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Revealed desirability: a novel instrument for social welfare

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Abstract
The note puts forward the idea of *revealed desirability*, a novel instrument, which like revealed preference is observable from choice and important for individual and social welfare. We provide the axiomatic underlying individual's choice model, preliminary experimental results that support the idea, and an appealing allocation rule that uses the revealed desirability information along with the revealed-preference information.

Keywords Revealed preference · Desirability · Nonforced choice · Allocation rules

1 Introduction

The conventional wisdom in the field of normative economics is that evaluating resource allocation of a society based solely on revealed preference information must involve some subjective judgment on how different values, such as efficiency and equality, are compromised. In effect, the Pareto criterion is still the common ground in such evaluations. This paper offers a novel instrument, called *revealed desirability*, which is observable from choice and can provide useful information for refining the set of Pareto optima.

The theory behind revealed desirability is that for each individual, options can be categorized as either desirable or undesirable in a meaningful way. Of course, desirable options are preferred to undesirable options; however, we claim that categorizing options in this manner provides additional useful information for welfare analysis. Consider, for example, the following judgment: $(*)$ a resource

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allocation is strictly socially preferred to another if only the former allows all individuals to obtain a desirable bundle.

A question that immediately arises is whether the aforementioned two categories are well defined. This is analogous to question one of the core assumptions of classical economics; that is, whether agents have well-defined, consistent preferences. We believe that in this respect, desirability is at least as an appealing concept as preference. For example, casual observations suggest that people change their preferences regarding, for instance, what they would like to eat for dinner, while the set of dishes they find desirable (or acceptable, see below) remains rather constant. In a preliminary experiment (presented in online Appendix C), we find evidence that agents are significantly more consistent with regard to what they report as (un)desirable compared to their stated preferences for the same set of products.[1]

A second possible answer to the question of the existence of well-defined preferences is that preferences are measured by choices, and as long as the latter are consistent, they should provide guidance for social planning, regardless of whether or not they reflect one's welfare preferences (Gul & Pesendorfer, 2008). Like the revealed-preference principle, which asserts that a preference between two objects can be revealed by offering a choice between them, we claim that desirability can be revealed from certain observable choices; specifically, choices from situations in which rejecting all the available options is possible (henceforth, nonforced-choice [NFC]). For example, consider Alice, who is taking a walk on the promenade and looking for a place to have dinner, realizing that only McDonald's is open, and deciding not to eat there. Alice's choice provides no information on her preferences, but it does tell us that she finds McDonald's undesirable. Contrasting, if Alice is at home and decides to drive for half an hour to eat at restaurant x, even though several closer eateries are open, she reveals that she finds x desirable. A preliminary experiment (reported in online Appendix B) shows that the NFCs' capture stated desirability well. Specifically, we find no significant difference between an NFC and stated desirability in predicting future stated desirability. This result is in line with the previous results on NFCs; for example, Zakay (1984) found that choosing an option in an NFC situation is negatively correlated with that option's distance from some "ideal option." As with revealed preference, revealed desirability through NFCs can be justified without relying directly on the concept of desirability. For example, judgment ($*$) can be rewritten as follows: a benevolent social planner should prefer allocations in which all agents obtain a bundle that they would choose for themselves in NFC situations, over allocations in which some agents receive a bundle that they would reject in NFC situations. Of course, not all choice situations allow for revealed (un)desirability. For example, if one is unable to leave one's home due to the coronavirus and is, therefore, faced with a limited number of options for dinner, then choosing an option in this forced-choice (FC) situation does not imply that it is desirable.

Revealed preference is almost always studied in an FC setting. Recently, however, Gerasimou (2018, Proposition 2) provides a model in which NFCs allow

[1] However, we acknowledge that the opposite conclusion may hold under different sets of products (see online Appendix C for details).

to reveal an individual's *desirability set* (i.e., the set of options she finds desirable) as well as her preferences between the options in that set. In this paper, we first axiomatically extend Gerasimou's model to include the standard domain of forced-choice (FC) situations to obtain, in addition to the agent's desirability set, her preferences over all choice objects, which is crucial for welfare analysis. Then, in the main section, we provide an example for the potential implications revealed desirability has on the theory of fair allocations. Specifically, we offer an allocation rule, called the *Disjunction Pareto* (DP) rule, which selects all the Pareto optimal allocations with the property that no unsatisfied agent can be better off without making an unsatisfied agent worse off (or turning a satisfied agent into an unsatisfied agent). We provide an axiomatization of this rule along with another normative motivation for considering it, based on its leximin properties.

The two allocation rules that dominate the literature are *free envy* (Varian, 1974; Cole & Tao, 2021) and *egalitarian equivalent* (Pazner & Schmeidler, 1978), both of which use only information on preferences. In fact, while NFCs have been extensively studied,[2] this is the first study to discuss their social implications.

We find an unexpected connection to List (2001) and Brams and Sanver (2009), whose data are mathematically equivalent to those considered by our model, but without any reference to choice. Motivated by a need to escape Arrow's impossibility theorem, List (2001) offers an axiom requiring that the social ordering be invariant to increasing transformations of the individual utility functions that do not distort the signs of the utility levels (i.e., positive, negative, or zero). Thus, the social welfare function may depend on individuals' preferences and a "zero line," which is mathematically equivalent to our setting. List (2001) shows that Arrow's dictatorship can be avoided in this setting, but anonymity remains impossible. This result motivates our focus on allocation rules rather than social welfare functions, as the former do not require a complete ranking of social alternatives.

Brams and Sanver (2009) data are mathematically equivalent to List's (2001) data. However, Brams and Sanver (2009) study voting choice rules, which select the most preferred candidate within each set of possible candidates. Because they study voting systems, their focus is different from ours. For example, their choice rule is nonaxiomatic. In addition, because their data are not observable from individuals' choices, they are susceptible to voting manipulation, and Brams and Sanver (2009) study how pools that precede actual voting may affect the social choice. In this context, they study the *preference-approval voting rule*, [3], which is closely related to the much-discussed concept of *approval voting* (Brams & Fishburn, 1978). Approval voting asks each voter to indicate the set of candidates that she finds acceptable and chooses the candidate with the greatest number of approvals. Similarly, the preference-approval voting rule selects the candidate with the most approvals when she is the only candidate receiving approval from the majority of

[2] From a behavioral perspective, NFCs have been studied extensively in both theoretical literature (e.g., Aĭzerman & Aleskerov, 1995; Clark 1995; Gaertner & Xu, 2004; Gerasimou, 2018), and in experimental literature (e.g., Zakay, 1984; Dhar & Sherman, 1996; Mochon, 2013; Costa-Gomes et al., 2019),

[3] Brams & Sanver, (2009) also study a different rule, which uses a somewhat different data than we consider.

voters or when no candidate receives the majority of approvals. When many candidates receive approval from the majority of voters, one of the majority-approved candidates is chosen based on voters' preference rankings. Thus, this rule prioritizes "approval" over preferences. In contrast, as will be clear from our axiomatization, the DP rule prioritizes the rankings of unsatisfied agents over those of satisfied agents. For example, our rule may not rank the unique majority-approved alternative higher than another alternative if some unsatisfied agents prefer the latter to the former.

The remainder of this note is organized as follows. Section 2 provides the choice theoretic foundations of our model, Sect. 3 discusses its social implications, and Sect. 4 informally discusses three open questions and possible solutions related to revealed desirability. Proofs are relegated to the Appendix.

2 Basic concepts

Let X be a set of alternatives and χ be the set of all nonempty subsets of X. The sets of FC and NFC menus are denoted $\mathcal{F}$ and $\mathcal{N}$, respectively, where both $\mathcal{F}$ and $\mathcal{N}$ are taken to be homeomorphic to χ. In addition, with a slight abuse of notation, we let χ denote $\mathcal{F} \cup \mathcal{N}$. *A (detailed) choice function* $c : \chi \to X \cup \{\emptyset\}$ is any function, such that $c(A) \in A$ for all $A \in \mathcal{F}$ and $c(A) \in A \cup \{\emptyset\}$ for all $A \in \mathcal{N}$.

Thus, $c(A) \neq \emptyset$ denotes the option chosen from menu A, while $c(A) = \emptyset$ means that menu $A \in \mathcal{N}$ is rejected entirely. The following notation will aid the exposition of our main definition: for any binary relation $\succeq$ on X (with asymmetric part $\succ$) and for any $A \in \chi$, let $\max(A, \succeq) := \{x \in A \mid y \succ x \text{ for no } y \in A\}$.

Definition 1 *A choice function* $c : \chi \to X \cup \{\emptyset\}$ *is* rationalizable *if there exist a linear preference relation* $\succeq$ *on* X *and a desirability set* $D \subseteq X$, *such that* $x \succeq y \in D$ *implies* $x \in D$, *and for all* $A \in \chi$:[4]

$$c(A) = \begin{cases} \max(A, \succeq) & \text{if } A \in \mathcal{F} or A \cap D \neq \emptyset, \\ \emptyset & \text{otherwise.} \end{cases}$$

Definition 1 describes an agent who is endowed with a linear preference relation $\succeq$ and a desirability set D, such that any alternative preferred to a satisfying option is itself satisfying. From all FC menus and all NFC menus including options in D, the agent chooses all the $\succeq -$maximal options; all the other menus are rejected.

It is well known that in a setting with only FC menus, WARP is necessary and sufficient for rationalizability.

Weak Axiom of Revealed Preference (WARP). For all $x, y \in X$, $x = c(A)$ and $y \in A$ for some $A \in \chi$ imply that if $y = c(B)$, then $x \notin B$.

[4] A linear order is a complete, transitive, and antisymmetric binary relation our assumption that individual preference are linear is made only for exposition simplicity, and all results can easily generalize to the case of complete order (formal statements and proofs are available on request)

WARP states that if x is revealed preferred to (i.e., chosen over) y, then y will never be chosen when x is available.

Together with WARP, the following axiom is necessary and sufficient to obtain Definition 1.

Weak Axiom of Revealed Desirability (WARD). For all $x \in X$, $x = c(A)$ for some $A \in \mathcal{N}$ implies that if $c(B) = \emptyset$, then $x \notin B$.

WARD states that if x is chosen in some NFC situation (i.e., x is desirable), then it will never be within a menu that is rejected entirely.

Theorem 1 *Let $c : \chi \twoheadrightarrow X$ be a choice function. Then, c satisfies WARP and WARD if and only if it rationalizable.*[5]

Theorem 1 gives us the characterization of rationalizable behavior in a setting that includes both FC and NFC menus; thus, it allows us to test rationalizability nonparametrically using the standard revealed-preference technique á la Samuelson. Two immediate corollaries of Theorem 1 are as follows:

$$x \in D \Leftrightarrow [x = c(A) \text{ for some } A \in \mathcal{N}].$$
$$x \succeq y \Leftrightarrow [x \in c(A) \text{ for some } A \ni y].$$

Thus, Theorem 1 allows us to reveal the agent's preferences and desirability set from her choice behavior. In the Sect. 3, we will utilize this information for the evaluation of social welfare.

3 Implications for social welfare

Let $N = \{1, 2, \ldots, n\}$ be the set of agents. *A society* is a profile $\mathscr{P} = (\succeq_i, D_i)_{i \in N}$, where each $\succeq_i$ and each D_i are as described in Definition 1. Our goal is to utilize the revealed desirability information to refine the set of Pareto optima:

$$O(\mathscr{P}) := \{x \in X \mid \nexists y \in X \backslash \{x\} \text{ such that: } [y \succeq_i x \quad \text{for all } i \in N].$$

For any $x \in X$, let $M(x) := \{i \in N \mid x \notin D_i\}$ denote the set of unsatisfied agents under allocation x. Then, the DP rule can be defined as follows:

$$\mathcal{D}(\mathscr{P}) := \{x \in O(\mathscr{P}) \mid \nexists y \in X \text{ such that: } [y \succeq_i x \quad \text{for all } i \in M(y)]\}.$$

The DP rule selects from the Pareto set all the allocations with the property that no unsatisfied agent can be better off without making an unsatisfied agent worse off (or turning a satisfied agent into an unsatisfied one).

When refining the set of Pareto optima, it is common practice to discuss under what cardinal assumptions on the agents' preferences, maximizing a leximin social welfare function results in the proposed refinement (see examples from the literature below). For this purpose, let a vector of utility functions $u = (u_i)_{i \in N}$ (with each

[5] Gerasimou (2018, Proposition 2) characterized Definition 1 on the domain including only NFC menus using WARP and other three axioms.

$u_i : X \rightarrow \mathbb{R}$) *represent* $(\succeq_i)_{i \in N}$, if for every i, we have $u_i(x) \geq u_i(y)$ if and only if $x \succeq_i y$, and let $\mathscr{U}$ be the set of representations u of $(\succeq_i)_{i \in N}$ with the property that

$$(*) \text{ For all } i, j \in N, \; x \in D_i \quad \text{and} \quad y \notin D_j \quad \text{imply} \quad u_i(x) > u_j(y).$$

Condition $(*)$ requires representation u to allocate higher utility to the satisfied agents than the unsatisfied agents.

Finally, given any representation u of $(\succeq_i)_{i \in N}$, let $L_u(X)$ denote the set of options selected from X by the *leximin criterion*. That is, $L_u(X)$ denotes the subset of X including the options that lexicographically maximize the values of u from the minimal to the maximal.

Proposition 1 *For any admissible profile* $\mathscr{P}$, $\mathcal{D}(\mathscr{P}) = \bigcup_{u \in \mathscr{U}} L_u(O(\mathscr{P}))$.

Proposition 1 states that the DP rule selects an allocation from the Pareto set if and only if it maximizes the leximin criterion for some representation u of $(\succeq_i)_{i \in N}$ that admits property $(*)$. Note that $(*)$ should not be interpreted literally as a measure of cardinal utility, similar to how the implicit cardinal assumptions in the envy-free concept (e.g., Varian, 1974) or in the egalitarian equivalent concept (Pazner & Schmeidler, 1978) should not be interpreted in this manner.[6] However, we find condition $(*)$ very appealing for distributional purposes; thus, Proposition 1 provides strong motivation to consider the DP rule.

The next example demonstrates the applications of the DP rule in a simple society.

Example 1 Let $N = \{1, 2\}$, $X = \{x, y, z\}$, $x \succ_1 y \succ_1 z$ and $z \succ_2 y \succ_2 x$. Then, the Pareto set is $O(\mathscr{P}) = \{x, y, z\}$. Assume further that $D_1 = \emptyset$ and $D_2 = \{z, y\}$. Then, the DP set is $\mathcal{D}(\mathscr{P}) = \{x, y\} \subset O(\mathscr{P})$. By contrast, if $D_1 = \{x, y\}$ (other things being equal), then $\mathcal{D}(\mathscr{P}) = \{y\}$. Finally, if instead, $D_1 = D_2 = \emptyset$, then $\mathcal{D}(\mathscr{P}) = O(\mathscr{P})$.

Two conclusions from Example 1 that can be easily generalized are as follows. First, when all agents have an empty desirability set, $\mathcal{D}$ coincides with the Pareto set. Second, when the set of alternatives with which all agents are satisfied is nonempty, it coincides with $\mathcal{D}$. Moreover, it is not difficult to show that in a standard economic environment, under mild conditions, the DP rule strictly refines the Pareto set, provided that some desirability sets are nontrivial (i.e., $D_i \neq \emptyset, X$ for some i).

3.1 Axiomatization for the DP rule

We provide the axiomatization for the DP rule in a slightly narrower setting. For any alternative $x \in X$, the projection of x for individual i is denoted by x_i.[7] *A society* is a profile $\mathscr{P} = (\succeq_i, D_i)_{i \in N}$, where each $\succeq_i$ is a linear order on $X_i := \{x_i \mid x \in X\}$, and $D_i \subseteq X_i$ is a desirability set. Thus, we assume here that an individual's preferences

[6] The implicit cardinal assumption in the envy-free concept is that the indirect utility of individuals is equal (at the price vector supporting the envy-free allocation), while in egalitarian equivalent concept, the cardinal utilities are equal on some allocation proportional to the vector of initial endowments.

[7] For example, in the standard economic environment, x_i the bundle received by agent i in allocation x.

are independent of the projections of options for the other individuals.[8] A profile $\mathscr{P}$ is *admissible* if any $(\succeq_i, D_i)$ that it contains satisfies $x_i \succeq_i y_i \in D_i$ implies $x_i \in D_i$.

Before providing the axiomatization of the DP rule, we remind the axiomatization of the Pareto rule given in Serrano and Volij (1998).[9] However, to utilize their axioms with those required to obtain the DP rule, we apply them to *a social rule*, which associates with each admissible profile $\mathscr{P}$, a (weak) social preference relation $R(\mathscr{P})$, such that $max(X, R(\mathscr{P})) = O(\mathscr{P})$ for all admissible $\mathscr{P}$. Formally, we assume that $R(\mathscr{P})$ is a binary relation on X with acyclic part $P(\mathscr{P})$.[10]

Axiom IR (*Individual Rationality*) For all $\mathscr{P}$ with $|N| = 1$, $xR(\mathscr{P})y$ if and only if $x_1 \succeq_1 y_1$.

Axiom IR states that in one-person society, the social rule coincides with the preferences of that person.

The next axiom requires some notation. A partition of the set of agents into $k \leq n$ disjoint subsets is denoted $(S_j)_{j=1,\ldots,k}$; given such a partition and an alternative $x \in X$, let x^j denote the projection of option x onto subset $S_j \subseteq N$. Each $\mathscr{P}_j := (\succeq_i, D_i)_{i \in S_j \subseteq N}$ is regarded as a subsociety with $X^j := \{x^j \mid x \in X\}$ being the set of feasible options.

Axiom C (*Consistency*) If $xR(\mathscr{P})y$, then for any $k \leq n$ and any partition of N into $(S_j)_{j=1,\ldots,k}$, we have $x^j R(\mathscr{P}_j) y^j$ for all j.

Axiom C is much discussed in the context of allocations rule (see Thomson (1996) and the reference therein). It uses the notion of partitions, where all the agents in a society are divided into subsocieties. The axiom states that if x is socially preferred to y, then for any division of N into subsocieties, the projection of x onto each subsociety should be socially preferred to the projection of y. As explained by Thomson (2012), the axiom is justified both on normative grounds (as it is linked to the solidarity fairness property) and on operational grounds.

Axiom CC (*Converse Consistency*) $xR(\mathscr{P})y$ if there exists a partition of N into $(S_j)_{j=1,\ldots,k}$, such that $x^j R(\mathscr{P}_j) y^j$ for all j.

Axiom CC states that if there is a partition, such that the projection of option x is more socially desirable than the projection of option y in all subsocieties, then x should be considered more socially desirable than y also in the grand society.

By *an allocation rule*, we refer to any mapping from the set of admissible profiles into χ.

Proposition 2 *(A restatement of Theorem 1 in Serrano and Volij (1998)) Let ϕ be an allocation rule. Then, $\phi = O$ if and only if there exists a social rule $R(\mathscr{P})$*

[8] We explicitly assume this very common and reasonable assumption here, because it is implicit in two of our axioms below (Axioms C and CC).

[9] The axiomatization of Serrano and Volij (1998) is a simple and classic result that seems to be forgotten. For more recent characterizations of the Pareto rule, see Duddy and Piggins (2020) and Kelly (2020).

[10] Acyclicity of $P(\mathscr{P})$ ensures that $\max(X, R(\mathscr{P}))$ is nonempty.

satisfying Axioms IR, C, and CC, such that $\phi(\mathscr{P}) = \max(X, R(\mathscr{P}))$ *for all admissible* $\mathscr{P}$.

To obtain our refinement of the Pareto set, we only need to replace Axiom IR with the following axiom.

Axiom IR' For all $\mathscr{P}$ with $|N| = 1$, $xR(\mathscr{P})y$ if and only if $[x_1 \succeq_1 y_1$ or $x \in D_1]$.

Axiom IR' weakens the conditions for a weak social preference; specifically, for x to not be dominated by any option, it is sufficient that $x \in D_1$. Recall that in this stage, we are only interested in axiomatizing our refinement, which utilizes the revealed desirability information to refine the Pareto set. Thus, when the single agent in the society is satisfied with x, the revealed desirability information cannot be used to eliminate x from the set of socially desired allocations.

Proposition 3 *Let* ϕ *be an allocation rule. Then,* $\phi = \mathcal{D}$ *if and only if there exists a social rule* $R(\mathscr{P})$ *satisfying Axioms IR', C, and CC, such that* $\phi(\mathscr{P}) = \max(O(\mathscr{P}), R(\mathscr{P}))$ *for all admissible* $\mathscr{P}$.

Proposition 3 characterizes the DP rule as a refinement of the Pareto set. This result shows that our refinement preserves the consistency axioms of the Pareto rule. We proceed with the main result of this section, which completely characterizes the DP rule.

Axiom PC (*Pareto Comparability*) For all $\mathscr{P}$, such that $M(x) \cup M(y) = \emptyset$, $xR(\mathscr{P})y$ if and only if $x \succeq_i y$ for all i.

Axiom PC states that when all agents are satisfied with both x and y (i.e., apart from their names, x and y are indistinguishable on the subsociety of unsatisfied agents), the social preference between x and y should coincide with the Pareto rule. The motivation for imposing Axiom PC is that when all agents are satisfied with both options, desirability information is silent and, hence, cannot be utilized to refine the Pareto set.

Axiom CU (*Consistency for the Unsatisfied*) If $xR(\mathscr{P})y$, then for any $k \leq n$ and any partition of $M(x) \cup M(y)$ into $(S_j)_{j=1,\ldots,k}$, we have $x^j R(\mathscr{P}_j) y^j$ for all j.

Axiom C requires that if x is not recommend as socially preferred to y in some society, then adding agents to that society cannot render x recommended over y in the new, larger, society (even if x is recommended as socially preferred to y in the society including the "new agents"). Axiom CU weakens this condition to hold only when the added agents are satisfied with both x and y. Because these agents are already satisfied with both options, their power over the remaining agents (whose some of them are unsatisfied) should be limited.[11] Thus, we find Axiom CU to be a very appealing weakening of Axiom C (see also the discussion after Theorem 2).

[11] Note that Axiom CU has no bite when all agents in the society are satisfied with both options.

Axiom CCP (*Converse Consistency with Priority*) $xR(\mathscr{P})y$ if there exists a nonempty partition of $M(x) \cup M(y)$ into $(S_j)_{j=1,\ldots,k}$, such that $x^j R(\mathscr{P}_j) y^j$ for all j.

Axiom CCP is a version of Axiom CC that prioritizes the preferences of unsatisfied agents. It states that if the unsatisfied agents can be partitioned into subsocieties, such that x is socially preferred to y in all subsocieties, then x should be socially preferred to y also in the grand society regardless of the preferences of the satisfied agents.

Theorem 2 *Let ϕ be an allocation rule. Then, $\phi = \mathcal{D}$ if and only if there exists a social rule $R(\mathscr{P})$ satisfying Axioms IR, PC, CU, and CCP, such that $\phi(\mathscr{P}) = max(X, R(\mathscr{P}))$ for all admissible $\mathscr{P}$.*

Theorem 2 provides axiomatization of the DP rule, which is based on the concept of prioritizing the preferences of the unsatisfied agents. This result, together with Proposition 1, provides a strong motivation for our allocation rule. In addition, Proposition 3 establishes that our rule satisfies both consistency axioms of the Pareto rule when it is viewed as a refinement. Only the combination of the Pareto rule and our refinement requires weakening Axiom C. We note that the axioms used in Theorem 2 are logically independent (see Appendix).

4 Open questions and possible solutions

In this section, we briefly discuss three issues that deserve some attention. The first discuss the case of *strategic rejections*; the second relates to *past-dependent* desirability, and the third to revealed desirability from field data.

Strategic rejections

One difficulty that emerges with the notion of revealed desirability is strategic rejection. In other words, an agent may reject an entire menu, not because no option in the menu is satisfying but because she is expecting with **high certainty** to face a menu that contains better options. This outcome may distort the relationship between NFC choices and desirability.

Note that this situation is also problematic for revealed preference. For example, an agent may choose a bundle that does not contain products of a certain category only, because she expects to be offered better products from that category. To deal with strategic rejection, for any sequence of dated menus with empty choices (A_i), $(A_{i+1},), \ldots, (A_{i+t})$, we can test whether WARP and WARD are consistent when the choice from (A_{i+t+1}) is regarded as a choice from $\bigcup_{i \leq t+1} A_i$. For some data sets, our axiom will hold when we treat each of $(A_i), \ldots, (A_{i+t})$ separately and when we treat them as a single menu; thus, we may find several possible desirability sets that are consistent with the data. This will result in an incomplete identification of the model, whereby not all options can be categorized as desirable or undesirable and where some options have “undetermined desirability.” There are several ways to extend the above analysis to allow for undetermined desirability; we think this is an interesting venue for future research.

Past-dependent desirability

Consider a child who is given as much ice cream for dinner as he wishes for a whole weak; subsequently, her set of desirable dishes would probably shrink. In online Appendix D, we generalize the model presented in Sect. 2 by allowing the threshold to depend on past consumption in a weakly monotonic way (i.e., "better past consumption" implies a weakly higher threshold). We show the existence of an *equilibrium desirability set,* which includes options with the property that, when an agent obtains them repeatedly, she finds them desirable. This equilibrium desirability set allows us to exploit revealed desirability for social welfare purposes, even when the threshold depends on the consumption history.

Revealed desirability from field data

In field data, we often observe bundles being rejected for a certain vector of prices and a certain budget. It is tempting to conclude that when an agent rejects an entire menu in this situation, the affordable bundles are undesirable. However, it may well be the case that rejections in this case are due to mental accounting.[12] Mental accounting is an individual's tendency to relate different expenses to different imaginary accounts, and to make decisions accordingly (Thaler 1999). Thus, an agent may decide to reject all the items in a store, not because all of them are undesirable, and not because she expects to find better prices for them elsewhere (i.e., due to strategic rejection), but because the desirable options in the store lie outside her mental budget set. As mental budgets are not observable, we may wrongly infer that the rejected items are undesirable. Barokas (2020) offered a way to test whether agents restrict their expenses via investment in illiquid accounts, which can also be used to test mental accounting. This may be another topic for future research.

Formal Proofs

Proof of the Theorem 1 The fact that rationalizabiltiy implies WARP and WARD is obvious. For the "only if part" of the theorem, let $\succeq$ on X be defined by the rule $x \succeq y$ if $x \in c(\{x,y\})$ for some $\{x,y\} \in \mathcal{F}$. $\succeq$ is by definition complete, to see that it is also transitive, take x, y, z, such that $x \succeq y \succeq z$, that is, $x \in c(\{x,y\})$ and $y \in c(\{y,z\})$. Now, consider the choice from $c(\{x,y,z\})$, if $z \in c(\{x,y,z\})$, then by WARP, $y \in c(\{x,y,z\})$, which then implies $x \in c(\{x,y,z\})$, for otherwise WARP would imply that $x \notin c(\{x,y\})$—a contradiction. Thus, $x \in c(\{x,y,z\})$ holds in any case (recall that $c(\{x,y,z\})$ is nonempty). This implies that $x \succeq z$ must hold, for otherwise $z = c(\{x,z\})$ would contradict WARP. This shows that $\succeq$ is transitive.

Next, define $D := \{x \in X \mid \{x\} = c(\{x,y\})$ with $\{x,y\} \in \mathcal{N}\}$. We now show that $x \succeq y \in D$ implies $x \in D$. $x \succeq y \in D$ implies $\{y\} = c(\{y\})$ for $\{y\} \in \mathcal{N}$ and $x \in c(\{x,y\})$ which imply, by WARD and WARP, respectively, that $[c(\{x,y\}) \neq \emptyset$ for any $\{x,y\} \in \chi]$ and $[y \in c(\{x,y\})$ implies $x \in c(\{x,y\})]$. Thus, $x \in c(\{x,y\})$ for any $\{x,y\} \subseteq \chi$ must be the case; otherwise, $c(x,y) = \emptyset$—a contradiction. However, $x \in c(\{x,y\})$ for $\{x,y\} \in \mathcal{N}$ imply by WARD that $\{x\} = c(\{x\})$ for $\{x\} \in \mathcal{N}$ and, hence, $x \in D$, as asserted.

We now show that Definition 1 holds for the constructed $\succeq$ and D. Assume that $c(A) \neq \max(A, \succeq)$ either for some $A \in \mathcal{F}$ or for some $A \in \mathcal{N}$ with $A \cap D \neq \emptyset$. Then, either we have (i) $x \in c(A)$ but $x \notin c(\{x,y\})$

[12] Of course, this is also problematic for revealed preference, where an agent may choose against what maximizes her welfare due to mental accounting.

for some $y \in A$ for $\{x,y\} \in \mathcal{F}$ or (ii) $x \notin c(A)$ but $x \in c(\{x,y\})$ for $\{x,y\} \in \mathcal{F}$ for all $y \in A$. However, (i) violates WARP, and provided that $c(A) \neq \emptyset$ so is (ii). Thus, assume that (ii) holds and that $c(A) = \emptyset$, then by $A \cap S \neq \emptyset$, we have a violation of WARD. Finally, assume that $A \in \mathcal{N}$ and that $A \cap D = \emptyset$. Then, by WARD, $c(A) = \emptyset$, and the proof is complete. □

Proof of the Proposition 1 For any $x \in \mathcal{D}(\mathscr{P})$, define $u \in \mathscr{U}$, such that $u_i(x) = a$ for all $i \in M(x)$, $u_i(x) = b > a$ for all $i \notin M(x)$, and $u_i(y) < a$ for all y such that $x \succeq_i y$. Now, if $x \notin L_u(P(\mathscr{P}))$, then there is $y \in O(\mathscr{P})$, such that

(i) $u_{j_1}(x) \leq u_i(y)$ for all i, and if $u_{j_1}(x) = u_{k_1}(y)$ for some k_1, then there exists j_2, such that $u_{j_2}(x) \leq u_i(y)$ for all $i \neq k_1$, and so on up to $u_{j_{n-1}}(x) \leq u_i(y)$ for $i = k_{n-1}, k_n$, and $u_{j_{n-1}}(x) = u_{k_{n-1}}(y)$ implies $u_{j_n}(x) < u_{k_n}(y)$.

(i) implies (ii) $u_i(y) \geq a$ for all $i \in M(x)$ and either [(iii) $u_i(y) > a$ for some $i \in M(x)$ or (iv) $u_i(y) \geq b$ for all $i \in N \backslash M(x)$ and $u_i(y) > b$ for some $i \in N$]. If (ii)–(iii) holds, then $x \notin D(\mathscr{P})$, and if (ii) and (iv) hold, then $x \notin O(\mathscr{P})$, so in any case, we have a contradiction.
For the other direction, assume that $x \notin \mathcal{D}(\mathscr{P})$, that is, there exists $y \in O(\mathscr{P})$, such that $y \succeq_i x$ for all $i \in M(y)$ and $y \succ_i x$ for some $i \in M(x)$. That is, for any representation u of $(\succeq_i)_{i \in N}$, we have $u_i(y) \geq u_i(x)$ for all $i \in M(y)$ and $u_i(y) > u_i(x)$ for some $i \in M(x)$. Note that if $u \in \mathscr{U}$, then $(*)$ implies that $min_{i \in N} u_i(x) \leq \min_{i \in N} u_i(y)$ and that if $min_{i \in N} u_i(x) = \min_{i \in N} u_i(y)$, then for $i_1(x) := \text{argmin}_{i \in N} u_i(x)$ and $i_2(x) := \text{argmin}_{i \in N \backslash \{i_1(x)\}} u_i(x)$, we have that $i_2(x) \in M(x)$. Thus, $\min_{i \in N \backslash \{i_1(x)\}} u_i(x) \leq \min_{i \in N \backslash \{i_1(y)\}} u_i(y)$ and if $\min_{i \in N \backslash \{i_1(x)\}} u_i(x) = \min_{i \in N \backslash \{i_1(y)\}} u_i(y)$, then defining $i_3(x)$ analogously, we find that $i_3(x) \in M(x)$ and so on, and because $M(x)$ is finite, it follows by $u_i(y) > u_i(x)$ for some $i \in M(x)$ that $x \notin L_u(O(\mathscr{P}))$. This completes the proof. □

Proof of Proposition 2 First note that for any binary relation $R^*(\mathscr{P})$ on X defined by the rule $xR^*(\mathscr{P})y \iff [x \succeq_i y$ for all $i]$, we have $O(\mathscr{P}) = max(X, R(\mathscr{P}))$. Now, the fact that R^* satisfies all the required axiom is straightforward. Conversely, the fact that $xR^*(\mathscr{P})y$ implies $xR(\mathscr{P})y$ follows directly from Axioms IR and CC. Finally, assume that $\neg(xR^*y)$ but xRy, then we have $y_i \succ_i x_i$ for some i and there is a partition with $S^1 = \{i\}$, and by Axiom C, we must have $y^1 R(\mathscr{P}_1)x$—this violates Axiom IR. □

Proof of Proposition 3 First, note that for the binary relation $R^*(\mathscr{P})$ on X defined by $xR^*(\mathscr{P})y \iff [x \in D_i$ or $x \succeq_i y]$ for all i, we have $\mathcal{D}(\mathscr{P}) = max(O(\mathscr{P}), R^*(\mathscr{P}))$ for all $\mathscr{P}$. Second, we show that $R^*(\mathscr{P})$, just defined, is transitive for all $\mathscr{P}$. Assume that $xR^*(\mathscr{P})yR^*(\mathscr{P})z$, and then, $x_i \notin D_i$ implies $x_i \succeq_i y_i$, which implies $y_i \notin D_i$, which in turn implies $y_i \succeq_i z_i$, and by the transitivity of $\succeq_i$, we find $x_i \succeq_i z_i$, and since this holds for all i, we have $xR^*(\mathscr{P})z$. Now, the fact that $R^*(\mathscr{P})$ satisfies all the required axioms is straightforward. For the other direction, assume that xR^*y (i.e., $[x_i \notin D_i$ implies $x_i \succeq_i y_i]$ for all i) and consider the partition of N into $|N|$ one-person subsocieties. Then, by IR', we have $x^i R(\mathscr{P}_i) y^i$ for all i, and by Axiom CC, $xR(\mathscr{P})y$. Finally, assume that $x_i \notin D_i \ni y_i$ for some i, but $xR(\mathscr{P})y$. Then, there exists a partition with $S^1 = \{i\}$, and by Axiom C, we must have $x^1 R(\mathscr{P}_1) y^1$—this violates Axiom IR'. □

Proof of Theorem 2 Let R^P denote the Pareto rule (i.e., xR^Py if $x \succeq_i y$ for all i), and note that for the relation $R^{**}(\mathscr{P})$ on X defined by $xR^{**}(\mathscr{P})y \Longleftrightarrow [xR^Py$ or $[xR^*(\mathscr{P})y$ and $M(y) \neq \emptyset]]$, (where $R^*(\mathscr{P})$ is as defined in the proof of Proposition 3), we have $\mathcal{D}(\mathscr{P}) = max(X, R^{**}(\mathscr{P}))$ for all $\mathscr{P}$. Second, we show that $P^{**}(\mathscr{P})$ is transitive. Define the binary relation Q on X by xQy if and only if $[xR^*(\mathscr{P})y$ and $x \succeq_i y$ for some $i \in M(y)]$, and note that $xP^{**}(\mathscr{P})y$ if and only if $[xR^Py$ or $xQy]$. Now, assume that $xP^{**}(\mathscr{P})yP^{**}(\mathscr{P})z$. Then, we have four cases: xR^PyR^Pz, $xQyQz$, xR^PyQz, and $xQyR^Pz$. The case of xR^PyR^Pz follows from the transitivity of R^P. We now show that $xQyQz$ implies xQz. Note that $xQyQz$ implies that if $x \notin D_i$, then $y \notin D_i$ and $z \notin D_i$. Now, because $xQyQz$ implies that $x \succeq_i y$ for all $x \notin D_i$, and $y \succeq_i z$ for all $y \notin D_i$, we find that $x \succeq_i z$ for all $x \notin D_i$. Thus, $x \succeq_i y$ for some $i \in M(y)$ implies both that $x \succeq_i z$ and that $i \in M(z)$. This together with $x \succeq_i z$ for all $x \notin D_i$ implies xQz. The fact that each of xR^PyQz and $xQyR^Pz$ implies xQz, follows in the same way and, thus, $P^{**}(\mathscr{P})$ is transitive. Being asymmetric and transitive $P^{**}(\mathscr{P})$ is also acyclic, as required.

It now follows directly that $R^{**}(\mathscr{P})$ satisfies Axioms IR, PC, and CCP. In addition, Axiom CU is satisfied, because $xR^{**}(\mathscr{P})y$ implies that $x_i \succeq_i y_i$ for all $i \in M(x) \cup M(y)$. For the other direction, assume that $xR^{**}(\mathscr{P})y$, and consider the case that $M(y) \neq \emptyset$. Then, there exists a nonempty partition of the agents in $M(y)$ into $|M(y)|$ subsocieties each including a single agent. By Axiom IR, we then have that $x^jR^{**}(\mathcal{P}_|)y^j$ for all j. Thus, Axiom CCP implies $xR(\mathscr{P})y$. Otherwise, $M(y) = \emptyset$ and $xR^{**}(\mathscr{P})y$ implies $x \succeq_i y$ for all i, and by Axiom PC, we obtain $xR^{**}(\mathscr{P})y$.

Finally, assume that $\neg(xR^{**}(\mathscr{P})y)$, and then, one of the following holds: (i) there exists $i \in M(y)$, such that $y \succeq_i x$, (ii) $M(y) = \emptyset$ and $y \succeq_i x$ for some i. However, case (i) implies that there is a partition with $S_1 = \{i\}$, and by Axiom IR, we have $y^1P(\mathcal{P}_|)x^1$. Thus, by Axiom CU, we find that $\neg(xR(\mathscr{P})y)$. In addition, case (ii) implies, owing to Axiom PC, that $\neg(xR(\mathscr{P})y)$. This completes the proof. □

For the independence of the axioms used in Theorem2, we give four examples; each provides a social rule $R(\mathscr{P})$ for which the resulting $max(X, R(\mathscr{P}))$ differs from $max(X, R^{**}(\mathscr{P}))$, and satisfies all our axioms but one. First, consider the Pareto rule $xR^P(\mathscr{P})y$ if and only if $x_i \succeq_i y_i$ for all i; this rule satisfies all our axioms but CCP. Second, the rule $xR(\mathscr{P})y$ if and only if $[[M(x) \neq \emptyset$ and $|M(x))| \geq |M(y)|]$ or $xR^Py]$ satisfies all our axiom but CU. Third, recall that $R^*(\mathscr{P})$ is defined by $xR^*(\mathscr{P})y$ if and only if $[x \in D_i$ or $x \succeq_i y]$ for all i, and consider the rule $xR(\mathscr{P})y$ if and only if $[[xR^*(\mathscr{P})y$ and $|N| \neq 1]$ or $[xR^P(\mathscr{P})y$ and $|N| = 1]]$, this rule satisfies all our axioms but PC. Finally, the rule $xR(\mathscr{P})y$ if and only if $[M(x) \subseteq M(y) \neq \emptyset$ or $[M(x) \cup M(y) = \emptyset$ and $xR^Py]]$ satisfies all our Axioms but IR.

Supplementary Information

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