OBITUARY

Peter C. Fishburn (1936–2021)

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We are sorry to report that Peter C. Fishburn died on June 10, 2021, in Racine, Wisconsin. He was born September 2, 1936, and is survived by his wife, Janet Forsythe Fishburn, three daughters, two grandsons, and three great grandchildren. He was a leading figure in decision theory, utility theory, and social choice, among many other topics.

Peter graduated from Pennsylvania State University in 1958 with a degree in Industrial Engineering and he received a Ph.D. in Operations Research in 1962 from Case Institute of Technology. He worked for the Research Analysis Corporation in McLean, Virginia (1962–1970), was a Research Professor at Penn State (1971–1978) and ended his career as a mathematics researcher at Bell Laboratories in New Jersey (1978–2001). He was also a Fulbright Professor at the Technical University in Copenhagen (1966) and a visiting member of the Institute for Advanced Study in Princeton, NJ (1970–1971).

Peter was amazingly prolific in several fields, including many of the topics covered by *Theory and Decision*: utility theory and expected utility, social choice and voting, decision trees, measurement, risk, fair division, game theory, etc. During his distinguished career, Peter authored eight books and co-authored a ninth, wrote over 500 journal articles with more than 80 collaborators, and received numerous awards and prizes, including the Frank P. Ramsey Medal for his work in decision analysis, and the John von Neumann Prize for his research in operations research and management science.

Peter's publications in *Theory and Decision*, which we will restrict our discussion to here, represent the wide swath of his research interests. Among those interests are axiom systems for different kinds of utility representations. For instance, paper (Fishburn & Edwards, 1997) formulates and axiomatizes utility models for denumerable time streams that make no commitment in regard to discounting future outcomes. The models address decision under certainty and decision under risk. Independence assumptions in both contexts lead to additive or multiplicative utilities over time periods that allow unambiguous comparisons of the relative importance of different periods. This discount-neutral feature is attained by restricting preference comparisons to outcome streams or probability distributions on outcome streams that differ in at most a finite number of periods. Paper (Fishburn & LaValle, 1996) studies self-reflecting signed orders that were introduced to account for negative as well as positive feelings about inclusion of items in potential subsets of choice. The paper extends the notion of signed orders to lotteries and describes axioms that simplify utility representations for preferences between lotteries. Paper (Fishburn & Roberts, 1978) studies expected utility theories. It discusses mixtures, which can be interpreted as lotteries that yield one prospect with probability α and another prospect with probability 1-α. This paper shows how axioms about mixtures that use a decision maker's indifference relation can be used to obtain general axiomatizations for the traditional linear utility form and a multilinear utility form.

Axiom systems are part of Fishburn's work in other applications as well. Paper (Fishburn, 1993) studies event ambiguity as a primitive concept. Axioms are described for a comparative ambiguity relation on an arbitrary event set that are necessary and sufficient for a representation of the relation by a functional that is

nonnegative, vanishes at the empty event, and satisfies complementary equality and submodularity. Uniqueness characteristics of representing functionals are discussed. The theory is extended to multifactor events, where marginal ambiguity and additive representations arise.

Two themes in the axiomatic theory of representational measurement that have been studied extensively in the past few years are numerical representations of nontransitive binary comparison structures and uniqueness in finite measurement systems. The first theme is exemplified by unique subjective probability on finite sets and the second by skew-symmetric bilinear generalizations of von Neumann-Morgenstern linear utility theory. Paper (Fishburn, 1990) brings the two themes together by exploring the solutions to a nontransitive, additive model that are unique up to multiplication by a positive constant. The model relates to various contexts including decision under risk, evaluation of objectives, comparative probability, and voting theory. The family of unique solutions for the model is shown to be extremely rich and varied. Paper (Gehrlein & Fishburn, 1977) is also interested in measurement, specifically measurement in the social sciences. Measurement in the social sciences often involves an attempt to completely order a set of entities on the basis of an underlying attribute. However, limitations of the measurement process often prevent complete empirical determination of the desired ordering. Nevertheless, the ordinal data obtained from the measurement process can be used in attempting to recover or construct more of the underlying order than is provided by the data. This paper examines a simple one-stage construction method, referred to as the "cardinal rule," that is fairly effective in correctly identifying ordered pairs in the underlying linear order that are not identified by the measurement process. It studies construction methods based on simple counting measures derived from the data, and it argues that it is the best one-stage method in this class when a natural monotonicity assumption holds for the measurement process.

Expected utility theory was a major part of Fishburn's work over his career. Several T*heory and Decision* papers deal with this topic. For example, papers (Fishburn, 1975) and (Fishburn, 1981) deal with subjective expected utility theory. Paper (Fishburn, 1981) reviews theories of subjective expected utility for decision making under uncertainty (up through 1980). It focuses on normative interpretations and discusses the primitives, axioms and representation-uniqueness theorems for a number of theories. Similarities and differences among the various theories are highlighted. The interplay between realistic decision structures and structural axioms that facilitate mathematical derivations is also emphasized. It includes theories developed by Ramsey; Savage; Suppes; Davidson and Suppes; Anscombe and Aumann; Pratt, Raiffa and Schlaifer; Fishburn; Bolker; Jeffrey; Pfanzagl; Luce and Krantz. Paper (Fishburn, 1975) shows how notions of vagueness in preferences and judgments of personal probabilities can be accommodated within an axiomatization of subjective expected utility by the use of extraneous scaling probabilities and gambles on consequences. Paper (Fishburn, 1974) studies the notion of stochastic dominance in expected-utility decision theory, which has been developed to facilitate the analysis of risky or uncertain decision alternatives when the full form of the decision maker's von Neumann-Morgenstern utility function on the consequence space is not completely specified. It discusses potential applications of

convex stochastic dominance such as analyses of simple-majority voting on risky alternatives when voters have similar preference orders on the consequences.

Other work deals with properties of risk and decision, utility, and lotteries. Paper (Fishburn, 1984) discusses a theorem of John Harsanyi that relates social welfare to the welfares of individual members of a social group. The theorem explores utility of lotteries and studies the structure of the utility function in the situation where the utilities of lotteries p and q are equal whenever they are equal under the utility for each individual in the group. In that paper, Fishburn gives a new proof of the theorem and explains issues in the literature with the correctness of the theorem. Paper (LaValle & Fishburn, 1987) studies decision trees as ways of identifying strategies optimal for a decider. It defines weak and strong forms of strategic equivalence of decision trees and partial normalization procedures by which all economically defined complete pure strategies may be identified for the decider and for "chance." It also studies characteristics of locally randomized "behavior" strategies for the decider as the degree of normalization is varied. Still a third paper, (Fishburn, 1979), starts with the well-known observation that comparisons of risky options that are based on the means and variances of the probability distributions on wealth that are associated with the options encounter serious logical difficulties unless suitable restrictions are imposed on the class of relevant distributions, on the range of possible outcomes, or on the shape of the decision agent's utility function for wealth in the event that congruence is desired between a mean–variance indifference map and expected utility maximization. The paper argues that there do not appear to be precise statements of the logical implications of several basic presuppositions of mean–variance analyses in conjunction with the very weak axiom that more money is preferred to less, and it aims to provide such statements.

Another major theme of Fishburn's work revolves around social choice, social welfare, elections, and voting. A series of papers in *Theory and Decision* reflect this theme. Paper (Campbell & Fishburn, 1980) provides a general formulation of anonymity and neutrality conditions for social choice functions. An alternative is viewed as the level (qualitative or quantitative) of a good assigned to an individual. A principal contribution of the paper is a notion of strong anonymity, which is based not only on permutations of preference relations among individuals but also on possibly different permutations or reassignments of levels of goods among individuals. Paper (Fishburn, 1972) discusses aspects of the theory of social choice when a nonempty choice set is to be determined for each situation, which consists of a feasible set of alternatives and a preference order for each voter on the set of nonempty subsets of alternatives. The individual preference assumptions include ordering properties and averaging conditions, the latter of which are motivated by the interpretation that subset A is preferred to subset B if and only if the individual prefers an even-chance lottery over the basic alternatives in A to an even-chance lottery over the basic alternatives in B.

There are numerous dimensions studied in the search for better election procedures, including the nomination process, agenda formation, candidate strategy, voter psychology and strategy, ballot forms and methods of aggregation, evaluative aspects of aggregation, incentive compatibility, costs and financing, and institutional effects. In paper (Fishburn, 1983), Fishburn comments on the history of the

search for better election procedures and discusses dimensions such as those above that are relevant to the evaluation and comparison of competing procedures. In paper (Fishburn & Gehrlein, 1982), Fishburn and Gehrlein discuss the propensities of different voting procedures to elect the simple majority candidate when one exists in a multicandidate election. They summarize this research area through a discussion of assumptions used in simulations and extend their interpretations to account for potential political realities that were not incorporated in the simulations.

Fishburn's research is characterized by his interest in a wide variety of topics. This is reflected in work on such issues as fair division and game theory. Paper (Brams et al., 2003) analyzes criteria of fair division of a set of indivisible items among people whose revealed preferences are limited to rankings of the items and for whom no side payments are allowed. The criteria include refinements of Pareto optimality and envy-freeness as well as dominance-freeness, evenness of shares, and two criteria based on equally-spaced surrogate utilities, referred to as maxsum and equimax. Maxsum maximizes a measure of aggregate utility or welfare, whereas equimax lexicographically maximizes persons' utilities from smallest to largest. The paper analyzes conflicts among the criteria along with possibilities and pitfalls of achieving fair division in a variety of circumstances. Paper (Fishburn & Kilgour, 1990) deals with noncooperative game theory. In this subject, there is a long tradition of analyzing conflicts using only ordinal information about players' preferences. This paper studies the 2×2 game, the simplest interactive decision model that portrays concerned decision makers with genuine choices. There are no restrictions on the players' preference relations over the outcomes, and they generalize special cases such as the strict ordinal 2×2 games where each player has a strict preference order over the four outcomes. Paper (Fishburn & Kilgour, 1990) enumerates the strategically distinct binary 2×2 games and identifies important subsets defined by the number of pure Nash equilibria and the occurrence of dominant strategies.

The variety of topics covered in Peter's *Theory and Decision* papers illustrates the significant impact his work has had on numerous areas of study, and this is just a hint at the many contributions that Peter made to decision theory, utility theory, social choice, and other subjects. He was an inspiration to us, and his many other colleagues, and will be sorely missed by many people.

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