata, and increase in the population of automata.

Most of the above apply when there is more than one automaton involved, and they exchange information. Many of the following sections concern particular cases, especially in the way the automata are organized and how they interact with their environment.

5.6 Agent-based models

Agent-based models (*ABMs*) represent a system as an ensemble of independent automata (autonomous *agents*), in which the outputs of one agent can become an input to another agent (Bonabeau 2002). Agent-based models have become increasingly popular for understanding complex systems because global features usually emerge bottom-up from interactions between simple agents (Odell 2000; Haynes and Alemna 2022). Often, the agents are embedded in an environment. Most of the modelling paradigms discussed below are variations on this idea, but put particular constructs on the agents.

Several questions have attracted much attention in agent-based modeling. One is the emergence of cooperation in groups. Game theory approaches look at the strategies that agents employ with the goal of achieving the best return. Prisoner's dilemma (Rapoport et al. 1965) poses this as a choice in which each of a pair of agents can either cooperate or defect. If they cooperate, then they each receive a small penalty. If one defects, then that agent receives no penalty and the other agent receives a heavy penalty. If both defect, then both receive a heavy penalty. The game can also be inverted, so that rewards are at stake, rather than penalties. *Iterated Prisoner's Dilemma* repeats the trial many times to discover what emerges as a population evolves. The outcome depends on the relative risks and rewards, but some scenarios make the emergence of social cooperation inevitable (Aydogmus et al. 2020; Fogel 1993).

The manner in which the states of agents change can have a big impact on the kinds of behaviour that emerges in a system (see Sect. 5.9).

One of the first agent-based models of living systems was of bumble-bee colonies (Hogeweg and Hesper 1983). This study showed that organisation of the colony emerged from simple TODO rules that govern individual behaviour. Similar models have since shown that in many organisms, group organisation emerges from sim-

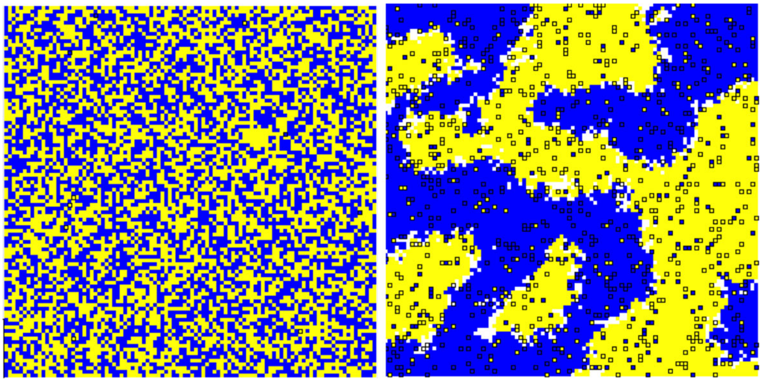

Fig. 7 A simple example of stigmergy (ant sort algorithm). From an initially random scatter of objects (left), order emerges via a combination of simple behaviour and positive feedback

ple rules of behaviour. In stigmergy, for instance, ants sort material in their nests by obeying simple rules (Dorigo et al. 2000), such as:

– *If you see an egg, pick it up*;
– *If you see a pile of eggs, drop the egg you are carrying*.

Positive feedback contributes to this process: larger piles tend to grow at the expense of smaller ones, until all the eggs are stored in one location (Fig. 7).

An important application of ABMs has been to study the movement of large groups of agents. For instance, a flock of birds emerges when individual birds follow simple rules, such as: (a) collision avoidance (avoid getting too close), (b) velocity matching (move in same direction and speed as those nearby) and (c) flock centering (stay close to flock) (Reynolds 1987). Models of self-organisation can explain how the shape and internal structure of groups emerge in flocks of birds and schools of fish (Hemelrijk and Hildenbrandt 2012).

When moving *en masse* in crowds, humans tend to behave in simple ways. Their behaviour causes spatial patterns to emerge, especially if barriers are involved. The dual problems of efficient movement of people, and preventing catastrophic jams during emergencies, have motivated a whole field of study (Helbing et al. 2000, 2005). In emergency situations, for example, exiting a building rapidly can make the difference between life and death. However, if a crowd of people tries to exit through a single door, then the exit becomes jammed and no one can move. Placing a small wall in front of the door forces people to move around it in singe file. This turns the crowd into two rapidly flowing streams.

One of the most widely used paradigms for modelling intelligent, autonomous systems is BDI agents (Rao et al. 1995). Here, BDI is short for Belief, Desire, and Intention. This approach arose in research on artificial intelligence and has been applied widely in programming autonomous agents, such as robotics, automated processing, traffic control systems, and control of electric power grids (De Silva et al. 2020; Cardoso and Ferrando 2021). The underlying idea is that an agent is programmed to decide what action to take in any situation. To do this, it has a *belief* (what it believes to be the current state of its environment, itself and other agents), a *desire* (the state that it wants to result), and an *intention* (the sequence of actions it plans to take to achieve the result). However, interaction between agents was not part of the original BDI paradigm, so it has not been so widely used in models of multi-agent systems (Cardoso and Ferrando 2021). However, the ability of BDI to embody cognitive reasoning has made it ideal for many financial applications, such as blockchain (Alaeddini et al. 2021).

ABMs have become increasingly popular in economics because of their ability to embody endogenous, bottom-up processes (Arthur 2006). Many of the studies cited in previous sections used ABMs, especially for dealing with feedback (Sect. 4.6), and entrainment (Sect. 4.7). In ABM studies of economics, the agents can represent individuals, companies or traders. Interactions between the agents are usually financial transactions, but can also be exchange of goods, or influence.

The need for agent-based models in economics was summed up succinctly by Gatti et al. (2010), who argued that traditional economics is reductionist: "*aggregates are just the sum of individual behaviours*", so "*... all explanations must be reduced to the more fundamental lower level, that is to microeconomics*". Instead, they proposed "*... a bottom-up approach: let us start from the analysis of the behaviour of heterogeneous constitutive elements (defined in terms of simple, observation-based behavioural rules) and their local interactions, and allow for the possibility that interaction nodes and individual rules change over time (adaptation).*"

Traditional economic models typically assume that players follow a single strategy, have perfect information, and that systems converge to an equilibrium state (Arthur 2021). Agent-based models typically relax all those assumptions, and allow behaviours to emerge out of interactions between agents (Arthur 2006). This break, from traditional numerical models to a computational approach, has been heralded as a new paradigm in economics (Odell 2000; Axtell 2007; Gatti et al. 2010; Bargigli and Tedeschi 2013).

ABM studies have shown how bottom-up interactions can lead to many kinds of emergence, including for example: macro-level phenomena (Ballot et al. 2015), rise of entrepreneurism (Yun et al. 2018), growth of specialization (Jung 2019), and the appearance of key players in markets (Galeotti 2006). Applications of ABMs to stock markets (Padgett and Powell 2012; Lussange et al. 2021; Cavalli et al. 2022) have helped to explain many kinds of emergence, such as price-taking behavior (Flåm 2020), market criticality (Harré 2018), crises (Kirsch and Rühmkorf 2017), and bubbles (White 1990; Rappoport and White 1993; Barbie and Hillebrand 2018).

5.7 Fractals

Fractals are patterns that emerge by the iteration of the same rules on different scales (Papentin 1980a; Mandelbrot 1982). The formation of fractal architecture has been shown to explain the emergence of metabolic networks (Aon et al. 2004) and the tightly paired emergence of airways and blood vessels in the lung (Glenny 2011). The emergence of nested clusters of cells during neuronal development shows fractal properties (Mir et al. 2014).

Because fractal patterns coincide with emergence in many contexts, fractal analysis has sometimes been proposed as a way to measure emergence in complex systems, such as the emergence of order in microbial communities (Balaban et al. 2018). Fractal properties have been widely used in conjunction with spectral properties to analyse time series (Gneiting et al. 2012) especially in association with transitions to chaos (Yu et al. 1990).

Despite early interest in fractal properties within economics (Peters 1994; Qian 1994; Corning 1995), fractal analysis has not been widely adopted. However, fractal patterns have been observed within the structure of supply networks (Hearnshaw and Wilson 2013) and in the patterns of transactions in bonds (Kim and Yoon 2004) and other financial markets (Peters 1994; Inaoka et al. 2004; Lussange et al. 2021). Greatest interest has been shown in the association of fractal patterns with chaotic market behaviour (Díaz 2015), especially when they are associated with market crashes (Shi et al. 2022).

5.8 Formal languages

Formal language models contain three elements: an *alphabet* to represent constants, variables and other symbols; *semantics* that define the meaning of the symbols; and a *syntax* to set rules for how expressions are formed.

Language based approaches to modelling complex systems are effective at capturing organization and patterns. However, they have not been widely adopted, at least in explicit form. Nevertheless, syntactic rules are implicit in many widely used applications, such as machine learning (Liang and de Rijke 2016), and agent-based models (Becerra-Bonache and Jiménez-López 2015) (see Sect. 5.6).

Syntactic models are very good at capturing modular processes. An excellent example of this are L-systems (named after their inventor Aristid Lindenmayer). They are formal languages that represent the organisation of growth (Lindenmayer 1968). Growth models are characterized by iteration of syntactic rules, in which constants correspond to discrete fixed units and variables denote growth elements (Herman and Rozenberg 1975). In plant growth, for instance, the iteration of similar elements often leads to emergent structures that are fractal in nature (Prusinkiewicz and Lindenmayer 2012).

A criticism labelled at early syntactic models was their over-reliance on first-order, context-free grammars. This made it impossible to deal with the context-sensitive nature of many kinds of growth and behaviour. In contrast, numerical models of growth use the paradigm of environmental constraints, such as biochemical gradients that control rate of growth of different organs during development. A solution, which combined both kinds of models, was to introduce parametric L-systems (Prusinkiewicz et al. 1999). These enable the introduction of context-sensitive rules, such as

$$S_0 : \text{condition} \rightarrow S_{\text{new}},$$

where condition relates to the values currently taken by some parameter. These parameter values can be determined by neighbouring units, or by conditions in an environment.

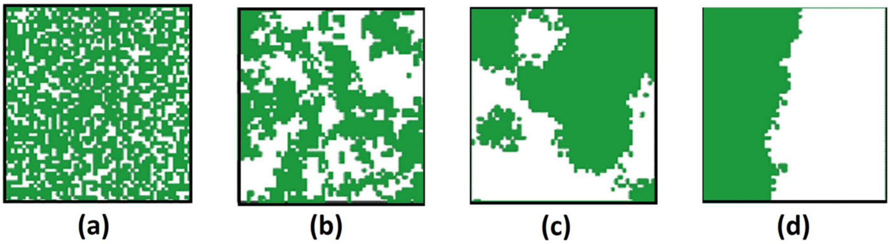

Fig. 8 Emergence of spatial pattern in vegetation. **a** Seeds disperse anywhere; **b** Local seed source; **c** As in **b**, but fires clear patches; **d** As in **c**, but an environmental gradient limits distributions (redrawn from Green et al. 2020a)

For instance, in plant growth, a bud may form on a branch only if there is sufficient sunlight and moisture to allow it to form. This is an example of clocked processing (see Sect. 5.9).

5.9 Cellular automata

A fruitful paradigm for studying emergence in computation is the *Cellular Automaton* (abbreviated "CA") (Wolfram 1984). A CA is a specialized agent-based model, in which the agents (cells) form a fixed grid. In addition, each cell has a neighbourhood, which is comprised of other cells, and accepts their states as inputs. All cells are identically programmed and update their states in a series of time steps, according to their programming. An early CA model was the Game of Life, introduced by Conway (1970) to demonstrate the way in which patterns can emerge in systems that obey simple rules.

Cellular automata have been used to simulate emergent patterns and behaviours in several kinds of real-world systems, especially where processes are spread across a surface (e.g. percolation) or landscape. Examples include fire spread, seed dispersal, and hydrodynamics. CA models have also been combined with other ABMs, with the CA being a substrate (e.g. landscape) on which the agents move (e.g. animals) (Fig. 8).

CAs, especially 1-Dimensional CAs, have also been used as vehicles for investigating properties of the state space of processes. For this purpose, the state of the CA grid is a vector comprised of the combined state of every cell in the grid. The states form a network, in which the nodes are states and the edges are transitions between states. In a grid of N cells, the number of possible states of a binary, deterministic CA is 2^N, which means that the CA must eventually return to a previous state in less than 2^N time steps. Hence it must ultimately fall into a fixed state, or a limit cycle. Given this network structure of the state space, three classes of behaviour emerge, depending on the degree of connectivity between the possible states of individual cells. When the connectivity is low, the resulting CAs quickly fall into a fixed state, or else cycle. If the connectivity is high, then fixed states are rare, and long cycles are common. Experiments show that the most "interesting" behaviour occurs when the connectivity of the state space lies in the critical region between low and high connectivity (the so-called "edge of chaos") (Langton 1990).

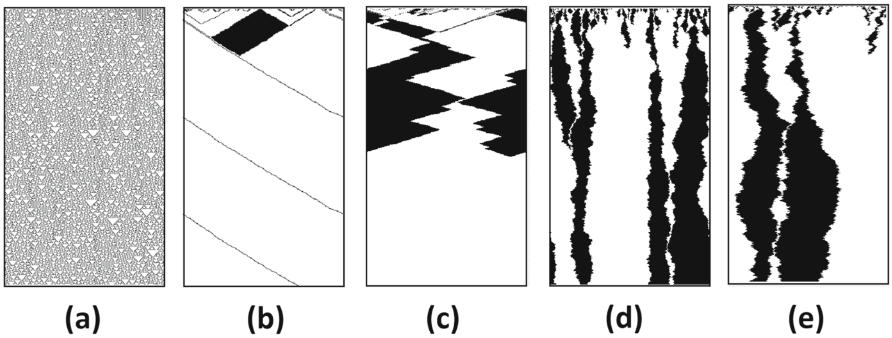

Fig. 9 Effects of processing order on emergent patterns. **a** Synchronous, **b** Cyclic (OAS), **c** Clocked (OAS), **d** Independent (RAS), **e** Random order (RAS). (Redrawn, based on Cornforth et al. (2005))

The emergence of patterns in CAs also depends on the manner of processing. Time is usually represented as a series of discrete steps, in which the state of every cell is updated in parallel, and synchronously. However, many real-world systems do not update synchronously. In plant growth, for instance, the rate at which new branches appear is limited by the supply of water, nutrients and light (see Sect. 5.8). Where these resources are not available uniformly, parts of the tree will grow faster than others. In social networks and markets, interactions between agents are usually asynchronous too.

We can highlight the impact of asynchronous processing using simple CAs, where different update schemes are applied (Fig. 9). Different patterns emerge, depending on the scheme applied. Several common processes have been identified (Cornforth et al. 2005):

- *synchronous*, where all events occur simultaneously;
- *random asynchronous* (RAS) where events happen in random order, such as transactions in a stock market;
- *ordered asynchronous* (OAS), where events are asynchronous but occur in a fixed order;
- *clocked scheme*, such as the example of plant growth above, where events are asynchronous, but occur locally at a fixed rate;
- *locally synchronizing*, events are initially random asynchronous, but synchronisation emerges from local interactions.

5.10 Boolean networks

Boolean networks (BNs) are computational structures in which a set of identically programmed processors (with just two states: ON or OFF) are linked into a network. The state of each processor at time $T + 1$ is determined by its prior state at time T, as well as the states of neighbouring processors. A cellular automaton is special case in which the processors form a grid. Boolean networks have been used to study social phenomena, such as the spread of beliefs, rumours, or consensus (e.g. Seeme et al. 2019; Green et al. 2014).

In an influential study, Kauffman (1969) applied the idea of a Boolean network to genetic regulation. This model treated genes as simple switches and the state of each gene affected changes in the state of other genes. These switching sequences form cycles. Surprisingly, although the number of potential cycles grows astronomically as the network grows, the length of cycles grows very slowly and corresponds to the way reproductive cycles grow with genome size.

6 Conclusion

In this survey of emergence in systems of simple agents, I have tried to present the core ideas, point to seminal studies, explain important issues that arise, and point to recent research. My overview summarizes several widespread mechanisms that contribute to emergence. However, as my account shows, emergence is often the result of several different mechanisms acting in concert.

It is difficult in a succinct account to do justice to the subject. Almost every topic covered here is now a vast field of research, with literally thousands of studies published each year. Moreover, they are often spread across many different fields, so it is difficult for readers to locate relevant research. Given this problem, one of my aims was to provide references and examples that help readers bridge the gulfs that now exist between fields.

I have used emergence in human activity, especially economics, as examples of several processes. As explained earlier, although humans are not simple agents, sometimes we do behave in very simple ways.

Finally, my account included description of modelling paradigms that are widely used to study complex systems in which emergence occurs. My aim here was to show ways in which we can interpret emergence, and to use those models to illustrate some aspects of it.

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