

# Ergodic Channel Capacity Analysis of Downlink in the Hybrid Satellite-Terrestrial Cooperative System

Yue Zhao<sup>1</sup> · Lei Xie<sup>1,2</sup> · Huifang Chen<sup>1,2</sup> · Kuang Wang<sup>1,2</sup>

Published online: 31 August 2017

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**Abstract** In this paper, the ergodic channel capacity of the downlink is analyzed for a hybrid satellite-terrestrial cooperative system, which consists of a satellite (the source), a mobile terminal (the destination), and several gap fillers (the relays) located at the ground. The links between the satellite and the relays and the link between the satellite and the destination experience independent shadowed Rician fading, and the links between the relays and the destination experience Rayleigh fading. The maximal ratio combining technique is used at the destination to combine the direct signal received from the satellite and the relayed signals from relays with different cooperative protocols, namely amplify-and-forward (AF) and decode-and-forward (DF). The moment generating function (MGF)-based approach is adopted to derive the closed-form expressions of the ergodic downlink channel capacity of the hybrid satellite-terrestrial cooperative system. The numerical results are compared with Monte Carlo simulations and numerical results calculated with the existing analytical expressions. Comparison results show that the analytical expression derived with the MGF-based approach can achieve a higher accuracy in the low signal-to-noise ratio (SNR) regime for the single relay scenario, and significantly reduce the computational complexity with a little loss of accuracy for multiple relays scenario. On the

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✉ Lei Xie  
xiel@zju.edu.cn

Yue Zhao  
zhaoy2013@zju.edu.cn

Huifang Chen  
chenhf@zju.edu.cn

Kuang Wang  
wangk@zju.edu.cn

<sup>1</sup> College of Information Science and Electronic Engineering, Zhejiang University, No. 38, Zheda Road, Hangzhou 310027, People's Republic of China

<sup>2</sup> Zhejiang Provincial Key Laboratory of Information Processing, Communications and Networks, No. 38, Zheda Road, Hangzhou 310027, People's Republic of China

other hand, the ergodic downlink channel capacity of the hybrid satellite-terrestrial DF cooperative system is generally higher than that of the AF cooperative system. Moreover, the ergodic channel capacity of the hybrid satellite-terrestrial cooperative system decreases as the number of participating relays increases, which can be overcome using the best relay selection strategy. In addition, the ergodic downlink channel capacity of the hybrid satellite-terrestrial cooperative system increases when the channel condition of the link between the satellite and the relay goes better, and is larger than that of the no relay land mobile satellite system when the transmitted SNR is below a certain value.

**Keywords** Hybrid satellite-terrestrial cooperative system · Ergodic channel capacity · Moment generating function · Amplify-and-forward · Decode-and-forward

## 1 Introduction

Because of the vast coverage, satellite communication system has been widely used in broadcasting, positioning, navigation, and so on [1]. Due to the mobility of user equipment, some problems may occur for land mobile satellite systems. The channel between satellites and users is not always line-of-sight any more. Multi-path and shadowing effects caused by bad weather, trees, buildings, etc. become even more severe [2].

The hybrid satellite-terrestrial cooperative system is an effective solution to expand the coverage of satellite when the channel condition is bad [1, 2]. It differs from the land mobile satellite system by deploying the gap fillers on the ground. There are two kinds of gap filling techniques, transparent gap filling and non-transparent gap filling. Transparent gap fillers, as relay nodes in networks, are responsible for forwarding received signal to users with amplify-and-forward (AF) or decode-and-forward (DF) protocol. Non-transparent gap fillers are different in that the air interfaces of the original system may be changed, and the radio resource management may be performed. Such case is also called integrated satellite-terrestrial system [1]. In this paper, the hybrid satellite-terrestrial cooperative system based on the transparent gap fillers and different forward protocols are considered.

In the hybrid satellite-terrestrial cooperative system, relay technique is of great importance. Diversity gain can be achieved through multiple paths, bringing higher data rate and improving the reliability of the communication system. In [3], the system performance of a hybrid satellite-terrestrial cooperative system, in terms of the outage probability, is analytically evaluated, where the shadowed Rician distribution for the satellite links and the Nakagami- $m$  distribution for the terrestrial links are considered. The impact of the shadowing conditions of both the satellite links and the terrestrial links on the system performance is analyzed. And the impact of the satellite elevation angle on the outage probability is also investigated. The system performance of the hybrid satellite-terrestrial AF cooperative system and DF cooperative system, in terms of the symbol error rate (SER), is analytically evaluated under different modulation schemes in [2, 4], respectively. The results indicate that at the most second order diversity and full order diversity can be obtained in the AF cooperative system and the DF cooperative system, respectively. In addition, the closed-form expressions of the outage probability and SER of the hybrid satellite-terrestrial AF cooperative system with multiple relay nodes are derived in [5]. All the related works in [2–5] show that, compared with the land mobile satellite

system, the hybrid satellite-terrestrial cooperative system performs well in achieving a higher reliability. In this paper, we will focus on the system performance of the hybrid satellite-terrestrial cooperative system, in terms of the ergodic channel capacity.

In [6, 7], approximated closed-form expressions of the ergodic and outage capacity of wireless relay systems under Rayleigh channels with the AF and DF protocols are derived. Modi et al. [8] extended the results in [6, 7] to the case of Rician channel, and used a novel approach to decrease the computational complexity, and guarantee the accuracy. However, these closed-form expressions of the ergodic channel capacity are derived under terrestrial fading channels. Up to now, few researchers have investigated the ergodic channel capacity of the land mobile satellite system, which is not easy to calculate directly since the channels are much more complex. Iqbal and Ahmed [9] analyzed the ergodic channel capacity of the hybrid satellite-terrestrial AF cooperative system using the second-order approximation of the Taylor expansion, and obtained the approximated closed-form expression and an upper bound. However, the approach used in [9] has a high computational complexity. Hence, it is necessary to find other approaches with low computational complexity and high accuracy to analyze the ergodic channel capacity of the hybrid satellite-terrestrial cooperative system.

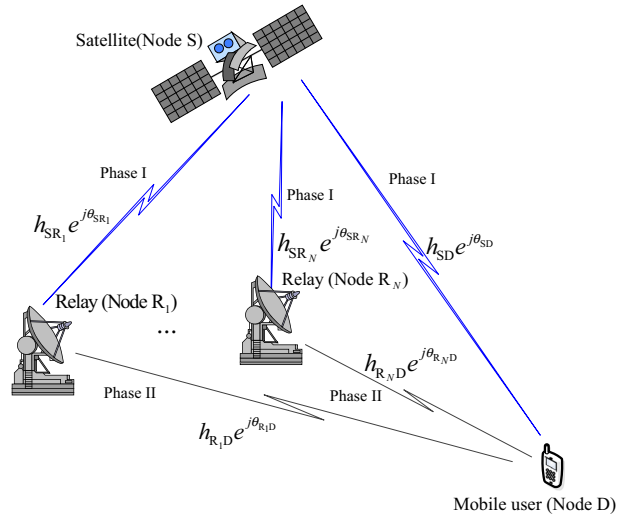
In this paper, we investigate the ergodic channel capacity of downlink in the hybrid satellite-terrestrial cooperative system, where the fading of the satellite channel follows shadowed Rician distribution [10] and the fading of the terrestrial channel follows Rayleigh distribution. The shadowed Rician model is an alternative to the well-known Loo's model [11], where the log-normal distribution in Loo's model is replaced by Nakagami- $m$  distribution. It has been shown that the model agrees with the experimental data well and offers significant advantages in theoretical and numerical analysis. Based on such model, the theoretical expressions of the ergodic downlink channel capacity of the hybrid satellite-terrestrial cooperative system with AF and DF relay protocols are derived using the moment generating function (MGF) approach. The main contribution of this work is that the closed-form expressions of ergodic downlink channel capacity are derived in hybrid satellite-terrestrial cooperative system. The numerical results calculated with the analytical expressions derived in this work are compared with the simulation results and the numerical results in [9] over a wide range of the transmitted signal-to-noise ratio (SNR). Furthermore, system performance with AF and DF cooperative protocols is analyzed. For the hybrid satellite-terrestrial cooperative system with multiple relay nodes, the relay selection strategy is addressed.

The remainder of this paper is organized as follows. Section 2 introduces the system model and channel model of the hybrid satellite-terrestrial cooperative system with AF and DF cooperative protocols. In Sect. 3, the MGF-based analysis approach is presented and the closed-form expressions of ergodic channel capacity are derived. Numerical and simulation results are given in Sect. 4. We conclude the paper in Sect. 5.

## 2 System Model

We consider a hybrid satellite-terrestrial cooperative system, as shown in Fig. 1, consisting of a satellite as a source node (node S), a mobile user as a destination node (node D) and an arbitrary number (say  $N$ ) of gap fillers as relay nodes (nodes  $R_1, R_2, \dots, R_N$ ). It is assumed that each node is equipped with a single antenna. In a real system scenario, the satellite will

**Fig. 1** The model of a hybrid satellite-terrestrial cooperative system with  $N$  relays



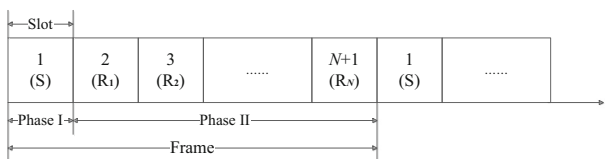
communicate with several mobile users on the land. However, the system model is still reasonable when multiple access schemes are adopted.

The communication from node S to node D is accomplished in two phases as shown in Fig. 1. In the phase I, node S broadcasts a signal to node D and all relay nodes. In the phase II,  $N$  relay nodes forward their signals to node S in order after simply amplified or successfully decoded, while node S keeps silent. Receiving different copies of one signal in two phases, node D combines them using the maximal ratio combining (MRC) technique. Generally the length ratio of phase I and phase II is  $1/N$ , as depicted in Fig. 2. Thus, it needs  $1 + N$  time slots to complete transmitting a frame from node S to node D.

It is also assumed that no information is exchanged among the relay nodes which operate in a time division duplex (TDD) mode, and the signals transmitted by node S and R are perfectly synchronized at node D.

Independent fading links exist between the satellite, the relay nodes and the destination. It is assumed that the link from node S to node D ( $S \rightarrow D$ ) and the link from node S to node  $R_i$  ( $S \rightarrow R_i$ ) experience shadowed Rician fading, and the link from node  $R_i, i = 1, 2, \dots, N$ , to node D ( $R_i \rightarrow D$ ) experiences the Rayleigh fading.

The hybrid satellite-terrestrial cooperative system is scalable from the single relay case. Without loss of generality, the number of relay equals to 1 in the following analysis and the corresponding results of multiple relays case will be only given as conclusions.



**Fig. 2** The TDMA frame structure

### 2.1 With Amplify-and-Forward (AF) Cooperative Protocol

With AF cooperative protocol, the signal received by node R is forwarded to node D after simply amplified in the phase II.

The signals received in nodes R and D,  $y_R$  and  $y_D$ , are

$$y_R = h_{SR}e^{j\theta_{SR}}x + n_R \tag{1}$$

and

$$y_D = h_{SD}e^{j\theta_{SD}}x + Gh_{RD}e^{j\theta_{RD}}y_R + n_D \tag{2}$$

respectively. Where  $x$  is the signal transmitted by node S.  $h_{SR}$  and  $\theta_{SR}$  denote the magnitude response and the phase response of  $S \rightarrow R$ ,  $h_{SD}$  and  $\theta_{SD}$  denote the magnitude response and the phase response of  $S \rightarrow D$ ,  $h_{RD}$  and  $\theta_{RD}$  denote the magnitude response and the phase response of  $R \rightarrow D$ .  $n_D$  and  $n_R$  are independent additive white Gaussian noise with zero mean and variance  $N_D$  and  $N_R$  at nodes D and R, respectively,  $n_D \sim \mathcal{N}(0, N_D)$ , and  $n_R \sim \mathcal{N}(0, N_R)$ .  $G$  denotes the gain factor of the AF cooperative protocol. If the channel state information (CSI) is available at node R,  $G = \sqrt{P_R / (h_{SR}^2 P_S + N_R)}$  [12], where  $P_S$  and  $P_R$  are the transmitted power at nodes S and R, respectively.

Based on (1) and (2), the total received SNR,  $\gamma$  can be derived with MRC as

$$\gamma_{AF} = \gamma_{SD} + \gamma_R = \gamma_{SD} + \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1} \tag{3}$$

where  $\gamma_{SD}$ ,  $\gamma_{RD}$  and  $\gamma_{SR}$  denote the instantaneous SNR of  $S \rightarrow D$ ,  $R \rightarrow D$  and  $S \rightarrow R$ , respectively.  $\gamma_{SD} = h_{SD}^2 P_S / N_D$ ,  $\gamma_{RD} = h_{RD}^2 P_R / N_D$ , and  $\gamma_{SR} = h_{SR}^2 P_S / N_R$ .

For the hybrid satellite-terrestrial cooperative system with  $N$  relays,  $\gamma_{AF}$  can be represented as

$$\gamma_{AF} = \gamma_{SD} + \sum_{i=1}^N \frac{\gamma_{SR_i} \gamma_{R_i D}}{\gamma_{SR_i} + \gamma_{R_i D} + 1} \tag{4}$$

Since  $h_{SR}$  and  $h_{SD}$  follow the shadowed Rician distribution, the probability density functions (PDF) of  $h_{SR}$  and  $h_{SD}$  are [10]

$$p(x) = \left( \frac{2bm}{2bm + \Omega} \right)^m \frac{x}{b} \exp\left(-\frac{x^2}{2b}\right) {}_1F_1\left(m, 1, \frac{\Omega x^2}{2b(2bm + \Omega)}\right) \tag{5}$$

where  ${}_1F_1(a, b, z)$  is the confluent hyper-geometric function with parameters  $(a, b, z)$ .  $m$  is the Nakagami multipath fading parameter.  $\Omega$  is the average power of the line-of-sight component and  $2b$  is the average power of the multipath component.

According to (5), the PDF of  $\gamma_{SD}$  and  $\gamma_{SR}$  can be represented as

$$p_{\gamma_{SD}}(x) = \frac{1}{2b_{SD}\bar{\gamma}_{SD}} \left( \frac{2b_{SD}m_{SD}}{2b_{SD}m_{SD} + \Omega_{SD}} \right)^{m_{SD}} \times \exp\left(\frac{-x}{2b_{SD}\bar{\gamma}_{SD}}\right) {}_1F_1\left(m_{SD}, 1, \frac{\Omega_{SD}x}{2b_{SD}\bar{\gamma}_{SD}(2b_{SD}m_{SD} + \Omega_{SD})}\right) \tag{6}$$

and

$$p_{\gamma_{SR}}(x) = \frac{1}{2b_{SR}\bar{\gamma}_{SR}} \left( \frac{2b_{SR}m_{SR}}{2b_{SR}m_{SR} + \Omega_{SR}} \right)^{m_{SR}} \exp\left(\frac{-x}{2b_{SR}\bar{\gamma}_{SR}}\right) \times F_1\left(m_{SR}, 1, \frac{\Omega_{SR}x}{2b_{SR}\bar{\gamma}_{SR}(2b_{SR}m_{SR} + \Omega_{SR})}\right) \tag{7}$$

where  $\bar{\gamma}_{SD}$  and  $\bar{\gamma}_{SR}$  denote the average transmitted SNR of  $S \rightarrow D$  and  $S \rightarrow R$ , respectively. If  $P_S$  is constant,  $\bar{\gamma}_{SD} = P_S/N_D$ .

Similarly, the PDF of  $\gamma_{RD}$  can be represented as

$$p_{\gamma_{RD}}(x) = \frac{1}{2b_{RD}\bar{\gamma}_{RD}} \exp\left(\frac{-x}{2b_{RD}\bar{\gamma}_{RD}}\right) \tag{8}$$

where  $\bar{\gamma}_{RD}$  denotes the average transmitted SNR of  $R \rightarrow D$ , and  $b_{RD}$  is a parameter of Rayleigh fading.

### 2.2 With Decode-and-Forward (DF) Cooperative Protocol

With the DF cooperative protocol, the relay node transmits received signal to node D in the phase II only when the received signal can be successfully decoded. For simplicity, a threshold,  $\gamma_{th}$ , is introduced to determine whether the decoding is successful or not by comparing  $\gamma_{SR}$  with  $\gamma_{th}$  [6]. Since  $\gamma_{SR}$  is a random variable, any change of  $\gamma_{th}$  will affect the probability of successful decoding. Generally,  $R_{th} = \frac{1}{2}\log(1 + \gamma_{th})$ , where  $R_{th}$  is the pre-determined requirement of the transmission efficiency. Hence,  $\gamma_{th} = 2^{2R_{th}} - 1$ .

The signals received by nodes R and D,  $y_R$  and  $y_D$ , are

$$y_R = h_{SR}e^{j\theta_{SR}x} + n_R \tag{9}$$

and

$$y_D = \begin{cases} h_{SD}e^{j\theta_{SD}x} + n_D, & \gamma_{SR} < \gamma_{th} \\ h_{SD}e^{j\theta_{SD}x} + h_{RD}e^{j\theta_{RD}x} + n_D, & \gamma_{SR} \geq \gamma_{th} \end{cases} \tag{10}$$

respectively. Where,  $P_R = P_S$ .

The total received SNR at node D is

$$\gamma_{DF} = \begin{cases} \gamma_{SD}, & \gamma_{SR} < \gamma_{th} \\ \gamma_{SD} + \gamma_{RD}, & \gamma_{SR} \geq \gamma_{th} \end{cases} \tag{11}$$

For the hybrid satellite-terrestrial cooperative system with  $N$  relays, the total received SNR at node D is

$$\gamma_{DF} = \begin{cases} \gamma_{SD}, & \mathcal{C} = \emptyset \\ \gamma_{SD} + \sum_{R_i \in \mathcal{C}} \gamma_{R_i D}, & \mathcal{C} \neq \emptyset \end{cases} \tag{12}$$

where  $\mathcal{C}$  is the set of relay nodes decoding the received signal successfully and transmitting signals in the phase II.

### 3 Ergodic Channel Capacity Analysis

Assuming that the CSI is available at the receiver and  $P_S$  is constant, the ergodic channel capacity of the system with one relay node is given as [13]

$$\frac{\bar{C}}{B} = \frac{1}{2 \ln 2} \int_0^\infty \ln(1+x) p_\gamma(x) dx \tag{13}$$

where  $\bar{C}$  is the ergodic channel capacity and  $B$  is the channel bandwidth.

After substituting (6), (7) and (8),  $p_\gamma(x)$  can be calculated with (4) and (12) for the AF cooperative protocol and the DF cooperative protocol, respectively. However, it is difficult to calculate (13) directly due to an integral of  $p_\gamma(x)$  derived by convolution of hypergeometric functions, which leads to high computation burden and inaccurate results in numerical calculation. Due to the high computational complexity for calculating (13), it imposes a practical limit in applying (13) directly at least when few efficient numerical methods are available. Hence, it is necessary to find a low computational complexity method, as well as guarantee the high calculation precision.

The moment generating function (MGF), an approach to describe the distribution property of a random variable, can reduce the computation burden when dealing with complex probability distributions. Hence, we adopt the MGF-based approach to resolve the problem of the computational complexity in this work.

The MGF of  $\gamma$  is defined as  $\phi(s) = \int_0^\infty e^{-sx} p_\gamma(x) dx$ ,  $x > 0$ . Hence, with (5), (6) and (7), the MGF of  $\gamma_{RD}$ ,  $\gamma_{SD}$  and  $\gamma_{SR}$  can be derived as

$$\phi_{\gamma_{RD}}(s) = \frac{1}{1 + 2b_{RD}s\bar{\gamma}_{RD}} \tag{14}$$

$$\phi_{\gamma_{SD}}(s) = \frac{(2b_{SD}m_{SD})^{m_{SD}}(2b_{SD}s\bar{\gamma}_{SD} + 1)^{m_{SD}-1}}{[(2b_{SD}m_{SD} + \Omega_{SD})(2b_{SD}s\bar{\gamma}_{SD} + 1) - \Omega_{SD}]^{m_{SD}}} \tag{15}$$

and

$$\phi_{\gamma_{SR}}(s) = \frac{(2b_{SR}m_{SR})^{m_{SR}}(2b_{SR}s\bar{\gamma}_{SR} + 1)^{m_{SR}-1}}{[(2b_{SR}m_{SR} + \Omega_{SR})(2b_{SR}s\bar{\gamma}_{SR} + 1) - \Omega_{SR}]^{m_{SR}}} \tag{16}$$

respectively.

According to the definition of MGF, (13) can be rewritten as [14]

$$\frac{\bar{C}}{B} = \frac{1}{2 \ln 2} \int_0^\infty \frac{e^{-s}}{s} [1 - \phi(s)] ds \tag{17}$$

#### 3.1 With AF Cooperative Protocol

As we know, for the MGF,  $\phi_C(s) = \phi_A(s)\phi_B(s)$  if  $C = A+B$ ,  $A$  and  $B$  are independent random variables.

For the hybrid satellite-terrestrial system with single relay and AF cooperative protocol, according to (3),  $\phi_\gamma(s) = \phi_{\gamma_{SD}}(s)\phi_{\gamma_R}(s)$ , where  $\phi_{\gamma_R}(s) = E[e^{-\gamma_R s}]$  and  $\gamma_R = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1}$ .

Although  $\phi_{\gamma_R}(s)$  is not easy to calculate, it is fortunate that the approximate solution to  $\phi_{\gamma_R}(s)$  can be easily derived with same approaches in [8, Eq. (11)] and [15, Eq. (8)] which have been proved feasible in performance analysis. Hence, when  $\gamma_R = \frac{\gamma_{SR}\gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1}$ ,

$$\phi_{\gamma_R}(s) \approx \phi_{\gamma_{SR}}(s) + \phi_{\gamma_{RD}}(s) - \phi_{\gamma_{SR}}(s)\phi_{\gamma_{RD}}(s) \tag{18}$$

Substituting (18) into (17), the ergodic downlink channel capacity of the hybrid satellite-terrestrial AF cooperative system with single relay is

$$\frac{\bar{C}}{B_{AF}} \approx \frac{1}{2 \ln 2} \int_0^\infty \frac{e^{-s}}{s} \left\{ 1 - \phi_{\gamma_{SD}}(s) \left[ \phi_{\gamma_{SR}}(s) + \phi_{\gamma_{RD}}(s) - \phi_{\gamma_{SR}}(s)\phi_{\gamma_{RD}}(s) \right] \right\} ds \tag{19}$$

From (19),  $\frac{\bar{C}}{B_{AF}}$  is derivable as long as the MGFs of the links,  $\phi_{\gamma_{SD}}(s)$ ,  $\phi_{\gamma_{SR}}(s)$  and  $\phi_{\gamma_{RD}}(s)$ , are known. The conclusion in (19) can extend to other similar relay systems where different channel fading is assumed.

For the ergodic downlink channel capacity of the hybrid satellite-terrestrial AF cooperative system with multiple relays, the ergodic downlink channel capacity is

$$\frac{\bar{C}}{B_{AF}} \approx \frac{1}{(1+N) \ln 2} \int_0^\infty \frac{e^{-s}}{s} \left\{ 1 - \phi_{\gamma_{SD}}(s) \left[ \prod_{i=1}^N \left( \phi_{\gamma_{SR_i}}(s) + \phi_{\gamma_{RD}}(s) - \phi_{\gamma_{SR_i}}(s)\phi_{\gamma_{RD}}(s) \right) \right] \right\} ds \tag{20}$$

Note that, (20) is an approximated expression and the accuracy is greatly influenced by  $N$ . The accuracy of (20) will be numerically analyzed further in Sect. 4.

### 3.2 With DF Cooperative Protocol

According to (11) and (13), the ergodic downlink channel capacity of the hybrid satellite-terrestrial DF cooperative system with single relay is

$$\frac{\bar{C}}{B_{DF}} = qE \left[ \frac{1}{2} \log_2(1 + \gamma_{SD} + \gamma_{RD}) \right] + (1-q)E \left[ \frac{1}{2} \log_2(1 + \gamma_{SD}) \right] \tag{21}$$

where  $q$ , the probability of successful decoding, can be calculated using the MGF-based approach with a given  $\gamma_{th}$  as

$$q = 1 - L^{-1} \left( \frac{\phi_{\gamma_{SR}}(s)}{s} \right) \Big|_{\gamma_{SR}=\gamma_{th}} \tag{22}$$

where  $L^{-1}(\cdot)$  is the inverse Laplace transform. There are many numerical techniques to calculate (22) efficiently in [16].

Substituting (22) into (21), the ergodic downlink channel capacity of the hybrid satellite-terrestrial DF cooperative system with single relay can also be derived using (17)

$$\frac{\bar{C}}{B_{DF}} = \frac{1}{2 \ln 2} \left\{ q \int_0^\infty \frac{e^{-s}}{s} [1 - \phi_{\gamma_{SD}}(s) \cdot \phi_{\gamma_{RD}}(s)] ds + (1-q) \int_0^\infty \frac{e^{-s}}{s} [1 - \phi_{\gamma_{SD}}(s)] ds \right\} \tag{23}$$

For the ergodic downlink channel capacity of the hybrid satellite-terrestrial DF cooperative system with multiple relays, it is assumed that  $\{h_{SR_i} | i = 1, 2, \dots, N\}$  are Independent Identically Distributed (i.i.d.) variables. That is,  $\phi_{\gamma_{SR_i}}(s) = \phi_{\gamma_{SR}}(s)$ ,  $i = 1, 2, \dots, N$ , and the probability of successfully decoding at each relay node is identical. Then, the ergodic



downlink channel capacity of the hybrid satellite-terrestrial DF cooperative system with  $N$  relays is derived as

$$\begin{aligned} \frac{\bar{C}}{B_{DF}} &= \sum_{i=0}^N \left\{ \binom{N}{i} (1-q)^{N-i} q^i E \left[ \frac{1}{(1+N)} \log_2 \left( 1 + \gamma_{SD} + \sum_{j=0}^i \gamma_{R_jD} \right) \right] \right\} \\ &= \frac{1}{(1+N) \ln 2} \sum_{i=0}^N \left\{ \binom{N}{i} (1-q)^{N-i} q^i \int_0^\infty \frac{e^{-s}}{s} [1 - \phi_{\gamma_{SD}}(s) \cdot \prod_{j=0}^i \phi_{\gamma_{R_jD}}(s)] ds \right\} \end{aligned} \quad (24)$$

## 4 Numerical and Simulation Results

In this section, we give the numerical and simulation results of the ergodic downlink channel capacity of the hybrid satellite-terrestrial cooperative system.

The fading property of shadowed Rician channel can be described by  $(m, b, \Omega)$ . Three different channel conditions are listed in Table 1 [10].

For performance evaluation, the channel conditions in the hybrid satellite-terrestrial cooperative system are assumed as follows:

1. It is assumed that  $S \rightarrow D$  is always frequent heavy shadowing (FHS). This assumption is reasonable in reality; otherwise, the deployment of node R will be of little significance.
2. For multiple relays, all of three channel conditions listed in Table I are considered for  $S \rightarrow R_i$  with identical parameters of  $\{h_{SR_1}, h_{SR_2}, \dots, h_{SR_N}\}$  based on the assumption in Sect. 2.
3. The transmitted power of the satellite and gap filler has a great impact on the channel capacity and should be chosen carefully. It is assumed that  $P_{R_i} = P_R = P_S$ , and thus  $\bar{\gamma}_{R_iD} = \bar{\gamma}_{RD} = \bar{\gamma}_{SD}$ ,  $i = 1, 2, \dots, N$  [2]. When  $P_S$  and  $P_R$  are constant, the transmitted SNR and the average SNR of one link are identical. With this assumption, the analysis will be simplified when we analyze the impact of  $\bar{\gamma}_{SR_i}$  on the system performance.

We first compare the numerical results of the ergodic downlink channel capacity calculated with the closed-form expressions derived in this work with the numerical results in [9] and the simulation results for the hybrid satellite-terrestrial AF cooperative system, as listed in Table 2.

From Table 2, the precision of the numerical results calculated with the analytical expression in this paper is higher than that in [9] in the low-SNR regime. As the number of relays increases, the precision of the numerical results in [9] gets better lightly, while the precision of the numerical results calculated with the analytical expression in this paper gets worse slightly. However, the difference between the numerical results calculated with the analytical expression in this paper and the numerical results in [9] is acceptable in the high-SNR regime. In addition, the numerical results agree with the simulation results well

**Table 1** Parameters of shadowed Rician channel

Channel condition	$b$	$m$	$\Omega$
Infrequent light shadowing (ILS)	0.158	19.4	1.29
Average shadowing (AS)	0.126	10.1	0.835
Frequent heavy shadowing (FHS)	0.063	0.739	$8.97 \times 10^{-4}$

**Table 2** Ergodic downlink channel capacity comparison of different numerical and simulation results with different channel parameters

$\bar{\gamma}_{SD}$ (dB)			0	10	20	30
$N = 1$	ILS	Simulation results	0.3388	1.4466	3.0060	4.6548
		Numerical results in this work	0.3343	1.4310	2.9867	4.6351
		Numerical results in [9]	0.3928	1.4693	3.0280	4.6778
	AS	Simulation results	0.2856	1.3262	2.8675	4.5140
		Numerical results in this work	0.2817	1.3102	2.8468	4.4926
		Numerical results in [9]	0.3386	1.3490	2.8878	4.5352
	FHS	Simulation results	0.1203	0.7616	2.1148	3.7294
		Numerical results in this work	0.1192	0.7511	2.0964	3.7096
		Numerical results in [9]	0.1414	0.7700	2.1226	3.7427
$N = 2$	ILS	Simulation results	0.3561	1.2567	2.3385	3.4432
		Numerical results in this work	0.3316	1.1937	2.2672	3.3709
		Numerical results in [9]	0.3954	1.2298	2.2972	3.4003
	AS	Simulation results	0.2995	1.1593	2.2346	3.3386
		Numerical results in this work	0.2798	1.0997	2.1652	3.2681
		Numerical results in [9]	0.3382	1.1346	2.1939	3.2961
	FHS	Simulation results	0.1046	0.6381	1.6141	2.7052
		Numerical results in this work	0.1021	0.6172	1.5812	2.6705
		Numerical results in [9]	0.1289	0.6301	1.5825	2.6707
$N = 3$	ILS	Simulation results	0.3453	1.0806	1.9020	2.7316
		Numerical results in this work	0.3075	0.9975	1.8107	2.6394
		Numerical results in [9]	0.3727	1.0444	1.8536	2.6819
	AS	Simulation results	0.2917	1.0017	1.8203	2.6496
		Numerical results in this work	0.2617	0.9245	1.7333	2.5615
		Numerical results in [9]	0.3221	0.9684	1.7731	2.6009
	FHS	Simulation results	0.0959	0.5563	1.3179	2.1410
		Numerical results in this work	0.0922	0.5291	1.2778	2.0987
		Numerical results in [9]	0.1197	0.5373	1.2721	2.0913

in most cases, which indicates that the approximate approach used in our analysis is reasonable, and the performance analysis is accurate.

On the other hand, for the analytical expression derived using the MGF-based approach in this paper, only an integral of a simple function is needed to calculate the ergodic downlink channel capacity. However, since the analytical expression derived using the second-order approximation of the Taylor expansion in [9] include infinite terms of Gauss hypergeometric functions with nested summations, it tends to be computationally intensive. The computational complexity depends on the numerical methods used. For example, when evaluating the hypergeometric function iteratively [17] and adopting various classical Gauss quadrature rules respectively, the number of the iterations for the analytical expression in [9] is at least at the scale of  $10^4$ , while the analytical expression in this paper only requires less than 200 iterations. In addition, in this paper, it is assumed that all the relay nodes are “identical” in statistics. However, in a general scenario where relay nodes

experience different fading, the computational burden of the analytical expression in [9] will become even larger, because the first and second moment of the hybrid path SNR need to be calculated for each relay. The method in this paper avoids the repeated computing and preserves almost constant iteration times as  $N$  increases.

The impact of various parameters on the system performance is analyzed. In Figs. 3, 4 and 5, the analytical results are illustrated with the colored curves while the simulation results are shown by ( $\times$ ) mark on all the curves for comparison. Axis  $x$  stands for the average transmitted SNR of  $S \rightarrow D$  and axis  $y$  denotes the normalized ergodic downlink channel capacity,  $\bar{C}/B$ .

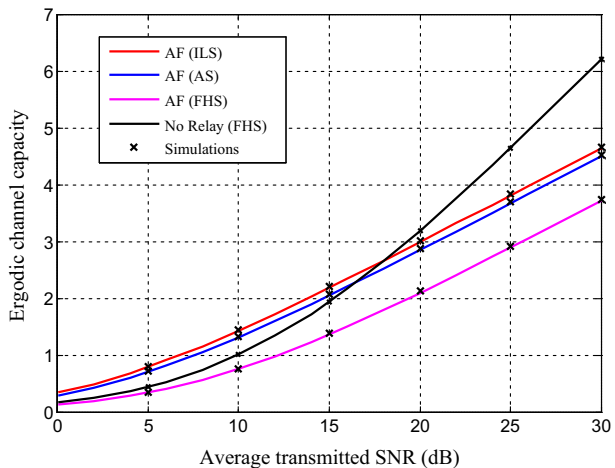
Figure 3 shows the impact of the channel condition of  $S \rightarrow R$  on the ergodic downlink channel capacity of hybrid satellite-terrestrial AF cooperative system, where  $\bar{\gamma}_{SR} = \bar{\gamma}_{SD}$  and  $N = 1$ . The ergodic downlink channel capacity of the no relay mobile land satellite system is also given for comparison.

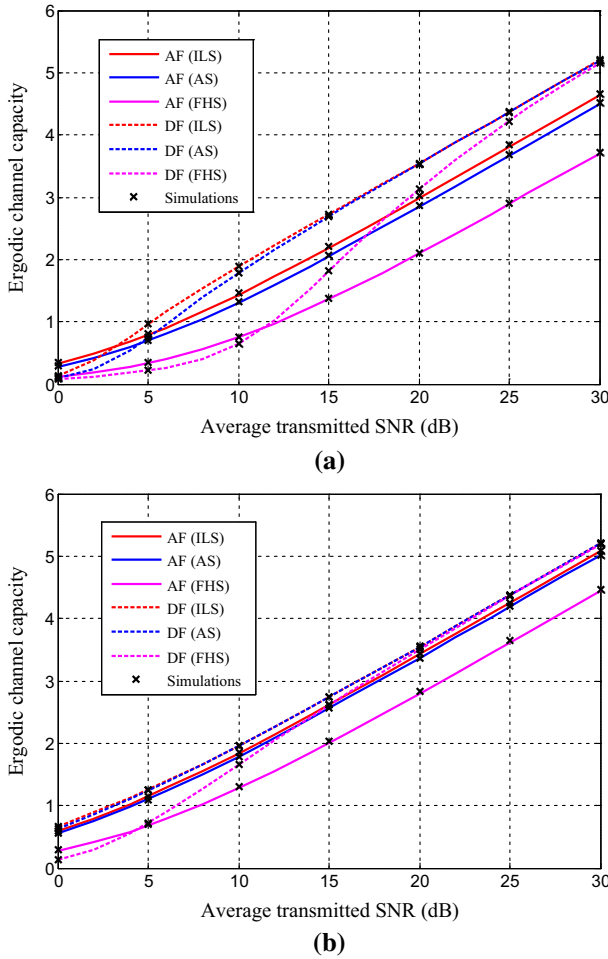
From Fig. 3, we observe that when the channel condition of  $S \rightarrow R$  gets better, the ergodic downlink channel capacity of the hybrid satellite-terrestrial AF cooperative system increases. However, the slopes of all three curves are identical when  $\bar{\gamma}_{SD}$  is high enough, which means that the channel condition of  $S \rightarrow R$  does not affect the asymptotic property of the ergodic channel capacity of the hybrid satellite-terrestrial AF cooperative system.

Moreover, the asymptotic property of the ergodic channel capacity of the no relay mobile land satellite system is better than that of the hybrid satellite-terrestrial AF cooperative system. When  $\bar{\gamma}_{SD}$  is low, the hybrid satellite-terrestrial AF cooperative system achieves a larger channel capacity due to the diversity gain achieved from the relay path. Nevertheless, the ergodic channel capacity of the no relay mobile land satellite system becomes better when  $\bar{\gamma}_{SD}$  exceeds a certain value. This is because we assume the hybrid satellite-terrestrial cooperative system adopts the time division multiple access (TDMA) scheme, which decreases the capacity efficiency, especially when the benefit of the relay path becomes less significant. Therefore, it is of great importance to deploy the gap filler in the ILS or AS case for a hybrid satellite-terrestrial cooperative system. In the FHS case, gap fillers will be less helpful.

Figure 4a, b show the ergodic channel capacity for  $\bar{\gamma}_{SR} = \bar{\gamma}_{SD}$  and  $\bar{\gamma}_{SR} = 10\bar{\gamma}_{SD}$ , where the relay node adopts the AF and DF cooperative protocols,  $N = 1$ , and  $R_{th} = 1$  bit/s/Hz.

**Fig. 3** Impact of channel condition of  $S \rightarrow R$  on the ergodic channel capacity, where  $\bar{\gamma}_{SR} = \bar{\gamma}_{SD}$ , the AF cooperative protocol is performed at the relay node





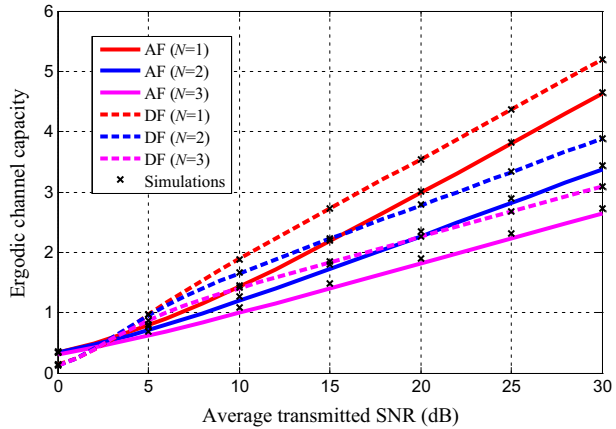
**Fig. 4** Impact of the cooperative protocol and  $\bar{\gamma}_{SR}$  on the ergodic channel capacity. **a**  $\bar{\gamma}_{SR} = \bar{\gamma}_{SD}$ , **b**  $\bar{\gamma}_{SR} = 10\bar{\gamma}_{SD}$

From Fig. 4a, it is observed that, compared with the hybrid satellite-terrestrial AF cooperative system, the hybrid satellite-terrestrial DF cooperative system achieves a relatively larger ergodic channel capacity except when the transmit SNR is below a certain value.

Furthermore, comparing the results in Fig. 4a, b, the performance gap between two cooperative protocols gets smaller as  $\bar{\gamma}_{SR}$  increases. Moreover, it is concluded that the channel condition of  $S \rightarrow R$  has great influence on the ergodic channel capacity especially for the DF cooperative protocol. In a real scenario, it is possible for  $S \rightarrow R$  to obtain a high  $\bar{\gamma}_{SR}$  when node R is located in a good environment with low noise power. If  $\bar{\gamma}_{SR}$  is high enough, the ergodic channel capacity of the hybrid satellite-terrestrial cooperative system is restricted by the channel condition of  $R \rightarrow D$ .

Figure 5 shows the impact of the number of relays on the ergodic downlink channel capacity of the hybrid satellite-terrestrial systems with AF and DF cooperative protocols, where the channel conditions of  $S \rightarrow R_i$  are assumed as ILS, and  $\bar{\gamma}_{SR_i} = \bar{\gamma}_{SD}$ .

**Fig. 5** The impact of the number of relays on the ergodic channel capacity, where  $\bar{\gamma}_{SR_i} = \bar{\gamma}_{SD}$



From Fig. 5, we observe that, as the number of relays increases, the ergodic channel capacity decreases. The reason for this phenomenon is that the hybrid satellite-terrestrial cooperative system adopts TDMA scheme. In the high-SNR regime, the ergodic channel capacity decreases heavily, especially when the satellite occupies fewer time resources in the multiple relay scenario. This is because node D tends to receive the same decoded data from multiple transmitters although the received SNR is improved due to the time diversity. Hence, it is not an efficient method to use multiple relays to forward signals especially in the scenario when high data rate is required. In that case,  $N = 1$  is preferred.

Since the system performance cannot obtain gain by increasing the number of relays in the hybrid satellite-terrestrial cooperative system. We investigate the ergodic channel capacity in the hybrid satellite-terrestrial cooperative system using different relay selection strategies. Figure 6 shows the impact of the relay selection strategy on the ergodic downlink channel capacity, where random relay selection strategy and the best relay selection strategy are compared. The channel conditions of  $S \rightarrow R_i$  are assumed as ILS, and  $\bar{\gamma}_{SR_i} = \bar{\gamma}_{SD}$ .

For random relay selection strategy, only one relay node is selected out of  $N$  relays randomly to transmit signal to node D in the Phase II with AF or DF cooperative protocol. For the best relay selection strategy, only one of the “best” relay node among  $N$  relays is selected to transmit signal to node D in the Phase II with AF or DF cooperative protocol. Here, based on the best node selection policy in [18], we present a modified node selection policy. That is, each relay estimates the instantaneous channel responses,  $\{h_{SR_i}, h_{R_iD} | i = 1, 2, \dots, N\}$ . For the AF cooperative protocol, the relay node is chosen in order to maximize the minimum of  $\{h_{SR_i}, h_{R_iD} | i = 1, 2, \dots, N\}$ . For the DF cooperative protocol, the relay node is chosen in order to maximize  $\{h_{R_iD} | i \in \mathcal{C}\}$ .

From Fig. 6, we observe that the ergodic channel capacity of the hybrid satellite-terrestrial cooperative system with the best relay selection strategy is larger than that of random relay selection strategy. Moreover, as the number of relays increases, the best relay selection strategy achieves better system performance. The reason for this phenomenon is that the best relay selection strategy always chooses the relay in the best channel conditions. However, to perform the channel estimation at each relay is a challenge for the best relay selection strategy.

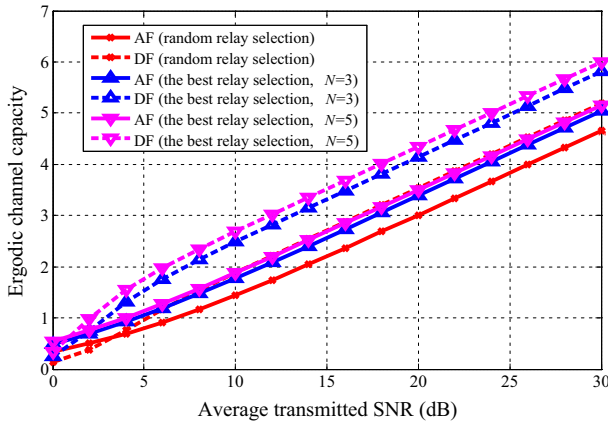


Fig. 6 The impact of the relay selection strategy on the ergodic channel capacity, where  $\bar{\gamma}_{SR} = \bar{\gamma}_{SD}$

### 5 Conclusions

In this paper, the MGF-based approach was adopted to derive the closed-form expressions of the ergodic downlink channel capacity in a hybrid satellite-terrestrial cooperative system, where both AF and DF cooperative protocols were considered. The comparison results showed that the analytical expressions derived using the MGF-based approach is accurate and has a lower computational complexity.

The impact of various parameters on the system performance is investigated. It is concluded that the ergodic downlink channel capacity of the hybrid satellite-terrestrial DF cooperative system is generally larger than that of the AF cooperative system under same channel conditions. And the performance of the hybrid satellite-terrestrial cooperative system outperforms the no relay land mobile satellite system when the average transmitted SNR of S–D link is below a certain value. The channel condition of S–R link also has great influence on the ergodic channel capacity of hybrid satellite-terrestrial cooperative system. And the ergodic channel capacity increase as the channel condition of S–R link gets better. Hence, the location selection of the gap filler is an important issue.

Furthermore, as the number of participating relays increases, the ergodic downlink channel capacity of the hybrid satellite-terrestrial cooperative system decreases because fewer time resources are available for the satellite. However, with the best relay selection strategy, the hybrid satellite-terrestrial cooperative system with multiple relays achieves a larger ergodic channel capacity, at the cost of performing the channel estimation at each relay.

**Acknowledgements** This work was partly supported by the National High Technology Research and Development Program (863) of China (No. 2012AA01A502), and National Natural Science Foundation of China (No. 61171087).

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**Yue Zhao** Ph.D. student in the College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou, China. He received the B.S. degree in Information and Communication Engineering from Zhejiang University, Hangzhou, China, in 2013. His current research interest is satellite mobile communication networks.



**Lei Xie** received the B.S. degree in Electronic Engineering, M.S. and Ph.D. degrees in Communications and Information Systems from Zhejiang University, Hangzhou, China, in 1994, 1997, and 2002, respectively. Now, he is an associate professor in the College of Information Science and Electronic Engineering, Zhejiang University, China. His current research interests include information theory and coding, coded modulation system, and wireless networks.



**Huifang Chen** received the B.S. degree in Electronic Engineering, M.S. and Ph.D. degree in Communications and Information Systems from Zhejiang University, Hangzhou, China, in 1994, 1997, and 2000, respectively. Now, she is an associate professor in the College of Information Science and Electronic Engineering, Zhejiang University, China. Her current research interests include information theoretic security and its applications, and wireless networks.





**Kuang Wang** received the B.S. degree in Electronic Engineering, and Ph.D. degree in Communications and Information Systems from Zhejiang University, Hangzhou, China, in 1989, and 1995, respectively. Now, he is a professor in the College of Information Science and Electronic Engineering, Zhejiang University, China. His current research interests include communication signal processing, coded modulation system, the resource management in wireless networks.