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Modeling Macroporous Soils with a Two-Phase Dual-Permeability Model

L. Stadler · R. Hinkelmann · R. Helmig

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Abstract Fast water infiltration during heavy rainfall events is an important issue for hillslope hydrology and slope stability. Most hillslopes are strongly heterogeneous and contain macropores and soil pipes, so that infiltrating water can bypass the soil matrix and reach rapidly deeper regions. Water infiltration into macroporous soils is usually simulated with dual-permeability models based on Richards equation (RDPM) which only describes water flow. In this article, we present a two-phase dual-permeability model (TPDPM) for simulating water and air flow in macroporous soils. Water and air flow are simulated in both domains and mass transfer for water and air between the domains is included with first-order transfer terms. The main objectives of this article are to discuss the differences between TPDPM and RDPM and to test the application of the TPDPM on the slope scale. First, the differences between RDPM and TPDPM were studied using a one-dimensional layered soil. For the chosen high infiltration rate, we observed significant differences in the macropore domain and small differences in the matrix depending on the transfer parameter. Second, we applied the model to simulate fast water infiltration and flow through an alpine hillslope, where the water flow mainly occurs in the macropore domain and the matrix domain is bypassed because it is low permeability. A good agreement of simulated and measured travel times of Wienhöfer et al. (Hydrol Earth Syst Sci 13(7):1145–1161, 2009) was obtained. Finally, we recommend using TPDPM for high infiltration in layered macroporous soils.

Keywords Dual-permeability model · Macropores · Two-phase flow

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1 Introduction

Most field soils are structured and contain large continuous pores and voids, so called macropores which are generated by soil mechanical processes like swelling and shrinking, soil flora (e.g., decayed plant roots), soil fauna (e.g., earthworm burrows) and further soil processes (Beven and Germann 1982). Because of their different origin, macropore texture in terms of surface properties, orientation, ramification, and connectivity may be manifold. The consequences of macropores on water flow and transport are important, as they can generate non-equilibrium and preferential flow (for a review see Šimůnek et al. 2003; Jarvis 2007). A typical phenomenon observed in macroporous soils is that water and solutes can bypass the matrix via macropores and rapidly reach deeper regions and shallow groundwater. Jarvis (2007) discussed the different effects for water flow and solute transport in detail. Further, macropore flow is responsible for fast infiltration of water after heavy rainfalls on hillslopes and thus can trigger landslides (Stadler et al. 2009; Ehlers et al. 2011). The initiation of macropore flow depends on initial conditions and rainfall intensity (e.g., Zehe and Flühler 2001; Weiler and Naef 2003). The mass transfer between matrix and macropores is controlled by the properties of the macropore surface and can be strongly inhibited, especially when the macropores are affected by lining and coating (Thoma et al. 1992; Gerke and Köhne 2002). All these complex processes should be covered by numerical models to simulate water flow in macroporous soils (macropore flow).

Water infiltration and flow in the unsaturated zone is usually simulated with the Richards equation (Richards 1931). Since water may rapidly bypass the soil matrix, continuum models that assume equilibrium flow in a single flow domain are not suitable for such soils. Thus, dual-porosity and dual-permeability models (see Barenblatt et al. 1960) based on the Richards equation (RDPM) (e.g., Gerke and van Genuchten 1993a) have been developed to simulate water flow in soils. Similar dual-permeability models for two-phase flow (TPDPM) have been applied for fractured rocks (e.g., Doughty 1999; Unger et al. 2004). The models are based on the separation of the soil pores into two different pore spaces, herein referred as domains. The first domain includes the small matrix pores and the second domain the macropores. While classical dual-porosity models assume that flow only occurs in the macropore domain, in dual-permeability models flow takes place in both domains. Dual-permeability models should be preferred for macroporous soils since flow will usually take place in matrix and macropores. Today, a wide range of numerical codes and software packages exists, that accounts for non-equilibrium water flow by applying dual-permeability or dual-porosity concepts for macroporous soils. Šimůnek et al. (2003) summarized the main concepts and models.

The soil air is usually neglected when simulating water infiltration processes in the unsaturated zone. Assuming that the soil air can easily escape, the pressure of the gas phase is atmospheric, and models based on Richards equation, which only describe water flow, are suitable. However, if surface ponding occurs or the flow in the soil is limited by low permeability layers, these simplifications often do not hold. Another aspect is that the soil air can transmit pressure reactions, which may be an issue for slope stability where pressure differences can trigger landslides (Schulz et al. 2009). The limitation of the Richards equation can be overcome using the two-phase flow equations (Bear 1972) for two immiscible fluids (water/air) instead of Richards equation. The two-phase flow equations will lead to a much more general dual-permeability model which describes water and air flow in the matrix and macropore domain. In principal this concept can also be chosen for other two-phase problems such as water and oil. Dual-permeability models based on the two-phase flow equations are rarely applied to study water infiltration in soils. Zimmerman et al. (1996) simulated water infiltration with a two-phase dual-porosity model to study mass transfer for well- defined matrix/macropore geometries. They reported that air flow had only little effect on water flow during their studies. TPDPMs have also been applied to simulate water infiltration into unsaturated fractured rocks (e.g.,Wu et al. 2002; Unger et al. 2004). Doughty (1999) used a two-phase dual-permeability model for the simulation of water infiltration into fractured rocks with relatively low infiltration rates. However, TPDPMs have not been applied for simulating flow and transport processes in macroporous soils.

In this article, we present a TPDPM which we have developed and implemented into the open-source software simulator Dumux (Flemisch et al. 2007). Further we developed one-dimensional finite-volume codes (C++/Python) to compare RDPM and TPDPM. A first-order Waren–Root-Type (Warren and Root 1963) mass transfer term (see also Gerke and van Genuchten 1993b; Ray et al. 2004) was chosen to compute mass transfer of water and air between matrix and macropore domain. As the exact geometry of pores and aggregates on the slope scale is not measurable and further effects like coating and mineralized layers can inhibit the exchange dramatically, the uncertainty of the larger scale and the error made by lumping different macropores into a domain is dominant, so that there are hardly advantages in using higher order transfer terms or a MINC model (Pruess 1985). This also implies that the coefficients of the transfer term can only be estimated and must be calibrated with measured data.

The objectives of this article are (i) to investigate when the flow of soil air is hindered so that differences between RPDPMs and TPDPMs occur and (ii) the application of the two-phase model for a macroporous part of the slow creeping hillslope Heumöser in the Austrian Alps to investigate the fast water infiltration and flow processes which were reported by Wienhöfer et al. (2009).

2 Conceptual Model and Implementation

All dual-permeability models are based on the separation of the soil pores into two different pore system—domains (e.g., Barenblatt et al. 1960; Gerke and van Genuchten 1993a; Šimůnek et al. 2003; Gerke 2006; Jarvis 2007; Šimůnek and van Genuchten 2008). Before discussing the model concept in detail, it is useful to point out that different types of macroporous soils exist. For example, in a three-dimensional volume of a structured soil, the matrix can be separated into soil blocks/aggregates and voids (macropores). When the soil saturation is low, the water flow mainly occurs between the aggregates and depends on the aggregate bridging (see Carminati et al. 2007). In less structured soils, macropores (e.g., earthworm burrows and decayed plant roots) may penetrate the matrix. The presented dual-permeability model is valid as long as a representative elementary volume (Bear 1972) can be found for matrix and macropores. The matrix domain consists of fine soil pores and the soil itself, while all macropores belong to the macropore domain. The definition of relevant pore diameters to distinguish between matrix pores and macropores is complex and not unique. Jarvis (2007) summarized that pore diameters larger than 0.03–0.05 mm are typically classified as macropores. The pores of both domains can be filled with water and soil air so that a separation of soil pores leads to two domains which represent separate two-phase flow systems that are connected over the macropore surface. It is assumed that matrix, macropores and the soil parameters do not change with time. If the flow velocities in the macropore domain exceed Reynolds number 1, Darcy's law is no longer valid and should be replaced by the Forchheimer law (see Bennethum and Giorgi 1997; Markert 2007). Jarvis (2007) discussed the physics of water flow in the macropore domain and the limitation of Darcy's law in his review in detail. In this article, we assume that Darcy's law is a valid approximation for both domains. In principal, all parameters related to the macropore domain must be considered under a certain reservation.

2.1 Balance Equations

2.1.1 Richards Equation

Splitting the soil pores into matrix pores and macropores leads to two separate overlapping pore domains d (I = matrix, II = macropore). A Richards dual-permeability model can be obtained when the Richards equation is applied for both domains (see Gerke and van Genuchten 1993a). Here in the one-dimensional formulation for the water content in the vertical z direction.

$$\frac{\partial \theta_{\mathrm{I}}}{\partial t} = \frac{\partial}{\partial z}\left[K_{\mathrm{I}}(\theta_{\mathrm{I}})\left(\frac{\partial h_{\mathrm{I}}}{\partial z}+1\right)\right] - \Gamma \tag{1}$$

$$\frac{\partial \theta_{\mathrm{II}}}{\partial t} = \frac{\partial}{\partial z}\left[K_{\mathrm{II}}(\theta_{\mathrm{II}})\left(\frac{\partial h_{\mathrm{II}}}{\partial z}+1\right)\right] + \Gamma \tag{2}$$

θ is the water content of a domain, t(s) is the time, z(cm) is the vertical direction, K(cm s^{-1}) is the saturated hydraulic conductivity, h is the pressure head (cm), and Γ(s^{-1}) is the water transfer term between both domains.

2.1.2 Two-Phase Flow Equations

Since water and air flow can take place in both domains, balance equations can be defined for each fluid in each domain. Applying the two-phase flow model concept to two immiscible fluids in porous media for a single domain results in two conservation equations, one for each fluid phase ($\alpha = w$ for the wetting phase and $\alpha = n$ for the non-wetting phase):

$$\phi \frac{\partial S_\alpha \rho_\alpha}{\partial t} - \mathrm{div}(\rho_\alpha \underline{v}_\alpha) - q_\alpha = 0 \tag{3}$$

in which $S(-)$ is the fluid saturation, ϕ(m^{3}m^{-3}) is the porosity, ρ_α(kg m^{-3}) is the phase density, t(s) is the time, $\underline{v}_\alpha$(m s^{-1}) is the Darcy velocity, and q(kg m^{-3} s^{-1}) is a sink/source-term. The splitting of the soil pores into two domains leads to conservation equations for each fluid in each domain d (I = matrix, II = macropore):

$$\phi_I \frac{\partial S_{w,\mathrm{I}} \rho_{w,\mathrm{I}}}{\partial t} - \mathrm{div}(\rho_{w,\mathrm{I}} \underline{v}_{w,\mathrm{I}}) - q_{w,\mathrm{I}} - \Gamma_w = 0 \tag{4}$$

$$\phi_{\mathrm{I}} \frac{\partial S_{n,\mathrm{I}} \rho_{n,\mathrm{I}}}{\partial t} - \mathrm{div}(\rho_{n,\mathrm{I}} \underline{v}_{n,\mathrm{I}}) - q_{n,\mathrm{I}} - \Gamma_n = 0 \tag{5}$$

$$\phi_{\mathrm{II}} \frac{\partial S_{w,\mathrm{II}} \rho_{w,\mathrm{II}}}{\partial t} - \mathrm{div}(\rho_{w,\mathrm{II}} \underline{v}_{w,\mathrm{II}}) - q_{w,\mathrm{II}} + \Gamma_w = 0 \tag{6}$$

$$\phi_{\mathrm{II}} \frac{\partial S_{n,\mathrm{II}} \rho_{n,\mathrm{II}}}{\partial t} - \mathrm{div}(\rho_{n,\mathrm{II}} \underline{v}_{n,\mathrm{II}}) - q_{n,\mathrm{II}} + \Gamma_n = 0 \tag{7}$$

These four conservation equations describe the fluid flow in macroporous soils. Fluid exchange (water/air) between the macropore/matrix interface is covered by the transfer terms

Γ_α(kg m^{-3} s^{-1}). By partitioning the pores into two domains, the variables of the conservation equations must be defined for each domain. For example, the porosity ϕ_{I} of domain I (matrix) is defined as $\phi_{\text{I}} = \frac{V_{\text{pore,I}}}{V_{\text{total,I}}}$. The saturation of a phase is given by the fluid volume divided by the available pore volume of a domain; for example for domain II (macropores):

$$S_{\alpha,\text{II}} = \frac{V_{\alpha,\text{II}}}{V_{\text{pore,II}}} \tag{8}$$

Since the whole pore volume of a domain is occupied by two fluid phases, the sum of the saturations of both phases in a domain d of the two-phase dual-permeability model is equal to one:

$$S_{w,d} + S_{n,d} = 1 \tag{9}$$

Fluid velocities for the two fluids in both domains are computed with the generalized Darcy law:

$$\underline{v}_{\alpha,d} = -\underline{\underline{K}}_d \frac{k_{r\alpha,d}}{\mu_{\alpha,d}} (\text{grad } p_{\alpha,d} - \rho_{\alpha,d}\, \underline{g}) \tag{10}$$

Here $\underline{\underline{K}}_d$(m^2) is the intrinsic permeability, $k_{r\alpha,d}(-)$ the relative permeability, and $\mu_{\alpha,d}$(Pa s) the dynamic viscosity. $p_{\alpha,d}$(Pa) is the phase pressure and $\underline{g}$(m s^{-2}) is the vector of gravity. The ratio of the relative permeability and the dynamic viscosity is called mobility $\lambda_{\alpha,d}$ (m s kg^{-1}).

The constitutive relationships describe the relation between the saturation and the capillary pressure and the relative permeability. The capillary pressure is defined as the pressure difference between the non-wetting (air) and wetting phase (water):

$$p_{c,d} = p_{n,d} - p_{w,d} \tag{11}$$

We have chosen the relationship after van Genuchten (1980) for the capillary pressure in combination with the model of Mualem (1976) to compute the relative permeability of the wetting phase and non-wetting phase. The compressibility of the gas phase was taken into account using the ideal gas law.

As mentioned earlier, the mass transfer Γ_α between matrix and macropores depends mainly on the macropore geometry, surface properties and the state of the system (non-equilibrium). Transfer formulations for dual-permeability models generally take various effects in a simplified manner into account to define an effective mass transfer. First-order terms (e.g., Warren and Root 1963; Gerke and van Genuchten 1993b) that describe the transfer as function of the pressure difference between both domains and a transfer coefficient are the simplest way to predict the water transfer. More accurate second-order terms which consider the matrix block size and its initial non-equilibrium might be better (e.g., Köhne et al. 2004; Zimmerman et al. 1996). However, the error when using a first-order equation instead of a second-order equation depends on the block size, and it decreases for small block sizes (Zimmerman et al. 1996). The order of transfer formulation will be of minor importance on larger scales where many pores are lumped and additional uncertainties will dominate the system, so that first-order terms seem to be accurate enough. On larger scales, transfer terms must be estimated or they should be fitted to experimental data. For a detailed review and discussion of the problems involved see Šimůnek et al. (2003) and Jarvis (2007). In this article, we used a first-order formulation which is quite similar to the one of Ray et al. (2004). The parameter $\beta(-)$ is the transfer coefficient for saturated conditions and λ_α the mobility, which is here computed with the relative permeability of the upstream domain. The transfer term Γ_α can be considered as an additional source term q_α(kg m^{-1} s^{-1}) that describes the fluid transfer over the macropore surface per cell volume

$$q_\alpha = -\beta\lambda_\alpha(p_{\mathrm{I},\alpha} - p_{\mathrm{II},\alpha})\ s\ \rho_\alpha \tag{12}$$

where s(m^{-1}) is a scaling parameter between soil volume and macropore surface.

The same transfer formulation was applied for our Richards model so that both models produce the same results for the water phases as long the soil air can freely escape.

2.2 Numerical Model

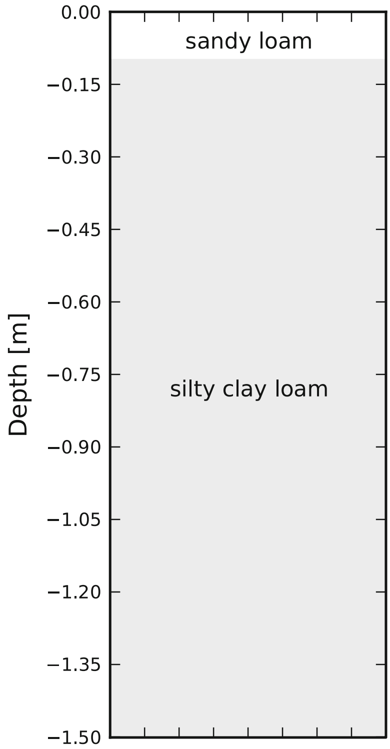
The TPDPM has been implemented into the open-source simulator Dumux which is based on the numerical toolbox DUNE (Distributed and Unified Numerics Environment) (Bastian et al. 2008a,b). The modular and generic concept of Dumux allows to apply different types of meshes (e.g., triangles, cubes) in different dimensions (two/three dimensions) for the implemented models. It is necessary to define four primary variables to solve the four balance equations. We have chosen a pressure/saturation formulation with the pressure of the gas phase and the saturation of the water phase as primary variables in each domain. A vertex-centered Finite-Volume method (box-method) (see Huber and Helmig 2000), which is a mixture of a finite-volume (FVM) and finite-element method, is used for spatial discretization. The temporal discretization is done with a fully implicit Euler scheme, using an adaptive time-stepping. The application of the TPDPM leads to a fully coupled system of four strongly coupled and nonlinear equations which are linearized with the Newton–Raphson method and solved with an inner BiCGSTAB solver. System dependent boundary conditions (Radcliffe and Šimůnek 2010; Stadler et al. 2011) have been implemented to simulate the in- and exfiltration through matrix and macropores at the surface. A detailed description of the underlying physical model concepts and numerical methods for Dumux can be found in (Helmig 1997; Hinkelmann 2005; Flemisch et al. 2007). In addition to the TPDPM that we implemented into Dumux, we have developed one-dimensional FVM codes (C++/Python) for RDPM and TPDPM to study the differences between both formulations in detail (see Sect. 3.3).

3 Simulations

Differences between RPDMs and TPDPMs only arise when the flow of soil air is hindered (e.g., discontinuous gas phase). In general, the macropores enforce the escape of soil air since the macropore domain is usually only partially saturated and the matrix air can escape via the macropores during water infiltration. If surface water ponds above a soil layer, the gas flow in the macropores is limited as it cannot freely escape. The escape of soil air can be further limited if the macropores are only partially connected to the soil surface.

In the following studies, we use the data and results of field and soil block experiments of Wienhöfer et al. (2009) which were conducted to investigate the fast water transport at the forested upper part of the Heumöser slope in Ebnit (Austria). A detailed description of the Heumöser slope can be found in Lindenmaier et al. (2005), Lindenmaier (2008) and Wienhöfer et al. (2009). An essential characteristic of the slope is that macropores and soil pipes are able to generate fast breakthrough curves of a tracer at a spring and on a cut-bank near a hiking trail (see Fig. 4). Wienhöfer et al. (2009) concluded that the processes are controlled by complex structures (macropores, soil pipes, soil layers) and their interactions. The soil matrix was classified as a silty clay loam and silt loam, with a shallow topsoil (∼10 cm)

Fig. 1 Model domain for the one-dimensional infiltration experiment



consisting of sandy loam. The macropores found by Wienhöfer et al. (2009) were mainly cracks and soil pipes with diameters up to several cm. The macropore walls had dark coatings, indicating their temporal persistence. We set the transfer coefficient β for the following studies equal to the matrix permeability and used the scaling parameter s to vary the fluid transfer.

3.1 Comparison of Richards and Two-Phase Flow Dual-Permeability Models

3.1.1 System and Boundary Conditions

Figure 1 shows a sketch of the idealized soil column that was used to study the differences between the RDPM and TPDPM. We adopted the soil properties form the Heumöser slope (Wienhöfer et al. 2009) for this study. The topsoil (10 cm) is a sandy loam, followed by a silty clay loam. We reduced the permeability of the macropore domain near the bottom, assuming that the macropores end 10 cm above the bottom. The soil parameters for the matrix were adopted from Carsel and Parrish (1988). We generated a one-dimensional cell centered finite-volume mesh with 75 cells for the discretization.

A Dirichlet boundary condition was set for the water phase (TPDPM)/head (RDPM) at the bottom of the domain to describe a saturated zone at the bottom. The gas flow was set to zero at the bottom to prevent the escape of soil air (TPDPM). System dependent boundary conditions were chosen at the top to describe the water infiltration caused by rain. The rainfall rate of 12 mm h^{-1} was equal to the sprinkling rate during the second field experiment of

Wienhöfer et al. (2009). The pressure of the soil air (TPDPM) was set to atmospheric pressure so that soil air can easily escape at the surface.

First, we set the scaling parameter s to 1 (s^{-1}) and determined a steady state saturation distribution as initial condition (no infiltration, only capillary rise of water from the bottom). The high matrix capillarity leads to a nearly saturated matrix. Second, we reduced the initial water saturation of the matrix domain to 0.75 (−). The resulting non-equilibrium leads to much higher transfer rates between both domains during water infiltration and could in principal be a result of root-water uptake by plants. As a further variation we increased the scaling parameter s to 5 (s^{-1}) to investigate the sensitivity of the fluid transfer.

3.1.2 Results and Discussion

Figure 2 shows the saturation during the infiltration in the matrix and macropore domain with the RDPM and TPDPM. The system is in equilibrium with the surface conditions at the top (no infiltration) and bottom (fully saturated) before the infiltration starts. In the first minutes, the water infiltrates into the matrix and the water begins to pile up above the lower permeable silty clay loam. The silty clay loam limits the further infiltration and the upper matrix gets fully saturated. The boundary condition for the matrix at the top switches to Dirichlet and the remaining water begins to infiltrate into the macropore domain. The water front in the macropores reaches the end of the macropores in a depth of −140 cm after 300 min and the saturation in the macropore domain begins to increase. The ponding is higher with the TPDPM (Fig. 2d). With the TPDPM, the high saturation of the matrix domain in the upper part limits the escape of soil air; whereas the water front infiltrates deeper for the RDPM, only limited by low conductivity of the silty clay loam. The difference is clearly visible after 500 min (Fig. 2d).

The surface of the macropore domain was fully saturated after 550 min with the TPDPM. This leads to a switch from a Neumann flux to a Dirichlet boundary condition for the water phase. A possible occurring water ponding on the surface due the reduced infiltration was neglected. For the RDPM, the macropore domain does not get saturated at the top during the infiltration. Water infiltration ended after 720 min. The differences between RDPM and TPDPM are still visible after 1,440 min (Fig. 2f) because the matrix infiltration in the RDPM is only restricted by the low matrix permeability, while the infiltration in the TPDPM is still limited by the high water saturations in the upper soil.

Overall, significant differences between both models occurred in the macropore domain, while the differences are minor in the matrix. The latter is caused by the fact that the system was initially in equilibrium with the boundary conditions and the matrix was almost saturated. The water infiltration with the TPDPM was limited because both domains were saturated at the surface after 555 min. We also investigated the influence of the transfer coefficient (not shown here) and observed only small differences when the scaling parameter s was increased. A further important result of this study is that the differences are stable over a long time period so that it will take a long time until the water saturations in the RPDM and TPDPM coincide again.

For the second example we changed the saturation of the matrix domain so that the initial system is no longer in equilibrium with the boundary conditions. A similar situation may occur if root water uptake reduces the matrix saturation in the silty clay loam and a wet period increases the saturation of the sandy loam. During the first 500 min, the water infiltration was similar as in the previous study with the exception that the higher saturation of the sandy loam caused an earlier initiation of macropore flow from the soil surface. The water ponding in the

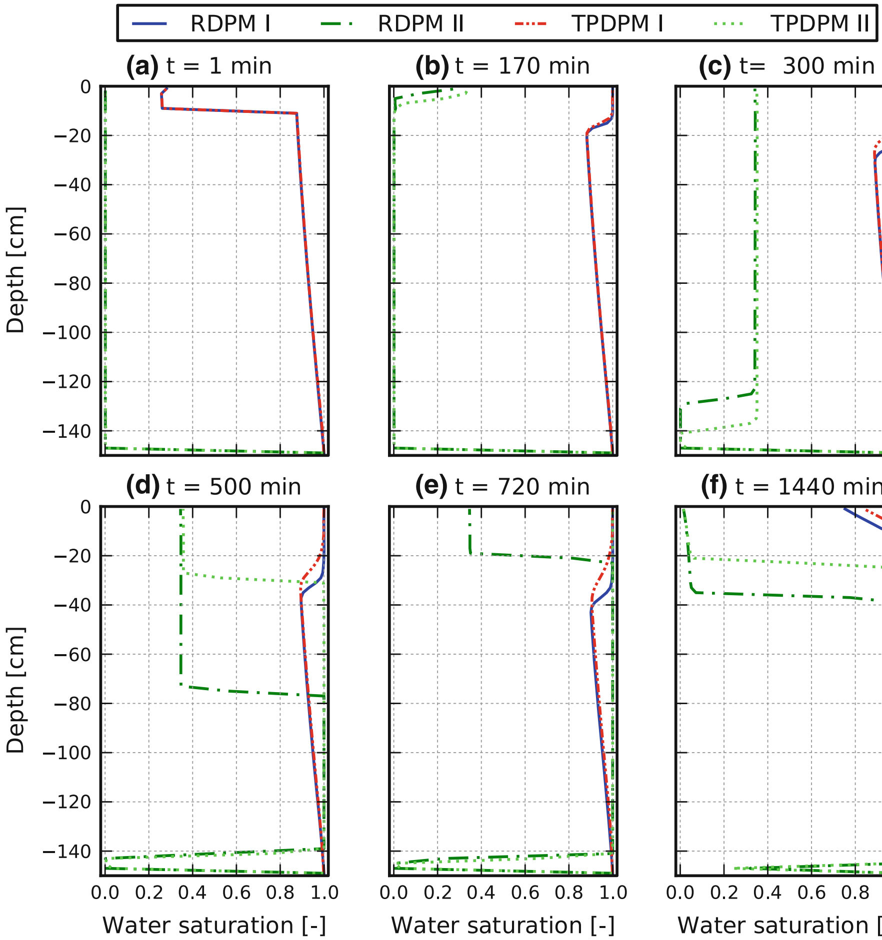


Fig. 2 Comparison of RDPM and TPDM for two-layered soil: equilibrium initial conditions and transfer parameter $s = 1$

macropore is lower when the initial saturation in the matrix is lower (Figs. 2d, 3a), also the matrix saturation in the silty clay loam is lower. Comparing the different initial conditions (Fig. 2d–f with Fig. 3a–c for $s = 1$) we see different results in the matrix as well as the macropore domain, even after 1,440 min.

Figure 3 shows the saturation in matrix and macropore domains during infiltration for two different transfer parameters s. An increased transfer (factor 5) leads to an increased transfer in parts where the macropore domain is saturated. This transfer reduces the height of the water ponding in the macropore domain above the part where the macropores end (-140 cm). With a higher transfer, the macropore domain of the TPDPM gets no longer fully saturated (Fig. 3b) and no surface runoff occurs. Further the higher transfer reduces the differences between RDPM and TPDPM in the upper part (matrix domain). The variation of the transfer parameter mainly changes the location where differences between RDPM and TPDPM occur. With a higher s the differences occur mainly in the lower parts, with a lower s in the upper part. The total differences in the water saturation are smaller with a smaller

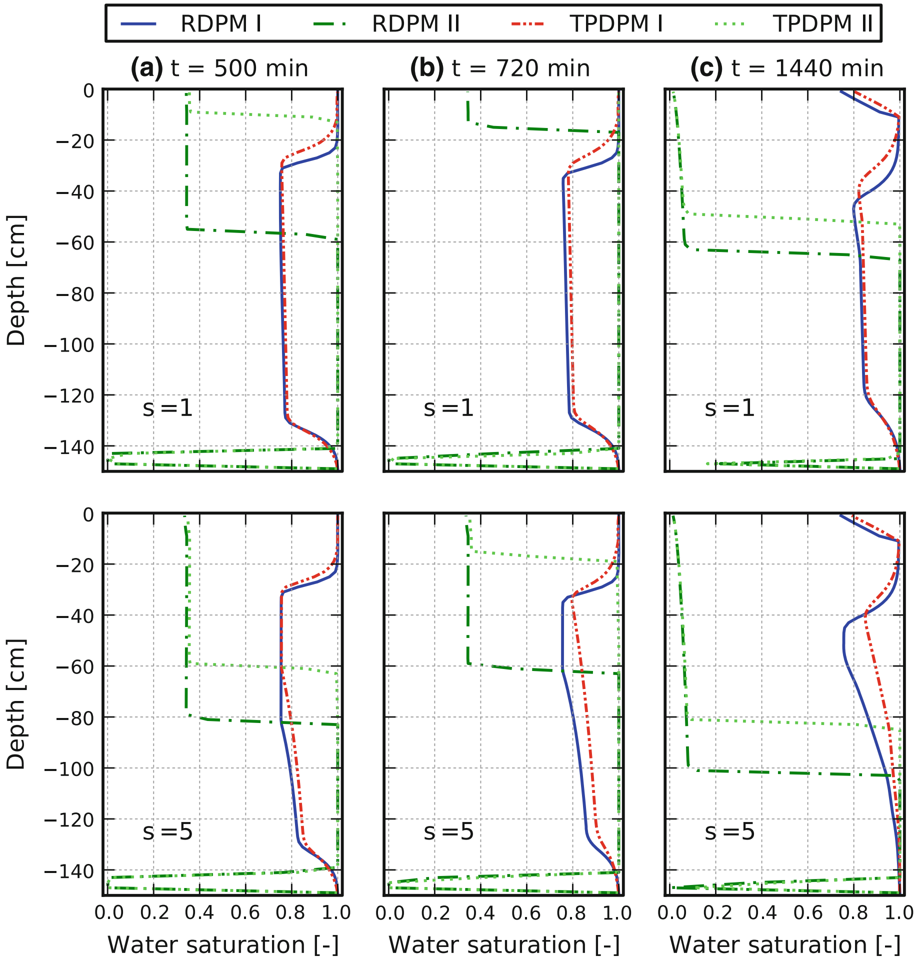


Fig. 3 Comparison of RDPM and TPDM for two-layered soil: non-equilibrium initial conditions and variation of transfer parameter *s*

transfer parameter *s* (Fig. 3b). As the water is not in equilibrium after the sprinkling, the differences between RDPM and TPDPM remain for a long time (Fig. 3c).

In our example, we observed significant differences between RPDPM differences are small in the matrix, but increases with increasing *s*. The differences were stable over a long time and when longer times will be investigated, evapotranspiration should be taken into account. We mention that we have chosen a very high rainfall rate and for smaller rainfall rates the differences between RPDPM and TPDPM are smaller (not shown here). Water flow was limited in all three studies by the low permeability of the matrix domain and water ponding occurred above the ending macropores in the macropore domains. This ponding plays an important role for the water transfer between matrix and macropores since it increases the pressure in the macropore domain. Based on these results, the water transfer between matrix and macropores is probably small for steep slopes where no significant ponding occurs and the water can flow rapidly in the macropore domain downwards on the bedrock surface. This will be investigated in next example.

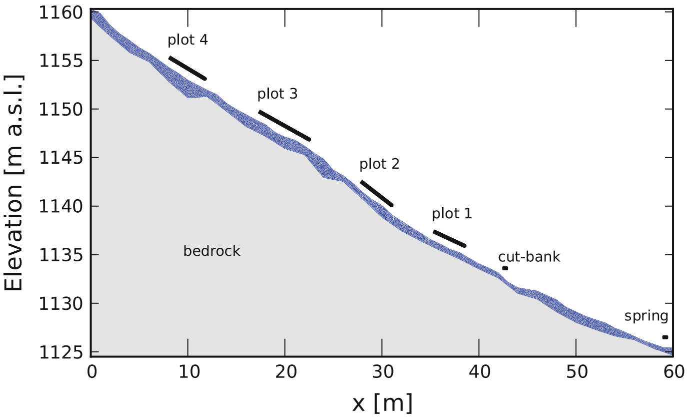


Fig. 4 Model domain with four sprinkling plots, cut-bank at the hiking trail and spring, after Wienhöfer et al. (2009)

3.2 Macroporous Hillslope

3.2.1 System and Boundary Conditions

The objectives of the second study was to simulate the water fast flow in the Heumöser slope which has been observed by Wienhöfer et al. (2009) during an artificial rainfall experiment where water (12 mm h^{-1}) was sprinkled on four different areas (plots 1–4).

Figure 4 shows the slope geometry and the position of the four sprinkling plots. Wienhöfer et al. (2009) reported that water outflow occurred at a cut-bank near a hiking trail 22 min after starting the sprinklers, reached nearly stead-state after 142 min, and ended 120 min after stopping the sprinkling. Although the soil was relatively shallow (Fig. 5) and the rainfall intensity was high, the macropores and soil pipes prevented surface runoff above the spring. We idealized the matrix domain for our study as a silty clay loam and used average parameters after Carsel and Parrish (1988). The macropore domain is characterized by a well connected network of macropores and soil pipes. We used the hydraulic capacity function of the van Genuchten model to estimate plausible van Genuchten parameters α and n. The macropore permeability was chosen to fit the observed initiation of water flow in the cut-bank. Considering the overall parameter uncertainty we used an idealized isotropic matrix and macropore domain. The soil parameters for the matrix (silty clay loam) and macropore domains (macropore slope) are given in Table 1.

We assumed an impermeable bedrock (zero flux) and used a triangle mesh with 3,089 nodes and 5,270 cells to set up an idealized two-dimensional model of the soil. We supposed that the observed outflow at the cut-bank was mainly influenced by the steep slope, the shallow soil and macropores that are ending at the soil surface. Exact geometries and properties of macropores and the soil at the cut-bank have not been determined. Since no outflow occurred for our idealized geometry, we compared the simulated discharge through the macropore domain at $x = 42$ m with the observed seepage at the cut-bank.

Three different variants were carried out to investigate the influence of the initial conditions and transfer parameters. For variant A we used the average yearly rainfall depth of 2,155 mm y^{-1} to compute the initial water saturation for the matrix and macropore domain. The high capillary suction of the silty clay loam lead to a nearly saturated soil matrix while

Table 1 Default soil parameters of matrix and macropore domains

	Sandy loam	Silty clay loam	Macropore column	Macropore slope
ϕ (−)	0.41	0.44	0.05	0.03
K (m^2)	1.23×10^{-12}	1.94×10^{-14}	1.2×10^{-11}	6.3×10^{-11}
VG_N (−)	1.89	1.23	4.5	4.5
VG_α (Pa^{-1})	7.64×10^{-4}	1.02×10^{-4}	0.08	0.08
Sw_r (m^{3}m^{-3})	0.16	0.21	0.0	0.0
Sn_r (m^{3}m^{-3})	0.0	0.0	0.0	0.0
β (−)	1.23×10^{-12}	1.94×10^{-14}	–	–

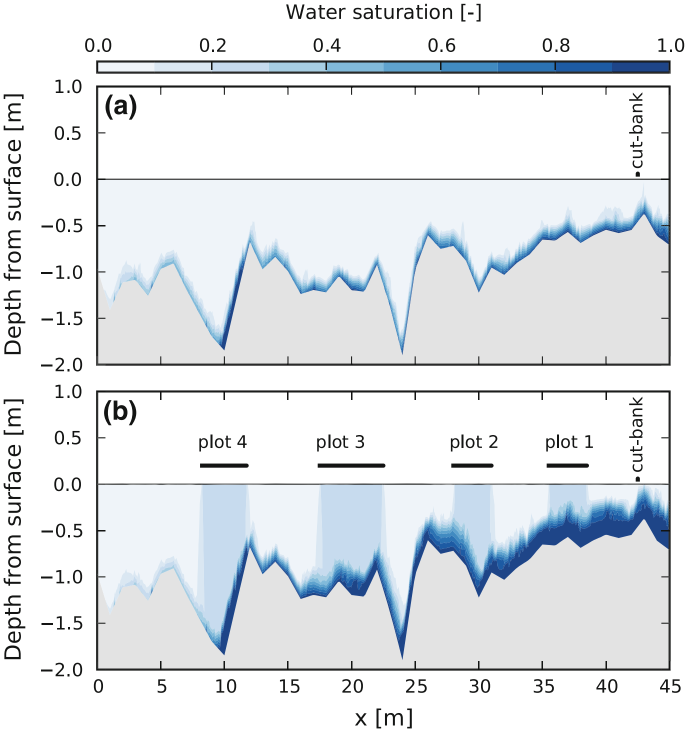


Fig. 5 **a** Initial water saturation and **b** water saturation after 150 min water sprinkling in the macropore domain for variant A; the height of the soil layer is plotted shown (in contrast to the coordinate system of Fig. 4

the macropore domain had very low saturation (see Fig 5a). Variant B and C had a reduced initial saturation (0.8 [–]) in the matrix, and a saturation of 0.1 [–] in the macropore domain. The scaling parameter s was set to $s = 2$ for variant A and B and set to 10 for variant C.

System dependent boundary conditions were implemented to simulate the sprinkling experiment with a constant rate of 12 mm h^{-1} on all four plots (see Fig. 2). The original infiltration time (see Wienhöfer et al. 2009) was reduced to 8 h. The pressure of the gas phase was set for both domains to atmospheric at the surface nodes (no ponding). It was assumed that all water infiltrates into the matrix until the infiltration capacity of the matrix was exceeded or the matrix gets saturated. If the matrix was saturated, the matrix boundary condition was switched to Dirichlet and the infiltration/exfiltration over the matrix was computed. The remaining water was infiltrated via the macropores. Surface runoff will only occur if the macropore domain is saturated. This happened only far below the cut-bank near the spring, so that surface runoff was not modeled in this study. At the bottom we set a Neumann boundary condition to describe the impermeable bedrock. A Dirichlet boundary condition for the water phase was set at the toe of the slope.

3.2.2 Results and Discussion

Figure 5 shows the initial saturation in the macropore domain for variant A where the averaged yearly rainfall was used to compute the initial water saturation. Note that in this figure the coordinate system was changed when compared to Figure 4. Here, the height of the soil layer (distance from the top to bedrock) is plotted along the x-direction. The constant rainfall and high matrix suction lead to an almost saturated soil matrix (not shown here). The macropore domain was only partially saturated, with large saturation at depressed areas (Fig. 5a). The sprinkling increased the macropore saturation at all four plots. When the water reached the bedrock the macropores above the bedrock developed a perched zone and a flow along the bedrock surface was initiated. At plot 1, the infiltrating water needed about 5–8 min to reach the bedrock and approximately 20 min to reach the cut-bank.

Figure 6 shows the water flow in the macropore domain through the soil divided by total applied influx (202.2 kg h^{-1}) near the cut-bank over time for the variants A, B, and C. The permeability of the macropore domain was fitted to the observed initiation of water outflow at the cut-bank, which occurred after 22 min (Wienhöfer et al. 2009). A change of the initial water saturation (variant A, B) caused only small differences in the travel times of the water front. However, the relative discharge was larger when the soil had higher initial saturation. For case A the matrix was initially oversaturated and transfer from the matrix to the macropores was dominant (Fig. 7), while water transfer from the macropores to the matrix was dominant when a initial saturation deficit existed (variant B, C). The flow rate increased in four steps, corresponding to the travel time of the different plots (variant A, B). The maximum discharge was obtained in variant A followed by B and then C. The higher transfer coefficient of variant C lead to a slower increase of the discharge. Wienhöfer et al. (2009) reported that seepage flow at the cut-bank was constant after 142 min. Variant A and B reached a constant discharge through the soil approximately at this time. The flow rate in variant C does not get a plateau over the whole sprinkling period so that the transfer coefficient seems to be too large. The discharge does not get equal to the infiltration rate since significant water transfer between macropores and matrix occurred over the whole time for all three variants (Fig. 7).

Seepage flow ended in the field experiment 142 min after stopping the sprinklers. The sprinkling during our numerical studies ended after 480 min and the flow through the soil at the cut-bank decreased after further 22 min stepwise. From that time on, only small differences were seen between the variants A, B and C. The flow rate became almost constant with a very low discharge after 590 and 610 min for variant B, C, and A, respectively. At this time, the remaining flow rate may no longer lead to seepage flow at the cut-bank.

Although many simplifications were made, the simulated water infiltration and flow through the Heumöser slope showed a good agreement of the observed travel times with these of Wienhöfer et al. (2009). The high permeability of the macropore domain enables to transport huge amounts of rainwater during rainstorm events and avoids surface runoff. Using a TPDPM for macroporous soils is recommended for special cases like high rainfall/infiltration, also for domains where macropores only occur in partial areas. There the water can

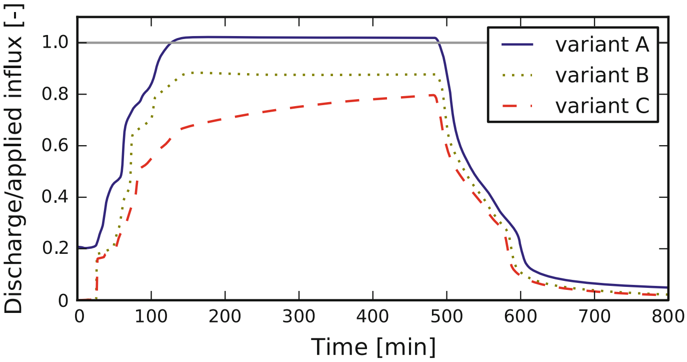


Fig. 6 Discharge through the macropore domain divided by the applied influx at the cut-bank for variant A, variant B with a lower initial saturation and variant C with lower initial saturation and higher transfer coefficient

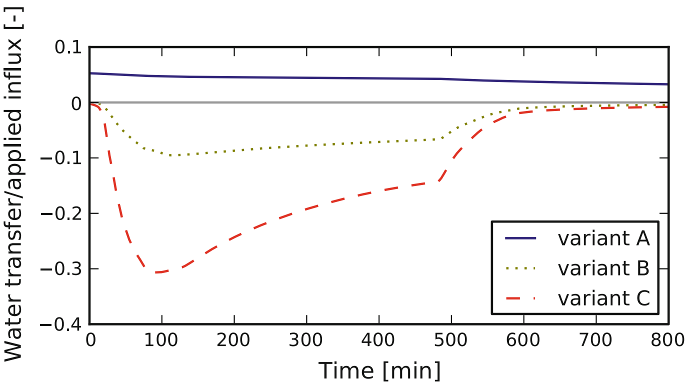


Fig. 7 Water transfer between matrix and matrix domain divided by the applied influx in the upper part above the cut-bank

rapidly infiltrate via the macropores and reach parts where no macropores exist, the further flow will then be hindered by the slow escape of the soil air via the matrix.

3.3 Model Performance

The model performance is an important aspect when simulating large system. On the one hand, the RPDM is less complex and faster since only two balance equations must be solved. On the other hand, we observed that the performance of our implemented RDPM (C++) and TPDPM (a C++ and a Python version) FVM was much lower than the performance of the Dumux version of the TPDPM. This is a result of the optimized C++ code of Dumux, so that the software quality may be often more important as the number of equations that must be solved.

However, a drawback of the higher complexity of Dumux occurs when the model becomes unstable. For example, we observed that the approximated results of the BiCGSTAB solver were sometimes not good enough to compute the correction for the Newton–Raphson method, even the absolute errors were small. This was especially the case if a significant amount of soil air was trapped and the regarded cell volumes were extremely small. Therefore we implemented the TPDPM in our simple C++ FVM code which was originally only developed to solve the RPDM. Later we found that the regularization of the constitutive relationships for the silty clay loam also must be improved. Thus, we implemented the TPDPM model with Python and used Scipy/NumPy (Oliphant 2007) to solve the equations with a direct solver and regularize the constitutive relationships with splines. We regularized the constitutive relationships for each soil independently and used Matplotlib (Hunter 2007) to visualize the differences between the original and regularized functions.

In practice, the cell sizes will be large and the speed of a well regularized model depends mainly on the possible time step size. In our slope studies, the time step size of the model was mainly restricted by the temporal change of the boundary conditions during the infiltration and not by the physical complexity (e.g., trapped air).

4 Conclusions

A TPDPM for macroporous soils has been presented and a one-dimensional infiltration study was conducted to investigate the differences between RDPM and TPDPM when using a linear mass transfer formulation for both fluids. Principal effects such as bypassing of water through macropores and initiation of macropore flow at the soil surface and above the low permeable layer were simulated. For our example we could demonstrate that using a TPDPM and accounting for air flow and transfer between matrix and macropores leed to different results when compared to RDPM. The differences occurred when the flow of soil air was hampered in both domains. We suggest that TPDPM should be preferred if flow of soil air may be hampered. This can be the case in layered macroporous soils in combination with high infiltration rates or soils where the escape of soil air via the macropores is restricted on the surface.

In a second study, we used the TPDPM to simulate fast water infiltration and flow on the bedrock in a macroporous hillslope. The results show a good qualitative agreement with observed travel times of Wienhöfer et al. (2009).

The additional computational effort of the presented TPDPM compared to a RDPM is small for most cases. However, a good regularization of the nonlinear constitutive relationships is necessary to ensure a stable solution. Further we observed that for some cases (very small cell sizes, entrapped air) it could be necessary to use a direct solver for the Newton–Raphson method. Overall, we think that the presented TPDPM is a suitable tool for modeling rainfall induced water infiltration into complex macroporous multi-layered aquifers such as macroporous hillslopes.

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