

# Energy Efficient Spectrum Auction Process with Utility Functions

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**Abstract** The concept of green payments together with single and multiple bidding processes for short-term spectrum auctions are compared based on two reference cases: (1) when the users and the auctioneer are aware of the value of the reserve price, and (2) when the value of the reserve price is known only to the auctioneer. This involves a novel concept known as the green payment. This concept is combined with the use of probabilities to determine the users participating in the auction process. The purpose of the green payment and the probability is to help in reducing the amount of energy wasted as a result of the auction process. The utility of each user and that of the wireless service provider with and without the green payment is also examined. The revenue obtained from each of the examined models is also compared to determine which model is more profitable for the WSP. This paper shows that the use of multiple bidding process for short-term spectrum auctions gives a better performance measure when compared to the single bidding process, more particularly when the value of the reserve price is known to the auctioneer and the users in the system. It also shows that using the proposed probability equation in combination with the concept of the green payment helps in reducing the amount of energy consumed by the system.

**Keywords** Spectrum pricing · Spectrum auction · Dynamic spectrum access · Green payment · Utility function

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## 1 Introduction

Demand for the radio spectrum is presently growing faster than its capacity [1, 2]. It is also envisioned that the future heterogeneous wireless devices shall require different bit rates and transmit power while satisfying the demands of the users [2]. Furthermore, as a result of the rapid growth in the applications requiring the use of the radio spectrum, *scarcity* of the most useful frequency bands for mobile communications is posing a problem [3]. This has led to the concepts of Dynamic Spectrum Access (DSA) as proposed in [4, 5], situations where DSA is delivered using a database as proposed in [6], the concept of the Cognitive Radio (CR) network as proposed in [7, 8], and short-term spectrum auction as proposed in [9]. Furthermore, due to a decline in the revenue of the wireless service providers (WSPs) in comparison with the growing number of devices seeking access to the radio spectrum, the concept of short-term spectrum auctions with a reserve price was proposed in [10]. In addition, spectrum auctions were introduced as a possible regulatory measure for user admittance in a wireless network, as a means of overcoming the perceived scarcity of the radio spectrum. In such a situation, an auction process allows users who can afford to pay the most to have access to the radio spectrum [11, 12]. An auction process is important because the price paid for the spectrum has over the years been based on potential price rather than allowing competition to reflect the actual price for the radio spectrum. This has resulted in a growth in demand for the radio spectrum without a corresponding growth in revenue [11]. The auction-based process provides an economic vehicle for achieving opportunistic spectrum access in order to mitigate the scarcity and the decline in revenue from the use of the radio spectrum. However, the primary users of the radio spectrum are still not willing to share the radio spectrum based on the concept of DSA [13]. This is because of concerns about interference from secondary users. Therefore, in order to encourage the efficient use of the radio spectrum by the secondary access, the authors of this paper had previously proposed the use of the green payments (GP) as an incentive for efficient use of the radio spectrum in [14]. Furthermore, an auction based balancing on revenue and fairness was proposed in [15]. In addition to the GP, [14] also proposed an auction process with a Reserve Price (RP) while allocating the spectrum dynamically with the help of a database which provides information about unused spectrum so that such bands can be allocated to the secondary users. This use of a RP can help in increasing the revenue obtained from the use of the radio spectrum, especially when demand is low as shown in [14]. However, the use of a RP, as proposed in the model in [14] highlights a number of problems. These include having a number of transmit channels available based on the information provided by the database, but the channels are not put to use after the auction and the allocation process. This is usually due to the fact that some of the offered bids being below the RP. Lowering the RP might be an option. However, this might lower the all-important revenue obtained for the use of the radio spectrum, or increase the demand for the spectrum based on the economic principle of demand and supply [11]. Therefore, the key focus of this paper is to examine how the short-term spectrum auction with GP that was proposed in [14] can be redesigned to reduce the amount of energy consumed during the bidding process while maximizing the use of the radio spectrum. This paper also examines the satisfaction of the secondary users and that of the WSP based on the proposed model. This paper is unique because it examines both the economic and the performance characteristics of the wireless system during and after the auction process.

Yuefei et al. [16] proposed a 2 dimensional (2D) auction process for mobile wireless users to enable exclusive access to the channels by users who desire to do so. The paper showed that optimal social welfare could be achieved; however, the performance of the system was not examined. Sheng et al. [17] proposed an auction-based spectrum allocation scheme based on location privacy preserving dynamic spectrum auction that can enable dynamic spectrum auction without leaking the position information of the user. The proposed scheme was based on Privacy Preserving Bid Submission Protocol (PPBS) and Private Spectrum Distribution Protocol (PSD) to help deal with the problem of privacy that may occur during the auction process. A dynamic auction with auction period optimisation was considered in [18]. A double-auction process for spectrum allocation for both the buyer-side and seller-side that achieves truthfulness, individual rationality and budget-balance was proposed in [19]. However, the proposed auction model did not take into consideration the need to reduce the amount of energy consumed by wireless users participating in the auction process, especially when some of the users are losing out as a result of the auction process. The work in [20] examined the impact of cognitive radio on the primary users quality of services by offering a means to compensate the primary users. It utilized the mean and variance of the return on investment while proposing a distributed market architecture that allows for a control channel in order to communicate price information between the spectrum buyer and the seller.

The remainder of this paper is organized as follows. Section II provides the system description by explaining each component of the model, while section III shows the system model when all components are put together with a modeling scenario. Section IV presents and discusses the result and finally section V provides the conclusions.

## 2 System Description

This paper investigates the use of single and multiple bidding processes in an auction-based DSA network. This is done by using two groups of users: the Low Powered Users (LPU) and the High Powered Users (HPU). Furthermore, a Spectrum Broker (SB), whose responsibility is to carry out the auction process, and the database that provides information to the spectrum broker regarding the available channels are also considered. The relationship between these three elements is described using Fig. 1.

In addition, two types of channels are assumed; one of the channels is dedicated to the auction process and the other is used for transmission. The allocation of channels to the users is carried out by the SB based on the bids submitted by the users and channel availability. We assume accurate channel detection after channel allocation.

The auction process used is depicted in Fig. 2. The system consists of  $N$  number of users and  $N_{TC}$  transmission channels in each cell. During an auction period ( $t$ ),  $N_{USA}$  users indicates interest in seeking access by submitting a bid as illustrated using the clockwise movement of A to B in Fig. 2. During each bidding period, the number of available channels ( $N_{AC}$ ) is provided by the database. After the auction process,  $N_{WU}$  winners emerge. Based on the received requests and received Signal to Noise Ratio (SNR), the SB applies the GP to the received bid, thereby subsidising the power efficient users and making an inefficient user to pay a tax as proposed in [14]. After the application of the GP, the users' bid is checked against the RP as depicted by the clockwise movement B to C in Fig. 2. Then the highest set of bidders with bids above the RP are offered the available channels as depicted by the clockwise movement C to D. If more users than the number of

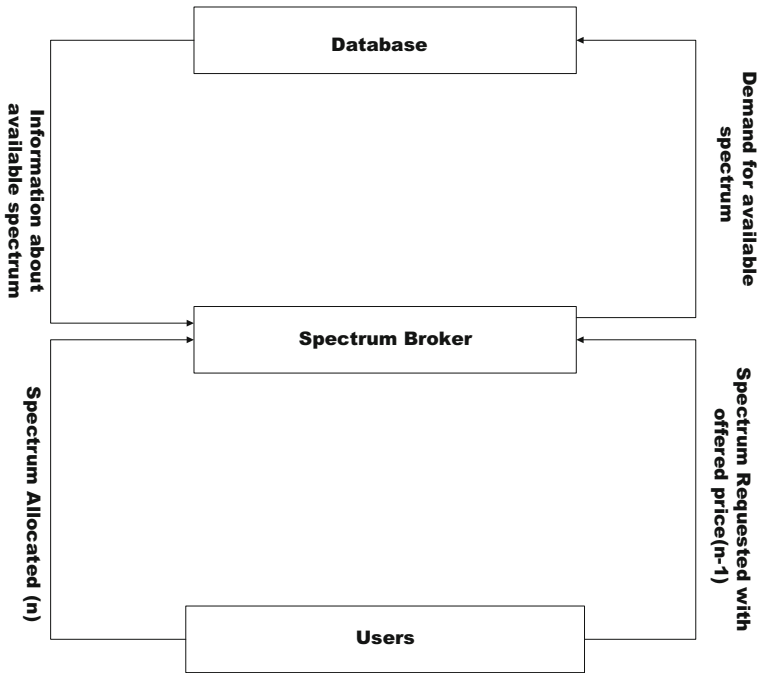


Fig. 1 The elements in the system

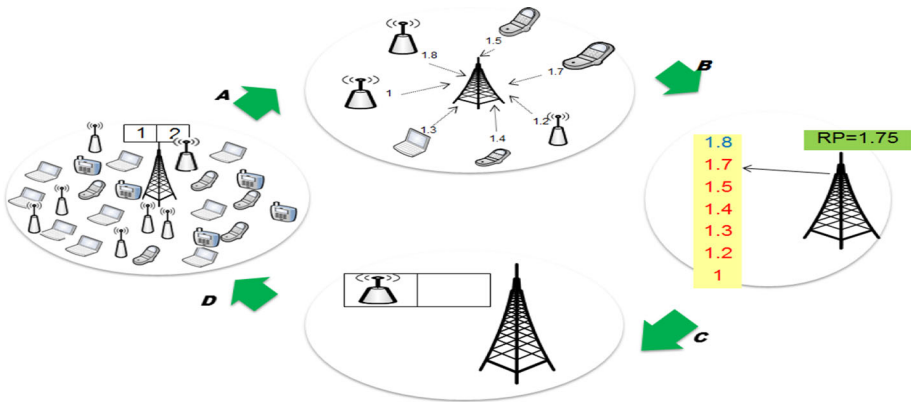


Fig. 2 General modelling scenario

available channels are offering a bid above the RP, then the number of users offered channel is equal to the number of available channels. After the auction process, the winning users are allocated a channel. However, not all winning users allocated a channel are able to transmit successfully because of inference or noise from adjacent users sharing same channel.  $N_{UT}$  represents the number of users who are able to transmit after the auction and the allocation process. However, a drawback of this approach is that two channels are available, but only one channel is allocated. This is because only one of the users is offering a price which is above the RP. Hence, this work aims at solving this

problem in order to maximize the use of the radio spectrum. This paper, therefore, proposes a model of multiple bidding process during a bidding period. A bidding period is a time window for users to submit their bids. This is further described later in this paper. Furthermore, in order to reduce the amount of energy wasted during the auction process, before submitting a bid, the user calculates the probability of being among the highest bidders based on the model described later in this paper. Based on this calculation, the user decides to either participate in the auction process or wait till the next bidding period.

The auction process being proposed is envisioned to be carried out based on an automated system in the user device with little human interaction. This should be interfaced with the user device such that the users can indicate their minimum price increment and maximum price for the auction.

## 2.1 The Users Bid

The users in this paper have a similar valuation for the radio spectrum, an assumption that is widely used in auction theory based models. The valuation of the users is drawn from a range of values represented as  $[V_{max}, V_{min}]$ . This is formulated based on the conventional settings in economics where users have a private valuation as done in [21]. Each user independently draws their bid with a probability density function as giving in Eq. (1).

$$f_V(v_i) = \frac{1}{V_{max}(i) - V_{min}(i)} \quad (1)$$

where  $V_{max}$  and  $V_{min}$  are the maximum and the minimum possible bid valuation of a user respectively in price units. The valuation ( $v_i$ ) depends on the user's budget per file. Hence, this valuation is always less than the maximum budget of the users. This work does not examine scenarios where the bidders run out of their budget; rather, the budget for all the users in the system is the same and specified in the parameters Table 1.

**Table 1** Parameters used

Parameters	Value
Interference threshold	-40 dBm
$[b_{min} \ b_{max}]$	[5 8]
$Thr_{max}$	4.5 bps/Hz
$\alpha$	0.65
$SNIR_{max}$	21 dB
$SNIR_{threshold}$	1.8 dB
$C_r$	0.5
Height of base station	15 m
Budget	100,000 Price Units
Transmit power [LPU, HPU]	[0.09, 1.09] W/bit
$[V_{min} \ V_{max}]$	[5 8]
Bid reduction	10%
Desired percentile	30
$\beta$	0.045
Number of cells ( $N_{oc}$ )	100
Processing time	0.25 ms

Each of the bidders generates a bid using the distribution shown in Eq. (2).  $b_i$  is the bid generated by user  $i$  and is always greater than zero ( $b_i > 0$ )

$$b_i(\text{Price Unit}) = \frac{(N_{\text{USA}} - N_{\text{AC}})V_i}{N_{\text{AC}}} + V_{\text{min}} \tag{2}$$

for  $i = 1, 2, \dots, N_{\text{USA}}$  and  $N_{\text{USA}} > N_{\text{AC}}$  where  $V_i$  is derived from Eq. (1),  $N_{\text{USA}}$  is the number of users seeking access to the radio spectrum during an auction period and  $N_{\text{AC}}$  is the number of available channels. Using Eq. (2), each of the users intending to transmit generates a bid within a given time window known as the bidding period  $t$ .  $N_{\text{USA}}$  is always greater than  $N_{\text{AC}}$  for an auction to take place. Equation (2) assumes that all the users have the knowledge of  $N_{\text{AC}}$  and  $N_{\text{USA}}$ , an assumption that is quite strong but reasonable.

In some parts of this paper, it is assumed that the users can estimate the value of RP based on information provided by the spectrum broker. Therefore, in such a scenario, it is assumed that the user generates a bid above the RP as shown in Eq. (3) for user  $i$ :

$$b_i = \frac{(N_{\text{USA}} - N_{\text{AC}})V_i}{N_{\text{AC}}} + r_{\text{EST}}, \quad \text{for } i = 1, 2, 3, \dots, N_{\text{USA}} \tag{3}$$

where  $r_{\text{EST}}$  is the estimated value of the RP.

### 2.2 The Reserve Price (RP)

The RP is the minimum price to be paid by any user intending to transmit before the spectrum is allocated to such a user. The RP is introduced because the demand for the radio spectrum is both time and space dependent. When the demand is low, the RP helps to retain the minimum selling price of the WSP as shown in [14]. In this paper, the RP is formulated by taking into account the current traffic load in the system, the frequency band in use, the total number of channels in the system and the number of channels in use as shown in Eq. (4):

$$RP(\text{Price Unit}) = C_f N_{\text{TC}} C_r \tag{4}$$

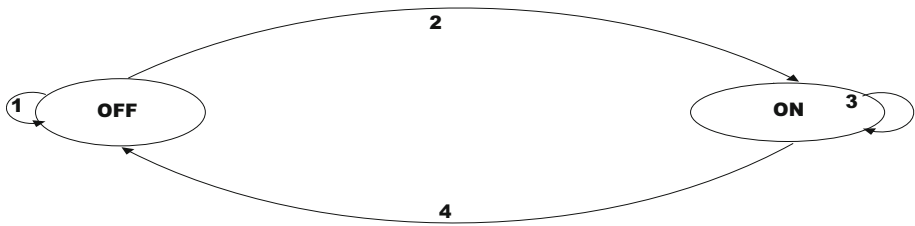
where  $C_r$  is a constant in price unit which is used to specify the value of a spectrum band in use.  $C_r$  is determined from the common knowledge regarding the price of the radio spectrum and it is specified in Table 1. Users also believe that the bigger the size of the network, the better the quality of service offered; hence, the total number of channels in the system is also taken into consideration when calculating the RP. The congestion factor ( $C_f$ ) is introduced because of the laws of demand and supply as explained in [22]. The congestion factor ( $C_f$ ) is the number of requesting users per channel during an auction period as shown below in Eq. (5):

$$C_f = \frac{N_{\text{USA}}}{N_{\text{AC}}} \tag{5}$$

$C_f$  is assumed to be the same for all the users who want to transmit within the same bidding period ( $t$ ).

### 2.3 The Energy Model

The energy model is represented using a 2 state Markov chain as shown in Fig. 3:



**Fig. 3** Energy and system model as a two-state Markov chain

1. A user who has file(s) to send moves into the OFF state and continues to be in this state until such user is among the winning bidders.
2. A user, who is among the winning bidders, moves from the OFF state to the ON state.
3. The user remains in the ON state until after transmitting (if transmission is successful) or until the user receives a failed signal either due to low offered bid compared to the RP or due to a poor quality channel.
4. After transmitting, the user moves back to the OFF state before shutting down if no file is to be sent again. However, if the user has another file to send, the user remains and attempts again in the off state.

In the ON state, it is assumed that a transmission is successful provided the Signal to Noise plus Interference Ratio (SNIR), and the RP are above the set thresholds, after which the user moves back to the OFF state. If the user moves from the ON state to the OFF state, and the thresholds are not met, then the energy consumed in processing the request of the user during the state transition is considered as energy wasted. A processing time, which is the time taken to process the received bid, is also assumed. All users that move from the ON state to the OFF state have the same processing time.

## 2.4 The Green Payment (GP)

The green payment was earlier formulated in our work in [23]. The GP is either in the form of a tax or a subsidy, and the main aim of introducing this is to allow the bids of the LPU to be subsidised, to allow the bids of the HPU to be taxed and to increase the probability of LPU winning the bid. This is because the assumed users in this work are opportunistic spectrum users (secondary), who need to reduce their interference to others, especially the primary users. This scheme should provide more confidence to the primary users and provide them more confidence in sharing the network with secondary user. This is because if low powered users are admitted into the system as secondary users, then, more often than not, the interference to primary users should not be too significant. The scheme allows the granting of access to the radio spectrum to the HPU only when the bid of the HPU after the tax is above the RP and above the bids of the LPU with the subsidy. This is allowed because sometimes due to the value of the money involved, low demand for the use of the spectrum or the importance of the application seeking the use, the HPU should be allowed to transmit after paying the price for using such transmit power. An equation for the GP derived from the inverse of the Truncated Shannon Bound (TSB) is obtained to either tax or subsidise the users [24]. The TSB represents the transmission rates that can be achieved in practice given an adaptive modulation scheme in a real world scenario depending on the SNIR of the user. The throughput (Thr) of the system is giving as:

$$Thr = \begin{cases} 0 & SNIR < SNIR_{\text{threshold}} \\ \alpha \cdot S(SNIR) & SNIR_{\text{threshold}} < SNIR < SNIR_{\text{max}} \\ Thr_{\text{max}} & SNIR > SNIR_{\text{max}} \end{cases} \tag{6}$$

$$S(SNIR) = \log_2(1 + SNIR)$$

where  $S(SNIR)$  is the Shannon bound and  $\alpha$  is the rate reduction factor as defined in the parameters Table,  $Thr_{\text{max}}$  is the maximum throughput for the codeset and  $Thr$  is the throughput of the system.  $Thr_{\text{max}}$  and  $SNIR_{\text{threshold}}$  are specified in the parameters table. The  $SNIR_{\text{threshold}}$  is the minimum threshold that allows the detection of the information at the receiver and  $SNIR_{\text{max}}$  is the maximum SNIR beyond which there is no change in throughput. TSB is used to derive the GP equation. The reason for using this equation is due to the fact that the transmission rate is an important parameter. The transmission rate is dependent on the SNIR which is dependent on transmit power and interference from other users sharing same channel. The derived GP equation is shown in Eq. (7) as formulated from the TSB:

$$GP(\text{Price Unit}) = \begin{cases} 2^{1+\beta\theta} - 1, & \text{If subsidy} \\ 2^{1+\beta\theta} + 1, & \text{If tax} \end{cases} \tag{7}$$

where  $\beta$  is the GP factor as derived later. The value of  $\beta$  is chosen in such a way that the GP does not introduce too much tax/subsidy into the system that could lead to delay or reduction in the system throughput, hence the reason for it being called the green payment factor.  $\theta$  is the absolute value of the linear difference between the SNR value of a user  $i$  ( $\psi_i$ ) and the value of the SNR of a set threshold ( $\psi_j$ )

$$\theta_i = |\psi_i - \psi_j| \quad \text{for } i = 1, 2, 3, \dots, N_{\text{USA}} \tag{8}$$

The set threshold ( $\psi_j$ ) is derived by first arranging the received SNR of the  $N_{\text{USA}}$  users who are seeking access to the radio spectrum at time  $t$  in an ascending order

$$\Psi_t = [\psi_1, \psi_2, \psi_3, \dots, \psi_{N_{\text{USA}}}] \tag{9}$$

Then to determine which of the SNR at time  $t$  is the set threshold, Eq. (10) is used, where  $P_c$  is the desired percentile.

$$|j| = \lfloor \frac{P_c N_{\text{USA}}}{100} \rfloor \tag{10}$$

Equation (10) gives an absolute value known as the percentile rank. This shows that whatever the value of  $j$ , the  $j$ th SNR in Eq. (9) is the set threshold ( $\psi_j$ ). For example, if  $j$  is 2, then the second SNR ( $\psi_2$ ) in Eq. (9) is the set threshold; therefore  $\psi_j = \psi_2$  with this example. The set threshold is not the same as or related to the SNIR threshold ( $SNIR_{\text{threshold}}$ ) in the TSB equation. The value of the percentile rank used has an effect on the total revenue needed to subsidise the bids of the users as seen in Eq. (10). As  $P_c$  increases the percentile ranks also increase.

### 2.5 The Congestion Charge

The HPU users are further charged a congestion charge which increases as the traffic load ( $L$ ) increases, as shown in Eq. (11), with the aim of regulating the congestion in the system.



The only scenario considered in this work allows the final bid value of any user to remain positive after the deduction of green tax and the congestion charge is only examined in this work.

$$b_i^{final} = \begin{cases} b_i + GP_i > 0 & \text{For LPU} \\ b_i - LGP_i > 0 & \text{For HPU} \end{cases} \quad (11)$$

For  $i = 1, 2, 3, \dots, N_{USA}$  where  $b_i^{final}$  is the final value after the deduction or addition of the congestion and green tax of the bid in price unit.  $L$  is the traffic load in Erlang and  $GP_i$  and the green payment for user  $i$ .

## 2.6 Probability of a Being Among the Highest $N_{AC}$ Bidders ( $P_{rN_{AC}}$ )

$P_{rN_{AC}}$  is introduced in this paper to prevent users who have a low probability of winning from attempting to transmit. The probability is dependent on the bid submitted by a user and the number of available channels in the system as shown in Eq. (12).

$$P_{rN_{AC}}(i) = \left( \frac{b_i - b_r}{V_{max} - b_r} \right)^{N_{USA} - N_{AC}}, \quad N_{USA} > N_{AC} \quad (12)$$

where  $b_r$  can be the value of the RP if known to the user, otherwise, it is the minimum possible bid by user  $i$  based on the user's budget.  $V_{max}$  is the maximum possible valuation for a user. The probability is calculated for all the users intending to transmit during any bidding period. If the value of the probability is greater than or equal to a set probability threshold  $P_{rThreshold}$  ( $P_{rN_{AC}}(i) \geq P_{rThreshold}$ ) the user is allowed to attempt; otherwise, the user stays out of the process.

## 2.7 The Auction Model

The auction model adopted in this work is a simultaneous first price sealed bid auction with a reserve price. In this type of auction, no bidder knows the bid of any other user in the system. The simultaneous process is adopted in order not to introduce significant additional delay into the system. To examine if keeping the knowledge of the RP private by the auctioneer is beneficial to either the auctioneer or the users, we examine two types of models: An auction model with public knowledge of RP, and an auction model without the public knowledge of the RP (having public knowledge of RP means that all the users are regularly updated with the current value of the RP after each auction process in order to guide any user bidding in the next auction round).

## 2.8 Energy Consumed Based on the Auction Model with and Without the Public Knowledge of the RP

A probability scheme can be used to analyse the energy consumed by the system when the RP is fed back to the users. Let  $P_r$  represents the probability of winning,  $1 - P_r$  represents the probability of not winning the bid and  $E_i$  represents the amount of energy consumed by user  $i$  in putting in a bid (The probability that is calculated here is not the same as  $P_{rN_{AC}}$ ). It is also assumed here that the user wins once the offered bid is above the RP and the trials come to an end (this assumption is reasonable because no user can transmit if the offered bid is below the RP). The total energy consumption with the probability is as shown in Eq. (13).

$$E_T = \sum_{i=1}^{N_{USA}} (1 - P_r)^{n-1} P_r(i) E_i \tag{13}$$

In the Eq. (13), the first part of the equation is 1 represents the situation if all the users win the auction process at the first attempt. The total energy consumed if the users do not win on the first attempt is the cumulative sum of all failures before success, assuming independent arrivals. As an example, if it takes 3 attempts before success, in the 4th attempt for all the users transmitting in the system, the cumulative energy consumed for  $P_r = 0.5$  and the total energy consumed by user  $i$  is represented as  $E_i$  for all trials up until the first success is as shown in Eq. (14).

$$E_T = \sum_{i=1}^{N_{UT}} \left( (1 - P_r)(i)^{1-1} P_r(i) + (1 - P_r)(i)^{2-1} P_r(i) + (1 - P_r)(i)^{3-1} P_r(i) + (1 - P_r)(i)^{4-1} P_r(i) \right) E_i \tag{14}$$

Substituting for  $P_r$  into Eq. (14) gives:

$$E_T = \sum_{i=1}^{N_{UT}} (0.5(i) + 0.25(i) + 0.125(i) + 0.0625(i)) E_i \tag{15}$$

From the Eq. (15) above, it can be seen that the energy consumed increases as the number of attempts before success. Hence, if a user does not attempt to participate in the auction process except when the user is sure that the bid offered is above the RP, then less energy is consumed. This shows theoretically that the public knowledge of the RP helps in reducing the amount of energy wasted as a result of the auction process and the RP. This is further examined in the modelling section. A user cannot be sure of offering a price above the RP if the RP is not a known to the user; hence the reason we examined an auction model with the public knowledge of the RP.

### 2.9 The Auctioneers Revenue

The revenue of the auctioneer does not include the tax because the tax paid by HPU is assumed to be used in subsidising the LPU. Hence, we assume that the tax paid goes into a separate account, which is where the subsidies are taken out from.

$$R_c(\text{Price Unit}) = \begin{cases} b_i - tax & \text{For HPU} \\ b_i + subsidy & \text{For LPU} \end{cases} \tag{16}$$

### 2.10 The Transmission Process

Two stages are assumed in this paper before a successful transmission as explained below:

*STAGE 1* A user places a bid in the OFF mode: Provided the user’s  $P_{rNAC}$  is above the set threshold, the user moves to the second stage ( $P_{rNAC}(i) > P_{r\text{threshold}}$ ).

*STAGE 2* The energy mode of the user is changed to the transmitting mode (ON) and the user is successful in the second stage if the offered bid is above the RP and the SNIR is above the SNIR threshold.

Comparing the bids of the two user groups, the following are the possible outcomes (PO) after the tax or subsidy is applied to the offered bids.

*PO 1: The final bid of the HPU is greater than that of the LPU; that is  $b_{\text{HPU}}^{\text{final}} > b_{\text{LPU}}^{\text{final}}$ . If this occurs then the subsidy has not had a significant impact or changed the winning bidder from a HPU to a LPU but has lowered the income of the auctioneer since  $b_{\text{HPU}}$  has been taxed.*

*PO 2: The final bid of the LPU is greater than that of the high-powered user: that is  $b_{\text{LPU}}^{\text{final}} > b_{\text{HPU}}^{\text{final}}$ . This means that the subsidy might have made a difference to the winning bid; if initially  $b_{\text{LPU}} > b_{\text{HPU}}$  before the green payment is applied. This has helped in increasing the revenue of the auctioneer.*

Under no circumstance is any user allowed to transmit if the final bid (after the tax or subsidy) is below the RP leading to possible outcomes 3 and 4.

*PO 3: The initial bid and the final bid of the HPU is greater than the reserve price ( $r$ ).  $b_i > r_i$ . Such users is allowed to transmit provided  $b_i$  is among the highest  $N_{\text{AC}}$  bids.*

*PO 4: The initial bid of the LPU is below the reserve price but after the subsidy the final bid is greater than the reserve price; that is  $b_i^{\text{final}} > r > b_i$ . In this case, the subsidy ensures the bid of user  $i$  is above the RP.*

## 2.11 The Utility Function

The utility function plays an important role in determining the achievable performance of a system. In wireless networks, the utility of the social welfare ( $U_{\text{SW}}$ ) is considered as more important than individual utility since the network is a shared resource. However, the utility of the social welfare is a function or the aggregate of the individual utility function of the group. According to [25], it is defined as:

$$U_{\text{SW}} = f(U_1, U_2, \dots, U_n) \quad (17)$$

where  $U_i$  is the utility function of user  $i$  where  $i = 1, 2, \dots, n$ .

Individual wireless users are known to act selfishly with the aim of maximising their utility. Defining a relative utility function can be complicated, but this paper uses the design objective of the system to determine the desired overall utility. Generally, when different performance metrics are used to determine the utility of a system, the weighted power is used to show the importance of each of the individual metrics that forms the overall utility function depending on the objective of the system. However, in the latter part of this paper, all the metrics are assigned the same weighted power. This is because a generic form of a utility function that can be modified based on the application of the system in the future is proposed. In this paper, the performance of the system, such as delay, data rate or probability of blocking due to price is used to determine the user's utility function.

*Data Rate* According to the TSB defined earlier, the data rate is dependent on the SNIR of the user and therefore, the data rate of the user is also a measure of the user's utility. The higher the data rate of a user, the higher the satisfaction of such user.

$$U_D = 2^{\frac{D_i}{D_{\text{max}}}} - 1$$

$$\text{For } i = 1, 2, 3, \dots, N \text{ and } D_i < D_{\text{max}} \quad (18)$$

$$0 \leq U_D \leq 1$$

$D_i$  is the data rate of user  $i$  and  $D_{\text{max}}$  is the maximum achievable data rate of the system.

*Delay ( $\Delta$ )* All wireless applications have a maximum delay that can be tolerated for the system to achieve the desired operation. Beyond such a threshold, the system is not feasible.

$$U_{\Delta} = \begin{cases} 2^{1-\frac{\Delta_i}{\Delta_{max}}} - 1 & \text{for } \Delta_i < \Delta_{max} \\ 0 & \text{for } \Delta_i > \Delta_{max} \end{cases} \tag{19}$$

$$0 \leq U_{\Delta} \leq 1 \quad \text{and} \quad \Delta_i < \Delta_{max}$$

where  $\Delta_i$  is the delay experienced by user  $i$  and  $\Delta_{max}$  is the maximum possible delay of the system to make the system feasible.

*Blocking Due to Price* A user whose bid is below the RP has a utility as shown in Eq. (20). Where  $B_p$  is the number of times a user is blocked due to price.

$$U_{B_p} = 2^{(1-B_p)} - 1; \quad (0 \leq U_{B_p} \leq 1) \tag{20}$$

*General Utility Before the Admission Process ( $U_G$ )* The general performance is analysed so as to provide a fair and balanced combination of the different individual performance metrics. We multiply the individual utility function together because if one of the components which form the utility function in this paper has a value of zero, the file transmitted by the user is not successful transmitted Hence, the reason for the multiplication. The general utility is defined as:

$$U_G = U_{\Delta} U_D U_{B_p} \quad (0 \leq U_G \leq 1) \tag{21}$$

From this, if any of  $U_{\Delta}$ ,  $U_D$ ,  $U_{B_p}$  is zero, then the overall general utility of the user is zero or the system is not feasible. The above utility function in Eq. (21) also shows that no user can be admitted into the system if the delay experienced cannot be tolerated by the application in demand, the data rate required by the application cannot be provided or the offered bid price is below the RP.

After analysing the general utility function, the utility in terms of price is also examined. The price utility function is separated because using an auction process, it works differently from other utility functions when combined as seen later.

*General Utility with Pricing ( $U_p$ )* Generally, the utility of a user decreases as the value of the offered bid increases. This is because users prefer to win the bid with a lower price in order to maximise their utility.

$$U_p = 2^{1-\frac{b_i}{b_{max}}} - 1 \quad (0 \leq U_p \leq 1) \tag{22}$$

where  $b_{max}$  is the maximum bid. No bidder can bid above the maximum bid value; therefore, a user bidding the maximum is deemed to have a utility of zero and  $b_i$  is the bid submitted by user  $i$  who is a winning bidder. Any bidder who is not among the winning bidders has a utility value of 1 in terms of price. It can be seen from the above Eq. (22) that when using an auction process the lower the value of utility in terms of price, the better it is for the user. This is because the price paid is always defined as a cost (i.e. negative) and users do not usually want to incur a high cost. However, in order to include the price function into the overall utility function, the equation is modified as shown later. Therefore, the overall utility of individual users is:

$$U = U_G - U_p \tag{23}$$

This shows that the winning users paying the least have the highest utility, provided the general utility of all users are the same. It is also worth pointing out that since an auction process is used, this allows the highest bidder to gain access to the spectrum. However, the price function considered in Eq. (23) examines the utility in terms of the price paid by the winning bidders because the users want to win with the least possible amount. Therefore, the user paying the least among the winning user has the best utility value.

The utility functions defined until this point represents the utility functions for the individual user. It is assumed that since the spectrum is a shared resource, the utility function of the social welfare is more important than that of the individual. However, as shown later in Eq. (24), the utility of the social welfare is a function of the individual utility; hence, the utility of the social welfare is defined.

### 2.11.1 Utility of Social Welfare

There are different objectives that can be met in the design of an auction such as efficiency. An efficient auction should maximise the social welfare. The utility of the social welfare is defined as the average of the total utility of all the users in the system. The individual utility contributes to the utility of the social welfare and, therefore, there must be some level of satisfaction from the individual user that maximises the utility of the social welfare.

$$U_{sw} = \frac{\sum_{i=1}^N U_i}{N} \quad (24)$$

To maximise the utility of the social welfare, there must be an optimal individual utility.

*Admission Process Utility ( $U_a$ )* This part shows how the price paid by each user affects the admission process. An expression for the utility of each of the users in terms of the GP is firstly defined, and then combined with the price utility to determine the utility in terms of the admission process.

*Utility of the GP ( $U_{GP}$ )* The GP is divided in two, the tax and the subsidy. If a user is taxed, the utility function is defined as:

$$U_{GP} = - \left( 2^{\frac{R_i^t}{R_{max}^t}} - 1 \right), \quad i = 1, 2, 3, \dots, N_{USA} \quad (25)$$

where  $R_i^t$  is the green tax paid by the user,  $R_{max}^t$  is the maximum tax paid by any of the users and  $N_{USA}$  is the total number of bidder paying a tax among the  $N_{USA}$  bidders who are attempting to gain access to the channel. From Eq. (25), it can be seen that a user paying a tax has a negative utility. The utility for a user receiving a subsidy is:

$$U_R = \left( 2^{\left( \frac{R_i^s}{R_{max}^s} \right)} - 1 \right), \quad i = 1, 2, 3, \dots, N_{USA} \quad (26)$$

where  $R_i^s$  is the subsidy paid by user  $i$ ,  $R_{max}^s$  is the maximum subsidy paid by any of the users and  $N_{USA}$  is the total number of bidder receiving a form of subsidy among the  $N_{USA}$  bidders who are attempting to gain access to the channel. Combining the utility for the price and the GP:

$$U_a = \frac{(1 - U_P) + U_{GP}}{2} \tag{27}$$

If the  $U_a$  values of all the bidders who wants to gain access to the channel at bidding round  $t$  is represented by set  $N_{USA}$  as giving in Eqs. (28) and (29) in descending order then a maximum of  $N_{AC}$  users who are having the highest  $U_a$  utility value is admitted into the system provided their bid is above the RP and can be represented by  $N_{N_{AC}}$  where  $N_{K_c} \subset N_{N_{USA}}$

$$N_{N_{USA}}(t) = [U_a^1 U_a^2 U_a^3 \dots U_a^{N_{AC}} U_a^{N_{AC}+1} U_a^{N_{AC}+2} \dots U_a^{N_{USA}}] \tag{28}$$

$$N_{K_c}(t) = [U_a^1 U_a^2 U_a^3 \dots U_a^{N_{AC}}] \tag{29}$$

From the above equations, the following can be concluded:

- A user, who is paying a low tax and has a high offered bid, has a high utility value and therefore, might be admitted into the system to transmit.
- A user, who is receiving a high value of subsidy and whose initial bid is high, has a very high probability of being admitted into the system.

### 2.11.2 Utility of the Wireless Service Provider

The utility of the WSP is determined by the number of users admitted simultaneously into the system, since spectrum re-use in adjacent cells is assumed. The utility of the WSP is given as:

$$U_{wsp}(t) = 2 \frac{\sum_{i=1}^{N_{oc}} N_{UT}(t)}{\sum_{i=1}^{N_{oc}} N_{TC}(t)} - 1 \tag{30}$$

where  $N_{oc}$  is the total number of cells in the system.  $\sum_{i=1}^{N_{oc}} N_{UT}(t)$  gives the cumulative sum of all channels in use in all the cells up to time period  $t$  and  $\sum_{i=1}^{N_{oc}} N_{TC}(t)$  gives the cumulative sum of all available channels in all the cells up to time period  $t$ .

## 3 System Model Description

One spectrum broker,  $N$  users and  $N_{TC}$  transmission channels in each cell are modeled in an infrastructure based uplink scenario, where each user is transmitting at a fixed power (high or low) level depending on the group the user belongs to. This work is based on the hexagonal cell structure with a fixed frequency reuse factor as specified in the parameters table. The flow chart from the user point of view is shown in Fig. 4a while Fig. 4b shows the flow chart based on the WSP's point of view.

The channel assignment scheme is based on the least interfered channel. A Poisson distribution process with arrival rate ( $\lambda$ ) and inter arrival rate described by an exponential distribution is assumed. Each user who wants to transmit at each auction period submits a uniform sealed bid  $b_i (i = 1, 2, 3, \dots, N_{USA})$  to the spectrum broker depending on the user's budget, based on Eqs. (2) or (3) as explained earlier. A bidder who loses a bid in a bidding round during time  $t$  increases the bid in the next bidding period ( $t + 1$ ) as shown in the modelling flow chart in Fig. 5. This process is repeated until a steady state is reached. The path loss is based on the WINNER II B2 model. A bid is submitted by a user after the

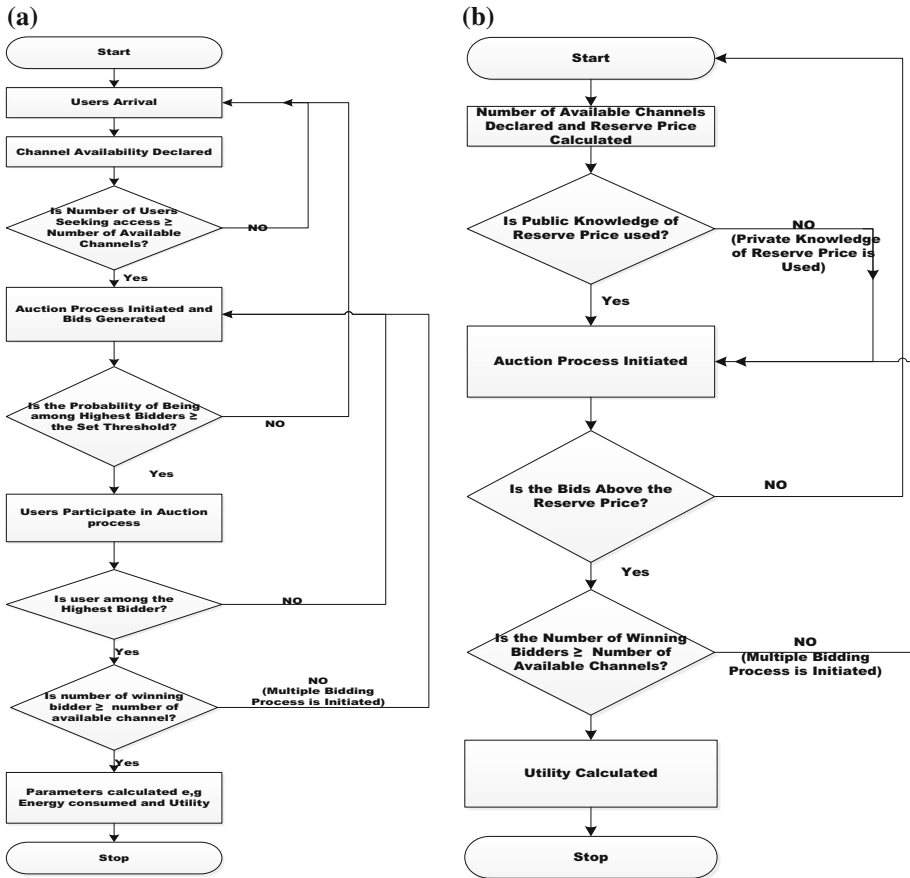
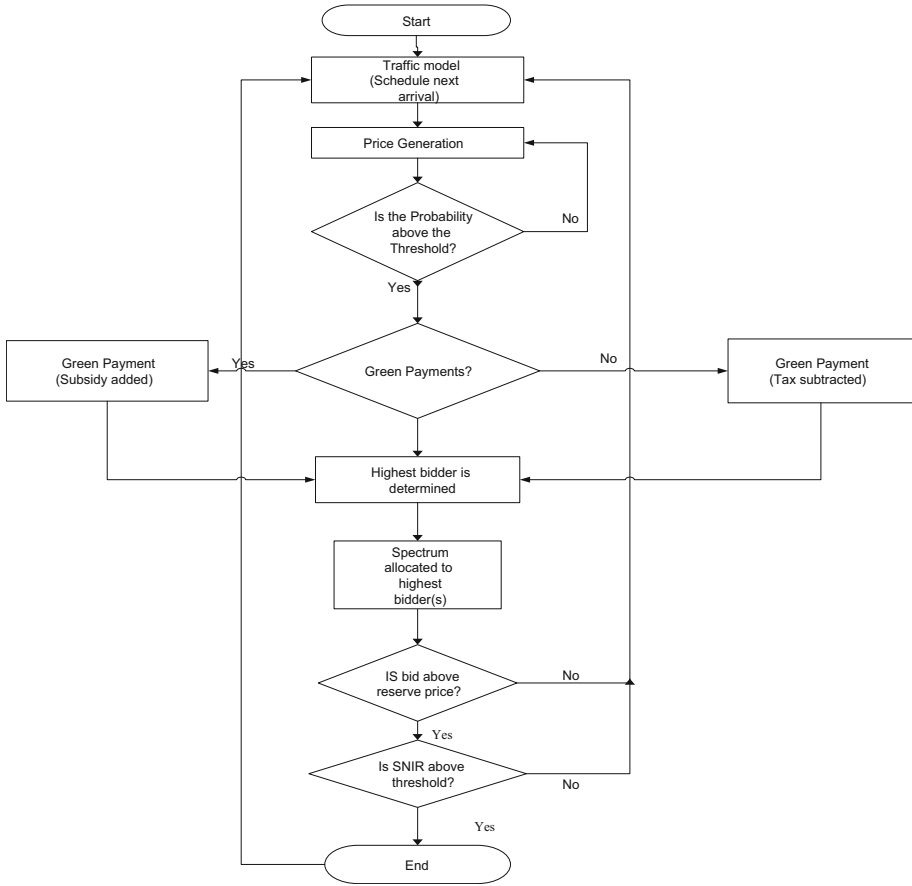


Fig. 4 a Flow chart from users point of view. b Flow chart from the WSP’s point of view

calculation of the probability of being among the highest  $N_{AC}$  bidder ( $P_{N_{AC}}$ ) as specified. If the probability calculated by a user is above the set threshold ( $P_{rN_{AC}}(i) \geq P_r(T_{threshold})$ ), then, the GP and congestion charge is applied as explained earlier. A user is assumed to have transmitted successfully, provided the SNIR and final bid price is above the SNIR threshold and the RP. After the auction process, the number of winning bidders that emerges is represented as  $N_{WU}$  and  $N_{UT}$  is the number of successful bidders.

This paper considers two different bidding scenarios: the Single Bidding Process (SBP) and the Multiple Bidding Process (MBP). These two bidding processes are illustrated using Fig. 6a, b. The SBP involves the users bidding in a single round before the transmission period. The implementation concept of the SBP can be described as shown in the example in Fig. 6a. In this example, it is assumed that 5 users are arriving out of the  $N$  possible users during each of the bidding periods. The users are represented as  $N_i^a$ , where the subscript  $i$  represents the user number and superscript  $a$  represent the packet number. Hence,  $N_1^2$  means that user number one is about to send packet number two. Three channels are assumed to be available in each bidding period (in our modeling process, the number of available channel varies). The RP is assumed and the spectrum broker is not aware of this value. The transmission period is represented as  $T$ . In the first bidding period, users 1–5

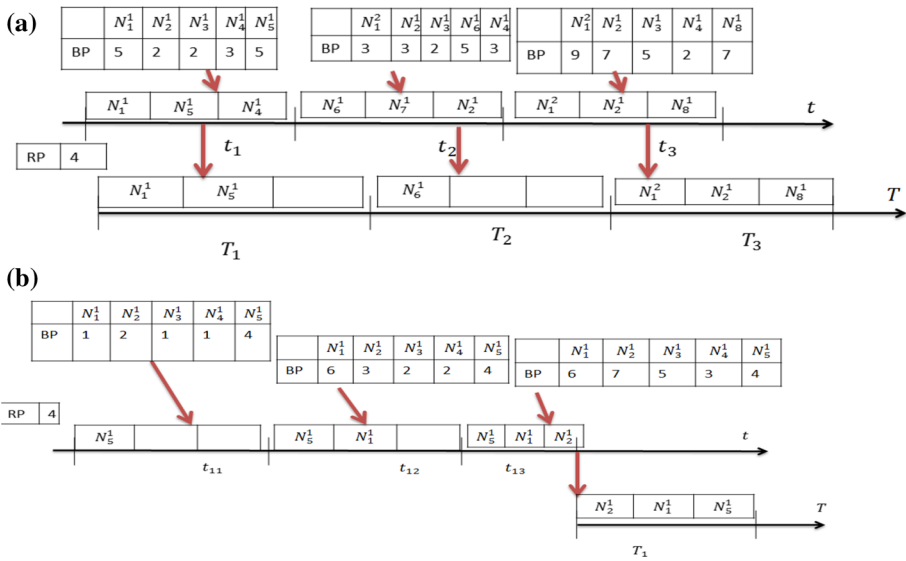


**Fig. 5** System flow chart

arrive into the system, each with a bidding value as indicated in the figure. Since 3 channels are available, the auctioneer picks the 3 users with the highest bidding values ( $N_1$ ,  $N_5$  and  $N_4$ ). From the example, it can be seen that there are times that channels are available but not used. To reduce the number of times when channels are available but not put to use, the concepts of MBP is examined. The MBP is introduced because despite having more than 3 users offering a bid above the RP during period  $t_3$  in the previous example with SBP, only 3 users were picked as winners and those 3 are allocated the transmitting channels during transmission period  $T_3$ .

The concept of MBP allows for a losing bidder to attempt again in the same bidding period as shown in Fig. 6b. Hence, it is assume that an instant feedback exists between the service provider and users. In this case, we assume that the auctioneer is aware of the RP because there is no way of carrying out a MBP without the auctioneer having such information. The auctioneer only sends bidders whose bids are above the RP to the WSP for the allocation of transmission channels. Here, the system is assumed to have a buffer to queue the winning bidders as they come into the system. The actual number of bidding rounds carried out is dependent on the number of winners emerging in each round and the number of channels available. The auction round only stops when  $N_{AC} = N_{WU}$ . Therefore,





**Fig. 6** a Single bidding process. b Multiple bidding periods

the GP can help solve the problem of channel quality by penalising the high powered users who are causing the interference to other users in the system. As an illustration for MBP, assuming  $x = 4$  and  $N_{USA} = 8$ . If in the first bidding round only 2 bidders out of the 8 bidders have probabilities above the set threshold, the two users are admitted and queued in the buffer while the remaining 6 bidders increase their bids accordingly.

### 4 Results and Discussion

This section shows and explains the results obtained from the simulation using 19 cells with cell radius of 2 km. We assume 200 users in each cell (100 HPU and 100 LPU), file size of 2 Gbits and a cell reuse factor of 3. Firstly,  $P_{rN_{AC}}$  is varied to determine the effect of the probability on the number of users admitted into the system. Based on this, the value for the threshold that helps in reducing the energy consumed by the system without reducing the other system performance metric significantly is determined. The remaining parameters used are given in the Table 1.

Figure 7 shows the probability of blocking due to price per files generated when the threshold ( $P_r(T_{threshold})$ ) of  $P_{rN_{AC}}$  is varied from 0 to 1. In general, for all the 3 scenarios examined, the number of users that are blocked due to price is reducing as the threshold is increasing. This is because when the threshold is 0 all the users entering the system are attempting, but a significant number of them are blocked because the offered bid price is below the RP. However, as the threshold is introduced and as it is increasing, some of the users whose probability of being among the highest  $N_{AC}$  bidders are no longer attempting, hence, the number of users that are blocked due to price is reducing with an increase in the threshold. When the threshold is set to a value of 1, all the users attempting are offering a bid value that is above the RP, resulting in none of the users being blocked. The MBP with Knowledge of RP (KRP) performs best because on the average, the users are placing only a

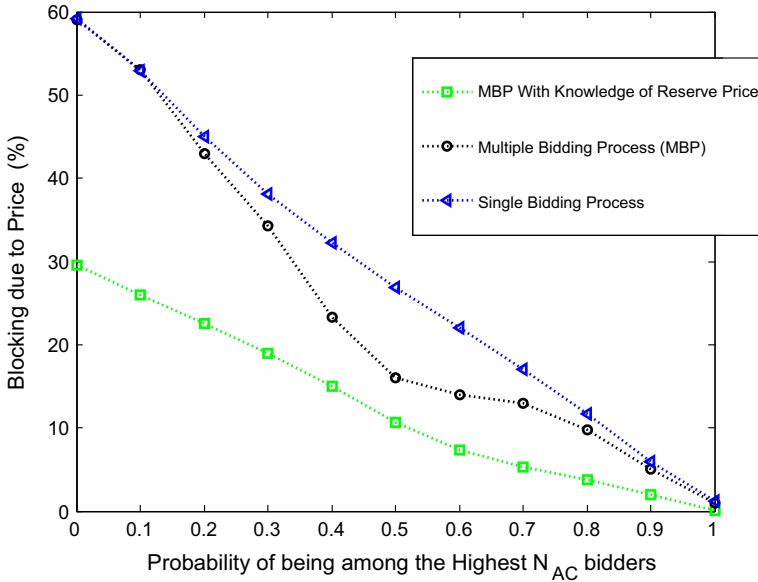
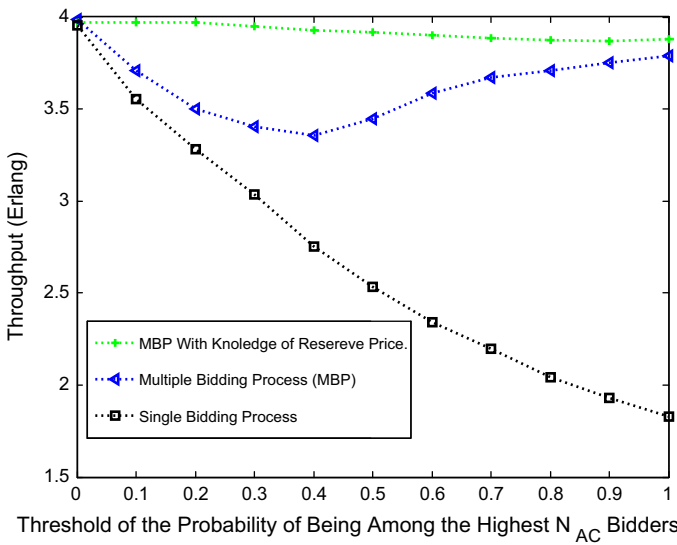


Fig. 7 Effects blocking due to price per file generated for SBP, MBP and MBP with KRP

bid that is above the RP. The MBP process without the KRP has no significant effect on the number of users who are blocked due to price when compared to the single bidding process at low values of the threshold. This is because the result examined here deals with only the number of users whose bid are above the RP, and it does not matter if the MBP or SBP is used, as long as the users are bidding below the RP. When the value of the set threshold is high, most of the users attempting are having a value that is below the set threshold, hence only a few users are attempting and fewer users are getting blocked due to price.

Figure 7 does not show if setting the value too high affects the performance of the system especially in terms of throughput. This is because the system might be operating below its capacity if the threshold is set too high.

Figure 8 shows the throughput against the threshold of  $P_{rN_{AC}}$ . A user is assumed to get through provided the user succeeds in having a probability value above the set threshold in both stages and the SNIR is above the SNIR threshold. It can be seen that with the SBP, the throughput reduces drastically with the increase in the threshold because as the threshold increases the number of admitted users reduces. This is due to fewer users having a probability above the threshold. Therefore, more channels are available than the number of admitted users after an auction round, leaving some of the transmit channels idle. With the MBP, the throughput reduces initially because at lower threshold values, some of the users actually get through stage 1 but fail at stage 2 due to the users offering a bid price below the RP. As the probability increases, more users are only attempting when their offered bid price is above the RP. With the KRP a user that gets through in stage one is more likely to have an offered bid that is above the RP especially as the threshold is increasing. However, the throughput reduces slightly as the threshold increases because only users offering bids close to  $V_{max}$  can move from stage one to stage two. As the threshold increases and the traffic loads are relatively constant, the system is sometimes loaded below its capacity.



**Fig. 8** System performance in terms of throughput for SBP, MBP and MBP with KRP

Hence, the reason for having a lower throughput than the maximum throughput that the system can support.

So far, the threshold helps in reducing the number of users that are not able to transmit after winning the auction process. Hence, it is important to determine the appropriate value for  $P_{r_{threshold}}$ . This value should take into account all the factors such as delay, throughput energy consumed and blocking probability in such a way that none of these performance metrics is badly affected. To determine the appropriate value for the probability threshold ( $P_{r_{threshold}}$ ) to be used in the future analysis, the deviation of the performance metrics (delay, energy consumed, throughput and probability of blocking) as shown in Eq. (31) is normalised and it is called the Normalised Difference (ND), where  $M_{V_i}$  is the value obtained at point  $i$  for the metric under consideration  $M_{V_{max}}$  and  $M_{V_{min}}$  are the maximum and minimum values respectively for the performance metric under consideration respectively.

$$ND = \frac{M_{V_{max}} - M_{V_i}}{M_{V_{max}} - M_{V_{min}}} \tag{31}$$

To balance all the performance parameters without trading off one of the other performance parameters too much, the midpoint between the crossover point is chosen. From Fig. 9, this is 0.55, and this is used as the probability threshold.

Figure 10 shows throughput against the traffic load. The MBP without the KRP performs slightly worse as expected. This is because with the MBP with the RP,  $N_{UT}$  is equal or almost equal to  $N_{AC}$ . This is unlike the SBP or the MBP without the KRP where the throughput of the system is significantly less than the traffic load because  $N_{UT}$  is not always or almost equal to  $N_{AC}$ . This shows that the MBP can only provide a better performance if the users have the KRP or, better still, if the users are aware of the GP if paying a tax.

However, the average delay per file sent as seen in Fig. 11 performs better with when using the probability to determine the admission process compared to the scheme with the GP alone. The delay with known KRP performs best of the three scenarios. The delay is

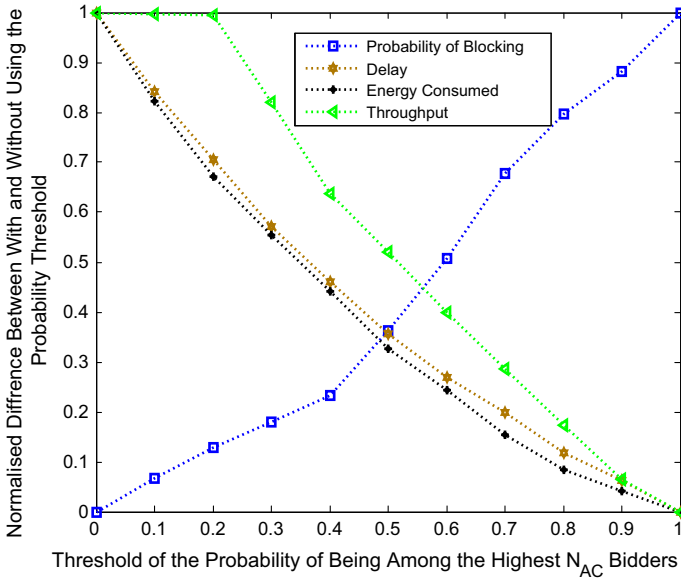


Fig. 9 The normalised difference for blocking, delay energy consumed and throughput

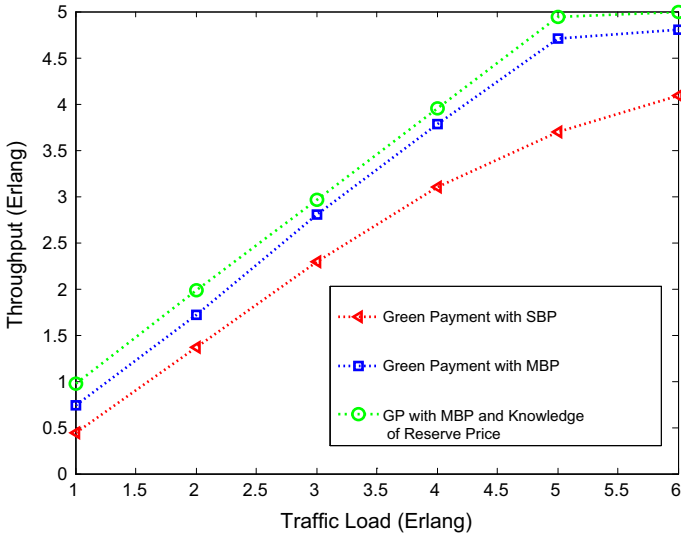
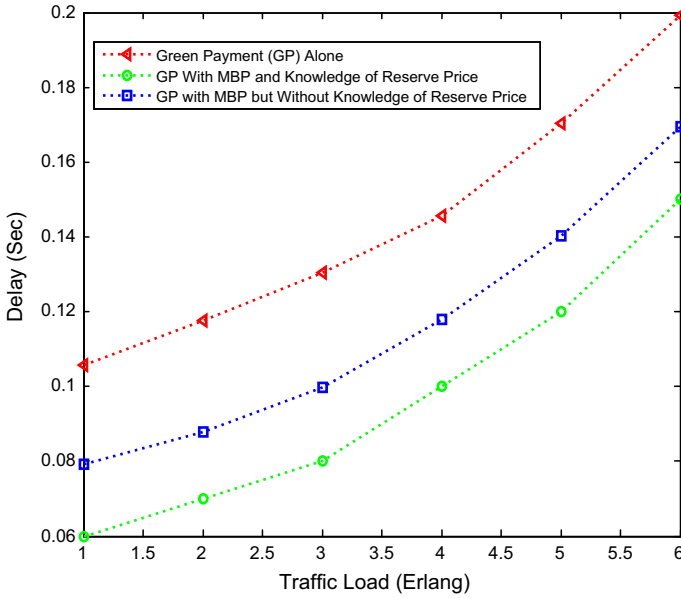


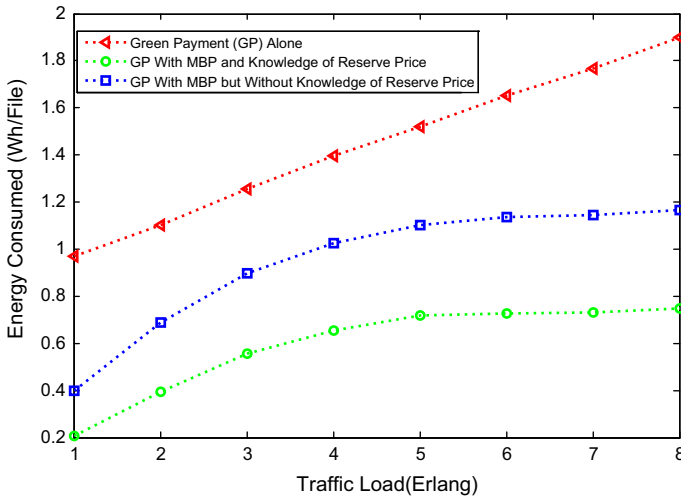
Fig. 10 The throughput of the system with GP, multiple bidding process and multiple bidding processes and KRP

increasing with the traffic load because as the traffic load increases the collision in the system also increases.

From Fig. 12, using the GP alone, the energy increases linearly with the traffic load. However, with the probability introduced, the increases can no longer be described as linear but similar to a parabola as the traffic load increases. As the traffic load increases,



**Fig. 11** The average delay experienced by the system for schemes with GP, GP with MBP and MBP and KRP



**Fig. 12** The energy consumption level of the system for schemes with GP, MBP and KRP

fewer go into the transmitting mode and are not able to transmit successfully. The KRP algorithm performs best because of the reasons stated earlier in Figs. 10 and 11.

Furthermore, Fig. 13 shows the average price paid per file sent. The result shows that with or without the KRP, the average revenue is almost the same. This shows that the price difference between having and not having the KRP is quite small. However, compared with the loss in terms of energy consumed and delay, having the knowledge of the RP is

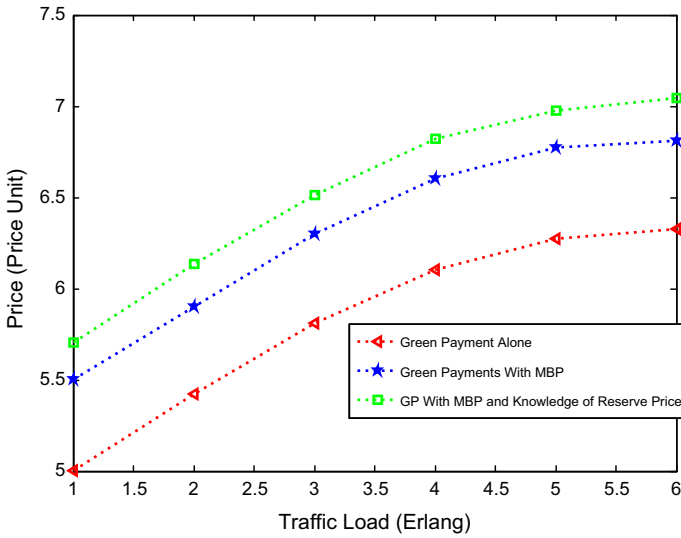


Fig. 13 The average revenues of the system for schemes with GP, GP with MBP and KRP

important. The average price also increases with the traffic load because the RP and the bids of the users increase the traffic load.

Figure 14a shows the general user utility against traffic load with and without the GP. The scheme with the GP involves MBP. The utility of the LPU without the GP decreases

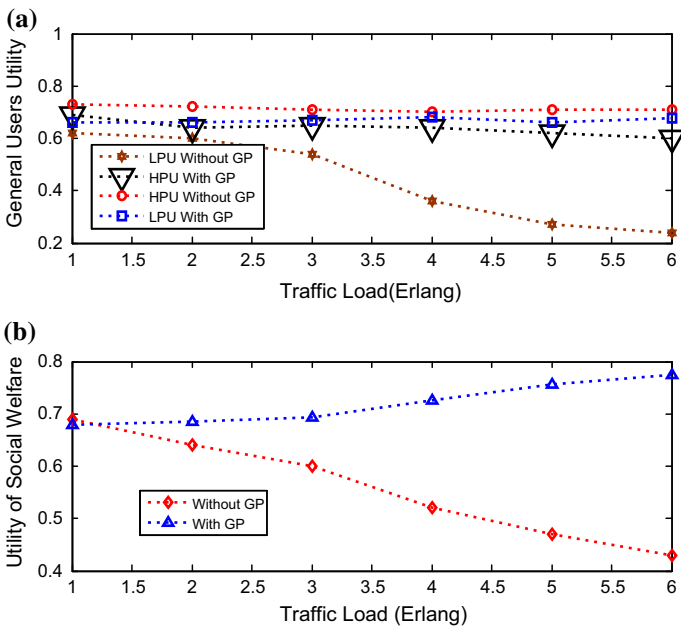


Fig. 14 a General utility of users for LPU and HPU with and without the GP. b Utility of the social welfare with and without GP

with increases in traffic load as a result of the interference caused by the HPU to the LPU. At low traffic loads, there are more channels available than required. Without the GP, the LPU receives no incentive and, therefore, their bids are sometimes rejected due to price. The utility of the LPU with the GP is relatively constant. This is because of the incentive received from the GP. The utility of the HPU with the GP decreases slightly as the traffic load increases. This is because they get squeezed out as the traffic load increases, thereby leading to more delay. Using the probability threshold with the MBP allows the utility of the HPU to be relatively high because the HPU only transmits when their calculated probability is above the threshold. This is due to calculating the utility for the delay and data rate being calculated only when a user is admitted into the system. However, without the GP, the general utility of the HPU is higher because they dominate the system. This is an undesirable effect because the utility of the social welfare should be of concern. This can be seen in Fig. 14b with the combined utility of all the users showing the utility of the social welfare. The scheme without the GPs is better than the scheme with the GP. This also shows that with the GP the HPU are disadvantaged, but the system gives a better performance as a whole. The utility without the GP falls with the traffic load because as the traffic load increases the LPU are performing worse. This is because they have a low value in all the utility factors (Delay, data rate and bid being below RP) that were taken into account while calculating the utility. However, with the GP, the HPU experiences more delay, but the higher data rate compensates for the loss due to the delay. The LPU has a lower data rate compared to the HPU but they make up for this with a shorter delay. The relative increases of the utility with the GP as the traffic load increases is because as the traffic load increases the HPU are squeezed out. Therefore, compared with the admitted users the LPU has a high utility for data rate and all the other factors. This is because the utility is calculated in terms of data rate and delay only if a user is admitted into the system.

## 5 Conclusions

In this work, we have designed an auction based dynamic spectrum access network for future heterogeneous wireless networks with secondary users as opportunistic users of the radio spectrum. The work proposed a green payment based single and multiple bidding processes for short-term spectrum auction and showed that the MBP is more energy efficient than the SBP. This work also showed the effects of using the probability of being among the highest  $N_{AC}$  bidder ( $P_{rN_{AC}}$ ) with the MBP and how it affects the system performance such as delay and throughput. It showed that if an appropriate value of the threshold of the probability is set, the amount of energy consumed by the system and the delay can be reduced. It also showed that the energy consumed by the system and other system performance measures can be improved when the users have the knowledge of the RP and the approximated tax to be paid as a result of the green payments. It showed that the knowledge of the RP has no effect on the revenue of the WSP since truthful bidding was assumed. Finally, the proposed scheme could be used to improve the congestion in the system and provide a better utility to optimise the social welfare of the users while also providing a better utility to the service provider.

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