Self-Localization of IoT Devices Using Noisy Anchor Positions and RSSI Measurements

Vikram Kumar¹ · Reza Arablouei²

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Abstract

Location-enabled Internet of things (IoT) has attracted much attention from the scientific and industrial communities given its high relevance in application domains such as agriculture, wildlife management, and infectious disease control. The frequency and accuracy of location information plays an important role in the success of these applications. However, frequent and accurate self-localization of IoT devices is challenging due to their resource-constrained nature. In this paper, we propose a new algorithm for self-localization of IoT devices using noisy received signal strength indicator (RSSI) measurements and perturbed anchor node position estimates. In the proposed algorithm, we minimize a weighted sum-square-distance-error cost function in an iterative fashion utilizing the gradient-descent method. We calculate the weights using the statistical properties of the perturbations in the measurements. We assume log-normal distribution for the RSSI-induced distance estimates due to considering the log-distance path-loss model with normally-distributed perturbations for the RSSI measurements in the logarithmic scale. We also assume normally-distributed perturbation in the anchor position estimates. We compare the performance of the proposed algorithm with that of an existing algorithm that takes a similar approach but only accounts for the perturbations in the RSSI measurements. Our simulation results show that by taking into account the error in the anchor positions, a significant improvement in the localization accuracy can be achieved. The proposed algorithm uses only a single measurement of RSSI and one estimate of each anchor position. This makes the proposed algorithm suitable for frequent and accurate localization of IoT devices.

Keywords IoT · Multilateration · RSSI · Self-localization · Weighted least-squares

✉ Vikram Kumar
vikram@iiitua.ac.in; er.vikram14@yahoo.com

Reza Arablouei
reza.arablouei@csiro.au; rezaa219@yahoo.com

¹ IIIT Una, Una, India

² Data61, CSIRO, Pullenvale, QLD 4069, Australia

1 Introduction

The location information plays an important role in the success of various IoT-based applications related to, for example, healthcare, agriculture, and disaster management [1–8]. The contact-tracing applications, which have emerged as helpful tools to contain the spread of COVID-19 pandemic, are also based on accurate and frequent location and distance information [9–11]. However, the self-localization of IoT devices is still challenging due to inherent characteristics of the pertinent applications such as the use of low-grade sensors, environmental disturbances, and lack of line of sight (LoS).

Global navigation satellite system (GNSS) receivers are generally used in IoT devices to obtain location information. However, these receivers are known for being power hungry and poor performance in indoor and dense outdoor areas such as forests and cities [12–15]. In addition, equipping each node with a GNSS receiver in mass IoT deployments may also be infeasible due to cost constraints. A number of alternative technologies such as cooperative localization (based on different ranging methods), use of environmental signals (magnetic, air pressure, etc.), and use of micro-electromechanical systems (MEMS) sensors have been proposed in the literature to address various issues around localization of resource-constrained devices [16–19].

In mass deployment scenario such as large-scale agriculture or wildlife monitoring, centimeter-level accuracy is often not required. Therefore, the alternative localization technologies such as cooperative localization using received signal strength indicator (RSSI) measurements have emerged as effective methods for localizing groups of battery-operated mobile nodes. They reduce the energy consumption by minimizing the use of expensive localization methods such as GNSS [20, 21]. In cooperative localization, any node interested in estimating its own position, referred to as the “blind node”, receives position and distance information from its neighboring nodes, called the “anchor nodes”, and estimates its position through multilateration.

In multilateration, a blind node requires the position of the anchor nodes and the distances to them in order to calculate its position. Given accurate anchor node positions and distances, the blind node position is the intersection of the circles with the origins located at the anchor node positions and the radii equal to the corresponding distances. The left plot in Fig. 1 shows three anchor nodes A, B, C with their positions given by coordinates (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , which are known exactly, and one blind node D with unknown position coordinate (x, y) . The exact distances between the anchor nodes and the blind node are represented as d_1 , d_2 , d_3 . Given the exact distances and anchor node positions, the blind node position can be calculated by intersecting the mentioned circles.

In real life, neither the distance estimates nor the anchor positions are error-free as the RSSI measurements are subject to perturbations arising from model inaccuracy, thermal noise, and measurement error while anchor node position information is usually corrupted by noise or error, especially when it is the product of a previous estimation process including the GNSS. Therefore, in practice, the left-hand side plot of Fig. 1 turns into the one on the right-hand side, where the intersection of circles has uncertainty. The solution to the blind node position estimation problem depends upon the nature of the perturbations assumed in the distance and anchor node position estimates.

The problem of coping with the adverse effects of perturbations in the RSSI measurements [9, 22–27] and the anchor node position information [28–39] has received significant attention from the research communities. Most of the existing proposed solutions rely on complex optimization techniques such as semidefinite programming (SDP) or second-order

Fig. 1 Left: multilateration given the perfect knowledge of anchor positions and distances. Right: multilateration when both anchor positions and distance measurements are perturbed

cone programming (SOCP) to mitigate the adverse effects of perturbations. However, SDP and SOCP-based methods are computationally expensive and generally unsuitable for localization of resource-constrained devices.

In this paper, we propose a self-localization algorithm suitable for resource-constrained IoT devices given perturbed anchor position information and noisy RSSI-based distance estimates. We tackle the considered self-localization problem by formulating a weighted least-squares cost function, which we minimize using the gradient-descent method. Each term in the cost function is the weighted square of the difference between the distance inferred from an RSSI measurement and the Euclidean distance between the blind node and the anchor node to which the RSSI measurement corresponds. We weight the square-error terms by their variances, which are estimated by taking into account perturbations in both RSSI measurements and anchor node position information. We use the well-known log-normal shadowing path-loss model as the radio propagation model. Therefore, we assume that the RSSI measurements in the logarithmic (dBm) scale are affected by normally-distributed perturbation and consequently the distance estimates inferred from the RSSI measurements have log-normal distribution.

We compare the performance of the proposed algorithm with that of an existing WLS-based algorithm [25] that only takes into account the perturbations in the RSSI measurements and has similar computational requirements. Our simulation-based evaluations reveal that significant improvement in location accuracy can be achieved by accounting for anchor position errors in the weight calculations. The proposed algorithm uses only a single RSSI measurement and position estimate corresponding to each anchor node. This makes the proposed algorithm suitable for frequent and accurate self-localization of IoT devices.

This paper is an extension of our previous work presented in [40]. The major additions to [40] are as follows:

1. In this paper, we investigate the performance of the proposed algorithm in both cases of anchor node position perturbations being homogeneous and heterogeneous.
2. We derive the Cramer-Rao lower bound (CRLB) for the problem in hand.

3. We investigate the impact of the perturbations in anchor node positions and RSSI measurements on the best achievable localization accuracy.
4. The experiments presented in this paper are more extensive and are entirely different from those in [40].
5. There are other substantial enhancements in various parts of the paper including introduction, literature review, algorithm description, and the general narrative.

2 Related Work

In this paper, we are interested in self-localization by a resource-constrained mobile node. Some example related applications are proximity services, mobile phone users tracking, and animal tracking [41]. The considered problem has the following distinct features compared to other localization problems, such as self-localization by a static blind node, target localization, and joint multiple blind and anchor node localization, considered in [22, 30, 34, 35, 42–46]:

- Unlike in target localization, the anchor nodes are not required to be in the communication range of each other.
- Limited resources entail the need for avoiding complex algorithms in performing multi-source or joint localization.
- The autonomous mobile nature of the nodes may lead to temporary grouping behaviors. The grouping interval sometimes may not be enough to receive or share the results of joint localization.

Substantial research efforts have been dedicated to developing localization algorithms that can compensate for the adverse effects of perturbations in the RSSI measurements and the anchor node position information [29, 31, 32, 34, 35]. Most of the proposed algorithms that can cope with perturbations in both RSSI measurements and anchor node position information assume zero-mean Gaussian perturbation in the distance measurements and use computationally demanding optimization techniques such as SDP or SOCP.

In [47–49], the authors propose an RSSI-based solution for location estimation of a source and multiple anchor nodes. They also provide the theoretical bounds for their solution as well as an interpretation of the theoretical bounds. The nature of the perturbations considered in [47–49] is the same as what we consider in this paper. However, their approach is based on using multiple RSSI and anchor position measurements. Nonetheless, our work is focused on energy-efficient outdoor localization of mobile nodes where the GNSS is the main source of anchor position information while being responsible for the majority of energy consumed. Therefore, the requirement of multiple measurements of a anchor position is not in line with our goal of energy/resource-efficient localization.

In [50], the authors explore the concept of collaborative GNSS duty cycling using Wi-Fi ad-hoc connectivity while ensuring application-specific error bounds. Whenever a node approaches the error bound, it first requests another (hopefully better) position estimate from its neighborhood and awaits for a response within a certain timeframe. In the case of a positive response, the node updates its position estimate. Otherwise, it acquires its own GNSS position coordinates and broadcasts them.

In [50], the authors present an algorithm for self-localization in resource-constrained environments but their main aim is to minimize the energy consumption. They

do not consider any perturbation in distance estimates or neighborhood position information. In [51], the authors aim at improving the Wi-Fi positioning in indoor environments by using the RSSI-based distances among neighbors. They calculate confidence scores as weights in deciding the positions of the neighbors. The confidence of the Wi-Fi-based position is a function of the standard deviation of the multiple Wi-Fi scans for the same point. A lower standard deviation means higher confidence on position and vice-versa. Similarly, the confidence score of the Bluetooth is assigned by RSSI modeling in different settings of the indoor environment. Lastly, game theory is used to determine the final position of the nodes. Similar to others, this work only considers the error in the RSSI measurements. A simple subspace-based algorithm for single mobile node positioning using time of arrival (TOA) measurements from three or more anchors with exact position information is given in [52].

In [25], the authors propose two WLS-based algorithms, called hyperbolic and circular, to localize a node in the presence of log-normal perturbations in the RSSI-based distance measurements. The hyperbolic algorithm linearizes the problem and solves it using the WLS method. The circular algorithm minimize the weighted approximation of the original non-linear sum-square-error cost function using the gradient-descent method. The circular algorithm performs better than the hyperbolic algorithm due to minimization of the original cost function by an iterative approach. The proposed algorithms match the low computational requirement of our applications of interest but do not consider perturbations in the anchor node positions. Our proposed algorithm is based on a similar approach and is of non-linear WLS type but assumes perturbations in the anchor positions as well.

In summary, the existing relevant works either assume the perturbations in both RSSI-based distance measurements and anchor positions to have normal distribution, which is not realistic given that the RSSI-based distance measurements follow log-normal distribution [53], or propose solutions that are based on complex optimization techniques such as SDP or SOCP. The applications of our interest fall into a category where a resource-constrained mobile node has access to perturbed anchor node positions and RSSI measurements and is interested in localizing itself in a resource-efficient way. We consider a more realistic log-normal perturbation for the RSSI-based distance measurements and normal distribution for anchor position perturbations. To meet the resource-efficiency requirement, we find a weighted nonlinear least-squares solution for the considered location estimation problem by minimizing a weighted sum-square error cost function using the gradient-descent method.

Notations: The symbol $\mathbb{N}^+$ denotes the set of positive integers and $\mathbb{R}$ denotes the set of real numbers. A lower-case letter, e.g., x , represents a scalar variable; a lower-case bold letter, e.g., $\mathbf{r}$, represents a vector; and an upper-case bold letter, e.g., $\mathbf{F}$, represents a matrix. The superscript $(\cdot)^T$ denotes the vector/matrix transpose and $(\cdot)^{-1}$ denotes the matrix inverse. A letter with a tilde accent, e.g., $\tilde{x}_i$, represents the noisy observation of the original variable, x_i ; a letter with a hat accent, e.g., $\hat{x}$, represents an estimated value; and a letter with an overbar, e.g., $\bar{z}$, represents an approximate value. The operator $\frac{\partial(\cdot)}{\partial v}$ is the partial derivative with respect to the variable v , $\text{Var}(\cdot)$ returns the variance of a scalar, $\text{Cov}(\cdot)$ returns the covariance matrix of its vector argument, $\mathbb{E}[\cdot]$ is the expectation operator, and $\text{Tr}\{\cdot\}$ is the matrix trace operator. The symbol $\mathcal{N}(\mu, \sigma)$ represents the normal (Gaussian) distribution with mean μ and standard deviation σ .

3 Problem Statement

We consider the problem of self-localization by a single node, referred to as the blind node, on a two-dimensional Cartesian plane. The blind node is interested in obtaining an estimate of its true position, denoted by (x_b, y_b) . There are $M \geq 3$ nodes, referred to as the anchor nodes, arbitrarily distributed within the communication range of the blind node at locations (x_i, y_i) , $i = 1, \dots, M$. The locations of the anchor nodes are known to the blind node only approximately as they are corrupted by random perturbations. The blind node estimates its distance from the anchor nodes using available RSSI measurements that are also subject to random perturbations. We denote the perturbed knowledge of the anchor node positions at the blind node by $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \dots, M$, and the corresponding perturbed RSSI measurements by $\tilde{p}_i$, $i = 1, \dots, M$, in the linear (mW) scale and by $\tilde{p}_{i(\text{dBm})}$, $i = 1, \dots, M$, in the logarithmic (dBm) scale.

We adopt the following common assumptions:

A1: The available knowledge of the position of the i th anchor node on x and y axes are corrupted by independent additive zero-mean Gaussian perturbations with standard deviation σ_{a_i} . The perturbations of different anchor node positions are independent of each other and the values of σ_{a_i} may not be the same for different anchor nodes. Therefore, we have

$$\tilde{x}_i = x_i + n_{x_i} \quad (1)$$

$$\tilde{y}_i = y_i + n_{y_i} \quad (2)$$

$$n_{x_i}, n_{y_i} \sim \mathcal{N}(0, \sigma_{a_i}). \quad (3)$$

A2: The path-loss model for the radio signal propagation is the log-normal shadowing model. Therefore, the RSSI measurement of the signal transmitted from the i th anchor node and received at the blind node has a nominal value of $\bar{p}_{i(\text{dBm})}$ in the logarithmic (dBm) domain. However, the actual measured value is a realization of the nominal value corrupted by a zero-mean Gaussian perturbation with standard deviation σ_{p_i} , i.e.,

$$\tilde{p}_{i(\text{dBm})} = \bar{p}_{i(\text{dBm})} + n_{p_i} \quad (4)$$

$$n_{p_i} \sim \mathcal{N}(0, \sigma_{p_i}). \quad (5)$$

According to the shadowing path-loss model, we have

$$\tilde{p}_{i(\text{dBm})} = p_{0(\text{dBm})} - 10\eta \ln \frac{d_i}{d_0} \quad (6)$$

where

$$d_i = \sqrt{(x_i - x_b)^2 + (y_i - y_b)^2} \quad (7)$$

is the distance between the blind node and the i th anchor node. In addition, d_0 , $p_{0(\text{dBm})}$, and η , are the reference distance, the received power at the reference distance, and the path-loss exponent, respectively. Therefore, given the perturbed value $\tilde{p}_{i(\text{dBm})}$, the RSSI-induced estimate for the distance between the blind node and the i th anchor node, denoted by $\tilde{d}_i$, is given by

$$\tilde{d}_i = d_0 10^{\frac{\hat{p}_i(\text{dBm}) - p_0(\text{dBm})}{10\eta}} \quad (8)$$

Furthermore, we assume that the blind node and the anchor nodes have limited computational and energy resources. Hence, at any particular instance of localization, only one RSSI measurement and position estimate from each anchor node is available to the blind node.

4 Proposed Algorithm

One can estimate the position of the blind node by minimizing the following sum-square-error cost function

$$c(x, y) = \sum_{i=1}^M e_i^2(x, y) \quad (9)$$

where

$$e_i(x, y) = \delta_i - d_i \quad (10)$$

and

$$\delta_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}. \quad (11)$$

Given an initial estimate, the blind node's position can be estimated using a gradient-descent method as follows:

$$\begin{aligned} \hat{x}_b^{(k+1)} &= \hat{x}_b^{(k)} - \alpha \frac{\partial c}{\partial x} \Big|_{x=\hat{x}_b^{(k)}} \\ \hat{y}_b^{(k+1)} &= \hat{y}_b^{(k)} - \alpha \frac{\partial c}{\partial y} \Big|_{y=\hat{y}_b^{(k)}} \end{aligned} \quad (12)$$

where $\alpha > 0$ is the step-size parameter and $\hat{x}_b^{(k)}$ and $\hat{y}_b^{(k)}$ are the estimate of x_b and y_b at iteration k , respectively.

However, we do not have access to the unperturbed values x_i , y_i , and d_i . Hence, we replace them with their corresponding perturbed observations $\tilde{x}_i$, $\tilde{y}_i$, and $\tilde{d}_i$ and approximate the i th error term with

$$\bar{e}_i(x, y) = \tilde{\delta}_i - \tilde{d}_i \quad (13)$$

where

$$\tilde{\delta}_i = \sqrt{(\tilde{x}_i - x)^2 + (\tilde{y}_i - y)^2} \quad (14)$$

In addition, to factor in the difference in the scale and statistical properties of the values associated with different anchor nodes, we weight each error term with the inverse of its standard deviation. Therefore, we minimize the following WLS cost function

$$\bar{c}(x, y) = \sum_{i=1}^M \frac{\bar{e}_i^2}{\text{Var}(\bar{e}_i)} \quad (15)$$

Since the perturbations of the anchor positions and RSSI-induced distances are independent of each other, each variance term $\text{Var}(\bar{e}_i)$ can be calculated as

$$\text{Var}(\bar{e}_i) = \text{Var}(\tilde{\delta}_i) + \text{Var}(\tilde{d}_i) \quad (16)$$

To calculate the first term on the right-hand side of (16), we note that $\tilde{\delta}_i$ is the Euclidean distance of the points (x, y) and $(\tilde{x}_i, \tilde{y}_i)$ where x and y are deterministic variables and $\tilde{x}_i$ and $\tilde{y}_i$ are independent stochastic variables that have Gaussian distributions with means x_i and y_i , respectively, and the same variance $\sigma_{a_i}^2$. Therefore, $\tilde{\delta}_i$ has a Rice distribution [54] with the variance

$$\text{Var}(\tilde{\delta}_i) = \delta_i^2 + 2\sigma_{a_i}^2 - \frac{\pi\sigma_{a_i}^2}{2} L_{1/2}^2 \left(-\frac{\delta_i^2}{2\sigma_{a_i}^2} \right) \quad (17)$$

where

$$\delta_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad (18)$$

and $L_{1/2}(\cdot)$ is a Laguerre polynomial expressed as

$$L_{1/2}(z) = \exp(z/2) \left[(1-z)I_0\left(-\frac{z}{2}\right) - zI_1\left(-\frac{z}{2}\right) \right] \quad (19)$$

with $I_0(\cdot)$ and $I_1(\cdot)$ being the modified Bessel functions of the first kind with order zero and one, respectively. The Laguerre polynomial $L_n(z)$, $n \geq 0$, is the solution of the following second-order linear differential Eq. [55, 56]

$$zp''(z) + (1-z)p'(z) + np(z) = 0. \quad (20)$$

Considering the assumption A2, the second term on the right-hand side of (16) is calculated as [25]

$$\text{Var}(\tilde{d}_i) = d_i^2 \left[\exp\left(2\sigma_{d_i}^2\right) - \exp\left(\sigma_{d_i}^2\right) \right] \quad (21)$$

where

$$\sigma_{d_i} = \frac{\ln 10}{10\eta} \sigma_{p_i}. \quad (22)$$

We estimate the blind node position by minimizing the formulated cost function (15) in an iterative manner using the gradient-descent method. Since the unperturbed anchor positions (x_i, y_i) , $i = 1, \dots, M$, are unknown, we replace δ_i in (17) with its approximate value of

$$\tilde{\delta}_i^{(k)} = \sqrt{\left(\tilde{x}_i - \hat{x}_b^{(k)}\right)^2 + \left(\tilde{y}_i - \hat{y}_b^{(k)}\right)^2} \quad (23)$$

where $(\hat{x}_b^{(k)}, \hat{y}_b^{(k)})$ is the most recent estimate of the blind node location. In addition, we replace the unknown values of d_i , $i = 1, \dots, M$ in (21) with their available approximate values $\tilde{d}_i$, $i = 1, \dots, M$. Therefore, we calculate the gradients as

$$\left. \frac{\partial \bar{c}}{\partial x} \right|_{x=\hat{x}_b^{(k)}} = -2 \sum_{i=1}^M \frac{(\bar{\delta}_i^{(k)} - \tilde{d}_i)(\tilde{x}_i - \hat{x}_b^{(k)})}{w_i^{(k)} \bar{\delta}_i^{(k)}} \quad (24)$$

and

$$\left. \frac{\partial \bar{c}}{\partial y} \right|_{y=\hat{y}_b^{(k)}} = -2 \sum_{i=1}^M \frac{(\bar{\delta}_i^{(k)} - \tilde{d}_i)(\tilde{y}_i - \hat{y}_b^{(k)})}{w_i^{(k)} \bar{\delta}_i^{(k)}} \quad (25)$$

where the weight $w_i^{(k)}$ that is an estimate of the variance of the error due to the measurements related to the i th anchor node is calculated at the k th iteration as

$$\begin{aligned} w_i^{(k)} = & \left(\bar{\delta}_i^{(k)} \right)^2 + 2\sigma_{a_i}^2 - \frac{\pi\sigma_{a_i}^2}{2} L_{1/2}^2 \left(-\frac{\left(\bar{\delta}_i^{(k)} \right)^2}{2\sigma_{a_i}^2} \right) \\ & + \tilde{d}_i^2 \left[\exp\left(2\sigma_{a_i}^2 \right) - \exp\left(\sigma_{a_i}^2 \right) \right] \end{aligned} \quad (26)$$

4.1 Computational Complexity

At each iteration, the proposed algorithm requires $16M$ multiplications and $4M$ divisions as well as M square-root and $4M$ exponentiation operations where M is the number of anchor nodes. Note that the required computations do not involve any complex number and are performed on real numbers. The required Bessel function values can be obtained using look-up tables, which only incur some extra storage requirement. Therefore, the proposed algorithm has a computational complexity of $O(IM)$ where I is the total number of iterations needed for convergence.

5 Cramer-Rao Lower Bound

Given an observation vector $\mathbf{w}$ in relation to an unknown parameter vector $\boldsymbol{\theta}$, the Cramer-Rao lower bound (CRLB) sets a lower bound on the covariance of any unbiased non-Bayesian estimator of $\boldsymbol{\theta}$. The CRLB is the inverse of the Fisher information matrix (FIM), denoted by $\mathbf{F}(\boldsymbol{\theta})$. Therefore, we have

$$\text{cov}(\hat{\boldsymbol{\theta}}) \geq \mathbf{F}^{-1}(\boldsymbol{\theta}) \quad (27)$$

The FIM represents the information provided by the observation $\mathbf{w}$ about the unobserved parameter vector $\boldsymbol{\theta}$ and is calculated as

$$\mathbf{F}(\boldsymbol{\theta}) = -\mathbb{E} \left[\frac{\partial^2 l(\boldsymbol{\theta} \mid \mathbf{w})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T} \right] \quad (28)$$

where $l(\boldsymbol{\theta} \mid \mathbf{w})$ is the log-likelihood function of $\boldsymbol{\theta}$ given $\mathbf{w}$.

In our self-localization problem, the observation vector $\mathbf{w}$ contains the perturbed anchor positions $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \dots, M$, and the perturbed RSSI measurements $\tilde{p}_i$, $i = 1, \dots, M$. Therefore, it can be written as

$$\mathbf{w} = [\tilde{p}_1, \tilde{x}_1, \tilde{y}_1, \tilde{p}_2, \tilde{x}_2, \tilde{y}_2, \dots, \tilde{p}_M, \tilde{x}_M, \tilde{y}_M]^T \quad (29)$$

Given (45) and (6), the probability density function (pdf) of $\tilde{p}_i$ is expressed as [23]

$$f_{\tilde{p}_i}(\tilde{p}_i) = \frac{10/\ln 10}{\sqrt{2\pi\sigma_{p_i}^2\tilde{p}_i}} \exp \left[-\frac{b_i}{8} \left(\ln \frac{d_i^2}{\tilde{d}_i^2} \right)^2 \right] \quad (30)$$

where

$$b_i = \left(\frac{10\eta}{\sigma_{p_i} \ln 10} \right)^2 \quad (31)$$

Our unknown parameters of interest are the position coordinates of the blind node (x_b, y_b) . However, since the RSSI values are functions of the unknown unperturbed anchor positions, (x_i, y_i) , $i = 1, \dots, M$, we include the unperturbed anchor positions as the nuisance parameters. Hence, the unknown parameter vector $\boldsymbol{\theta}$ is

$$\boldsymbol{\theta} = [x_b, y_b, x_1, y_1, x_2, y_2, \dots, x_M, y_M]^T \quad (32)$$

The perturbed anchor node positions $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \dots, M$, are statistically independent of each other as well as the RSSI values. Therefore, the log-likelihood function can be written as

$$\begin{aligned} l(\boldsymbol{\theta} \mid \mathbf{w}) &= \sum_{i=1}^M \ln f_{\tilde{p}_i}(\tilde{p}_i \mid x_b, y_b, x_i, y_i) \\ &+ \sum_{i=1}^M \ln f_{\tilde{x}_i}(\tilde{x}_i \mid x_i) + \sum_{i=1}^M \ln f_{\tilde{y}_i}(\tilde{y}_i \mid y_i) \end{aligned} \quad (33)$$

We derive the second-order partial derivatives of $l(\boldsymbol{\theta} \mid \mathbf{w})$ with respect to the entries of $\boldsymbol{\theta}$, required for the calculation of the FIM, in the "Appendix". Using [23]

$$\mathbb{E} \left[\ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \right] = 0 \quad (34)$$

together with the results in the "Appendix", we can express the FIM as

$$\mathbf{F}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{12}^T & \mathbf{F}_{22} \end{bmatrix} \quad (35)$$

where

$$\mathbf{F}_{11} = \sum_{i=1}^M \frac{b_i}{d_i^4} \mathbf{H}_i, \quad (36)$$

$$\begin{aligned} \mathbf{F}_{12} &= - \left[\frac{b_1}{d_1^4} \mathbf{H}_1, \dots, \frac{b_M}{d_M^4} \mathbf{H}_M \right], \\ \mathbf{F}_{22} &= \text{blockdiag} \left\{ \frac{b_1}{d_1^4} \mathbf{H}_1 + \frac{1}{\sigma_{a_1}^2} \mathbf{I}_2, \dots, \frac{b_M}{d_M^4} \mathbf{H}_M + \frac{1}{\sigma_{a_M}^2} \mathbf{I}_2 \right\}, \\ \mathbf{H}_i &= \begin{bmatrix} (x_i - x_b)^2 & (x_i - x_b)(y_i - y_b) \\ (x_i - x_b)(y_i - y_b) & (y_i - y_b)^2 \end{bmatrix}, \end{aligned} \quad (37)$$

and $\mathbf{I}_2$ is the 2×2 identity matrix. Consequently, we obtain a lower bound on the localization root-mean-square error (RMSE) of any unbiased estimator of the blind node location as below

$$\sqrt{\mathbb{E} \left[(\hat{x}_b - x_b)^2 + (\hat{y}_b - y_b)^2 \right]} \geq \sqrt{\text{tr} \left\{ (\mathbf{F}_{11} - \mathbf{F}_{12} \mathbf{F}_{22}^{-1} \mathbf{F}_{12}^T)^{-1} \right\}}$$

6 Performance Evaluation

This section is focused on performance evaluation of the proposed algorithm. We first describe the algorithm in comparison followed by the results of some experiments performed to find the minimum number of anchor nodes required to obtain a satisfactory localization accuracy in the presence of perturbations. Then, we describe the simulation setup and different scenarios simulated to evaluate the performance. Lastly, we present the simulation results and our key observations.

6.1 Algorithm in Comparison

The performance of the purposed algorithm is compared with the weighted circular algorithm proposed in [25]. Similar to our assumptions, the authors of [25] assume the presence of log-normally distributed perturbations in the RSSI-induced distance estimates. However, they assume the availability of noise-free anchor position information. They also rely on minimizing a weighted sum-square-distance-error cost function in an iterative fashion using the gradient-descent method. However, their weight calculation only considers the perturbations in the RSSI measurements. The high level of similarity in assumptions and approach to solve the problem in hand makes the algorithm proposed in [25] a suitable candidate for algorithm in comparison. Moreover, to the best of our knowledge it is the only relevant algorithm in the literature that has a computational complexity comparable to that of the proposed algorithm.

6.2 Cramer-Rao Lower Bound Experimental Insights

The number of anchor nodes required to perform self-localization by a blind node in a group of resource-constrained nodes is an important factor for determining the usefulness

of any localization algorithm. This is because the anchor nodes may also be resource-constrained in nature and frequent use of their positioning sensors, such as the GNSS receiver, will drain the battery quickly. This defeats the major purpose of energy-efficient self-localization for resource-constrained devices.

In a perturbation-free environment, we require at least three anchor nodes to localize a blind node in 2D. However, three anchor nodes may not be sufficient to achieve the required level of accuracy in the presence of perturbations. Therefore, we perform some experiments to find the minimum number of nodes required to achieve a reasonable localization accuracy in the presence of perturbations in both anchor positions and RSSI measurements.

In the first experiment, we simulate a scenario in which initially only three anchor nodes and one blind node are present in a $35\text{m} \times 35\text{m}$ region. The three sub-figures in Fig. 2a show three different node placement arrangements selected for evaluation. The three anchor nodes and one blind node used in this experiment are labelled as “anchor 1-3” and “blind node”, respectively. For now, let us ignore the anchor nodes labeled as “anchor 4-6” and “anchor 7-12”. We consider the standard deviation of the perturbation in the anchor position σ_{a_i} to be equal to 1m, 3m, and 5m. We vary the standard deviation of the RSSI perturbations σ_{p_i} between 1dB and 10dB. We evaluate the CRLB for two different perturbation scenarios, i.e., when the perturbations are only present in the RSSI measurements and when the perturbations are present in both RSSI measurements and anchor position information. For the former, we use the CRLB derived in [23] and label the results as “CRLB-R-1,3,5m”. For the latter, we use the CRLB derived in Section 5 while “CRLB-RP-1m”, “CRLB-RP-3m”, and “CRLB-RP-5m” represent the results when σ_{a_i} is equal to 1m, 3m, and 5m, respectively.

The three sub-figures of Fig. 2b present the results for the three corresponding node placements given in Fig. 2a. The following observations are made from Fig. 2b:

- As expected “CRLB-R-1,3,5m” do not vary with the level of perturbation in the anchor position information.
- The presence of perturbation in anchor position information as $\sigma_{a_i} = 1\text{m}$ represented by “CRLB-RP-1m” does not increase the CRLB noticeably compared to “CRLB-R-1,3,5m”. This is possibly due to the perturbation in RSSI dominating the localization accuracy. Hence, both algorithms (proposed and the algorithm in comparison) perform similarly in such perturbation scenarios.
- For σ_{a_i} being equal to 3m and 5m, the CRLB rises to 25m, which is considerably high for any practical purpose. Hence, we conclude that three anchor nodes are not sufficient to have a reasonably good localization estimate in the presence of perturbations.

In the second experiment, we keep everything the same as the previous experiment except for adding three more anchor nodes to the scenario. The newly added anchor nodes in this experiment are shown as “anchor 4-6” in 2a. Let us ignore the anchor nodes labeled as “anchor 7-12” for now. Therefore, in this experiment, the blind node receives the RSSI and anchor position information from six anchor nodes compared to three in the first experiment. The three sub-figures of Fig. 2c present the results for the corresponding three node placements shown in Fig. 2a. The following observations are made from Fig. 2c:

- The addition of three anchor nodes reduces the CRLB drastically compared to the CRLB values in 2b.
- The upper limit of the CRLB in all the three sub-figures is only around 10m.

Fig. 2 (a) Three stages of the network used in three experiments with a fixed blind node position (the green dot). The first experiment consists of three anchors represented by squares, the triangles represent the added three anchors in the second experiment, and the stars represent the six anchors added in the third experiment; (b) the CRLB of localization RMSE of the network simulated in the first experiment; (c) the CRLB of localization RMSE of the network simulated in the second experiment; (d) the CRLB of localization RMSE of the network simulated in the third experiment

In the third experiment, we keep everything the same as the second experiment except for adding six more anchor nodes to the scenario. Therefore, the blind node receives the RSSI and anchor position information from twelve anchor nodes compared to six in the second experiment. The anchor nodes added in this experiment are labeled as “anchor 7-12”). The three sub-figures of Fig. 2d present the results for the corresponding three node placements shown in Fig. 2a. The following observations are made from Fig. 2d:

- The CRLB using twelve nodes is considerably low compared to using three anchor nodes.
- The CRLB gap using six and twelve anchor nodes has a maximum value around 20% in all the cases, whereas from an energy consumption perspective, the latter case requires about twice the energy. A similar observation of insignificant performance improvement by adding more anchor nodes has been made in [26].

In light of the above observations, we use six anchor nodes in all our experiments, described in the next section, to evaluate the performance of the proposed algorithm.

6.3 Simulation Details and Results

We simulate a scenario where six anchor nodes and one blind node are present in a $35\text{m} \times 35\text{m}$ area. We vary the RSSI measurement errors, σ_{p_i} , ranging from 0dBm to 5dBm in all the simulations. We use $d_0 = 1\text{m}$, $p_{0(\text{dBm})} = -33.44$, and $\eta = 3.567$ in our simulations, which are based on the results reported in [57]. We tune the step-size parameter of the gradient-descent algorithm to obtain convergence within 300 iterations. We use the same step-size and maximum number of iterations in the implementation of both algorithms.

We use the root mean square error (RMSE) as the performance criterion. The results presented are averaged over 1000 independent trials for each simulation. We also compare our results to the CRLB values, when appropriate, as the estimates of both algorithms may have bias in some scenarios due to the nature of the perturbations. We perform multiple simulations varying the anchor position errors σ_{a_i} and the arrangement of the nodes to evaluate the proposed algorithm as described next.

First, we simulate a scenario in which the position error, σ_{a_i} , is the same for all anchor nodes. Figure 3 shows the results for σ_{a_i} being equal to 1, 3 and 5m in an arbitrary node arrangement as in Fig. 3a. The performance of the proposed algorithm is comparable to that of the existing algorithm for low perturbation levels in anchor position information, i.e., σ_{a_i} being 1 or 3m. However, for $\sigma_{a_i} = 5\text{m}$, the proposed algorithm performed considerably better than the existing algorithm when the perturbation level in the RSSI measurements is moderate, i.e., when σ_{p_i} is in the range of 1dBm to 3dBm. Note that the CRLB values in Fig. 3d for $\sigma_{a_i} = 5\text{m}$ are higher than the corresponding RMSE values in Fig. 3c. This can be justified considering the fact that the estimates produced by both proposed and

Fig. 3 Homogeneous σ_{a_i} and fixed nodes; (a) the network topology: the stars are the anchor nodes, the circle is the true position of the blind node, the triangle is the initial estimate for the blind node position; (b) the RMSE of the existing algorithm for different values of σ_{p_i} and σ_{a_i} ; (c) the RMSE of the proposed algorithm for different values of σ_{p_i} and σ_{a_i} ; (d) the corresponding CRLB values

Fig. 4 Homogeneous σ_{a_i} and fixed nodes; (a) the network topology: the stars are the anchor nodes, the circle is the true position of the blind node, the triangle is the initial estimate for the blind node position; (b) the RMSE of the existing algorithm for different values of σ_{p_i} and σ_{a_i} ; (c) the RMSE of the proposed algorithm for different values of σ_{p_i} and σ_{a_i} ; (d) the corresponding CRLB values

Fig. 5 Heterogeneous σ_{a_i} and fixed nodes; (a) the network topology: the stars are the anchor nodes with $\sigma_{a_i} = 6m$, the squares are the anchor nodes with $\sigma_{a_i} = 3m$, the circle is the true position of the blind node, the triangle is the initial estimate for the blind node position; (b) the RMSE of the proposed and existing algorithms as well as the corresponding CRLB for different values of σ_{p_i} ; (c) the localization error histograms of the proposed and existing algorithms when $\sigma_{p_i} = 2dBm$

existing algorithms may be biased. We make a similar observation for another arbitrary node arrangement given in Fig. 4a.

In practise, different anchor nodes may have different values of σ_{a_i} based on the quality of their GNSS receivers or their location affecting the number of visible satellites at the time of positioning. Hence, we simulate a scenario in which half of the anchor nodes have one value for σ_{a_i} and the other half have another value for σ_{a_i} . The evaluation results of this scenario for two arbitrarily selected node arrangements are presented in Figs. 5 and 6. In this scenario, the proposed algorithm results in significantly (up to 30%) lower values of RMSE compared to the existing algorithm. The error profile of the proposed algorithm is also better than that of the existing algorithm as shown in Figs. 5c and 6c.

It is known that the performance of the RSSI-based localization highly depends on the network geometry [49, 58]. To generalise the applicability of the evaluation results to a wider range of physical node arrangements, we simulate a scenario in which we assign an independent region for the physical location of the anchor nodes, the true position of the blind node, and the initial estimate of the blind node position. This means that in each simulation run (total 1000 runs), the physical node arrangement and the initial estimate of the blind node position change while they were fixed in the previous simulations.

Fig. 6 Heterogeneous σ_{a_i} and fixed nodes; (a) the network topology: the stars are the anchor nodes with $\sigma_{a_i} = 4\text{m}$, the squares are the anchor nodes with $\sigma_{a_i} = 2\text{m}$, the circle is the true position of the blind node, the triangle is the initial estimate for the blind node position; (b) the RMSE of the proposed and existing algorithms as well as the corresponding CRLB for different values of σ_{p_i} ; (c) the localization error histograms of the proposed and existing algorithms when $\sigma_{p_i} = 2\text{dBm}$

Same as earlier, we first simulate a scenario in which all the anchor nodes have the same value for their position error σ_{a_i} . The two considered arbitrary region-based node arrangements are shown in Figs. 7a and 8a. In the former arrangement, σ_{a_i} is 5m for all anchor nodes and in the latter one, σ_{a_i} is 3m. The superior performance of the proposed algorithm in terms of lower RMSE is noticeable in both considered arrangements.

Lastly, we simulate a scenario in which the anchor nodes in one region have one value for σ_{a_i} and the anchor nodes in another region have another value for σ_{a_i} . The results for this scenario are given in Figs. 9 and 10. The proposed algorithm achieves up to 25% lower RMSE compared to the algorithm in comparison. The error histogram of the proposed algorithm for $\sigma_{p_i} = 3\text{dBm}$ is also more compact than that of the algorithm in comparison.

In summary, the proposed algorithm offers significant improvements over the existing algorithm in the realistic scenario of heterogeneous GNSS error. However, its advantages when the GNSS error is homogeneous are less pronounced.

Fig. 7 Homogeneous σ_{a_i} and region-based random node placement; (a) the network topology: the grey rectangles contain the anchor nodes with $\sigma_{a_i} = 5\text{m}$, the green rectangle contains the true position of the blind node, the yellow rectangle contains the initial estimate for the blind node position; (b) the RMSE of the proposed and existing algorithms; (c) the localization error histograms of the proposed and existing algorithms when $\sigma_{p_i} = 3\text{dBm}$

Fig. 8 Homogeneous σ_{a_i} and region-based random node placement; (a) the network topology: the grey rectangles contain the anchor nodes with $\sigma_{a_i} = 3\text{m}$, the green rectangle contains the true position of the blind node, the yellow rectangle contains the initial estimate for the blind node position; (b) the RMSE of the proposed and existing algorithms; (c) the localization error histograms of the proposed and existing algorithms when $\sigma_p = 3\text{dBm}$

7 Conclusion

We investigated the problem of self-localization of resource-constrained IoT devices given noisy anchor position information and imperfect RSSI-based distance estimates. We experimentally analysed the values of CRLB in different situations to gain insights into the impacts of anchor position errors and the number of anchors on the localization accuracy. We showed that the presence of perturbation in anchor position information in addition to the perturbations in the RSSI measurements can severely degrade the self-localization accuracy of the blind node. Our analysis also revealed that the presence of noisy anchor nodes beyond a certain number may not improve the localization accuracy. We proposed a self-localization algorithm that can localize an IoT device in the presence of perturbations in both anchor positions and RSSI measurements. Our simulation results show that the proposed algorithm can yield significantly lower localization error compared to an existing related algorithm.

Fig. 9 Heterogeneous σ_{a_i} and region-based random node placement; (a) the network topology: the grey rectangle contains the anchor nodes with $\sigma_{a_i} = 5\text{m}$, the blue rectangle contains the anchor nodes with $\sigma_{a_i} = 1\text{m}$, the green rectangle contains the true position of the blind node, the yellow rectangle contains the initial estimate for the blind node position; (b) the RMSE of the proposed and existing algorithms; (c) the localization error histograms of the proposed and existing algorithms when $\sigma_p = 3\text{dBm}$

Fig. 10 Heterogeneous σ_{a_i} and region-based random node placement: **(a)** network topology: the grey rectangles contain the anchor nodes with $\sigma_{a_i} = 4\text{m}$, the blue rectangle contains the anchor nodes with $\sigma_{a_i} = 1\text{m}$, green rectangle contains the true position of the blind node, the yellow rectangle contains the initial estimate for the blind node position; **(b)** the RMSE of the proposed and existing algorithms; **(c)** the localization error histograms of the proposed and existing algorithms when $\sigma_p = 3\text{dBm}$

Appendix

The second-order partial derivatives of the log-likelihood function $l(\theta | \mathbf{w})$ with respect to the entries of its argument θ , which are required for the calculation of the FIM, are computed as in the following

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_b^2} = \sum_{i=1}^M \left\{ -\frac{b_i(x_i - x_b)^2}{d_i^4} + b_i \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \left[\frac{(x_i - x_b)^2}{d_i^4} - \frac{1}{2d_i^2} \right] \right\} \quad (38)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial y_b^2} = \sum_{i=1}^M \left\{ -\frac{b_i(y_i - y_b)^2}{d_i^4} + b_i \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \left[\frac{(y_i - y_b)^2}{d_i^4} - \frac{1}{2d_i^2} \right] \right\} \quad (39)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_b \partial y_b} = \sum_{i=1}^M \left\{ \frac{b_i(x_i - x_b)(y_i - y_b)}{d_i^4} \left[\ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) - 1 \right] \right\} \quad (40)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_b \partial x_i} = \frac{b_i(x_i - x_b)^2}{d_i^4} - b_i \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \left[\frac{(x_i - x_b)^2}{d_i^4} - \frac{1}{2d_i^2} \right] \quad (41)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_b \partial y_i} = \frac{b_i(x_i - x_b)(y_i - y_b)}{d_i^4} \left[1 - \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \right] \quad (42)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial y_b \partial x_i} = \frac{b_i(x_i - x_b)(y_i - y_b)}{d_i^4} \left[1 - \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \right] \quad (43)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial y_b \partial y_i} = \frac{b_i (y_i - y_b)^2}{d_i^4} - b_i \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \left[\frac{(y_i - y_b)^2}{d_i^4} - \frac{1}{2d_i^2} \right] \quad (44)$$

for $i = j$:

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_i \partial x_j} = -\frac{b_i (x_i - x_b)^2}{d_i^4} - \frac{1}{\sigma_{a_i}^2} + b_i \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \left[\frac{(x_i - x_b)^2}{d_i^4} - \frac{1}{2d_i^2} \right] \quad (45)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial y_i \partial y_j} = \frac{b_i (y_i - y_b)^2}{d_i^4} - \frac{1}{\sigma_{a_i}^2} + b_i \ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) \left[\frac{(y_i - y_b)^2}{d_i^4} - \frac{1}{2d_i^2} \right] \quad (46)$$

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_i \partial y_j} = \frac{b_i}{d_i^4} (x_i - x_b)(y_i - y_b) \left[\ln \left(\frac{d_i^2}{\tilde{d}_i^2} \right) - 1 \right] \quad (47)$$

for $i \neq j$:

$$\frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_i \partial x_j} = \frac{\partial^2 l(\theta | \mathbf{w})}{\partial y_i \partial y_j} = \frac{\partial^2 l(\theta | \mathbf{w})}{\partial x_i \partial y_j} = 0. \quad (48)$$

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Vikram Kumar received the Bachelor of Technology degree in Information Technology from the University Institute of Information Technology, Himachal Pradesh University, India in 2006. He received his PhD degree from The University of Queensland, Australia in 2018. Currently, he is working as Assistant Professor in Indian Institute of Information Technology Una, India. Mailing address: School of Computing, Indian Institute of Information Technology, Una, India

Reza Arablouei received his PhD degree in telecommunications engineering from the University of South Australia in 2013. He is currently a research scientist at the Commonwealth Scientific and Industrial Research Organisation. His current research interest are in machine learning and statistical signal processing. Mailing address: CSIRO's Data61, Pullenvale QLD 4069, Australia