



# Correction to: Statistical, time series and fractal analysis of full stretch of river Yamuna (India) for water quality management

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As corresponding author, while going through higher research in this area, it is found that the formula for Hurst exponent given in equation (13) on page no. 401 is wrongly written. Accordingly, following corrections suggested:

## ABSTRACT

- On page 397, in Abstract **All water quality parameters at every site follows persistent behaviour**

instead of

Brownian motion (true random walk) behavior exists at different sites for BOD, AMM, and total Kjeldahl nitrogen (TKN).

- Hurst Exponent is defined as slope of regression line of log-log curve of any time series with time instead of the slope of the regression line. Hurst exponent (H) can be computed on the basis of power law decay i.e.**

$$p(k) = Ck^{-\alpha} \text{ where } C - \text{constant}; p(k)$$

—autocorrelation function with lag  $k$

**Hurst exponent is related to the exponent  $\alpha$  by a relation  $H = 1 - \frac{\alpha}{2}$**

**On page 401**

- Delete equation (13)
- Replace  $p(k) = Ck^{-\alpha}$  as Equation (13) instead of equation (14)
- Replace  $H = 1 - \frac{\alpha}{2}$  as Equation (14) instead of equation (15)
- Replace  $D = 2 - H$  as Equation (15) instead of equation (16)
- Replace  $D(H) = -\lim_{\varepsilon \rightarrow 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon}$  as Equation (16) instead of equation (17)
- Replace  $PI = 2|D - 1.5|$  as Equation (17) instead of equation (18)

Reference [1]–[3] clearly states the above-mentioned formula. If  $0 < H < 0.5$ , then anti-persistence behavior and if  $0.5 < H < 1$ , then persistence behavior exists. At  $H = 0.5$ , Brownian motion exists.

## Results and Discussions:

- On page 401, in Results and Discussions: **Table 2 gives regression equation, coefficient of determination of water parameters. Table 3 depicts, Hurst exponent, fractal dimension, and predictability analysis behavior for  $S_1, S_2, S_3, S_4, S_5$ . By using equations (1)–(17), the following are observed:**

instead of

Table 2 gives regression equation, coefficient of determination, Hurst exponent, fractal dimension, and predictability index, and Table 3 depicts fractal and predictability analysis behavior for  $S_1$ – $S_2, S_1$ – $S_3, S_1$ – $S_4, S_1$ – $S_5, S_2$ – $S_3,$

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$S_2-S_4$ ,  $S_2-S_5$ ,  $S_3-S_4$ ,  $S_3-S_5$ , and  $S_4-S_5$ . By using equations (1)–(18), the following are observed:

- On page 412, **Table 2: Regression equations, coefficient of correlation of water parameters at different sites of River Yamuna**

instead of

Table 2 Fractal analysis of water quality parameters

- Also, the values of **H (abs)**, **D (Fractal)** and **PI** from **Table 2 to be deleted as Fractal Dimension (D) is based on a single time-series thus, no correlation exists and fractal statistics are calculated for single time-domain only.**
- On page 413, **Table 3 Fractal Analysis of Water Quality Parameters for Different Sites of Yamuna River (AP- Anti persistence, P- Persistence, B- Brownian time series motion) to be deleted.**
- Add Table 3: Fractal Analysis of Water Quality Parameters for Different Sites of Yamuna River. (Table 3 enclosed)**
- On page 402, **pH: From Table-3, it has been observed that persistent behaviour exists for pH at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

Anti-persistence behavior exists at all sites except for  $S_2-S_5$  which shows persistence behavior.

- On page 404, **COD: From Table-3, it has been observed that COD follows persistent behaviour at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

It is observed that persistence behavior exists for  $S_1-S_2$ ,  $S_1-S_3$ , and  $S_1-S_4$ ; anti-persistence for  $S_2-S_3$ ,  $S_2-S_4$ ,  $S_2-S_5$ ,  $S_3-S_4$ ,  $S_3-S_5$ , and  $S_4-S_5$ ; and Brownian time series (true random walk) for  $S_1-S_5$ .

- On page 404, **BOD: From Table-3, it has been observed that BOD follows persistent behaviour at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

It is observed that anti-persistence behavior exists for  $S_2-S_3$ ,  $S_2-S_4$ ,  $S_2-S_5$ ,  $S_3-S_4$ ,  $S_3-S_5$ , and  $S_4-S_5$ ; and Brownian time series (true random walk) for  $S_1-S_2$ ,  $S_1-S_3$ ,  $S_1-S_4$  and  $S_1-S_5$ .

- On page 404, **AMM: From Table-3, it has been observed that AMM exhibits persistent behaviour at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

It is observed that persistence behavior exists for  $S_1-S_5$ ,  $S_3-S_5$ , and  $S_4-S_5$ ; anti-persistence for  $S_2-S_3$ ,  $S_2-S_4$ ,  $S_2-S_5$  and  $S_3-S_4$ ; and Brownian time series (true random walk) for  $S_1-S_2$ ,  $S_1-S_3$  and  $S_1-S_4$ .

- On page 404 and 407, **TKN: From Table-3, it has been observed that AMM exhibits persistent behaviour at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

It is observed that persistence behavior exists for  $S_1-S_5$ ; anti-persistence for  $S_2-S_3$ ,  $S_2-S_4$ ,  $S_2-S_5$ ,  $S_3-S_4$ ,  $S_3-S_5$  and  $S_4-S_5$  and Brownian time series (true random walk) for  $S_1-S_2$ ,  $S_1-S_3$  and  $S_1-S_4$ .

- On page 408 and 409, **DO: From Table-3, it has been observed that DO observes persistent behaviour at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

It is observed that persistence behavior exists for  $S_1-S_2$ ,  $S_1-S_3$ ,  $S_1-S_4$ ,  $S_2-S_5$  and  $S_3-S_5$ ; anti-persistence for  $S_1-S_5$ ,  $S_2-S_3$ ,  $S_2-S_4$ ,  $S_3-S_4$  and  $S_4-S_5$ .

- On page 410 and 411, **WT: From Table-3, it has been observed that persistent behaviour exists for WT at all sites  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ .**

instead of

It is observed that anti-persistence behavior exists for  $S_1-S_2$ ,  $S_1-S_3$ ,  $S_1-S_4$ ,  $S_1-S_5$ ,  $S_2-S_3$ ,  $S_2-S_4$ ,  $S_2-S_5$ ,  $S_3-S_4$ ,  $S_3-S_5$  and  $S_4-S_5$ .

### Conclusion:

- Also, on page 413 in Conclusion, **Persistent Behaviour exists for all water quality parameters at all sites of Yamuna river; therefore long term behaviour of water quality trend is predictable.**

instead of

Brownian motion (true random walk) behavior exists at different sites for BOD, AMM, and TKN; therefore, water quality trend is unpredictable.

### References

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**Table 3** Fractal analysis of water quality parameters for different sites of Yamuna River

Water Parameters	Sample sites	Hurst Exponent	Fractal Dimension	Predictability Index (PI)	Behaviour
pH	Hathnikund (S <sub>1</sub> )	0.6583	1.3417	0.3166	Persistent
	Nizamuddin (S <sub>2</sub> )	0.6657	1.3343	0.3314	Persistent
	Mazawali (S <sub>3</sub> )	0.5998	1.4002	0.1996	Persistent
	Agra DS (S <sub>4</sub> )	0.5934	1.4066	0.1868	Persistent
	Juhika (S <sub>5</sub> )	0.6681	1.3319	0.3362	Persistent
COD	Hathnikund (S <sub>1</sub> )	0.5801	1.4199	0.1602	Persistent
	Nizamuddin (S <sub>2</sub> )	0.5763	1.4237	0.1526	Persistent
	Mazawali (S <sub>3</sub> )	0.6108	1.3892	0.2216	Persistent
	Agra DS (S <sub>4</sub> )	0.6604	1.3396	0.3208	Persistent
	Juhika (S <sub>5</sub> )	0.5681	1.4319	0.1362	Persistent
BOD	Hathnikund (S <sub>1</sub> )	0.6441	1.3559	0.2882	Persistent
	Nizamuddin (S <sub>2</sub> )	0.6093	1.3907	0.2186	Persistent
	Mazawali (S <sub>3</sub> )	0.5982	1.4018	0.1964	Persistent
	Agra DS (S <sub>4</sub> )	0.6486	1.3514	0.2972	Persistent
	Juhika (S <sub>5</sub> )	0.6210	1.379	0.242	Persistent
AMM	Hathnikund (S <sub>1</sub> )	0.6349	1.3651	0.2698	Persistent
	Nizamuddin (S <sub>2</sub> )	0.6316	1.3684	0.2632	Persistent
	Mazawali (S <sub>3</sub> )	0.6471	1.3529	0.2942	Persistent
	Agra DS (S <sub>4</sub> )	0.6346	1.3654	0.2692	Persistent
	Juhika (S <sub>5</sub> )	0.6064	1.3936	0.2128	Persistent
TKN	Hathnikund (S <sub>1</sub> )	0.6436	1.3564	0.2872	Persistent
	Nizamuddin (S <sub>2</sub> )	0.6102	1.3898	0.2204	Persistent
	Mazawali (S <sub>3</sub> )	0.6497	1.3503	0.2994	Persistent
	Agra DS (S <sub>4</sub> )	0.6512	1.3488	1.6512	Persistent
	Juhika (S <sub>5</sub> )	0.6329	1.3671	0.2658	Persistent
DO	Hathnikund (S <sub>1</sub> )	0.6127	1.3873	0.2254	Persistent
	Nizamuddin (S <sub>2</sub> )	0.5643	1.4357	0.1286	Persistent
	Mazawali (S <sub>3</sub> )	0.5955	1.4045	0.191	Persistent
	Agra DS (S <sub>4</sub> )	0.5775	1.4225	0.155	Persistent
	Juhika (S <sub>5</sub> )	0.5831	1.4169	0.1662	Persistent
WT	Hathnikund (S <sub>1</sub> )	0.5520	1.448	0.104	Persistent
	Nizamuddin (S <sub>2</sub> )	0.5580	1.442	0.116	Persistent
	Mazawali (S <sub>3</sub> )	0.5631	1.4369	0.1262	Persistent
	Agra DS (S <sub>4</sub> )	0.5724	1.4276	0.1448	Persistent
	Juhika (S <sub>5</sub> )	0.5426	1.4574	0.0852	Persistent

3. Kirichenko L, Radivilova T, Bulakh V (2019) Machine learning in classification time series with fractal properties. Data 2019, 4:5. <https://doi.org/10.3390/data4010005>

**In test add Table 3**

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