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Mathematical Methods for Medical Imaging

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Computational anatomy is an emerging discipline at the interface of geometry, statistics and image analysis which aims at modeling and analyzing the biological shape of tissues and organs. The goal is to estimate representative organ anatomies across diseases, populations, species or ages, to model the organ development across time (growth or aging), to establish their variability, and to correlate this variability information with other functional, genetic or structural information.

Geometry, and especially differential geometry is a natural foundation of the shape modeling. Dealing with data as extracted from medical images, makes the necessity of handling statistical modeling. Hence, the mathematical foundation of computational anatomy, seeks to unify statistics, and geometry. The aim, that methods may serve the computational anatomy emphasized the need for numerical methods.

D'Arcy W. Thomson (1860–1948) used transformations of the underlying space to equalize the form of hand drawn biological objects. In some examples factoring out an affine transformation, in other examples projective transformations, and in a single example a non-linear transformation containing singularities when transforming a *Scarus Sp.* into a *Pomacanthus*.

David G. Kendall created, used originally in archeology to argue if historic landmarks was more than accidentally aligned, what is now called Kendall's shape space, by factoring out similarity transformations of a labeled point sets. He introduced Procustes analysis and the invariant quotient metric to equip the shape space with a metric and a measure.

Timothy F. Cootes and co-workers introduced the active shape models, ignoring the intrinsic curvature of the space of Procustes-aligned shapes, used linear statistics and thereby made computations straightforward. This has had a immense impact on the computational modeling with now close to 5,000 citations.

Much work has followed these contributions in creating shape spaces equipped with (invariant) metrics, using transformations to align shapes, equipping the group of transformations with metrics, etc. and to reformulate mathematical theories in a computational tractable manner.

This special issue follows this line of research. Five papers have been included in this special issue. Of these, three papers deal with diffeomorphic mappings and metric constructions of these (originating from the Large Diffeomorphic Metric Mapping (LDDMM) framework), one with a novel shape descriptor, and one with making statistics in curved spaces.

The paper by Lorenzi et al. "Geodesics, parallel transport & one-parameter subgroups for diffeomorphic image registration" underpin the mathematical rigor of using the one-parameter sub-groups of the LDDMM framework. These stationary velocity fields show to be of good approximation to the LDDMM for smaller deformations while larger inter-subject registrations have substantially different geodesics than the full LDDMM implementations.

The paper by Günther et al. "Flexible shape matching with finite element based LDDMM" decouples the current and deformation discretization by using conforming adaptive finite elemts. This leads to more flexibility in the formulation as illustrated for example by incorporating multiple scales.

The paper by Modin et al. "Geodesic warps by conformal mappings" studies the shape equivalence under planar

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conformal mappings. The space of planar conformal mappings is equipped with a metric and a numerical discretization is developed. Computational examples are illustrated on artificial examples.

The paper by Zeng et al. "Teichmüller shape descriptor and its application to Alzheimer's disease study" studies the family of non-intersecting closed 3D curves. It is applied to MRI data form the Alzheimer's Disease Neuroimaging Initiative (ADNI) distinguishing normal controls from Alzheimer's patients by the shape of the cortical surface. The Teichmüller space is a quotient space of conformal mappings; two surfaces in between which a conformal mapping exists have the same representation in Teichmüller shape space. The space is equipped with a metric. By solving a Ricci flow, the Teichmüller representation may be found.

The paper by Fletcher et al. "Geodesic regression and the theory of least squares on Riemannian manifolds" introduces the geodesic regression as a mapping between a real-valued parameter to be regressed to a manifold-valued dependent random variable. The key idea is that regressed curves must be geodesics on the manifold. Existence, uniqueness, and maximum likelihood criteria are developed. Also here brain shape is analyzed: the relation of the shape of corpus callosum to age.

All these are aspects of the fundamental property that shapes do not live in Euclidean space. Since the pioneering work of D'Arcy Thomson equaling shapes by transformations, David Kendall showing that shapes represented as labeled landmarks live in complex projective spaces, and Timothy Cootes reformulating this in a computational simple manner, the work has continued developing representations and appropriate metrics to allow for statistical modeling. This special issue continues this development bridging statistics, geometry, and computational science.