



# Comments on “A Three-Dimensional Geometrical Scattering Model for Cellular Communication Environment”

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## Abstract

Primarily, this comment first focuses on a critical review of a geometric channel model and incorrect formulas presented by M. Riaz, M. M. Khan, and Z. Ullah. The adopted solution is a simplified case of a model that has already presented in a literature. Next, we give a proposal to improve the analyzed problem and expected results for this model.

**Keywords** Three dimensional · Angle-of-arrival · Cellular mobile communications · Channel modeling · Semi-ellipsoid

## 1 Introduction

A three dimensional (3D) geometric channel model proposed in [1] is a simplified case of models presented in [2–4]. In previous models proposed by Riaz et al. [2–4], we may notice an increase in their complexity. In contrast, the newest model presented in [1] is a trivial simplification of these models. Although, the models in [2–4] concern a mobile-to-mobile (M2M) scenario, while the models in [1] is dedicated to a fixed-to-mobile (F2M) scenario. However, this fact does not justify the presentation of the simplified model.

In [1], numerous errors in the description, introduced symbols, and equations occur there. Hence, the obtained results are erroneous and make it impossible to correctly interpret a propagation phenomenon occurring in a real environment. A detailed analysis of the models in [1] and [2–4] shows that most of the formulas presented in [1] are simplified versions of equivalents of the previous works. However, the transformation of the models from [2–4] is carried out negligently. Hence, many of the symbols used in [1] do not have their counterparts in the analyzed solution. As a result, the presented formulas contain the fundamental errors that undermine the credibility of this paper.

In Sect. 2, a detailed list of the errors that occur in the model description and used formulas is presented. A solution of the analyzed problem is the topic of the next section.

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Section 4 includes summary, which shows a critical evaluation of using this model in a practice.

## 2 Review of Commented Paper

In this section, we present a justification for the critical assessment of [1]. For a clear presentation of the objections, below, we list the most important errors. The fundamental errors occurring among them are the reason for writing this comment. They are the following:

1. In abstract of [1], the authors wrote ... *expressions for the joint and marginal probability density functions for angle-of-arrival and time-of-arrival in azimuth and elevation planes are derived.* .... However, in abstracts of [3] and [4], there are ... *expressions for the joint probability density function of angle-of-arrival and time-of-arrival in azimuth and elevation planes are derived.* .... In [3] and [4], the expressions and analysis of time-of-arrival really are presented, while in [1], this issue is not raised at all. Therefore, the cited extracts show that [1] is an nonsolid copy of the previous work.
2. Admittedly, in [1, Sect. 2], it is written ... *To model scattering environment around the MS along roads, street and canyons, Riaz et al. introduced 3D spatial channel models for M2M communication environment in [21, 22, 25].* ...., where cited [21,22,25] are [2, 4], and [3] in this paper, respectively. A detailed analysis of the models presented in [1–4] shows their close dependences. However, the newest of these paper [1] represents a trivial simplification of earlier work.
3. In [1, Sect. 3], there is ... *This geometry and the work carried out for M2M communication environment in [21, 22, 25] motivated us to propose a 3D semiellipsoid geometrical channel model for F2M communication scenario. Such geometrically-based scattering channel modeling approach is useful in designing and modeling of wireless networks as presented in [26].* ...., where cited [26] is [5] in this paper. In [5], the wireless networks concern mobile ad-hoc networks (MANETs). The authors of [1] do not indicate a strict justification for using the presented model for MANET. Typically, MANET refers more to M2M environments than F2M. Therefore, this conclusion is unjustified. Furthermore, in [5], the issue of channel modeling is not at all brought up.
4. In [1, Sect. 4], there is ... *Main research contributions in this paper are as follows; ...*
5. *To validate our proposed model, we compare it with the existing models in the literature.* .... In the topic related to a presentation of new channel models, the comparative assessment of the proposed solution with models available in a literature is accepted as standard, especially for probability density function (PDF) of angle of arrival (AoA) models. In this case, measurement results available in a literature, e.g., [6–11], are commonly used for this evaluation. Examples of properly conducted comparative analysis of the PDF of AoA models include in [12–15]. In this assessment, different measures are used, e.g., the least-square error (LSE), standard deviation, Kolmogorov–Smirnov, and Cramer–von Mises statistics. Furthermore, if a model is fitted to empirical data, optimal parameters of this model should always be given. In [1], the model validation and its comparison with other models are not presented.
6. In [1, Sect. 5], in [1, Fig. 2], the authors did not introduce the relevant symbols used in the further description of the model, e.g.,:
  - The orientation of the Cartesian coordinate system is not shown.

- A sample scattering point, S, is marked in a wrong location. In this case, S is outside of the scattering area bounded by semi-ellipsoid. The position of analogous S in [3] is presented correctly.
  - Some symbols, i.e.,  $\theta$ ,  $x_m$ ,  $y_m$ ,  $z_m$ ,  $\phi_{t1}$ ,  $\phi_{t2}$ ,  $\beta_{t1}$ ,  $\beta_{t2}$ ,  $r_m$ ,  $P_1$ , and  $P_2$ , are not shown in the figure, so understanding the model description is very difficult.
  - BS is located at a ground level, no at  $h$  height.
7. In [1, Sect. 5], the authors wrote ... *The model is made more flexible by introducing rotation of the semi-ellipsoid around the vertical z-axis, this rotation is symbolized as  $\theta$ .* ... The symbol,  $\theta$ , does not appear in the model description anymore and this rotation is not actually included! However, in the previous models [2–4], the possibility of rotating the scattering area by using the appropriate angle,  $\theta$ , is introduced.
  8. In [1, Sect. 5], in sentence ... *Let the MS is placed at the origin point in space and the BS is placed some distance D from the MS at point (d,0,0) in the Cartesian coordinates system.* ..., the authors introduce two symbols,  $D$  and  $d$ . In our opinion, these symbols represent the same parameter.
  9. In [1, Sect. 5], below ([1], 1), there are three formulas, i.e.,

$$“ \dots x_m = x_m + D, y_m = y_m, z_m = z_m \dots ”.$$

The first equation is incorrect. The other two are identities and contribute nothing.

10. In [1, Sect. 5], the description from the paragraph ... *The scatterers that contribute in the arriving of signals at the MS are confined in a scattering region and named as partition  $P_j$ .* ... to formula ([1], 5) is unrelated to this model. This may fit into the description of the angular dispersion seen on the base station (BS) side rather than the mobile station (MS). A similar description with partitions,  $P_1$ , and  $P_2$ , and adequate equations are shown in the more complex models [3, 4]. In addition, this part of the model description contains many errors, i.e.,:
  - [1, Fig. 3] is unnecessary because it does not bring anything new. In addition, the case presented here only applies to case for  $\beta = 0^\circ$ . Otherwise, the position of the scatterer, S, will be located outside of the scattering area bounded by the semi-ellipsoid.
  - There is written ... *This scattering partition is identified by looking at azimuth angle,  $\phi$  (i.e.,  $\phi_{t1} \leq \phi \leq \phi_{t2}$ ) and elevation angle,  $\beta_{t1}$  (i.e.,  $\beta \leq \beta_{t1}$ ).* ... and next in ([1], 3) and ([1], 4), relationship between  $\phi_{t1}$  and  $\phi_{t2}$  is completely different, i.e.,  $\phi_{t2} \leq \phi \leq \phi_{t1}$ .
  - Formula ([1], 5) is unclear. Firstly, if  $h^2 d^2 \Omega = d^2 h^2 \Omega$ , so  $2h^2 d^2 \Omega$  should be in formula instead of two the same elements. Secondly, the elements of this formula have different dimensions (i.e.,  $h^2 d^2 \Omega(m^6)$ ,  $\Omega^2(m^4)$ , and  $h^2 d^2(m^4)$ ), so the argument of the arctan function is not dimensionless.
11. In [1, Sect. 5], formula ([1], 6) is incorrect. According to [1, Fig. 2], for corresponding  $\phi$  and  $\beta$ ,  $r_1$  should be equal to proper semi-axes of the semi-ellipsoid, i.e.,  $r_1 = a$  for  $\beta = 0^\circ$  and  $\phi \in \{0^\circ, 180^\circ\}$ ,  $r_1 = b$  for  $\beta = 0^\circ$  and  $\phi \in \{90^\circ, 270^\circ\}$ ,  $r_1 = c$  for  $\beta = 90^\circ$  and  $0^\circ \leq \phi \leq 360^\circ$ . As it shows, any conditions are not met based on ([1], 6).
12. In [1, Sect. 6], the authors wrote ... *The joint PDF, of propagation distance  $r_1$ , elevation  $\beta$  and azimuth  $\phi$  angles, can be usually be expressed as,* ... and then, in formula ([1], 7) and the continued description, they introduced the symbol  $r_m$  instead of  $r_1$ .

13. In [1, Sect. 6], in formulas ([1], 7), ([1], 8), and ([1], 10) symbol  $r_m$  is instead of  $r_1$ .
14. In [1, Sect. 6], formula ([1], 11) is incorrect, because this function is not normalized. Each probability density function should be normalized. Considering the 10th note, if we integrate ([1], 11) over  $\beta$  and  $\phi$ , we will not get value "1".
15. In [1, Sect. 6], it is written ... *If we integrate (11) over  $\beta$  then we can find marginal PDF of azimuth AoA as follows,*

$$p(\phi) = \int_0^{\pi/2} p(\phi, \beta) d\beta \quad ([1], 12)$$

*Similarly, PDF of elevation AoA can be found by integrating the joint PDF in (11) over  $\phi$  as given by,*

$$p(\beta) = \int_0^{2\pi} p(\phi, \beta) d\phi \quad ([1], 13)$$

.... In this case, the authors do not find marginal PDFs and these formulas show only the properties of the marginal PDFs. According to cited sentence from abstract in first above note, the expressions for the marginal PDFs should be given.

16. In [1, Sect. 7], The authors should present the results and comparative analysis of the proposed solution with other models. The proper methodology of presenting this analysis is described in 4 note. To proper presenting results, values of model parameters should always be given. In previous models proposed by Riaz et al. [2–4], this approach is used. In this case, values of  $a$ ,  $b$ , and  $c$  are not known for the graphs shown in [1, Figs. 4–7].
17. In [1, Sect. 7], the results shown in [1, Figs. 4–6] are incorrect because the formula ([1], 11) on which they are based is incorrect (see 13 note). In [1, Figs. 4–6], new symbols,  $\beta_1$  and  $\phi_1$ , are used that are not explained in the paper. Additionally, the graphs presented in [1, Figs. 5–6] do not represent PDFs, because  $p(\phi_1, \beta_1 = \text{const.}) \neq \int_0^{\pi/2} p(\phi_1, \beta_1) d\beta_1$ . Additionally, the area under the PDF curve should always be normalized, i.e., equals 1 for PDF support. These graphs are not all the more the marginal PDFs. According the formulas ([1], 12) and ([1], 13), the marginal PDFs depend only on one selected angle. The legend in these figures shows something else. These graphs represent the cross-sections of the 3D surface from [1, Fig. 4].
18. In [1, Sect. 7], it is written ... *In order to validate the obtained results using the proposed 3D geometrical channel model, we compare the spatial characteristics of our model with the experimental data in [27] as shown in Fig. 7. It can be seen from these curves that the results obtained from our developed model are well-matched with the experimental results. ....*, where cited [27] is [6] in this paper. In [1, Fig. 7], the authors show only one curve. Whereas, in [6], two measurement scenario, i.e., for Aarhus and Stockholm, are presented. In this case, the selected scenario for empirical data is not known. It is not possible to objectively evaluate the fit of the proposed model to empirical data, and especially, to compare it with other models, if value of comparative measure, e.g., LSE, is not given (see note 4). The authors also did not provide a methodology for optimizing the model parameters while fitting it to empirical data.

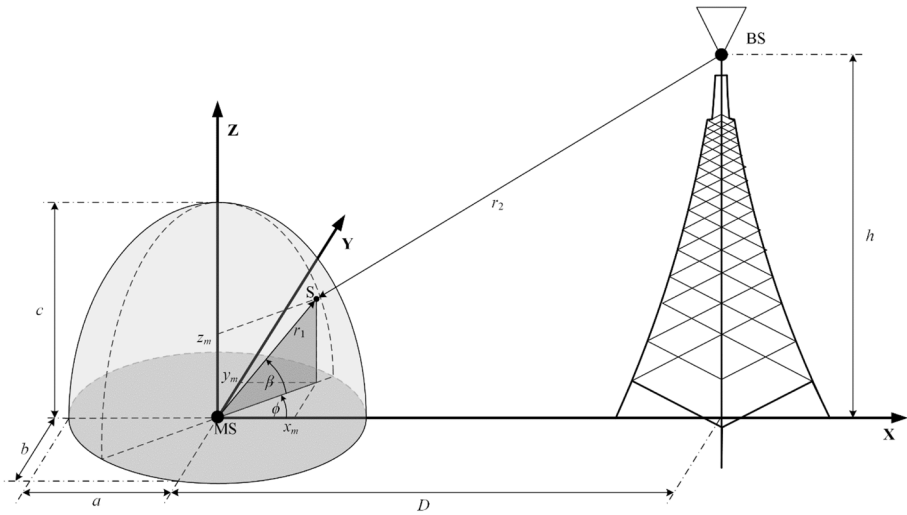


Fig. 1 Geometry of the proposed channel model

Additionally, it is not possible to obtain the presented graph for this model. Riaz et al. [1, Fig. 7] is a copy of [4, Fig. 22], which is obtained for a case, where scattering occur around a transmitting antenna, i.e., BS. The proposed model only considers scattering occur around a receiving antenna, i.e., MS.

- 19. General remark. If the authors introduced the PDF abbreviation for the probability density function, it is accepted that they should only use this abbreviation in the rest of the paper. Whereas, they use interchangeably the PDF or pdf acronyms and full name.

Based on the analysis of [1], we can conclude that the proposed model is a trivial and nonsolid simplification of the previous models proposed by Riaz et al. [2–4]. This triviality consists in the fact that this model considers only so-called local scattering around MS. In this case, MS could represent only a receiver. As a result of so presented problem, the PDFs of AOA seen at MS do not depend on the distance  $D$  between MS and BS and the BS height,  $h$ . In the description, the PDF of AOA seen at BS is not presented at all.

### 3 Solution of Problem for Scenario Presented in Commented Paper

In this section, we present a proposal to improve the description of the model shown in [1].

Figure 1 shows a geometry of the channel model proposed in [1]. MS and BS are located in the origin of the coordinate system and at the point  $(D, 0, h)$ , respectively, where  $h$  and  $D$  are the BS height, and distance between MS and BS, respectively. Signal scattering occur only around a MS antenna. A location of an exemplary scatterer  $S, (x_m, y_m, z_m)$ , is bounded by a semi-ellipsoid

$$\frac{x_m^2}{a^2} + \frac{y_m^2}{b^2} + \frac{z_m^2}{c^2} \leq 1 \tag{1}$$

where  $a, b,$  and  $c$  are semi-axes of the semi-ellipsoid, so-called the semi-major and semi-minor axes, and height of the semi-ellipsoid, respectively.

The relations between the semi-axes of the semi-ellipsoid and  $h$ , and  $D$ , should be determined as  $b \leq a \leq c < h \ll D$ .

Location of S can be expressed in Cartesian,  $(x_m, y_m, z_m)$ , or spherical,  $(r_1, \phi, \beta)$ , coordinates, where  $r_1$ ,  $\phi$ , and  $\beta$  are radius, azimuth, and elevation angles, respectively. The Cartesian and spherical coordinates are related as follows [16]

$$x_m = r_1 \cos \phi \cos \beta, \quad y_m = r_1 \sin \phi \cos \beta, \quad z_m = r_1 \sin \beta \tag{2}$$

where variation ranges of the spherical coordinates are  $r_1 \in \langle 0, r_{\text{imax}} \rangle$ ,  $\phi \in \langle -\pi, \pi \rangle$ , and  $\beta \in \langle 0, \pi/2 \rangle$ , respectively.

The maximum radius,  $r_{\text{imax}}$ , can be determined on the basis of

$$r_{\text{imax}} = \frac{abc}{\sqrt{\frac{1}{2}c^2[a^2 + b^2 + (b^2 - a^2) \cos 2\phi] \cos^2 \beta + a^2 b^2 \sin^2 \beta}} \tag{3}$$

Hence, for  $\beta = 0$  and  $\phi \in \{0, \pi\}$ , we have  $r_1 = a$ , for  $\beta = 0$  and  $\phi \in \{\pi/2, 3\pi/2\}$ , we have  $r_1 = b$ , and for  $\beta = \pi/2$  and  $\phi \in \langle 0, 2\pi \rangle$ , we have  $r_1 = c$ .

The authors of the proposed model assumed the uniform distribution of the scatterers inside the semi-ellipsoid, so the density of the scatterers in Cartesian coordinates is equal

$$f(x_m, y_m, z_m) = \frac{1}{V} \tag{4}$$

where  $V$  is the semi-ellipsoid volume, i.e., [16]

$$V = \frac{2}{3} \pi abc \tag{5}$$

The joint PDF,  $p(r_1, \phi, \beta)$ , of the radius, elevation, and azimuth is expressed as [17]

$$p(r_1, \phi, \beta) = \frac{f(x_m, y_m, z_m)}{|J(x_m, y_m, z_m)|} = f(x_m, y_m, z_m) |J(r_1, \phi, \beta)| \tag{6}$$

where  $J(x_m, y_m, z_m)$  and  $J(r_1, \phi, \beta)$  are the Jacobians of coordinate transformations, i.e., [16]

$$J(r_1, \phi, \beta) = \frac{1}{J(x_m, y_m, z_m)} = \begin{vmatrix} \frac{\partial x_m}{\partial r_1} & \frac{\partial x_m}{\partial \phi} & \frac{\partial x_m}{\partial \beta} \\ \frac{\partial y_m}{\partial r_1} & \frac{\partial y_m}{\partial \phi} & \frac{\partial y_m}{\partial \beta} \\ \frac{\partial z_m}{\partial r_1} & \frac{\partial z_m}{\partial \phi} & \frac{\partial z_m}{\partial \beta} \end{vmatrix} = r_1^2 \cos \beta \tag{7}$$

Thus, after substituting (4), (5), and (7) to (6), we obtain

$$p(r_1, \phi, \beta) = \frac{3r_1^2 \cos \beta}{2\pi abc} \tag{8}$$

If we integrate (8) over  $r_1$  in range from 0 to  $r_{\text{imax}}$ , we get the joint PDF for the elevation and azimuth angles

$$p(\phi, \beta) = \int_0^{r_{\text{imax}}} p(r_1, \phi, \beta) dr_1 = \frac{r_{\text{imax}}^3 \cos \beta}{2\pi abc} \tag{9}$$

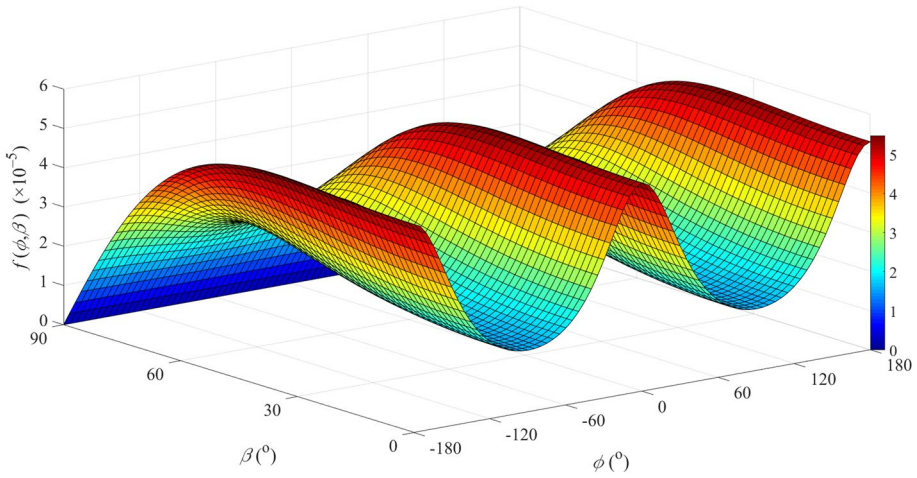


Fig. 2 Joint PDF of AOA for  $a = 1.5b$  and  $c = 2b$

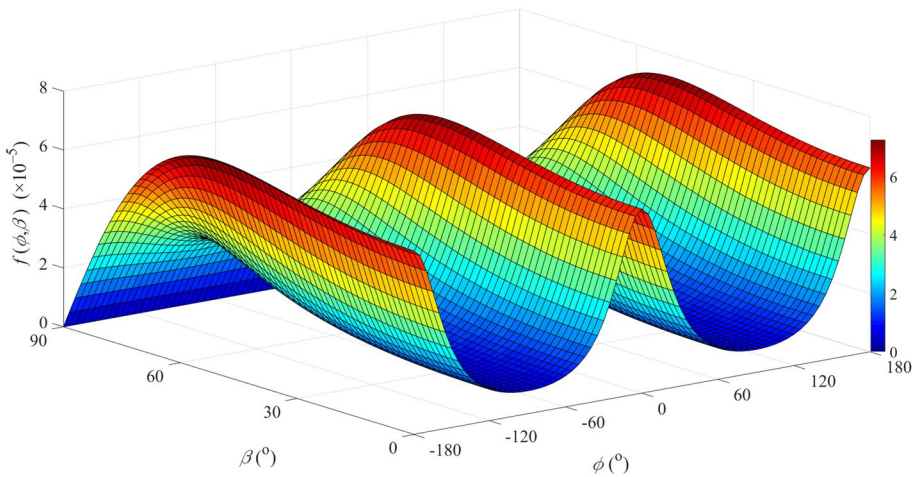


Fig. 3 Joint PDF of AOA for  $a = 2b$  and  $c = 3b$

where  $r_{1\max} = r_{1\max}(\phi, \beta)$  is given by (3).

Based on the marginal PDF properties and using numerical integrating (9) over  $\beta$  or  $\phi$

$$p(\phi) = \int_0^{\pi/2} p(\phi, \beta) d\beta \tag{10}$$

$$p(\beta) = \int_{-\pi}^{\pi} p(\phi, \beta) d\phi \tag{11}$$



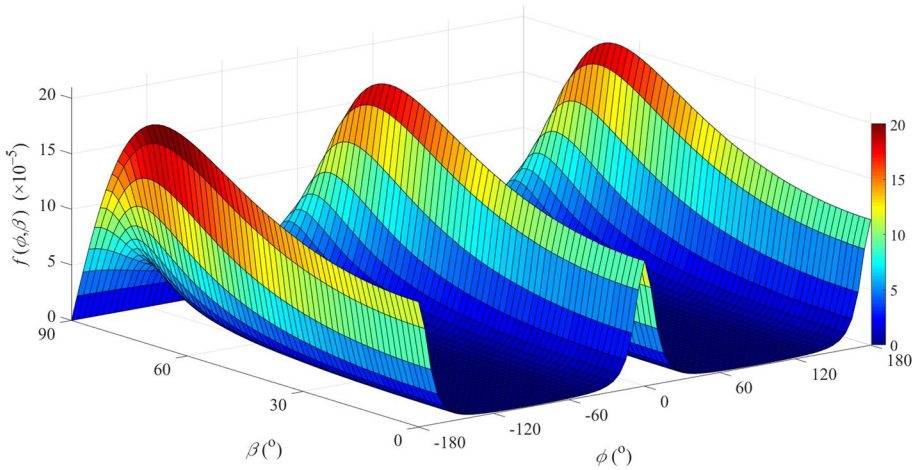


Fig. 4 Joint PDF of AOA for  $a=5b$  and  $c=10b$

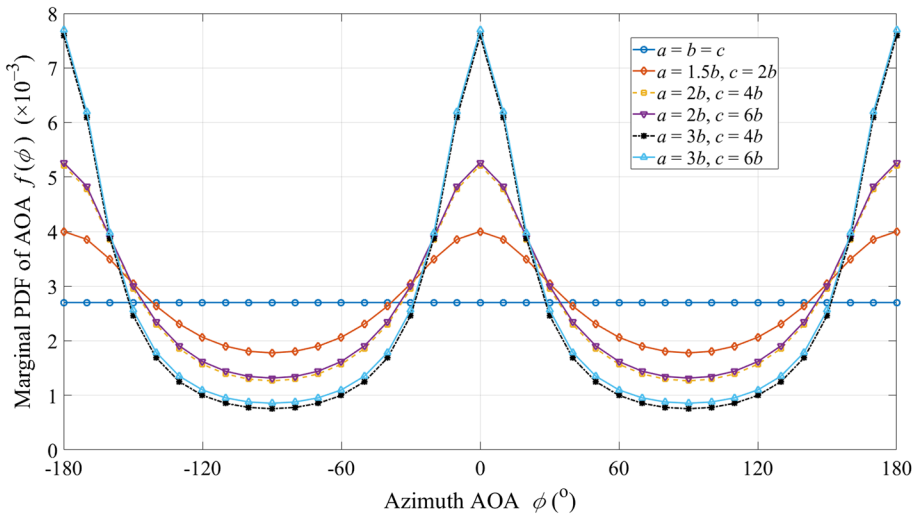


Fig. 5 Marginal PDF of azimuth AOA for selected semi-ellipsoid parameters

we can determine the PDFs of azimuth and elevation AOA, respectively. Exemplary joint PDFs of AOA are determined for different parameters of the semi-ellipsoid. The obtained results are shown in Figs. 2, 3, 4 for  $(a = 1.5b, c = 2b)$ ,  $(a = 2b, c = 3b)$  and  $(a = 5b, c = 10b)$ , respectively.

The characteristic feature of the joint PDFs is the presence of two maxima for  $\phi = 0$  and  $\phi = 180^\circ$ , respectively. Changing the semi-ellipsoid parameters causes modifications of a height and width of these maxima.

The numerical calculations for (10) and (11) give the possibility of determining the marginal PDFs of AOA shown in Figs. 5 and 6 for the azimuth and elevation, respectively. These graphs are presented for the selected semi-ellipsoid parameters.



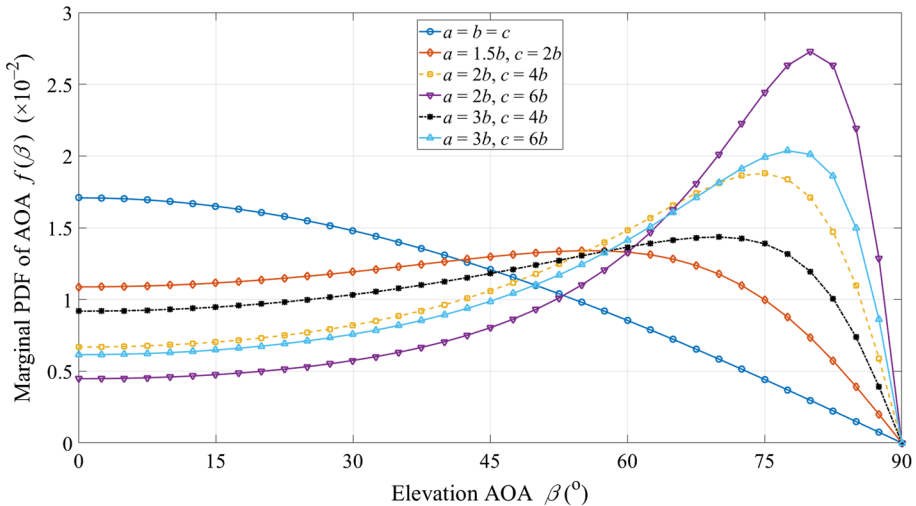


Fig. 6 Marginal PDF of elevation AOA for selected semi-ellipsoid parameters

Based on the obtained results, we can conclude that the shapes of the joint and marginal PDFs of AOA do not depend on the absolute value of the semi-ellipse parameters, but only on their mutual relations. This means that graphs of  $p(\phi, \beta)$ ,  $p(\phi)$ , and  $p(\beta)$  are the same for, e.g.,  $(a = 10 \text{ m}, b = 5 \text{ m}, c = 20 \text{ m})$  and  $(a = 20 \text{ m}, b = 10 \text{ m}, c = 40 \text{ m})$ , because relations between the parameters are constant, i.e.,  $a = 2b$  and  $c = 4b = 2a$ . Additionally, in the case of the marginal PDF of azimuth AOA, only the ratio between  $a$  and  $b$  influences on the shape of this characteristic. Thus, the parameter  $c$  can be arbitrary. This results in overlapping the graphs in Fig. 5 for  $(a = 2b, c = 4b)$  and  $(a = 2b, c = 6b)$ , or  $(a = 3b, c = 4b)$  and  $(a = 3b, c = 6b)$ , respectively. For the same values of the semi-ellipsoid parameters, i.e.,  $a = b = c$ , this distribution is uniform. This effect results from (10), i.e., the integration  $p(\phi, \beta)$  over the elevation.

Analogously as in the case of the joint PDF, the marginal PDF of azimuth AOA has two maxima for  $\phi = 0$  and  $\phi = 180^\circ$ , respectively. Hence, the proposed model in the azimuth plane can only be used to map bimodal AOA distributions. So, such distributions can occur in propagation environment for a typical street canyon. Whereas, empirical distributions presented in [6–11] are unimodal, therefore they cannot be mapped by this model.

### 4 Conclusion

In this comments, we present the review of [1] and proposal to improve the proposed channel model. Although introducing appropriate changes in the analytical description of the model, this model does not provide an opportunity to adequately reflect the propagation properties of the environment.

The main reason for our comments on [1] is the presentation of the channel model, which is the trivial and nonsolid simplification of the previous models proposed by Riaz et al. [2–4]. This triviality consists in the fact that this model considers only the local scattering around MS, which could represent only the receiver. Therefore, the PDFs of AOA seen at MS do not depend on the distance between MS and BS and the BS height.

Additionally, this model does not depend on the absolute values of the semi-ellipsoid parameters but only on the relations between these parameters.

This model incorrectly maps the propagation conditions that exist in the real environment. Proof of this is incorrect approximation of measurement data. Using the proposed model, approximation of typical empirical distributions of AOA, presented in [6–11], is not possible. It shows the distributions obtained from this model.

In addition, the numerous errors, especially in the entered formulas and symbols, are in the model description. The erroneous application of mathematical rules makes it impossible to assess the utility of the proposed model. In abstract of [1], the authors declare the derivation of ... *expressions for the joint and marginal probability density functions for angle-of-arrival and time-of-arrival in azimuth and elevation planes ...*, but only the equation on the joint PDF is given. In Sect. 4 of [1], the authors promise to validate the model on the basis of measurement data and compare it with other models, but these are also not presented, although this approach is typical for papers describing new channel models. The parameters for the obtained results and used optimization method for these parameters to match the model to empirical data also are not presented in the paper.

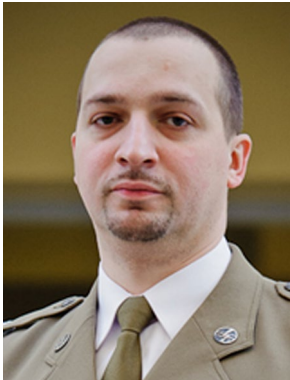
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