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A Novel Double Threshold-Based Spectrum Sensing Technique at Low SNR Under Noise Uncertainty for Cognitive Radio Systems

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Abstract
Cognitive Radio is a novel concept that has invoked a paradigm shift in wireless communication and promises to solve the problem of spectrum underutilization. Spectrum sensing plays a pivotal role in a cognitive radio system by detecting the vacant spectrum for establishing a communication link. For any spectrum sensing method, detection probability and error probability portray a significant part in quantifying the detection performance. At low SNR, it becomes cumbersome to differentiate noise and signal due to which sensing method loses robustness and reliability. In this paper, mathematical modeling and critical measurement of detection probabilities has been done for energy detection-based spectrum sensing at low SNR in uncertain noisy environment. A mathematical model has been proposed to compute double thresholds for reliable sensing when the observed energy is less than the uncertainty in the noise power. A novel parameter "Threshold Wall" has been formulated for optimum threshold selection to overcome sensing failure. Comparative simulation and analytical result measurements have been presented that reveals improved sensing performance.

Keywords Spectrum measurement · Energy detection · Noise uncertainty · Dynamic threshold · Threshold wall · Probability of detection · Probability of error

1 Introduction

In the present pandemic situation due to the spread of COVID 19 virus worldwide, where millions of people are forced to stay at home with schools/ colleges/ offices/ entertainment theatres closed, people are now spending more of their times online. The internet usage and its consumption has surged significantly and has pushed people of all ages to online mode for work, play and education. With the rise in social distancing norms, users are seeking new ways to connect mostly through internet according to the recent collated reports from telecom industry [1].

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The main problem encountered while rendering such wireless services is the spectrum scarcity. Research studies carried out by FCC state that the use of fixed spectrum allocation policies makes the wireless spectrum underutilized [2]. The measurement of spectrum usage portrays the underutilization as well as overcrowding of few chunks of RF spectrum [3, 4]. The reason behind spectrum scarcity is the inefficient existing scheme of spectrum allocation. In the present situation, the spectrum scarcity problem has aggravated and needs attention. The issue of spectrum crunch is of utmost significance and needs immediate action for future networks.

For efficient spectrum usage, un-licensed secondary users (SU) can share and access the spectrum along with licensed primary users (PU) with the help of Dynamic Spectrum Allocation via Cognitive Radio [5]. Cognitive Radio (CR) has invoked a paradigm shift in wireless communication era and has the intelligence to detect the vacant band of spectrum and adapt its parameters accordingly [6, 7]. For cognitive radio systems, identification of spectrum availability in the vicinity is the most crucial task which is implemented through spectrum sensing. For spectrum sensing in CR the primary objective is to achieve the desired Receiver Operator Characteristics (ROC) in terms of detection probability P_D, false alarm probability P_{FA}, number of samples N_s, sensing duration at an acceptable level of SNR (Signal to noise ratio).

The main contribution of this work is given as follows: firstly, a novel mathematical framework for computing the “Threshold Wall” has been proposed and simulated to overcome a sensing failure without additional hardware or an extra detection stage. To further enhance the system performance a novel concept of adaptive double threshold has been proposed to make the sensing reliable. The variation in the adaptive thresholds with respect to noise uncertainty has been quantified and formulated as the “Threshold Wall” to avoid sensing error at low SNR in presence of noise uncertainty. The computation of double thresholds prevents detection malfunction and hence reduces the overall error probabilities. This work presents a mathematical model to compute double thresholds for reliable sensing and a new constraint to avoid failure with a computed “Threshold Wall”.

The rest of the paper is organized as: Sect. 1.1 describes the recent related work in the field of spectrum sensing in cognitive radio networks. Section 2 presents the conventional system model in a spectrum measurement process and the exhibits the shortcomings as well. Section 3 proposes a novel method of spectrum sensing through adaptive double thresholds. Section 4 gives the simulation results and presents a comparative analysis of the proposed technique with the existing ones. The paper has been summarized and concluded in Sect. 5.

1.1 Related Work

Over two decades, researchers worldwide explored upon various methods for spectrum sensing. Based on the literature survey on spectrum sensing various methods like: Energy detection (ED) [8–11]; Cyclo-stationary feature detection [12–14]; Matched filter detection [15, 16]; Statistical covariance-based detection [17, 18]; Cooperative sensing [19, 20]; compressed sensing [21–23] were proposed to examine the spectrum occupancy under different scenarios. Considering the research on spectrum sensing methods; energy detection draws the attention of the researchers for its lesser computational and operational complexity [24]. Energy Detection doesn’t require a priori knowledge about the primary user (PU) signal being sensed but is significantly affected by noise conditions [25]. The major constraint that puts a check on the performance of the energy detection (ED) method is selection of threshold at low SNR (signal-to-noise-ratio). Fixed single threshold concept at low SNR prolongs the sensing time. The situation worsens with flickering noise power and leads to sensing failure below a certain SNR (termed as "SNR wall") which is characterized by low P_D and increased probability of error or error rates The noise power fluctuation causes degradation in the performance of a spectrum sensing method which can be improved by using dynamic threshold factor [26–28]. The authors of [29] have presented a double threshold-based sensing technique that narrows down the uncertainty zone to increase the robustness of the system against noise power fluctuations. Several adaptive double threshold-based detection methods have been proposed which incorporates an adaptive factor in computation of the decision threshold to improve the robustness of the system against noise [30–32]. The authors in [33] have proposed a two-stage scheme, fitted to perform in TV bands. It promises to offer a good detection of the occupied bands with desirable false alarm rates (Stage 1) and an exact measurement of the real occupied spectrum (Stage 2). A new adaptive sensing time technique-based spectrum sensing that depends on the SNR has been given in [34] to increase the literal spectrum usage and the achievable throughput. Apart from using double threshold, some methods employ machine learning or optimization algorithms to achieve desirable sensing performance and prevent sensing failure [35, 36]. Authors in [37, 38] have proposed the use of artificial neural networks (ANN) to implement hybrid spectrum sensing. However, all these methods require extra implementation cost and increased hardware complexity. Our previous work in [39] aimed at introducing a novel mathematical model for energy detection-based sensing. It was re-modeling of the existing mathematical system to achieve desirable performance metrics without changing the sensing algorithm. The concept of cooperative communication has been used for spectrum sensing in [40] to enhance the detection performance of an energy detector for VANET. The region of uncertainty is addressed with the help of a Fusion center that makes a cooperative decision on the presence of an available band. Some of the latest research as reported in [41–43], the extensive use of cooperative sensing clubbed with deep learning techniques promises to enhance the system performance and robustness. Furthermore, the sensing technique with the use of double threshold can be made more robust with optimization techniques as illustrated in [44].

All the methods discussed above, aim at reducing the error probability or increasing the detection probability as well as throughput. None of the methods, aimed at inclusion of a minimum threshold value that should be taken into consideration at low SNR. Moreover, adaptive double threshold not only divides the sensing scenario in two regions of high and low SNR but also varies it according to the changing SNR.

Therefore, this paper aims at spectrum measurement at low SNR in the presence of uncertainty in noise with the help of double thresholds that can adapt its value according to the varying noise. A novel mathematical model to compute the double threshold based on the dynamic threshold factor and noise uncertainty has been proposed to offer robustness and higher detection probability even in the region of uncertainty without incorporating extra relays, fusion center or deep learning. Therefore, the presented work aims at simplifying the detection problem by addressing the mathematical formulations that relate the performance metrics. The obtained detection probability and error rate through the simulation depicts improvement as well as reduction in the width of uncertainty zone at low SNR using the proposed scheme of double threshold-based spectrum sensing.

2 System Model

Spectrum sensing via energy detector involves observation of received signal energy over a certain time interval [7]. Signal detection is tested by formulation of two hypotheses with the following conditions:

$$\left\{ \begin{array}{l} \boldsymbol{H}_0 \ : X(n) = N(n) \\ \boldsymbol{H}_1 \ : X(n) = S(n) + N(n) \end{array} \right\} \tag{1}$$

where *X(n)* is the received signal samples at CR node, *S(n)* is the primary signal of interest and *N(n)* is the noise samples. Assume that the noise is Gaussian with zero mean and band limited power spectral density. H_0 and H_1 are the hypothesis denoting absence/presence of a PU signal respectively.

Suppose total number of samples observed during a spectrum sensing time is N_S, average power of received signal *X(n)* is known and noise has a constant variance σ_n^2. The test statistic D(X) is given as follows:

$$D(X) = \frac{1}{N_S} \sum_{n=0}^{N_s-1} X(n)^2 \tag{2}$$

Without noise uncertainty, the central limit theorem gives performance analysis parameters P_{FA} and P_D as per [9] is given below:

$$P_{FA} = Q\left(\frac{\lambda - \sigma_n^2}{\sqrt{\frac{2}{N_S}\sigma_n^2}} \right) \tag{3}$$

$$P_D = Q\left(\frac{P - \sigma_n^2}{\sqrt{\frac{2}{N_S}\left(P + \sigma_n^2\right)}} \right) \tag{4}$$

In Eqs. (3) and (4), λ is the detection threshold, Q(.) is the standard Gaussian complementary function and $SNR = P/\sigma_n^2$. The total error rate can be expressed as sum of error probabilities: false alarm (P_{FA}) and Probability of missed detection (P_{MD}):

$$P_{MD} = 1 - P_D \tag{5}$$

$$P_e = P_{FA} + P_{MD} \tag{6}$$

To compute the number of samples N_S, the detection threshold λ is eliminated from Eqs. (3) and (4) and expressed as:

$$N_s = \frac{2\left[Q^{-1}\left(P_{FA}\right) - Q^{-1}\left(P_D\right)(1 + SNR)\right]^2}{SNR^2} \tag{7}$$

According to Eq. (7), if the noise variance is completely known and constant then signal detection is possible even at low SNR just by increasing N_S. However, number of samples $N_S \to \infty$ when SNR < < 1 (less than 0 dB). In such a scenario, detection of signal becomes difficult with a fixed threshold for all SNR levels [40].

(A) With noise uncertainty and fixed threshold

In previous section, the general model for spectrum sensing assumed constant noise with zero mean and no uncertainty. In real world, model uncertainties cannot be completely ignored since it affects the reliability of the entire system under observation. The noise uncertainty factor ρ is introduced in the noise model $\rho \geq 1$ and for practical requirements the σ_n^2 in (3) and (4) now lie between $(\sigma_n^2/\rho, \rho\sigma_n^2)$ as presented in [9]. At low SNR and uncertain noise, the selection of threshold (λ) by each SU becomes a crucial task to avoid missed detection or raised false alarm. In such a scenario sensing capability of the detection scheme fails. Taking noise uncertainty factor ρ into account in the noise model, Eqs. (3) and (4) are modified as follows:

$$P_{FA} = Q\left(\frac{\lambda - \rho\sigma_n^2}{\sqrt{\frac{2}{N_S}}\rho\sigma_n^2}\right) \tag{8}$$

$$P_D = Q\left(\frac{\lambda - \left(P + \frac{\sigma_n^2}{\rho}\right)}{\sqrt{\frac{2}{N_S}\left(P + \frac{\sigma_n^2}{\rho}\right)}}\right) \tag{9}$$

The uncertainty in the noise power is distributed in the interval $[\sigma_n^2/\rho, \rho\sigma_n^2]$ and can be quantified as shown in Eq. (10). The abovementioned probabilities P_D and P_{FA} are used to compute the total error probability P_E, which is the sum of probability of false alarm and probability of missed detection P_{MD} $(1 - P_D)$:

$$P_E = P_{FA} + P_{MD} \tag{10}$$

The error rate or error probability P_E should be as low as possible for better sensing performance. In [27] the authors have presented the role of "SNR Wall", and its influence on detection robustness. SNR wall has been defined as the minimum required SNR for proper detection in presence of noise uncertainty. It has been explained as the margin below which the detector cannot robustly sense the presence or absence of the signal when the signal power is less than the uncertainty in noise i.e. $P \leq (\rho - 1/\rho)$.

$$SNR_{wall} = \rho - \frac{1}{\rho} \tag{11}$$

It is considered that when $\rho = 1$, the noise has constant power and when $\rho > 1$, the noise model exhibits noise power fluctuations. While designing the sensing-error tradeoff problem, SNR wall is taken as a major constraint that leads to failure of the detection capability. The sensing tradeoff issue signifies the situation when the detector fails to detect or sense the spectrum and does not give any decision on the spectrum availability. This state arises when the SNR at which the measurement is being made is lower than the threshold SNR or SNR wall and is a serious degradation factor for spectrum sensing performance. Such a sensing tradeoff problem holds great significance while deciding a threshold for signal detection as well as threshold for SNR to avoid any sensing failure. At low SNR, it is assumed that the factor $(1 + \mathrm{SNR})\ 1$ and N_s can be now expressed as below:

$$N_s = \frac{2\left[\rho Q^{-1}\left(P_{FA}\right) - \left(\frac{1}{\rho} + SNR\right)Q^{-1}\left(P_D\right)\right]^2}{\left[SNR - \left(\frac{\rho - 1}{\rho}\right)\right]^2} \tag{12}$$

From Eq. (12) it can be inferred that when SNR becomes less than the uncertainty in noise (11), then $N_S\ \infty$ and sensing failure occurs as shown in Fig. 1 [28, 29].

Setting the threshold within the uncertainty region leads to detection failure no matter how large the sensing time is or how large is number of samples (N_s). Thus, selection of detection threshold in the noise uncertain region remains a problem.

(B) With noise uncertainty and dynamic threshold

The performance of a detection technique degrades significantly in a noise uncertain environment at very low SNR. The authors in [28] suggest the use of dynamic threshold for spectrum sensing in presence of noise uncertainty since fixed detection threshold leads to non-robust and unreliable performance. A dynamic threshold factor ρ' in the range $[\lambda/\rho', \rho'\lambda]$ is introduced in detection probabilities and λ is modified to get following equations:

$$\mathrm{P_{FA}} = \mathrm{Q}\left(\frac{\rho'\lambda - \rho\sigma_n^2}{\sqrt{\frac{2}{N_S}}\rho\sigma_n^2}\right) \tag{13}$$

$$\mathrm{P_D} = \mathrm{Q}\left(\frac{\frac{\lambda}{\rho'} - \left(\mathrm{P} + \frac{\sigma_n^2}{\rho}\right)}{\sqrt{\frac{2}{\mathrm{N_S}}}\left(\mathrm{P} + \frac{\sigma_n^2}{\rho}\right)}\right) \tag{14}$$

Again, eliminating λ from the Eqs. (13) and (14), we get N_S as:

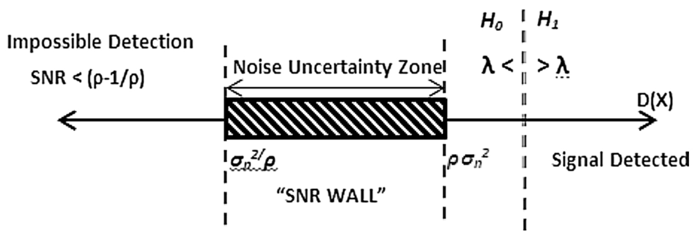

Fig. 1 Testing of Hypotheses H_0 and H_1

$$N_s = \frac{2\left[\left({}^{\rho}/_{\rho'}\right)Q^{-1}(P_{FA}) - \rho'\left({}^{1}/_{\rho} + SNR\right)Q^{-1}(P_D)\right]^2}{\left(\rho' SNR + \frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right)} \tag{15}$$

In Eq. (15) it can be observed that if the noise uncertainty factor is equal to dynamic threshold factor numerically ($\rho = \rho'$), the expression will become like that without noise uncertainty as given in Eq. (7). Thus, it can be interpreted that the effect of noise uncertainty can be nullified by the dynamic threshold factor ρ'. But if the threshold is kept fixed then sensing might become unreliable due to uncertainty in noise at low SNR according to Eq. (15).

3 Proposed Mathematical Model

In this section, the proposed model for spectrum sensing using dynamic double threshold concept has been presented and a new term "Threshold wall" has been coined to account for the ambiguous decision zone due to noise uncertainty at low SNR. The case when the received energy lies in the region of confusion between the two thresholds at low SNR is considered and a novel method is proposed. Furthermore, a comprehensive comparison of the proposed spectrum sensing technique with few of the existing schemes has been presented to have a better insight into the improvement in spectrum sensing performance measurement proposed by the novel method.

(C) Threshold wall

As discussed previously, "SNR wall" deters the selection of threshold within the noise uncertainty zone and renders unreliable performance. This may result in false alarm or missed detection and could disrupt the PU communication system. Such undesirable consequences can be prevented by dynamically adjusting the threshold according to the varying uncertainty in noise. The uncertainty in noise can be compensated by introducing dynamicity/variation in the detection threshold, expressed as below:

$$\text{Variation in } \lambda = \left(\frac{1}{\rho'} - \rho'\right) \tag{16}$$

From Eq. (16) it can be observed that when the uncertainty in noise is at the lower limit (σ_n^2/ρ), it can be balanced by dynamicity in detection threshold ($\rho'\lambda$). Similarly, when the noise uncertainty is at the upper limit ($\rho\sigma_n^2$), its effect can be annulled by reducing the threshold level by (λ/ρ'). Considering the degrading effect of uncertainty in noise and the exalting repercussion of dynamic threshold factor on sensing performance, a new parameter "Threshold Wall" can be quantified through Eq. (17):

$$THRESHOLD_{wall} = \left(\frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right) \tag{17}$$

Threshold wall can be defined as the check parameter to keep the performance metrics within desirable limits by knowing the actual change in the detection threshold value with respect to the noise uncertainty. It can be regarded as the edge where the performance of the detector in terms of number of samples improves drastically at very low SNR. The proposed mathematical expression for threshold wall solves the problem of threshold selection in uncertain noise region. It is observed in Eq. (15) N_s won't tend towards infinity even at low SNR with suitable selection of the threshold wall value.

Previously, it was observed in Sect. 2 that threshold selection below the minimum required SNR ("SNR wall") leads to a situation where N_s increases infinitely. It ultimately ends up in a detection failure with impractical value of probability of error ($P_e \geq 1$). In such a scenario, sensing failure can be eschewed by using the threshold wall (17) for making a decision and varying ρ' in accordance with the ρ when SNR is less than the uncertainty in noise. Grounded on the threshold wall, a novel sensing method based on double dynamic thresholds has been proposed in the next sub section.

(D) Dynamic double threshold

In [31–33] two threshold based energy detection model has been proposed to mitigate the effect of noise uncertainty by maximizing the P_D and minimizing the error probabilities (P_e). The prediction is based on PU activity profile and switching the detection threshold dynamically between the two limits. The upper threshold to achieve higher P_D is taken as λ_{HIGH} and lower threshold for reduced P_{FA} is considered as λ_{LOW}. In this work, a novel mathematical model has been proposed for enhanced detection by considering the following conditions and illustrated in Fig. 2:

$$P_{FA} = P\left(D(X) > \lambda | H_0\right) \tag{18}$$

$$P_D = P\left(D(X) > \lambda | H_1\right) \tag{19}$$

Region of confusion occurs in uncertain low SNR environment and raises the probability of missed detections or false alarm. According to the proposed technique, the detection threshold is switched between the two suggested thresholds in accordance with the variation in noise uncertainty when the detection enters the region of confusion. The concept of threshold wall as proposed in Sect. 3A and as proposed in [31, 39] is taken as the basis for formulation of mathematical expressions of double dynamic thresholds and justified below:

$$\lambda_{HIGH} = \frac{\rho' \lambda}{\rho} \tag{20}$$

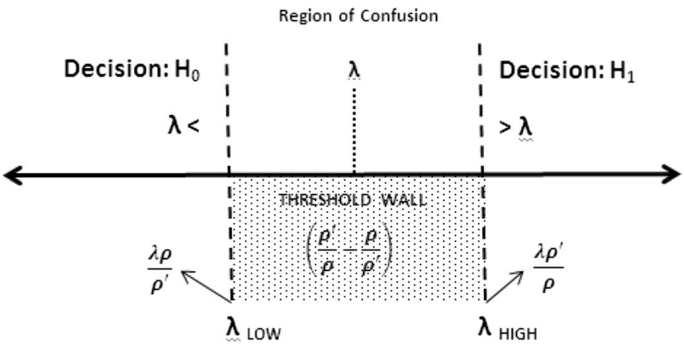

Fig. 2 Proposed Dynamic Double Threshold concept for Region of Confusion

$$\lambda_{LOW} = \frac{\rho\lambda}{\rho'} \tag{21}$$

The two thresholds expressed in Eqs. (20) and (21) depict the two extreme points of the region of confusion. These thresholds help in achieving desired detection probability at varying levels of SNR. At low SNR the lower threshold λ_{LOW} is considered and at higher values of SNR, the second threshold λ_{HIGH} is taken. In both the cases, the effect of noise uncertainty ρ is compensated by a nullifying effect of dynamic threshold factor ρ′. Therefore, the dynamic double threshold can be varied according to the change in noise uncertainty factor at low or high SNR. This not only increases the system robustness but also reduces the probability of error. The proposed method outperforms fixed threshold detection, double threshold and cyclo-stationary feature detection at lower values of SNR. Furthermore, the detection threshold λ is the mid-point of the double thresholds. The difference in the double thresholds can be computed as below:

$$\begin{aligned} \nabla\lambda &= \lambda_{HIGH} - \lambda_{LOW} \\ &= \frac{\rho'\lambda}{\rho} - \frac{\rho\lambda}{\rho'} \\ &= \lambda\left(\frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right) \end{aligned} \tag{22}$$

In Eq. (22), $\left(\frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right)$ is the detection threshold wall according to Eq. (17). Appropriate optimization of this factor helps in attaining desired P_D, lower P_e and aims at finding the targeted number of samples N_S. The two novel thresholds λ_{HIGH} and λ_{LOW} are termed as "*dynamic*" as they depend on the variable uncertain noise (ρ) and a dynamically changing parameter (ρ′). Apart from these two factors, it is dependent on λ which is in turn reliant on noise variance σ_n^2. The proposed technique exhibits enriched detection probability when ρ′ > ρ. The main aim to lower the false alarm and raise detection probability at low SNR is thereby achieved. The novel double threshold-based method performs appropriately at smaller values of SNR and ensures improved sensing performance even with increasing uncertainty in noise as confirmed by the simulation results in Sect. 4. To validate the improvement in performance of a sensing technique with the proposed method, a comprehensive comparison of the few existing sensing methods and their parameters is given in Table 1. For cyclo-stationary detection method, the test statistic is the autocorrelation coefficient assuming OFDM (Orthogonal Frequency division multiplexed) signals with T_d as the number of data symbols; T_C is the number of symbols in cyclic prefix and SNR is signal to noise ratio. The basic concept of cyclo-stationary feature detection is extraction of statistical properties of the signal. In [14] OFDM modulation is considered due its popularity and the correlation structure of the signals with cyclic prefix. Table 1 represents the test statistic parameter used and decision threshold mathematical expressions for different detection techniques like Matched filter detection, cyclostationary feature detection, conventional energy detection with fixed threshold, existing double thresholds, available adaptive double thresholds, and the proposed energy detection with dynamic double thresholds. The parameters from Table 1 have been utilized to obtain simulations results depicting different sensing methods as shown in Fig. 8 to showcase their comparative analysis and the enhancement achieved through the proposed detection method.

Table 1 Comparison of different sensing methods

Sensing technique	Test statistic D(X)	Threshold λ
Energy detection with fixed threshold λ [7]	$D(X) = \frac{1}{N_S}\sum_{n=0}^{N_{S-1}} X(n)^2$	$\lambda = Q^{-1}\left(P_{FA}\right)\sigma_n^2\sqrt{\frac{2}{N_s}} + \sigma_n^2$
Energy detection with double threshold λ_1, λ_2 [28]	$D(X) = \frac{1}{N_S}\sum_{n=0}^{N_{S-1}} X(n)^2$	$\lambda_1 = (\rho - 1)\lambda$ $\lambda_2 = (\rho + 1)\lambda$
Energy detection with adaptive double threshold λ_1, λ_2 and λ^* [30]	$D(X) = \frac{1}{N_S}\sum_{n=0}^{N_{S-1}} X(n)^2$	$\lambda_1 = (\rho - 1)\lambda$ $\lambda_2 = (\rho + 1)\lambda$ $\lambda^* = \left(\lambda_2 - \lambda_1\right)\alpha + \lambda_1$
Cyclo-stationary feature detection [12]	$D(X) = \frac{T_D}{T_D+T_C}\cdot\frac{SNR}{1+SNR}$	$\lambda = \frac{1}{\sqrt{N_S}}erfc^{-1}\left(2P_{FA}\right)$
Proposed method with double dynamic threshold	$D(X) = \frac{1}{N_S}\sum_{n=0}^{N_{S-1}} X(n)^2$	$\lambda_{HIGH} = \frac{\rho'}{\rho}\lambda$ $\lambda_{LOW} = \frac{\rho}{\rho'}\lambda$

4 Simulation Results and Discussion

This section provides simulation results and comparative analysis to validate the proposed mathematical model in the Sect. 3. The results have been obtained through MATLAB simulations and its analysis has led to a conclusion that the proposed method offers better sensing performance in terms of increased detection probability and reduced error probability at low SNR in presence of noise uncertainty. The channel is assumed to be AWGN with noise variance $\sigma_n^2 = 1$, N_S is varied between 200 and 2000 samples, SNR is set low between − 20 and 10 dB, noise uncertainty factor is taken as: $1 < \rho < 1.09$ and dynamic threshold factor is varied between $1 < \rho' < 1.7$. The probability of false alarm is kept low at $P_{FA} = 0.1$ and high detection probability is aimed [38, 39]. Table 2 depicts the simulation parameters and its values in a tabular form.

At low SNR (− 20 to − 2 dB),when signal power is less than the uncertainty in noise $P < \left(\rho - {}^{1}/_{\rho}\right)$, then detection becomes impractical as the number of samples ($10\log_{10}N_S$) increases infinitely as shown in Fig. 3. It denotes the location of SNR wall with a dotted line in dB below which detection does not takes place for uncertainty factor of $\rho = 1.002$, 1.02, 1.04 and 1.06 respectively. It clearly depicts that with increasing noise uncertainty factor ($\rho > 1$), the SNR wall shifts on the higher side and makes the signal presence very vague which ultimately leads to sensing failure. In other words, it depicts that there exists a minimum value of SNR at which detection takes place and is termed as SNR wall [27].

Table 2 List of simulation parameters

S. no.	Simulation parameter	Value/range
1	N_s (number of samples)	200–2000
2	SNR (Signal to Noise ratio)	− 20–10 dB
3	ρ (noise uncertainty factor)	1–1.09
4	P′ (dynamic threshold factor)	1–1.7
5	P_{FA} (Probability of false alarm)	0.1

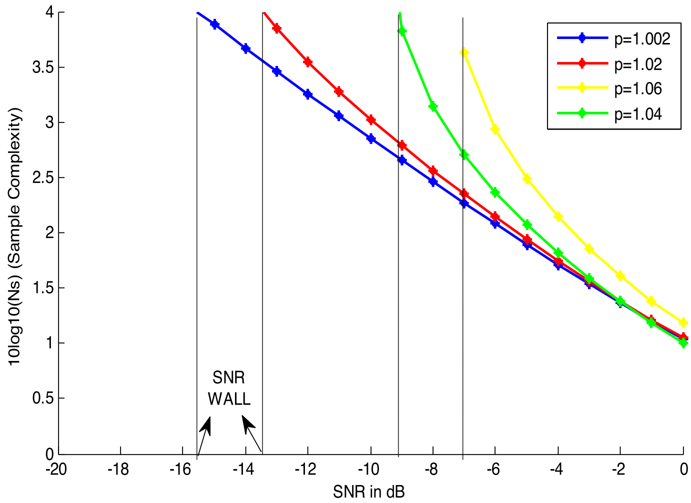

Fig. 3 Increase in N_s at low SNR with increasing noise uncertainty ρ

The signal cannot be detected unless SNR becomes greater than the SNR wall no matter for how long the channel is sensed. The sample complexity $10\log_{10}N_s$ increases at low SNR and thus makes the system performance error prone. The number of samples and SNR has been expressed in dB; performance metrics are taken as $P_{FA}=0.01$, $P_D=0.9$, and $1.002 < \rho < 1.4$.

Figure 4 illustrates the concept of SNR wall as discussed in Sect. 2A. It depicts SNR as a function of noise uncertainty ($10\log_{10}\rho$) defined by (10) and depicts the position of SNR wall for $\rho=1.02$ [27].

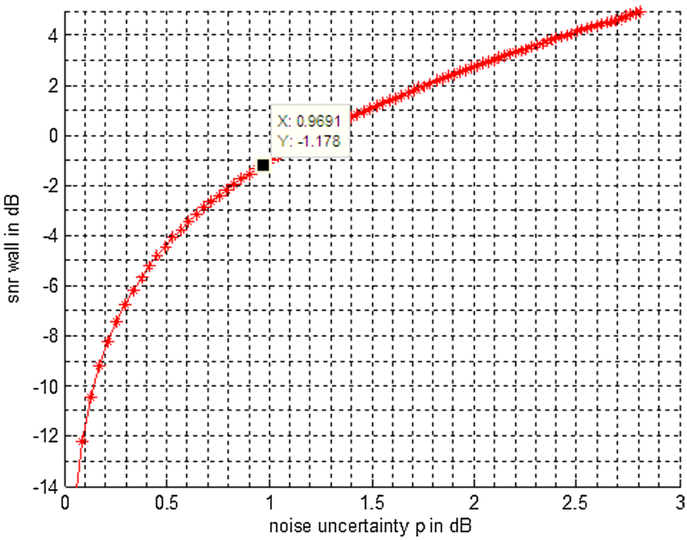

Fig. 4 "SNR Wall" as a function of noise uncertainty [25]

The effect of noise uncertainty on error rate (P_E) can be seen in Fig. 5 using the Eqs. (10), (11) and (12). Without considering noise uncertainty, the performance of the system in terms of error rate and number of samples seems acceptable (as per Eqs. (8), (9) and (10)). Though it requires higher number of samples to obtain lower error rate. However, error rate increases impractically (beyond 1) at low SNR in presence of noise uncertainty according to Eqs. (10) and (12). A significant enhancement in the sensing performance can be observed when dynamic threshold factor (Eq. (15)) is introduced and increased with respect to ρ (ρ′ = 1.3, 1.5). It can be observed in Fig. 5 that when dynamic threshold factor is introduced the error rate reduces at lower value of N_s. With ρ′ = 1.5, there is a reduction of about 20% in the error rate at same N_s as compared to error rate at ρ′ = 1.3. Similarly, in Fig. 6 a graph is plotted between error rate and detection threshold in presence and absence of noise uncertainty ρ using the same set of equations and by varying the detection threshold this time. It can be inferred from the plot that as dynamic threshold factor ρ′ is increased in accordance with ρ, the error rate improves significantly thereby making the system robust.

Finally, in Fig. 7, the proposed model for "threshold wall" has been analyzed and validated. It demonstrates the concept of "SNR wall" for fixed threshold method and "Threshold wall" for dynamic double threshold method. The later one shows significant enhancement in sensing performance that helps in overcoming the sensing failure at low SNR in noise uncertain zone. It indicates that with proper selection of the threshold wall parameter, P_e can be reduced with respect to number of samples at low SNR. As dynamic threshold factor is introduced, it compensates for the performance deterioration due to noise uncertainty factor, thereby promising better performance with reduced error probability.

In Fig. 8, a comparative analysis of the sensing performance in terms of ROC (Receiver Operator Characteristics) curve is shown. The plot between P_D and P_{FA} signifies the enhancement in the performance of the energy detection-based spectrum sensing method using proposed dynamic double threshold as compared to few of the existing

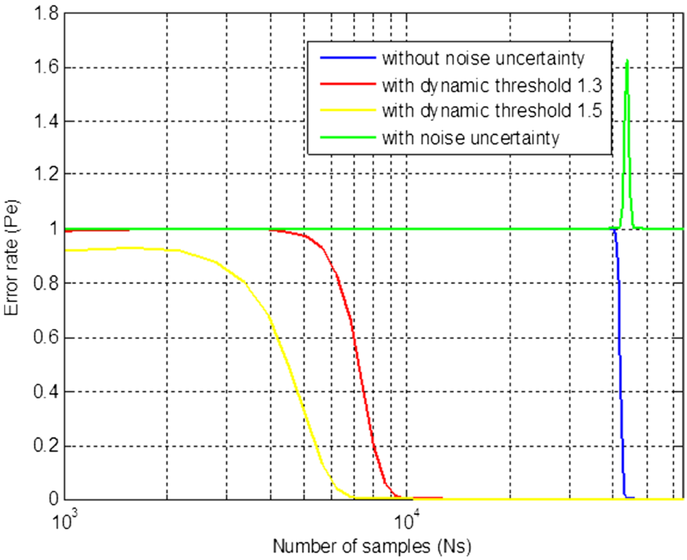

Fig. 5 P_e Vs N_s with fixed and dynamic threshold in presence and absence of ρ

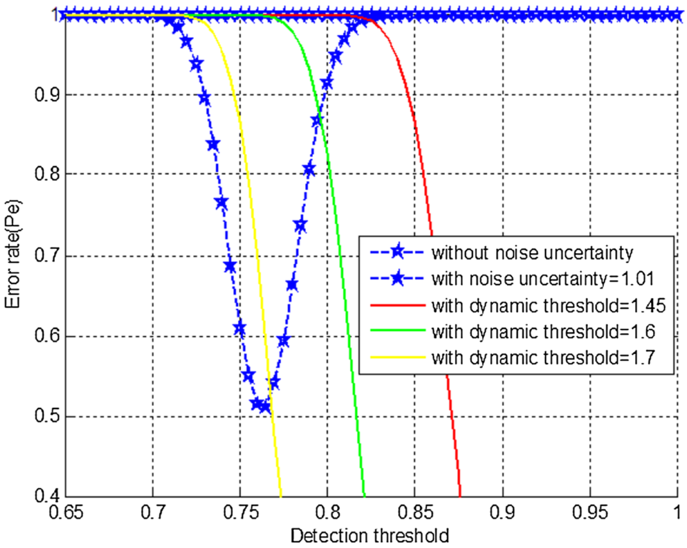

Fig. 6 P_e Vs detection threshold with increasing dynamic threshold factor ρ′ in presence of ρ

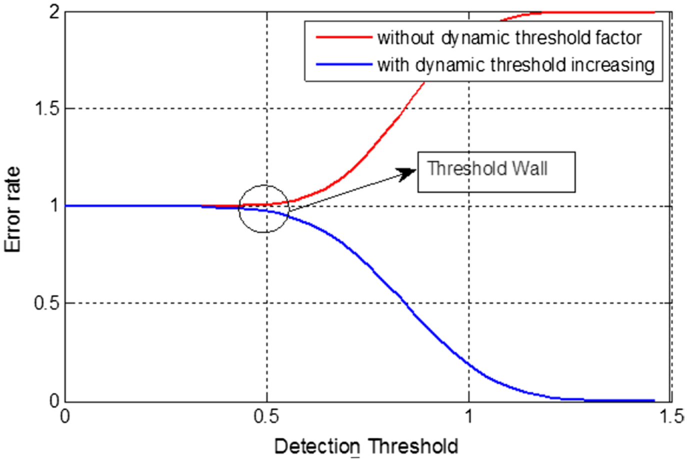

Fig. 7 "Threshold wall" to overcome detection failure

techniques mentioned in Table 1 of the previous section. It can be observed in Fig. 8 that the proposed method using dynamic threshold factor ρ′ = 1.6 for an uncertainty in noise ρ = 1.04, outperforms the fixed threshold, double threshold [29], adaptive double threshold [31] and cyclo-stationary feature detection [12] at SNR = – 10 dB. Thus, the proposed method exhibits robust and reliable sensing performance using dynamic double threshold. Using double thresholds that adapts itself with respect to the uncertainty in noise at low SNR would enhance the sensing performance in terms of increased detection probability.

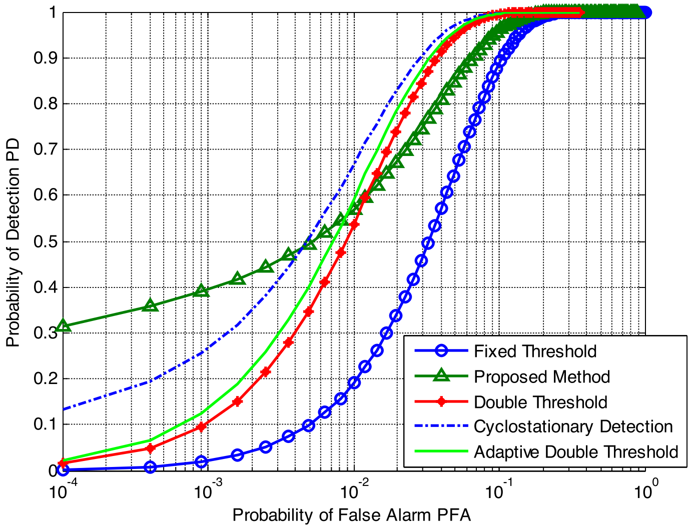

Fig. 8 Comparison of ROC for different sensing methods as per Table 1

5 Conclusion

Energy detector-based spectrum sensing technique does not provide robust and reliable performance at low SNR in presence of noise uncertainty. It leads to sensing failure and urges for a minimum SNR value over which it can perform appropriately. However, SNR wall does not provide a method to select appropriate detection threshold in uncertainty zone and results in increased error rates. A novel parameter "Threshold Wall" along with its mathematical model has been proposed that alleviates the effect of noise uncertainty by using dynamic threshold factor. The dynamicity in the threshold level with respect to noise uncertainty has been quantified as the "threshold wall" and it represents the compensation offered by the dynamic threshold factor in the sensing performance to noise uncertainty factor. To further enhance the performance the concept of dynamic double threshold is considered and a significant reduction in error rate is observed through simulation measurements by using two dynamic thresholds. The role of threshold wall for double thresholds has been analyzed based upon the newly calibrated expressions. Simulated results show that error rates decrease by increasing dynamic threshold factor with respect to noise uncertainty. The proposed dynamic double threshold concept improves the sensing performance and makes it robust at low SNR in presence of noise uncertainty.

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Declarations

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