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CORRECTION

# Correction to: Multi-relay selection in energy-harvesting cooperative wireless networks: game-theoretic modeling and analysis

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## Correction to: Telecommunication Systems https://doi.org/10.1007/s11235-019-00611-6

Unfortunately, the original publication contains production errors. We would like to correct the errors as given below:

(a) The fourth author email address should read as "almubarak.13@osu.edu" instead of "Almubarak.13@osu.edu.kw."
(b) The following equations 5, 14, 17, 19, 25, 26, 27, 32, 36, 40, 42, 46, 48, 49 should read as below.
(c) The equations in the algorithm 2,3 should read as below.

The original article has been updated.

$$p_{\xi_{r_k}^{\zeta}}(\xi) \triangleq \mathbb{P}\left[\xi_{r_k}^{\zeta} = \xi\right] = \sum_{m=0}^{\infty} \mathbb{P}\left[\xi_{r_k}^{\zeta} = \xi | m\right] \cdot \mathbb{P}\left[\mathcal{N}_{r_k}^{\zeta} = m\right], \tag{5}$$

$$\mathcal{R}_i^{\zeta}\left(\mathbf{E}_{C_i}^{\zeta}, \boldsymbol{\mathcal{I}}_i^{\zeta}\right) = \frac{1}{N+1}\log_2\left(1 + \frac{E_{B_i}^{\zeta}|h_{i,d}^{\zeta}|^2}{N_0} + \sum_{k=1}^{K}\mathcal{I}_{i,k}^{\zeta}\frac{E_{B_i}^{\zeta}E_{C_{i,k}}^{\zeta}|h_{i,k}^{\zeta}|^2|h_{k,d}^{\zeta}|^2}{N_0\varrho_N\left(E_{B_i}^{\zeta}|h_{i,k}^{\zeta}|^2 + E_{C_{i,k}}^{\zeta}|h_{k,d}^{\zeta}|^2 + N_0\right)}\right), \tag{14}$$

$$\mathcal{R}_i\left(\mathbf{E}_R^{\zeta}, \mathbf{n}_R^{\zeta}\right) = \frac{1}{N+1}\log_2\left(1 + \frac{E_{B_i}^{\zeta}|h_{i,d}^{\zeta}|^2}{N_0} + \sum_{k=1}^{K}\mathcal{I}_{i,k}^{\zeta}\frac{E_{B_i}^{\zeta}\left(\frac{E_{r_k}^{\zeta}}{n_{r_k}^{\zeta}}\right)|h_{i,k}^{\zeta}|^2|h_{k,d}^{\zeta}|^2}{N_0\varrho_N\left(E_{B_i}^{\zeta}|h_{i,k}^{\zeta}|^2 + \left(\frac{E_{r_k}^{\zeta}}{n_{r_k}^{\zeta}}\right)|h_{k,d}^{\zeta}|^2 + N_0\right)}\right), \tag{17}$$

$$\Delta\mathcal{R}_{i,k}\left(E_{r_k}^{\zeta}, n_{r_k}^{\zeta}\right) = \frac{1}{N+1}\log_2\left(1 + \frac{\left(\frac{E_{r_k}^{\zeta}}{n_{r_k}^{\zeta}}\right)\cdot\Omega_{i,k}^{\zeta}}{\left(\frac{E_{r_k}^{\zeta}}{n_{r_k}^{\zeta}}\right) + \Upsilon_{i,k}^{\zeta}}\right), \tag{19}$$

$$\beta_{i,k}^{\zeta}(\theta_k)\frac{p_{\mathcal{E}_{r_k}^{\zeta}}\left(\phi_{i,k}^{\zeta}|\theta_{r_k}\right)p_{r_k}^{\zeta-1}(\theta_{r_k})}{\sum_{\theta_{r_k}\in\Theta}p_{\mathcal{E}_{r_k}^{\zeta}}\left(\phi_{i,k}^{\zeta}|\theta_{r_k}\right)p_{r_k}^{\zeta-1}(\theta_{r_k})}, \ \forall\theta_{r_k}\in\Theta, \tag{25}$$

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$$p_{r_k}^{\zeta}(\theta_{r_k}) = \frac{1}{N}\sum_{i=1}^{N}\left[\mathcal{I}_{i,k}^{\zeta}\beta_{i,k}^{\zeta}(\theta_{r_k}) + \left(1-\mathcal{I}_{i,k}^{\zeta}\right)p_{r_k}^{\zeta-1}(\theta_{r_k})\right], \tag{26}$$

$$p_{r_k}^{\zeta}\left(\theta_{r_k}\right) = p_{r_k}^{\zeta-1}\left(\theta_{r_k}\right) + \frac{1}{N}\sum_{i=1}^{N}\mathcal{I}_{i,k}^{\zeta}\left(\frac{p_{\mathcal{E}_{r_k}^{\zeta}}\left(\phi_{i,k}^{\zeta}|\theta_{r_k}\right)}{\Lambda\left(\phi_{i,k}^{\zeta}\right)}\right)p_{r_k}^{\zeta-1}\left(\theta_{r_k}\right), \quad \forall\theta_{r_k}\in\Theta \text{ and } \forall k\in\{1,2,\ldots,K\}, \tag{27}$$

$$\mathbb{U}_{i,k}\left(\mathbf{p}_{r_k}^{\zeta}, n_{-i,k}^{\zeta}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{i,k}\left(\xi, n_{-i,k}^{\zeta}+\mathcal{I}_{i,k}^{\zeta}\right)p_{\xi_{r_k}^{\zeta}}\left(\xi|\theta_{r_k}\right)p_{r_k}^{\zeta}\left(\theta_{r_k}\right), \tag{32}$$

$$\mathbb{U}_{i,k}|_{\boldsymbol{\mathcal{I}}_i^{\zeta}=\boldsymbol{\omega}_q} = \omega_{q,k}\sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{i,k}\left(\xi, n_{s_i,r_k}^{\zeta}+\mathcal{J}_{i,k}^{\zeta}|_{\boldsymbol{\mathcal{I}}_i^{\zeta}=\boldsymbol{\omega}_q}+\omega_{q,k}\right)p_{\xi_{r_k}^{\zeta}}\left(\xi|\theta_{r_k}\right)p_{r_k}^{\zeta}\left(\theta_{r_k}\right). \tag{36}$$

$$\boldsymbol{\mathcal{I}}_i^{\zeta,*}\left(\mathbf{p}_R^{\zeta}, \mathbf{n}_{-i}^{\zeta}\right) = \sum_{k=1}^{K}\underset{\mathcal{I}_{i,k}^{\zeta}\in\{0,1\}}{\operatorname{argmax}}\ \mathcal{I}_{i,k}^{\zeta}\cdot\mathbb{U}_{i,k}\left(\mathbf{p}_{r_k}^{\zeta}, n_{-i,k}^{\zeta}\right). \tag{40}$$

$$\mathbb{U}_{i,k}\left(\mathbf{p}_{r_k}^{\zeta}, n_{s_i,r_k}^{\zeta}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{i,k}\left(\xi, n_{s_i,r_k}^{\zeta}+\mathcal{J}_{i,k}^{\zeta}|_{\mathcal{I}_{i,k}^{\zeta}=1}+1\right)p_{\xi_{r_k}^{\zeta}}\left(\xi|\theta_{r_k}\right)p_{r_k}^{\zeta}\left(\theta_{r_k}\right). \tag{43}$$

$$\Delta\mathcal{R}_i\left(\mathbf{E}_R^{\zeta}, \mathbf{n}_R^{\zeta}\right) = \frac{1}{N+1}\left[\log_2\left(1+\frac{E_{B_i}^{\zeta}|h_{i,d}^{\zeta}|^2}{N_0} + \sum_{k=1}^{K}\mathcal{I}_{i,k}^{\zeta}\frac{E_{B_i}^{\zeta}\left(\frac{E_{r_k}^{\zeta}}{n_{r_k}^{\zeta}}\right)|h_{i,k}^{\zeta}|^2|h_{k,d}^{\zeta}|^2}{N_0\varrho_N\left(E_{B_i}^{\zeta}|h_{i,k}^{\zeta}|^2+\left(\frac{E_{r_k}^{\zeta}}{n_{r_k}^{\zeta}}\right)|h_{k,d}^{\zeta}|^2+N_0\right)}\right) - \log_2\left(1+\frac{E_{B_i}^{\zeta}|h_{i,d}^{\zeta}|^2}{N_0}\right)\right]. \tag{46}$$

$$\mathbb{U}_{i,k}\left(\mathbf{p}_{r_k}^{\zeta}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{i,k}\left(\xi, \mathcal{I}_{i,k}^{\zeta}\right)p_{\xi_{r_k}^{\zeta}}\left(\xi|\theta_{r_k}\right)p_{r_k}^{\zeta}\left(\theta_{r_k}\right). \tag{48}$$

$$\mathbb{U}_{i,k}\left(n_{s_i,r_k}^{\zeta}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{i,k}\left(\xi, n_{s_i,r_k}^{\zeta}+\mathcal{I}_{i,k}^{\zeta}\right)p_{\xi_{r_k}^{\zeta}}\left(\xi|\theta_{r_k}\right)p_{r_k}^{0}\left(\theta_{r_k}\right), \tag{49}$$

**Algorithm 2 : Best-response with constrained selections BR_CS $\left(i, \mathbf{n}_{s_i}^{\zeta}, \mathbf{p}_R^{\zeta}\right)$**

IF $i \neq N$
  FOR $q = 1 : Q$
    $\left(\boldsymbol{\mathcal{I}}_{i+1}^{\zeta}, \boldsymbol{\mathcal{J}}_{i+1}^{\zeta}\right) \leftarrow$ BR_CS $\left(i+1, \mathbf{n}_{s_i}^{\zeta}+\boldsymbol{\omega}_q, \mathbf{p}_R^{\zeta}\right)$;
    $\boldsymbol{\mathcal{J}}_i^{\zeta} \leftarrow \boldsymbol{\mathcal{J}}_{i+1}^{\zeta}+\boldsymbol{\mathcal{I}}_{i+1}^{\zeta}$;
    Calculate $\mathbb{U}_i|_{\boldsymbol{\mathcal{I}}_i^{\zeta}=\boldsymbol{\omega}_q} = \sum_{k=1}^{K}\mathbb{U}_{i,k}|_{\boldsymbol{\mathcal{I}}_i^{\zeta}=\boldsymbol{\omega}_q}$;
  END FOR
  $\boldsymbol{\omega}^* = \operatorname{argmax}_{\boldsymbol{\omega}_q\in\boldsymbol{\Omega}}\mathbb{U}_i|_{\boldsymbol{\mathcal{I}}_i^{\zeta}=\boldsymbol{\omega}_q}$;
  IF $\mathbb{U}_i|_{\boldsymbol{\mathcal{I}}_i^{\zeta}=\boldsymbol{\omega}^*} > 0$;
    $\left(\boldsymbol{\mathcal{I}}_{i+1}^{\zeta}, \boldsymbol{\mathcal{J}}_{i+1}^{\zeta}\right) \leftarrow$ BR_CS $\left(i+1, \mathbf{n}_{s_i}^{\zeta}+\boldsymbol{\omega}^*, \mathbf{p}_R^{\zeta}\right)$;
    $\boldsymbol{\mathcal{I}}_i^{\zeta} \leftarrow \boldsymbol{\omega}^*$;
    $\boldsymbol{\mathcal{J}}_i^{\zeta} \leftarrow \boldsymbol{\mathcal{J}}_{i+1}^{\zeta}+\boldsymbol{\mathcal{I}}_{i+1}^{\zeta}$;
  ELSE
    $\boldsymbol{\mathcal{I}}_i^{\zeta} = \mathbf{0}$;
    $\boldsymbol{\mathcal{J}}_i^{\zeta} \leftarrow \boldsymbol{\mathcal{J}}_{i+1}^{\zeta}$;
  END IF
ELSE
  FOR $k = 1 : K$
    Calculate $\mathbb{U}_{N,k}\left(\mathbf{p}_{r_k}^{\zeta}, n_{-N,k}^{\zeta}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{N,k}\left(\xi, n_{-N,k}^{\zeta}+1\right)p_{\xi_{r_k}^{\zeta}}\left(\xi|\theta_{r_k}\right)p_{r_k}^{\zeta}\left(\theta_{r_k}\right)$;
  END FOR
  $\mathcal{K} = \{k_1, k_2, \ldots, k_\kappa\} \leftarrow \underset{k\in\{1,2,\ldots,K\}}{\overset{(\kappa)}{\operatorname{argmax}}}\left\{\mathbb{U}_{N,k}\right\}$;
  FOR $k = 1 : K$
    IF $\mathbb{U}_{N,k}\left(\mathbf{p}_{r_k}^{\zeta}, n_{-N,k}^{\zeta}\right) > 0$ AND $k \in \mathcal{K}$
      $\mathcal{I}_{N,k}^{\zeta} \leftarrow 1$;
    ELSE
      $\mathcal{I}_{N,k}^{\zeta} \leftarrow 0$;
    END IF
  END FOR
  $\boldsymbol{\mathcal{J}}_N^{\zeta} = \mathbf{0}$;
END IF

**Algorithm 3 : Best-response with unconstrained selections BR_US** $\left(i, n^{\zeta}_{s_i,r_k}, \mathbf{p}^{\zeta}_{r_k}\right)$

IF $i \neq N$
  $\left(\mathcal{I}^{\zeta}_{i+1,k}, \mathcal{J}^{\zeta}_{i+1,k}\right) \leftarrow$ BR_US $\left(i+1, n^{\zeta}_{s_i,r_k}+1, \mathbf{p}^{\zeta}_{r_k}\right)$;
  $\mathcal{J}^{\zeta}_{i,k} \leftarrow \mathcal{J}^{\zeta}_{i+1,k} + \mathcal{I}^{\zeta}_{i+1,k}$;
  Calculate $\mathbb{U}_{i,k}\left(\mathbf{p}^{\zeta}_{r_k}, n^{\zeta}_{s_i,r_k}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{i,k}$
  $\left(\xi, n^{\zeta}_{s_i,r_k} + \mathcal{J}^{\zeta}_{i,k} + 1\right) p_{\xi^{\zeta}_{r_k}}\left(\xi|\theta_{r_k}\right) p^{\zeta}_{r_k}\left(\theta_{r_k}\right)$;
  IF $\mathbb{U}_{i,k}\left(\mathbf{p}^{\zeta}_{r_k}, n^{\zeta}_{s_i,r_k}\right) > 0$
    $\mathcal{I}^{\zeta}_{i,k} \leftarrow 1$;
  ELSE
    $\left(\mathcal{I}^{\zeta}_{i+1,k}, \mathcal{J}^{\zeta}_{i+1,k}\right) \leftarrow$ BR_US $\left(i+1, n^{\zeta}_{s_i,r_k}, \mathbf{p}^{\zeta}_{r_k}\right)$;
    $\mathcal{J}^{\zeta}_{i,k} \leftarrow \mathcal{J}^{\zeta}_{i+1,k} + \mathcal{I}^{\zeta}_{i+1,k}$;
    $\mathcal{I}^{\zeta}_{i,k} \leftarrow 0$;
  END IF
ELSE
  Calculate $\mathbb{U}_{N,k}\left(\mathbf{p}^{\zeta}_{r_k}, n^{\zeta}_{s_N,r_k}\right) = \sum_{\theta_{r_k}\in\Theta}\sum_{\xi=0}^{\infty}\mathcal{U}_{N,k}$
  $\left(\xi, n^{\zeta}_{S_N,r_k} + 1\right) p_{\xi^{\zeta}_{r_k}}\left(\xi|\theta_{r_k}\right) p^{\zeta}_{r_k}\left(\theta_{r_k}\right)$;
  IF $\mathbb{U}_{N,k}\left(\mathbf{p}^{\zeta}_{r_k}, n^{\zeta}_{-N,k}\right) > 0$
    $\mathcal{I}^{\zeta}_{N,k} \leftarrow 1$;
  ELSE
    $\mathcal{I}^{\zeta}_{N,k} \leftarrow 0$;
  END IF
  $\mathcal{J}^{\zeta}_{N,k} \leftarrow 0$;
END IF