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ORIGINAL RESEARCH

Bolzano's Tortoise and a loophole for Achilles

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Abstract
This paper discusses a novel response to two closely related regress arguments from Bolzano's Theory of Science and Carroll's What the Tortoise Said to Achilles. Bolzano's argument aims to refute the thesis that full grounds must include propositions involving notions such as entailment, grounding or lawhood which link the respective grounds to their groundee. This thesis is motivated, Bolzano's argument is reconstructed, and a response based on self-referential linking propositions is developed and defended against objections concerning self-reference and Curry's paradox. Finally, the idea is applied to a reading of Carroll's dialogue and a corresponding solution to the so-called infinite regress problem of inference is proposed.

Keywords Grounding · Self-reference · Explanation · Curry's paradox · Infinite regress problem of inference · Bolzano · Carroll

1 Introduction

This paper discusses the grounding role of grounding propositions and laws of metaphysics, Bolzano's (1837b, §199) regress argument, and a novel kind of response to both this and analogous arguments from the discussion of Carroll (1895).

Fixing some terminology: Instances of grounding involve something that is grounded (i.e. a groundee P), a plurality of grounds (Γ), and a grounding proposition such as $\Gamma \prec P$ (for partial ground) or $\Gamma < P$ (for full ground).[1] Additionally, it has been suggested in the literature that instances of grounding are intimately related

[1] A note on conventions: I primarily use Fine's (2012) notation to talk about grounding. Where the context suffices for disambiguation, I sometimes use formulae as names for corresponding propositions, and I will mostly write as if assuming the relata of grounding to be propositions rather than facts, but I intend to remain agnostic about the matter here.

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to laws of metaphysics.[2] Using "linking proposition" as an umbrella term for propositions like these (that in some sense link the grounds to the groundee), we can then ask the following about their grounding role:

1. Do linking propositions (of some kind) in general *ground* the corresponding groundees? E.g., does $\Gamma \prec P$ ground P?
2. Are linking propositions (of some kind) generally required in full grounds? E.g., do the Γ in $\Gamma < P$ contain a suitable linking proposition?

An early discussion of the matter is due to Bolzano (1837b, §199), who first offers a motivation to answer (1) and (2) for a certain kind of linking proposition affirmatively, but then provides a compelling regress argument against the idea. Versions of this argument have been used to argue for a tripartite conception of explanation and explanatory notions (e.g. grounding) according to which the roles of reasons why something obtains (e.g. grounds) and explanatory links or principles (e.g. grounding propositions or laws of metaphysics) must be strictly distinguished (for a recent example of such a theory and further references see Skow (2016); for uses of the Bolzanian argument and applications of the arising conception of explanation see for example Kappes and Schnieder (2016, p. 558, fn 34), Kappes (2023, Chap. 1), and De Rizzo (manuscript).

Section 2 of this paper clarifies the issue and reconstructs the Bolzanian consideration in favor of answering questions 1 and 2 affirmatively. This is further motivated using considerations about the nature of explanation (cf. Kappes, 2022), as well as adapting considerations by Bennett (2017) and Frugé (2023) for the present context. Additional motivation is provided by considerations by Litland (2018) and Berker (2018). Section 3 reconstructs Bolzano's regress argument, generalizes it, and provides an argument for why the resulting regress is indeed (as Bolzano simply contends) unacceptable. Section 4 identifies a loophole in the Bolzanian argument. Section 5 defends this proposal against several objections, including an argument by Berker (2018). Section 6 identifies an interesting relation the proposal bears to Curry-style paradoxes and discusses whether this amounts to a problem for it.

One way to read this main discussion of the paper is as a defense and development of the conception of grounding motivated in Sect. 2 against the Bolzanian argument. Alternatively, readers ultimately unsympathetic of the developed response can read this paper as a completion of Bolzano's argument: Sect. 3 reconstructs Bolzano's argument in contemporary terms and argues why the regress is indeed unacceptable, and Sects. 3–6 can be understood as drawing out problematic consequences of the loopy account proposed to avoid the regress.

Finally, Sect. 7 turns to Carroll's (1895) famous dialogue *What the Tortoise Said to Achilles*, which contains a regress strikingly reminiscent of Bolzano's argument.[3] I discuss the significance of the loophole idea for Carroll's dialogue and propose a loopy solution to what Rosa (2019) calls the "infinite regress problem of inference". (This section can be understood largely independently of Sects. 5 and 6, so readers may skip ahead if they wish.)

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[2] Two prominent proponents are Schaffer (2017, 2018) and Kment (2014). A related proposal concerns essence, e.g. Dasgupta's (2014).

[3] Cf. Rusnock and George (2014, xli).

2 Motivating the issue

Section 2.1 reconstructs Bolzano's motivation for why one might think that a certain kind of linking proposition (in his case one involving the notion of logical consequence) needs to be part of any full ground, and Sect. 2.2 discusses some problems for Bolzano's motivation. Section 2.3 turns to linking propositions involving ground or metaphysical law and argues that affirmative answers to questions 1 and 2 arise from a certain intuitive conception of the relation between grounding, metaphysical laws, and explanation (that Bolzano's motivation may have been getting at). I further argue that we should take this conception seriously by identifying closely related ideas by Bennett (2017), Frugé (2023), Berker (2018), and Lawler (2018).

2.1 Bolzano's idea

In §199 of his *Wissenschaftslehre*, Bernard Bolzano considers the following: Suppose some propositions P_1, P_2, ... both entail and at least partially ground a proposition Q. Must then a proposition stating the correctness of a rule governing the inference from P_1, P_2, ... to Q be added to P_1, P_2, ... in order to obtain a *complete* ground of Q? Note: The question is further refined below. I will understand 'complete ground' as expressing what is nowadays known as *full* ground (Fine, 2012, p. 50); for this paper, I will stick to Bolzano's terminology. Given this clarification, here is an example for what Bolzano is asking: P (if true) both entails and at least partially grounds $P \vee Q$. Must then, in order to obtain a complete ground of $P \vee Q$, a proposition be added to P that states the validity of a rule to the effect that disjunctions are entailed by their disjuncts?

Having set up and motivated the question, Bolzano offers his regress argument for a negative answer. But before, he (1837b, §199) professes sympathy for a consideration in favor of the idea that if P_1, P_2, ... both entail and partially ground Q, a corresponding inference rule must be added to P_1, P_2, ... to obtain a complete ground of Q:

> [If] the rule according to which we [wanted] to deduce certain propositions M, N, O, ... from others A, B, C, D, ... [were] incorrect, then it is obvious that we [could not] claim that the propositions M, N, O, ... are truths which *follow* from the truths A, B, C, D, ... Reflecting on this, one could get the idea that the rule, according to which propositions M, N, O, ... are deducible from propositions A, B, C, D, ..., ought to be envisaged as a *truth* which must be added to the truths A, B, C, D, ... in order to give the *complete* ground of the truths M, N, O, ...[4]

Some clarifications are in order: First, Bolzano (in the translation) uses 'follows' to express grounding. Second, as we can see, Bolzano's conception of ground is many-many. For simplicity's sake and since nothing in his argument turns on this, I will stick with many-one formulations. Third, Bolzano distinguishes a *material* notion of deducibility (named simply 'deducibility' or 'Ableitbarkeit') from a (to the present

[4] Where indicated, I have modified the translation slightly to better capture the grammatical mood of the original.

reader perhaps more familiar) *logical* notion of deducibility (Bolzano, 1837b, §223; Morscher, 2018, 3.8). To avoid complications that lead away from the purpose of this paper, I assume that Bolzano's argument is concerned with logical deducibility. Moreover, for present purposes, we can substitute our contemporary notion of logical consequence (or entailment) for this notion. Fourth, according to Bolzano (1837b, §199), for each instance of deducibility there exists a rule of of inference that "describes it, i.e. a proposition which indicates what attributes the premises A, B, C, D, ... and the conclusions M, N, O, ..., which follow from them, must have." This formulation suffices for now, but we will come back to Bolzano's understanding of rules of inference when discussing his argument proper.

As to the reasoning he considers, Bolzano appears to first identify a relation of counterpossible dependence between the correctness of rules of inference and corresponding grounding propositions. For instance, it seems that Bolzano would suggest that if (the rule of) disjunction introduction were incorrect, then P would not ground $P \vee Q$ (even assuming P's truth). Bolzano then suggests that this counterpossible dependence relation might move us to assume that disjunction introduction should be added to P to obtain a complete ground of $P \vee Q$.

2.2 Problems for Bolzano's idea

Let us discuss some complications for Bolzano's idea: First, if rules of inference have a quantified (or schema-like) character, then one might wonder whether if only instances of a rule of inference were to fail, instances of grounding that correspond to other instances of the rule could perhaps still hold. Since, as we will see momentarily, Bolzano understands rules of inference as having a non-quantified conditional form, we will set this issue aside.[5]

Second, we will focus on cases involving just a single rule of inference corresponding to an instance of ground. Otherwise, Bolzano's counterpossible reasoning is a non-starter, since only *one* suitable rule of inference seems sufficient to maintain the corresponding instance of grounding. In other words, in cases were there are two independent rules of inference which we can use to deduce M, N, O, ... from A, B, C, D, ... , Bolzano's following contention loses plausibility: "[If] the rule according to which we [wanted] to deduce certain propositions M, N, O, ... from others A, B, C, D, ... [were] incorrect, then it is obvious that we [could not] claim that the propositions M, N, O, ... are truths which *follow* from the truths A, B, C, D, ...".

Third, a technical problem: if P and Q are true, P both entails and partially grounds $(P \wedge Q) \vee (R \vee \neg R)$. Yet assuming that P neither grounds nor is identical with R or $\neg R$, any complete ground involving P plausibly has to involve Q as well (or at least something that grounds Q).[6] Now, there is one inference rule corresponding to the entailment between P (and by extension P and Q) and $(P \wedge Q) \vee (R \vee \neg R)$,

[5] We can also note that people have argued that instances of grounding must correspond to principles of a general form. See deRossett (2013).

[6] For example in the logic of Fine (2012). For discussion see Glazier (2017) and Kappes (2020). Note that even if $R \vee \neg R$ is zero-grounded, as the latter considers, it does not follow that P can be a complete ground of $(P \wedge Q) \vee (R \vee \neg R)$. Thanks to an anonymous referee for discussion here!

namely a rule that allows us to derive the logical theorem $R \vee \neg R$ and then introduce an arbitrary disjunct. On the other hand, in the case of the entailment between P, Q and $(P \wedge Q) \vee (R \vee \neg R)$, there is an additional corresponding inference rule, namely one that allows us to conjoin P and Q and then introduce an arbitrary disjunct (in this case $R \vee \neg R$).

It seems that (given Bolzano's reasoning), only the latter rule is a good candidate to form a complete ground of $(P \wedge Q) \vee (R \vee \neg R)$ together with P and Q, since the grounding relation between P, Q and $(P \wedge Q) \vee (R \vee \neg R)$ corresponds to the entailment between P, Q and $(P \wedge Q) \vee (R \vee \neg R)$—it just so happens that P also entails $(P \wedge Q) \vee (R \vee \neg R)$, because the latter is already entailed by the theorem $R \vee \neg R$.

My suggestion to help out Bolzano here is two-fold: First, when he considers partial grounds that entail their groundees in this context, he seems to be thinking of grounds that are complete with the possible exception of rules of inference linking grounds with groundees and which might have to be added to obtain a complete ground. Let us call grounds like this 'quasi-complete'. This excludes the above example: P is not a quasi-complete ground of $(P \wedge Q) \vee (R \vee \neg R)$, since any complete ground of the latter containing P also contains Q (or at least grounds of Q).[7]

Second, Bolzano appears to have in mind only entailments and inference rules that (in some sense) tightly correspond to instances of grounding: For quasi-complete grounds Γ that entail their groundee P, he is concerned with rules that license inferring P from Γ. Indeed, as we will see in the next subsection, Bolzano could have plausibly considered instances of grounding or corresponding laws of metaphysics instead of rules of inference as additional grounds.

Interestingly, Bolzano's suggestion is that the counterpossible relation between rules of inference and instances of grounding might motivate the inclusion of the rules of inference among the grounds of the corresponding groundees. Here, one might perhaps rather have expected a counterpossible relationship between rules of inference (i.e. the candidate grounds) and the *groundee* in question. For example, one might think that had disjunction introduction been incorrect, then there should be P and Q such that P is true, Q is false and $P \vee Q$ is also false. But this idea is questionable due to the modal character of rules of inference: In the closest worlds in which disjunction introduction fails, it may not fail because of goings-on on at those worlds, but rather because of goings-on at more remote worlds.

Aside from the notoriety of counterpossibles and the somewhat undermotivated step from the counterpossible relation between rules of inferences and instances of grounding to the inclusion of the rules among the grounds in the latter, the reasoning suffers from the general issue that counterfactual relations regularly fail to entail corresponding explanatory relationships such as grounding or causation. For example, and pertinent to Bolzano's reasoning, if $P \vee Q$ were false, then P would not ground $P \vee Q$, but of course we should not conclude from this that $P \vee Q$ partially grounds itself.

[7] It may be possible to spell out the kind of inference Bolzano is concerned with using some combination of his notion of exact inference (cf. Roski, 2017, 2.3.5) and a notion of direct inference, but we will need the notion of a quasi-complete ground below anyhow. Thanks to Stefan Roski for discussion here!

2.3 Improving upon Bolzano: explanation and contemporary considerations

Bolzano's consideration can be understood as pointing to a certain intuition about the relation between grounds and either instances of grounding or laws of metaphysics. To see what I have in mind, note that explanations are often assumed to have a tripartite structure (see e.g. Schaffer, 2017): Grounding explanations consist of a groundee, grounds, and a grounding proposition (alternatively a law of metaphysics) that links the grounds to the groundee, while causal explanations consist of an effect, causes, and an instance of causation (or a law of nature) that links the causes to the effect.

Now, while Bolzano asks whether in cases where P_1, P_2, ... both entail and partially ground Q, a corresponding *inference rule* must be added to P_1, P_2, ... to obtain a complete ground of Q, related questions arise with respect to the linking propositions involved in explanations: Must complete grounds involve grounding propositions or laws of metaphysics that link the grounds to the groundee?[8] More precisely, call 'quasi-complete' any grounds that are complete with the possible exception of grounding propositions (or laws of metaphysics) that link their grounds to their groundee. We can then ask whether a complete ground of Q involving a quasi-complete ground P_1, P_2, ... must involve a grounding proposition (or law of metaphysics) linking P_1, P_2, ... to Q.

While the following discussion will mostly stick to grounding, analogous concerns may arise for instances of causation and laws of nature as well: Do they, in addition to the causes proper, also cause effects? Can causes only be complete if they involve corresponding instances of causation or laws of nature that cause the effect together with the causes proper? And if laws are not causes, are they at least reasons why the effects in question occur?[9]

Now, I take it that there is some intuitive pressure to answer these questions in the affirmative (and this might have been what Bolzano was grasping at). To see this, note first that the counterfactual relation between P's grounding Q and Q is more straightforward than in Bolzano's case: Ignoring the usual complications like overdetermination and supposing that P grounds Q, it appears indeed plausible that had it been the case that P but not that P grounds Q, then it would not have been the case that Q.

Second, while identifying the preceding counterfactual relation at best amounts to fallible evidence that there exists a corresponding grounding relation too, P's grounding Q is not only counterfactually related to Q, but also *explanatorily*: After all, P's grounding Q appears to explain (together with P) Q. Indeed, in some sense, Q appears to explanatorily depend not merely on P, but also on P's grounding Q. Since what we are dealing with here is metaphysical explanation, it is a fair question whether this explanatory relation is grounding—does the grounds' grounding the groundee always also ground the groundee? Is the grounds' grounding the groundee a required part of any complete ground of the groundee?

The intuitive conception underlying this can be brought out further as follows: Sometimes, the tripartite structure of explanation is elucidated with a mechanistic

8 For accounts of the notion of a law of metaphysics see Kment (2014) and Schaffer (2018).

9 For a recent discussion of whether instances of causation can stand in causal relations themselves see Kovacs (2022, Sect. 2).

metaphor according to which explanation is like a machine: It involves an explanatory link or mechanism which takes causes (grounds) as inputs and puts out effects (or groundees). It is possible (although presumably not mandatory) to construe the metaphor in a way that suggests that the relation between explanatory link and effect is one of causation (or by analogy grounding) as well: After all, the machine *somehow* figures in the causes of its outputs.

Now, on a conception of explanation like Skow's (2016), which holds that laws and instances of grounding (or causation) are not in general reasons why the corresponding groundees (or effects) obtain, these considerations should be resisted and it maintained that they are not grounds (nor causes). But while Skow may develop a neat picture, he does not provide a lot of argument beyond this. Indeed, Lawler (2018) argues against him that laws can indeed occur as reasons why in the way he objects to. The following can be understood as the development and discussion of a conception in the latter's spirit (for further discussion from a different angle see Kappes (2023, Chap. 1) and De Rizzo (manuscript)).

Further motivation for the following discussion stems from the literature on what grounds instances of grounding themselves. First, Bennett (2017, 207f) argues that whatever grounds a grounding link whose groundee is P (e.g. $\Gamma \prec P$) must also ground P. Then she uses this principle together with a regress argument a lot like Bolzano's to argue that a class of views about what grounds grounding claims (she calls these 'connectivism') fail. While I cannot treat Bennett's discussion in detail here, the discussion below can both serve as refinement of the regress argument in question, and provide the connectivist with further resources to withstand it.

Second, Frugé (2023) argues (contra, e.g., Bennett, 2011) that the regress arising from the thesis that every instance of grounding is grounded is indeed vicious. In Kappes (2022) I argue that Frugé's position threatens to run into its own kind of regress, namely Bolzano's. What follows can thus be understood both as a completion of that argument, and the development of a conception of grounding (i.e. the loopy one) that might be of help to Frugé's approach.

Further questions closely related to my present concerns have arisen in the grounding literature: Litland (2018) argues that instances of grounding *sometimes* ground the corresponding groundees, and Berker (2018) considers whether instances of grounding or metaphysical laws that help ground the corresponding groundees could help solve an issue concerning explanation in metaethics. The present discussion can be understood as a more general approach to these questions (we will return to Berker in Sect. 5).

While I take it that all of the preceding considerations may be reasonably resisted (maybe even independently of Bolzano's objection that we will turn to momentarily), I believe that they hint at an intuitive conception of the grounding role of grounding link (or a more generally the relation between explanatory links and explanandum) that deserves more attention, if only to ultimately reveal it to be untenable (and thus help support accounts of that relation the vein of Skow (2016). See Kappes (2023, Chap. 1) for further discussion and literature). In this spirit: While I proceed to identify a loophole in Bolzano's argument, the reader might in the end decide to take my ensuing discussion as revealing theoretical costs sufficient to plug it.

3 Bolzano's Tortoise

Let us now turn to Bolzano's (1837b, §199) argument proper:

> Assume that the complete ground of the truths $M, N, O, \ldots$ includes, besides truths $A, B, C, D, \ldots$, from which they are deducible, also the rule which allows the deduction of propositions $M, N, O, \ldots$ from propositions $A, B, C, D, \ldots$. This amounts to saying that propositions $M, N, O, \ldots$ are true only because this rule of inference is correct and because propositions $A, B, C, D, \ldots$ are true; hence it is tantamount to the following inference: 'If propositions $A, B, C, D, \ldots$ are true, then propositions $M, N, O, \ldots$ are also true; but propositions $A, B, C, D, \ldots$ are true, therefore propositions $M, N, O, \ldots$ are true.' But since every inference has its rule of inference, this one does too. If we abbreviate the first of the above propositions by X, and the second by Y, then the rule can be put in this way: 'If propositions X and Y are true, then propositions $M, N, O, \ldots$ are also true.'—Now if it was required in the first place that the complete ground of the truths $M, N, O, \ldots$ must include, besides the truths $A, B, C, D, \ldots$ also the rule of their deduction, then it must be required, for the same reason, that the *second* rule of inference also be added to the ground on the same footing as the first, since we can also say of the second rule: the truths $M, N, O, \ldots$ could not follow if it were invalid. We can see at once that this type of argument can be repeated ad infinitum and therefore that, if it were legitimate to add one rule of inference to the ground of the truths $M, N, O, \ldots$, an *infinite* number of them could be claimed to belong to this ground, which seems absurd.[10]

As this passage makes clear, Bolzano appears to think of rules of inference as conditionals corresponding to instances of deducibility. Therefore, for now, we will reconstruct Bolzano's target using our notion of logical consequence (expressed by the connective '$\Rightarrow$') as follows (grounding and metaphysical laws will be considered momentarily):

(Entailment as Ground) Any complete ground Ω of P that contains a quasi-complete ground Γ of P which entails P must include the proposition $\Gamma \Rightarrow P$.

Here, let us extend our notion of a quasi-complete ground of P to cover grounds of P that are complete with the possible exception of linking propositions such as $\Gamma \Rightarrow P$.[11] The formulation seems to come close to what Bolzano has in mind, and we will see that his argument easily generalizes to related principles that he could have had in mind. (Note that while it suits our present purpose, I will argue in the next section that **Entailment as Ground** captures the motivating desideratum imperfectly and amend it there.)

We now reconstruct the argument as a reductio of **Entailment as Ground**. To do so, consider some propositions Γ that entail P and are a quasi-complete part of a complete

[10] Mentioning Carroll (1895), Bennett (2017, p. 207) offers a related argument directed at certain conceptions of the grounds of ground.

[11] Note that in calling propositions like $\Gamma \Rightarrow P$ "linking propositions", I do not want to suggest that they can be explanatory links in the sense introduced above.

ground Ω of P (following Fine (2012), we use '$<$' to express complete ground):

$$\underbrace{\Gamma, \ldots}_{\Omega} < P$$

By **Entailment as Ground**, $\Gamma \Rightarrow P$ is among the Ω:

$$\underbrace{\Gamma, (\Gamma \Rightarrow P), \ldots}_{\Omega} < P$$

But Γ and $\Gamma \Rightarrow P$ are also a quasi-complete ground of P and entail it, so by **Entailment as Ground** again:

$$\underbrace{\Gamma, (\Gamma \Rightarrow P), ((\Gamma, (\Gamma \Rightarrow P)) \Rightarrow P), \ldots}_{\Omega} < P$$

Thus, we embark on a regress that Bolzano takes to be absurd.

To give a concrete example, consider a complete ground of $P \vee Q$ that contains the quasi-complete ground P:

$$P, \ldots < P \vee Q$$

By **Entailment as Ground** we first get

$$P, (P \Rightarrow P \vee Q), \ldots < P \vee Q$$

And then it again leads to a more complex ground and into the regress:

$$P, (P \Rightarrow P \vee Q), (P, (P \Rightarrow P \vee Q) \Rightarrow P \vee Q), \ldots < P \vee Q$$

Now, could one perhaps bite the bullet and declare the plurality of all the grounds constructed by the regress to be a full ground Ω of P? Two initial obstacles are these: First, the regress does seem to introduce a questionable kind of infinity and complexity into complete grounds. Second, it appears congenial to the motivating thoughts above that every initial segment of the regress requires the help of the next element in order to even partially ground the groundee—only via the grounding link constituting the next element of the regress does any initial segment partially ground the groundee. But if so, the regress looks vicious: Some grounding work to be done is always deferred to the next element of the regress.[12]

More importantly though, it may be possible to substantiate Bolzano's contention that the regress is indeed unacceptable, because the reasoning that leads into the regress

[12] Contrast the situation with the following candidate for an infinite complete ground: $0 = 0, 1 = 1, 2 = 2, \ldots < 0 = 0 \wedge 1 = 1 \wedge 2 = 2 \wedge \ldots$. The partial grounds completely ground the conjunction only taken together, but no initial segment of the complete ground partially grounds the conjunction only by help of the next elements. This appears to hold even if we consider non-factive ground (see Fine (2012, p. 48) and Sect. 5 below for an introduction of this notion).

seems to reveal that Ω cannot be a full ground of P either: In Ω, there is no grounding fact that takes us from all the grounds in Ω to P! Yet, this is what **Entailment as Ground** seems to require. One might now consider whether a full ground of P could be obtained from Ω by some transfinite construction similar to how Ω was constructed, but as long as the result is such that we can say something that amounts to those grounds (i.e. those resulting from the construction) grounding P, it looks like we can apply **Entailment as Ground** and to reveal a missing grounding fact that should be part of the full ground of P but has not been constructed. Thus, unless declaring full grounds to be ineffable and giving up talking about them like above is considered an option, I conclude that Bolzano's regress must be avoided.

Now, Bolzano's argument generalizes straightforwardly to notions besides logical consequence: As long as the assumption that complete grounds must involve certain propositions linking grounds and groundee is sufficiently general, virtually the same argument can be run by substituting the propositions involving logical consequence by propositions concerning a generic conditional like Bolzano's, statements about the validity of general rules of inference, or indeed metaphysical laws or grounding propositions as considered in the previous section. For example, consider this principle:

(Grounding as Ground) Any complete ground Ω of P that contains a quasi-complete ground Γ must include a grounding proposition linking Γ and P (e.g. $\Gamma \prec P$ or $\Gamma < P$).[13]

To construct the regress, consider a complete ground of P containing a quasi-complete part Γ:

$$\Gamma, ... < P$$

Applying **Grounding as Ground** once and sticking to formulation involving partial ground we get:

$$\Gamma, \Gamma \prec P, ... < P$$

Applying it again, we embark on the Tortoise's journey once more:

$$\Gamma, \Gamma \prec P, (\Gamma, (\Gamma \prec P) \prec P), ... < P$$

Bolzano's argument identifies a general problem for the idea that complete grounds must contain propositions linking the corresponding grounds and their groundees. If sound, it supports the thought that while both grounds and linking propositions (grounding propositions or metaphysical principles) play their respective role when it comes to grounding and grounding explanations, these roles should be sharply distinguished.[14]

[13] I follow Fine (2012, p. 53) in using '$\prec$' to express partial ground.

[14] See Skow (2016) and Schaffer (2017) for examples of accounts which extend this claim to causation and explanation in general.

4 A loophole in the argument

Pace Bolzano, there is—at least in principle—a way to avoid the regress and thus allow for finite complete grounds while demanding that a linking proposition must be part of any complete ground. This can be achieved by using linking propositions of a *loopy*, self-involving form.[15] We will first look at our rendering of Bolzano's idea that groundee-entailing grounds need to include propositions capturing the relation of logical consequence between grounds and groundee and then turn to the idea that a grounding-link proposition must be part of any complete ground. We will see that the idea applied to logical consequence may face a problem related to Curry's paradox, while the idea applied to grounding does not.

Now suppose that $\Gamma \Rightarrow P$ and that the Γ are a quasi-complete ground of P. Our task then is to find a substitute for '...' that does not give rise to the Tortoise's regress:

$$\Gamma, \underbrace{\ldots}_{?} < P$$

To find the substitute, consider first the instance of logical consequence corresponding to our grounding statement:

$$\Gamma, \underbrace{\ldots}_{?} \Rightarrow P$$

Here, we can find a substitute for '...' such that the resulting proposition may complete our ground above, namely the following:

$$(C) \qquad \Gamma, C \Rightarrow P$$

Here, let 'C' stand for the proposition expressed by '$\Gamma, C \Rightarrow P$'. The proposal is that C is the missing partial ground:

$$\Gamma, C < P$$

Since C just is $\Gamma, C \Rightarrow P$ (and assuming a sufficiently coarse-grained notion of grounding), this amounts to

$$\Gamma, (\Gamma, C \Rightarrow P) < P$$

No regress arises from the demand that a proposition expressing the relation of logical consequence between Γ, C and P has to be part of P's complete ground: This proposition would be $\Gamma, C \Rightarrow P$ and hence C, but that proposition is already part of the ground!

Now, while the proposal satisfies the desideratum (motivated in Sect. 2) that the relation of logical consequence between the ground of P and P must be captured by

[15] My work on this paper has been prompted by Meinertsen (2018, Chap. 10) which contains a structurally related proposal aimed at avoiding Bradley's regress.

a proposition that is part of P's ground, it does not satisfy **Entailment as Ground**: According to the latter, for any P-entailing, quasi-complete ground Γ of P, $\Gamma \Rightarrow P$ must be part of any complete ground of P that contains Γ, but $\Gamma \Rightarrow P$ is not part of the ground suggested above, namely Γ, C. My response here is that **Entailment as Ground** captures the desideratum inadequately: In essence, the desideratum requires that the relation of logical consequence between ground and groundee be captured by part of the ground, and the loopy proposal shows that this might be achieved in a way not anticipated by **Entailment as Ground**. A better way to make the desideratum more precise may be this (already somewhat loopy-looking) principle:

(Entailment as Ground*) Any complete ground Ω of P which entails P must include a proposition that captures the entailment between Ω and P.

Now to the idea that grounding links must be part of complete grounds: Suppose Γ are a quasi-complete part of a full ground of of P. The suggestion is that this ground includes a loopy grounding proposition as follows:

$$\Gamma, G < P \tag{G}$$

Here, 'G' stands for $\Gamma, G < P$, so assuming a suitably coarse-grained notion of grounding again, we get:

$$\Gamma, (\Gamma, G < P) < P$$

Thus, the added ground is identical to the proposed grounding proposition itself. Roughly put, the Γ completely ground P together with a grounding proposition that says that it itself together with the Γ completely ground P. This stops the Tortoise: According to **Grounding as Ground**, any complete ground of P containing both the quasi-complete ground Γ and G must include a grounding proposition linking Γ and P. The original idea was that this be $\Gamma \prec P$ or $\Gamma < P$, but $\Gamma, G < P$ also links Γ (albeit together with G) to P. Now since G just is $\Gamma, G < P$ and hence also links Γ, G and P, **Grounding as Ground** is satisfied by $\Gamma, G < P$ without giving rise to the regress.

5 Against the loopy proposal?

In this section we discuss some potential problems for the loopy proposal concerning (1) the complexity of the proposed grounds, (2) the self-involving form of the loopy propositions, (3) the logic of ground, (4) an "unpacking" regress, and (5) Berker's (2018) argument. The following section discusses worries stemming from the relation the loopy proposal bears to Curry's paradox.[16]

[16] I set aside two possible lines of objection for another occasion: First, the loopy proposal is incompatible with the possibility of *zero-ground* (cf. Fine, 2012). Second, considerations concerning the alleged purity of the fundamental could spell trouble, although see Correia (2023) and Barker (forthcoming) for objections to the principle.

Note: The last section on Carroll's dialogue and a loopy solution to the infinite regress problem of inference contained therein can be understood largely independently of this and the following section, so the reader may skip ahead if desired.

5.1 Complexity

It is sometimes suggested that grounds must be less conceptually complex than their groundees. If so, C and G may be problematic because the respective grounds can seem more complex than the groundees, since the latter seem to figure in the linking part (i.e. C and G) of the respective grounds of P. Today, while considerations from complexity are sometimes used to inform what grounds what, the idea that grounds *must* be less conceptually complex than their groundees is often rejected. Most pertinent here is perhaps that a posteriori grounding theses such as the idea that truths about water are grounded in truths about H_2O, truths about consciousness in truths about physical reality, or truths about value in non-normative truths, are widely assumed to be sensible.[17]

5.2 Self-involvement and self-reference

Second, one might take issue with the self-involving form of the loopy propositions. For example, if one assumes that propositions are mereological compounds, one might think that (problematically) a proposition like G must have itself as a mereological constituent.[18] As it stands, this objection also leaves room for responses: Propositions might not be mereological compounds (or set-theoretic constructions that give rise to analogous problems); if they are, it is unclear whether the propositions in question really require non-well-founded mereology (perhaps a proposition can be involved in itself partially in virtue of it being merely an improper, rather than a proper, part of itself); and even if so, given that such mereologies have been developed (e.g. Cotnoir & Bacon, 2012), it is unclear whether this would be objectionable.[19]

In any case, there are two ways in which this problem can be avoided.[20] The first is to adopt a predicational (instead of operator-based) formulation of grounding and the loopy principles, thereby allowing to refer to the proposition in question using a singular term (such as a name, description, or deictic device), e.g.:

$(G_P RED)$ $\gamma_1, \ldots, \gamma_n$, this proposition ground p

[17] Indeed, Bolzano (cf. Roski, 2017, p. 115) restricts the complexity constraint to conceptual truths. Now, perhaps a case could be made that if Γ and P are conceptual truths, then $\Gamma < P$ or G are conceptual truths as well, although Bolzano does not use this idea in his §199.

[18] For Bolzano (cf. Bolzano, 2017, p. 22), propositions are structured complexes, and indeed, he (1837a, §19, p. 79) would object to a complex like this having itself as a constituent.

[19] See Kearns (2011) for an argument that given an understanding of propositions as mereological wholes, we should accept that certain (otherwise unproblematic) self-referential propositions are parts of themselves.

[20] Both are congenial to what Bolzano (1837a, §19) has to say about the liar sentence "This is false": According to him (and contra Savonarola against whom Bolzano argues there), the sentence expresses a proposition, but one that does not contain itself as a constituent. Rather, it merely contains as such a singular concept of itself. Thanks to Stefan Roski and Jan Claas here!

Here, $\gamma_1, ..., \gamma_n$ and p are propositions and 'this proposition' is intended to refer to the proposition expressed by the sentence labeled 'G_{PRED}' (we could also have introduced a name for that proposition, as is customary in the literature on the semantic paradoxes).

Alternatively, we can stick to an operator view of grounding, employ a truth-predicate as follows and use 'this proposition' analogously:

$$(G_T) \qquad \Gamma, T(\text{this proposition}) < P$$

Perhaps a case can be made that propositions of the form $T(p)$ and P (where 'p' refers to the proposition P) are strongly equivalent in a sense such that what we wanted to express with 'G' can simply be less problematically expressed with 'G_T'.[21] Otherwise, we may assume that G_T grounds T(this proposition) (where 'this proposition' refers to G_T).[22] Then, transitivity of partial ground allows us derive from that G_T partially grounds P. While we do not quite reach G this way, the proposal seems worth investigating further as an alternative.

Some might object against propositions like G_{PRED} and G_T as well, but similar propositions are at least taken seriously by many philosophers and logicians interested in self-reference and related phenomena.[23] In what follows, I will stick to the original operator-formulation, but I presume that much of what I say could be translated into the predicational formulation, if required.

5.3 Logic of ground

If we allow for self-involving propositions in general, propositions like $A \vee P$, where A is this very proposition are troublesome for some widely accepted principles about grounding: If P is true, then $A \vee P$ is true and grounded in P. But then A is true and because disjunctions are grounded in their true disjuncts, A grounds $A \vee P$ and hence itself.

This problem is related to the paradoxes of ground for example discussed in Fine (2010) and Krämer (2013); I therefore suggest that one of the solutions to these problems might help in our case as well. In any case, an analogous problem arises using a truth predicate '$T()$' and a singular term for reference to propositions, as witnessed by $T(a) \vee P$ (here, let 'a' refer to the proposition expressed by the formula it occurs in). As noted above, the existence of propositions like this is normally granted despite of the problem, it is thus unclear why we should reject the existence of propositions like $A \vee P$ due to the analogous problem.[24]

[21] Relatedly, one could consider using a pro-sentential device instead of 'T(this proposition)'.

[22] This is an instance of Aristotle's insight, cf. Correia and Schnieder (2012, p. 26).

[23] For example, see the investigation of Curry-paradoxes, cf. Shapiro and Beall (2018). See also Whittle (2017) on self-referential propositions.

[24] Thanks to Julio De Rizzo here!

5.4 Another regress?

Since G just is $\Gamma, G < P$, it seems to follow that G also just is $\Gamma, (\Gamma, G < P) < P$ and $\Gamma, (\Gamma, (\Gamma, G < P) < P) < P$, etc. Thus, the loopy proposal can seem to involve its own regress. My response is that '$\Gamma, G < P$' is the most fundamental and perspicuous formulation of G—while the sentence expressing this proposition might be "unpacked" in the above fashion, this is but an unproblematic consequence of its self-involvement.

In his discussion of the grounding role of moral principles, Berker (2018, Sect. 4) considers a proposal very much like the present one. Like I suggested above, Berker argues that we should not prematurely dismiss such proposals merely due to their commitment to self-involving or self-referential propositions, but continues to argue against them. His first (2018, p. 915) argument is very close to the problem just responded to (he is focusing on an alleged fact F which is stipulated to be identical to [F grounds G]):

> If we substitute F into itself an infinite number of times, the identity statement by which we introduced F becomes $F =$[[[... grounds G] grounds G] grounds G]. (See Figure 4.) But now we have cause to be worried. How can such an endless quicksand of iterated grounding relations, never leading back to an independent grounder for the entire sequence, be the case?

We should respond as before: There is a perspicuous formulation of the grounding fact in question, namely '[F grounds G]'. It is neither clear that Berker is entitled to "substitute F into itself an infinite number of times", nor that the strangeness of the result is inherited by the perspicuous self-referential formulation: There is no objectionable "endless quicksand of iterated grounding relations", just a single instance of grounding that relates itself (in similar instances together with other grounds) to what it grounds.[25]

Moreover, given what I have argued above, the ground constructed by Bolzano's regress (or a transfinite continuation thereof) does not even amount to a full ground by the lights of **Entailment as Ground** or **Ground as Ground**): This amounts to an argument for the viciousness of the regress. In contrast to this, the loopy proposal affords a finite and fully perspicuous formulation of full grounds, e.g. '$\Gamma, G < P$', that satisfy the idea that linking propositions linking the full ground in question to its groundee must be part of the full ground.[26]

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[25] For what it is worth, it may be possible to use the metaphor of grounding as related to construction work (cf. Bennett, 2017) to argue that the proposed self-involvingness of grounding facts need be nothing objectionably strange. For think of grounding as a crane that stacks physical objects (e.g. concrete blocks) on top of each other: The crane itself is a physical object and (given the right shape) may stack concrete blocks on top of itself.

[26] What about variants of the loopy proposal? While Berker's case seems to show that switching to a predicational idiom does not immediately block the substitution move, we could consider variants of the loopy proposal that use (as linking propositions to form full grounds) laws of metaphysics that quantify over themselves instead of the loopy grounding facts. I suspect that such a proposal, if properly formulated, blocks the substitution move. Thanks to an anonymous referee and Jan Claas for discussion here!

5.5 Berker's bootstrapping argument

Berker (2018, 915f.) offers a second argument:

> Specifying G helps us see how implausible it is to think that a fact with this form obtains. Suppose we let
>
> G =[Some self-referential fact makes something the case].
>
> Then we have:
>
> F =[F makes it the case that some self-referential fact makes something the case]
> ...
>
> This choice of G is just about the best-case scenario for a value of G that would allow F to obtain. But even here we should be skeptical. This version of F is something like a ground-theoretic analogue of the truth-teller sentence "This sentence is true": if F obtains, then it is a self-referential fact that makes something the case (because all facts make something the case), so it makes it the case that some self-referential fact makes something the case, and hence its content is true; but if F does not obtain, then none of this holds, so its content is not true; and there are no independent facts that settle which of these is the case. Just as we should be skeptical that the truth-teller sentence is true, so too should we be skeptical that F obtains.

Now, I agree with Berker that a close relation between the loopy proposal and a familiar paradox exists, but the paradox I will discuss below is Curry's. As to Berker's objection, note first that even if we agree that something is problematic about the particular case he considers, he does not argue that the problem generalizes to the loopy grounding claims proposed, and it is not clear how it should. Thus one may well suspect that the problem (if there is one) concerns the particular kind of case he considers.

Admittedly, I am not sure that I see what exactly is supposed to be objectionable about Berker's case. In that regard, it is unlike other cases of (arguably) worrisome bootstrapping, such as cases of reflexive or symmetric grounding, or the cases I consider in my (2022) (for related considerations see Hicks (2020)): In those, a grounding claim connects a ground to itself: P grounds Q, and this grounding fact itself is identical to Q.

In the remainder of this section, I will (1) consider another candidate for a ground of [Some self-referential fact makes something the case] that suggests that either F is unproblematic or the problem is specific for that case, (2) suggest that what Berker considers to be an objectionable kind of bootstrapping is indeed an unproblematic phenomenon that occurs in other contexts too, and (3) use considerations involving zero-ground to argue that proponents of the loopy proposal have principled reason to affirm F.

First, assuming there are self-referential facts, according to Berker, [Some self-referential fact makes something the case] is grounded in some such self-referential facts. Consider the metaphysical or grounding law L (conceived of a kind of generalized quantified proposition) that corresponds to these cases of grounding: While it may not be self-referential in the strict sense, L quantifies over itself and can thus be considered self-referential in a loose sense (alternatively, we can consider [Some fact that quantifies over itself makes something the case] instead of [Some self-referential fact makes something the case]). Given Berker's reasoning, L grounds [Some self-referential fact makes something the case].

But this does not give rise to a problematic kind of bootstrapping, not even if we assume that L is fundamental (i.e. ungrounded) and that instances of grounding are grounded in corresponding laws plus grounds, and thus that (analogously to what Berker observes about F) there is no fact independent of L that settles either L or L's grounding [Some self-referential fact makes something the case].

Second, consider laws of metaphysics or grounding conceived as before, for example a law $L_\vee$ governing the grounding of disjunctions in each of their disjuncts. Now, since every true proposition grounds something, so does this law. The law thus does not only quantify over itself, it also governs an instance of itself grounding something, e.g. $L_\vee < (L_\vee \vee \bot)$ (where $\bot$ is some arbitrary logical falsehood). But now note that just like the previous case, this does not give rise to an objectionable kind of bootstrapping! Moreover, this appears to hold even if we assume that the law is fundamental and that instances of grounding are grounded in corresponding laws plus the involved grounds: Once more, analogously to what Berker observes about F, there is no independent fact to settle whether $L_\vee \vee \bot$ and $L_\vee$ is the case, and whether the latter grounds the former—there is just $L_\vee$ grounding $L_\vee \vee \bot$ and $L_\vee < (L_\vee \vee \bot)$.

I take this to suggest on the one hand that there is no objectionable kind of bootstrapping in Berker's case, and on the other hand that variants of the loopy proposal that concern metaphysical laws that are self-referential (or rather quantifying over themselves) are immune to Berker's worry (although I lack the space to develop such a proposal in any detail here).

Third, at Berker does not show is how his consideration is supposed to generalize to "realistic" examples for [F grounds G] (where this fact is identical to F)—in particular the loopy analogues of zero-grounding facts, i.e. facts that amount to something being grounded, but not grounded in anything (see for example Fine (2012) and Litland (2017) for introductions to this notion).

On the loopy proposal, there are no zero-grounding facts, because at least the loopy grounding fact itself is required as a ground. Thus, a candidate zero-grounding fact like that amounting to the existence of the empty-set being zero-grounded, i.e.:

$$< \exists x (x = \emptyset)$$

would have something like the following analogue on a loopy proposal (again, in loopy fashion, the following fact is identical to F; for convenience sake I repurpose 'F' as a letter for a sentence expressing the fact F):

$$F < \exists x (x = \emptyset)$$

It is hard to see how this sentence shares the allegedly truth-teller-ish quality of Berker's example (although again, we are going to discuss the loopy proposal vis-à-vis Curry's paradox momentarily). Moreover, it is not clear why there would have to be an "independent fact that settles whether F does or does not obtain": To require this seems to be as misplaced as it would be in the original zero-grounding case.

Now, cases like this give the proponent of the loopy proposal a principled reason to accept Berker's F . Given widespread and non-loopy assumptions, there plausibly are self-referential zero-grounded facts like (here, let 'ϕ' be some expression that established self-reference), for example:

$$\exists x(x = \emptyset) \vee \phi$$

This fact is zero-grounded given the above assumptions that its first disjunct is zero-grounded, that disjunctions are grounded in their true disjuncts, and that transitivity holds. But given Berker's reasoning quoted above, the following should then hold too:

$$(\exists x(x = \emptyset) \vee \phi) < \text{[some self-referential fact makes something the case]}$$

But by transitivity we then get that [some self-referential fact makes something the case] is zero-grounded:

$$< \text{[some self-referential fact makes something the case]}$$

Hence, given what I said before, the loopy theorist aiming to endorse analogues of zero-grounding facts within their framework should endorse the corresponding loopy fact, which is Berker's F:

$$F < \text{some self-referential fact makes something the case}$$

This concludes my discussion of the first few potential problems for the loopy proposal.

6 Curry

While I believe that we should not dismiss the loopy proposals on the basis of the problems addressed in the previous section, there remains reason to suspect that the self-involving propositions required are problematic due to their relation to Curry-style paradoxes.

Section 6.1 discusses sentences of the form of C vis-à-vis Curry-style paradoxes. This is not only relevant with respect to Bolzano's original idea (whose reconstruction involves sentences of the form of C), but also for the application of the loopy idea to Carroll's regress in Sect. 7 below. Section 6.2 then discusses sentences of the form of G vis-à-vis Curry-style paradoxes.

6.1 Curry and C

One might think that because sentences of the form of C (i.e. $\Gamma, C \Rightarrow P$) are problematic because they are closely related to a variant of Curry's paradox that employs the notion of logical consequence (the so-called "validity Curry" or "v-Curry", see Shapiro and Beall (2018, Sect. 6)). For assume

$$C \tag{1}$$

This just is

$$\Gamma, C \Rightarrow P \tag{2}$$

Now assume

$$\Gamma \tag{3}$$

From 1, 2, and 3 due to a modus ponens principle for $\Rightarrow$:

$$P \tag{4}$$

In steps 1–4 we have derived P from Γ and C, so given a principle of conditionalization for $\Rightarrow$ we can derive $\Gamma, C \Rightarrow P$ independently of any assumption:

$$\Gamma, C \Rightarrow P \tag{5}$$

Since this just is C, we can then derive P assuming only Γ and using modus ponens for $\Rightarrow$:

$$P \tag{6}$$

Since Γ and P can be chosen arbitrarily, paradox ensues.

Here is an instructive alternative derivation: Choose Γ and P arbitrarily and assume $\neg C$ and hence $\neg(\Gamma, C \Rightarrow P)$. If we understand $\Rightarrow$ as a strict conditional using logical possibility, it follows that it is logically possible that $\Gamma \wedge C \wedge \neg P$, so C is logically possible. But assuming S5 modal logic for logical possibility, C follows with contradiction, so we have "proven" $\Gamma \wedge C \wedge \neg P$ with arbitrarily chosen Γ and P, which in turn seems to allow us to derive arbitrary P from arbitrary Γ.

To attack the loopy proposal, Bolzano might then argue that since thinking about instances of C can lead to paradox, even those instances that correspond to legitimate inference principles (i.e. those required by the loopy proposal) are suspect. While the original Tortoise argument results in a dubious regress, according to this objection the loopy suggestion falters because its reliance on candidate propositions whose close relationship to paradox makes them dubious.

I think this response would go too far: Many instances of C are of course unacceptable, but those instances that correspond to valid inferences (e.g. $P, C_\vee \Rightarrow P \vee Q$, where $C_\vee$ is this very proposition) are not obviously defective. Indeed, a case can be made for their truth: Assuming that P entails $P \vee Q$ and given the monotonicity of entailment, P entails $P \vee Q$ together with any arbitrary proposition. Thus, if $C_\vee$ expresses a proposition, P entails $P \vee Q$ together with $C_\vee$ (see below for a variant of this argument and an application to v-Curry sentences with a logically true consequent). It is unclear whether this consideration alone would be sufficient, but if a principled ground to differentiate the proper instances from the improper ones were available, a response to the above worry can be given.[27]

In contrast to such a *Curry-incomplete* approach, *Curry-complete* approaches to the paradox admit of the problematic propositions and aim to avoid paradox by providing a suitably modified logic (cf. Shapiro & Beall, 2018, Sect. 4). This is where things quickly get quite complicated.[28] Since a detailed discussion is beyond the scope of this paper, I hope to provide an idea of how such a discussion could start. Now, Curry-complete solutions modify the logic of the conditional (or conditional-like connective) and fall roughly into two categories: *Contraction-free* responses deny the rule of contraction and *detachment-free* responses deny a version of modus ponens sometimes called detachment. Both responses come in a weak and a strong version (for details see Shapiro & Beall, 2018, Sects. 3 and 4), but the weakly contraction free approach and the strongly detachment free approach are both implausible as solutions to the v-Curry (Shapiro & Beall, 2018, Sect. 6). This leaves the strongly contraction free approach and the weakly detachment free approach.

There is some reason to suspect that the loopy proposal involving logical consequence cannot be made to work on the strongly contraction free approach: This is because on this approach, logical consequence is sensitive to the number of times a premise is used in a derivation. Thus, a conclusion might follow from a premise taken twice, but fail to follow from the same premise taken once. Since propositions of the form of C are supposed to express instances of logical consequence, they should be formulated accordingly. Now, one *might* think that in deriving P from Γ and C (i.e. $\Gamma, C \Rightarrow P$), the latter is used *twice*: Once as antecedent, and once in its conditional-like capacity. Accordingly, C would not correctly capture the corresponding entailment fact, since its antecedent only contains C twice. At this point, one might consider a revised version of C:

$$(C^?) \qquad \Gamma, C^?, C^? \Rightarrow P$$

It seems that the corresponding argument would have to have $C^?$ as a premise *twice*. But in the corresponding derivation, it appears $C^?$ would be used *thrice*: Twice as antecedent, and once in its conditional-like capacity. If this reasoning is correct, it is hard to see how the required loopy propositions could look like on strongly contraction free approaches.

Ripley's (2013) weakly detachment free approach avoids such considerations, but for our purposes, there is a problem with how he diagnoses Curry sentences within his framework: It seems that according to him (2013, 14f.), *all* Curry sentences (and presumably sentences of the form of C as well) should neither be asserted nor denied. But of course, the loopy proposal requires some sentences of the form of C to be true and assertible. Luckily, as far as I can see, Ripley's framework allows us to hold that

[27] An account according to which some Curry-sentences are acceptable that might be useful here is Rosenkranz and Sarkohi (2006).

[28] If so inclined, the reader may skip ahead to either Sect. 6.2 or Sect. 7.

some v-Curry sentences (and sentences of the form of C) should be asserted (and not denied), despite what he seems to claim in the passage cited above.

For in the bilateralist framework that Ripley (following Restall, 2005) employs, logical consequence is understood in terms of constraints on assertion and denial: Some positions that one might take (i.e. ordered pairs $< \Gamma, \Delta >$ of assertions Γ and denials Δ) are *in bounds* and some are *out of bounds*. Roughly, the notion of being in bounds can be glossed as "logically (or conceptually) coherent", while the notion of being out of bounds can be glossed as "logically (or conceptually) incoherent". (Ripley, 2013, p. 141) then understands logical consequence via the notion of an out-of-bounds position and reads $\Gamma \vdash \Delta$ as "the claim that the position $< \Gamma, \Delta >$ is out of bounds", i.e. logically (or conceptually) incoherent.

Now consider a v-Curry sentence with a logically true consequent, e.g.:

$$(C_{LEM}) \qquad C_{LEM} \Rightarrow P \vee \neg P$$

Understanding logical consequence as above, we can read this as "The position $< C_{LEM}, P \vee \neg P >$ (i.e. the position that asserts C_{LEM} and denies $P \vee \neg P$) is out of bounds (i.e. logically incoherent)". But now one can reason as follows: Since $P \vee \neg P$ is a logical truth, denying it should be logically incoherent irrespective of what one may additionally assert. Therefore, it seems simply true that the position $< C_{LEM}, P \vee \neg P >$ is logically incoherent: Irrespective of what the assertion of C_{LEM} may contribute, denying $P \vee \neg P$ seems sufficient to make for an incoherent position.[29] Hence, it should be admissible to assert C_{LEM}, and (by plausible extension) some sentences of the form of C.

6.2 Curry and *G*

Let us now turn to the loopy proposal involving sentences of the form of G and consider Curry-like paradoxes vis-à-vis the grounding connective '$<$'.

Several conditional-like connectives allow for a greater variety of solutions to Curry's paradox than $\Rightarrow$. Some of these are the material conditional, Nolan's (2016) modal conditional, and the grounding connective '$<$': As is the case for Nolan's conditional, no analogue to the rule of conditionalization (or conditional proof) seems to hold for '$<$' (meaning for them a weakly contraction free solution is available). For example, having derived P from $\neg\neg P$, conditionalization allows us to derive $\neg\neg P \rightarrow P$ while discharging the assumptions that $\neg\neg P$ depended on. In contrast, '$<$' cannot satisfy this rule because $\neg\neg P$ does not ground P (rather, it is P that grounds $\neg\neg P$). Hence, no arbitrary propositions of the form $\Gamma, G < P$ can be derived along the first schema presented in Sect. 6.1.[30]

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[29] We assume that $P \vee \neg P$ is indeed logically true.

[30] Conditionalization already fails because of the factivity of '$<$': Even if we could in principle derive $\Gamma, G < P$ after having derived P from Γ, G, it seems factivity would demand that we cannot discharge the assumptions that Γ depends on. Since I am unsure how much we should rely on this, I will focus on aspects pertaining to both factive and non-factive ground here.

Likewise, the negation of $\Gamma, G < P$ does not entail the possibility of $\Gamma \wedge G \wedge \neg P$, since grounding claims (non-factive and factive) can fail for different reasons than their left-hand side being true and their right-hand side being false, like their relata not exhibiting the right kind of metaphysical priority relation or explanatory connection. Hence, no arbitrary instance of $\Gamma, G < P$ can be derived along the second schema presented in Sect. 6.1. Thus, worries related to Curry's paradox appear to gain no immediate purchase on **Grounding as Ground**.

In the remainder of this section, I discuss a complication stemming from Litland's (2017) calculus for explanatory arguments. The calculus involves a conditionalization-like introduction rule for non-factive grounding that allows us to introduce $\Gamma <_{NF} P$ while discharging the assumptions Γ depends on, *if* the derivation of P from Γ has been explanatory.[31] Now, ironically, it might seem like the derivation of P from Γ and $\Gamma, G < P$ is explanatory. To see this, consider first the (non-factive) grounding Curry-sentence ('P' is arbitrarily chosen):

$$(G^*) \qquad G^* <_{NF} P$$

Assuming G^*, we then get $G^* <_{NF} P$ by substitution, and a suitable detachment principle gives us P. If this is an explanatory argument, then '$<_{NF}$' is not Curry-resistant in the envisaged sense given a conditionalization-like introduction rule like Litland's, because such a rule would allow to then derive G^* as a theorem.

Now, it seems that what characterizes explanatory arguments (of the grounding variety) is that each of their steps tightly corresponds to an instance of non-factive grounding (at least this appears to hold for Litland's calculus). Therefore, one might argue that in assuming G^* and hence $G^* <_{NF} P$, we have effectively assumed that an argument from G^* to P is explanatory. Moreover, according to the tripartite account of explanation and especially according to the motivating idea that in grounding explanations, grounding propositions play an explanatory role akin to ordinary grounds, grounds Γ and grounding proposition $\Gamma <_{NF} P$ together explain P. Therefore, the inference from Γ and $\Gamma <_{NF} P$ may appear like an explanatory one. Since the inference of P from G^* and $G^* <_{NF} P$ appears to be an instance of this schema, it may look explanatory as well.

What, if anything, is wrong with this reasoning? Explanatory intuition first: Suppose P is the proposition that the moon is rising. Then a case can be made that arguments of the above form are not in general explanatory: That the moon is rising is not *explained* by the curious proposition M: $M <_{NF}$ The moon is rising. Or consider propositions of the form of G, such as the propositions SM: Singleton Socrates exists, $SM <_{NF}$ The moon is rising. This seems false, since Singleton Socrates' existence is explanatorily

[31] Litland uses '$\Rightarrow$' to express non-factive ground; we will use '$<_{NF}$'. For an argument to be an explanatory argument, the premises must actually stand in an explanatory (if non-factive) relationship to the conclusion. In this respect, 'explanatory argument' behaves like 'valid argument'.

irrelevant to the moon's rising. Hence, the corresponding argument does not seem to be explanatory.[32]

Let us therefore try to diagnose where the Curry-like reasoning for G^* above goes awry. First, we should deny that assuming $G^* <_{NF} P$ as a premise of an argument forces us to consequently admit of a corresponding explanatory rule of inference (according to which the inference of P from G^* is explanatory). Likewise, assuming falsities about what entails what as premises in an argument does and should not force us to change our calculus accordingly.

As to the other consideration (i.e. the inference of P from G^* and $G^* <_{NF} P$ looking like an instance of a more general explanatory schema): If we were to assume that grounding propositions do not in general play the role of premises in explanatory arguments (but rather correspond to the involved explanatory rules of inference), then an easy fix would be possible: For ordinary conditionals (e.g. material, strict, counterfactual), detachment is not an explanatory inference; likewise, if grounding propositions do not in general play the role of premises in explanatory arguments, there appears no reason to assume that the inference from Γ and $\Gamma <_{NF} P$ to P is explanatory in general.

Alas, for our purposes, this response does not suffice, for it seems in the spirit of the motivating idea from Sect. 2 that grounding propositions often (if not always) play the role of explanatory premise in corresponding explanatory arguments (after all, according to the motivating idea, they partially ground the corresponding groundee)!

Here, I believe, we should steadfast: Even if we want to allow for the idea that many arguments of the form $\Gamma, \Gamma <_{NF} P \therefore P$ are explanatory, we should deny that all are. As my reflection on cases suggests, merely having $\Gamma <_{NF} P$ as a premise of an argument with conclusion P does not forge an explanatory connection between Γ and P. Consider for example the proposition that the moon is rising: I believe we should not say that the argument from Singleton Socrates' existence and the propositions that (Singleton Socrates exists $<_{NF}$ The moon is rising) to the moon's rising is explanatory—while according to the second premise, the first premise would non-factively ground the conclusion, it does not in fact do so, and this is what matters for whether or not the argument actually is explanatory in the sense I aim at.[33] So, the explanatory connections that explanatory arguments correspond to are real (if non-factive). At least, it seems plausible that notions of ground and explanatory arguments exist (perhaps in addition to more permissive notions) that suit our purpose (their Curry-resistance may additionally speak in their favour).

Given these notions, the derivation of arbitrary grounding Curry-sentences and sentences of the form of G can be blocked because modus-ponens-like arguments of the form $\Gamma, \Gamma <_{NF} P \therefore P$ are not in general explanatory and hence the conditionalization for explanatory arguments does not apply to them. Hence, the loopy grounding

[32] Possible response: While Singleton Socrates' existence does seem to be irrelevant to the moon's rising, the singleton's existence *together* with SM can still be relevant to the moon's rising. Rejoinder: Maybe, but the consideration of the last paragraph of this section seems to hold either way.

[33] Moreover, if all arguments of the form Γ, $(\Gamma <_{NF} P) \therefore P$ were explanatory, the notion of an explanatory argument would violate certain structural rules that we might desire. For example, it allows for circular arguments to be explanatory.

proposal involving sentences of the form of G can be defended against the worry from Curry-like problems.

This concludes my discussion of the grounding role of linking propositions, Bolzano's argument, and the loopy proposal.

7 A loophole for Achilles?

In this section, I show how an analogue of the loopy idea developed above can be applied to the discussion of Carroll's (1895) regress in which his characters Achilles and the Tortoise get entangled in.

Since what exactly this regress is supposed to show is a matter of debate, it is not immediately evident how the existence of the loophole in Bolzano's argument may bear on Carroll's structurally analogous regress. The suspicion is that Achilles could have offered the Tortoise a loopy premise of the kind considered above and thereby stop the interrogation, but what exactly would this show? Below I will first develop my own, modest, reading of the dialogue and apply the loopy response to it. Then I will offer a loopy response to the infinite regress problem of inference as formulated in Rosa (2019).[34] Initially, we will suppose that the Curry problem can be solved for '$\Rightarrow$'; later, we will discuss other connectives.

In the dialogue, Achilles' task is to force the Tortoise to accept the conclusion Z of a modus ponens argument with premises A and B. The Tortoise initially does not accept (or at least claims as much) that if A and B are true, then Z must be true. For our purposes, we will assume that the Tortoise initially does not accept that A and B *entail* Z. However, the Tortoise is willing to grant Achilles any premise he requires, once he makes it explicit (presumably, the argument must not be circular):

> "I'm to force you to accept Z, am I?" Achilles said musingly. "And your present position is that you accept A and B, but you don't accept the Hypothetical—"
> "Let's call it C," said the Tortoise.
> "—but you don't accept
> (C) If A and B are true, Z must be true."
> "That is my present position," said the Tortoise.
> "Then I must ask you to accept C."
> "I'll do so," said the Tortoise, "as soon as you've entered it in that note-book of yours. What else have you got in it?"

The regress arises because whenever Achilles adds a premise and thus produces a new argument, the Tortoise insists that a premise is still missing which states that the new argument's premises entail its conclusion. Thus, the dialogue eventually continues as follows:

> [Achilles:"] Logic would tell you 'You ca'n't help yourself. Now that you've accepted A and B and C and D, you must accept Z!' So you've no choice, you see."

[34] My discussion is thus restricted to these two readings of Carroll's dialogue.

> "Whatever *Logic* is good enough to tell me is worth writing down," said the Tortoise. "So enter it in your book, please. We will call it
> (E) If A and B and C and D are true, Z must be true. Until I've granted that, of course I needn't grant Z. So it's quite a necessary step, you see?"

It appears that the Tortoise demands that the logical relation between an argument's premises and conclusion should be stated explicitly by (some of) the argument's premises. On a modest reading, the regress thus aims to establish that the following assumptions cannot hold together:

(Consequence among Premises) For an argument to be fully explicit, the relation of logical consequence holding between its premises and conclusion must be captured by (some of) its premises.

(Finite Premises) There are fully explicit arguments that have finitely many finite premises.

The dialogue also suggests a stronger reading, according to which only fully explicit arguments can be sufficiently convincing or rationally compelling, and there may be better ways than **Finite Premises** to spell out the viciousness of the regress, but **Consequence among Premises** and **Finite Premises** will do for our purposes. To illustrate how the regress is supposed to show that these principles cannot hold together, consider the argument schema of modus ponens:

$$\frac{\begin{array}{c}P\\ P \rightarrow Q\end{array}}{Q}$$

According to **Consequence among Premises** (ordinary) instances of this schema are not fully explicit. Thus, Achilles attempts to supply the missing premise, the corresponding argument schema being this:

$$\frac{\begin{array}{c}P\\ P \rightarrow Q\\ (P, P \rightarrow Q) \Rightarrow Q\end{array}}{Q}$$

But the Tortoise complains: This still violates **Consequence among Premises**, because now no premise captures that P, $P \rightarrow Q$ and $(P, P \rightarrow Q) \Rightarrow Q$ together entail Q. **Finite Premises** then models the idea that Achilles cannot succeed by simply supplying additional linking premises ad infinitum.

The existence of the loophole now shows that the infinite regress may be avoided and both **Finite Premises** and **Consequence among Premises** be satisfied, if we accept the relevant linking propositions to be loopy:

$$\frac{\begin{array}{c}P\\ P \rightarrow Q\\ (P, P \rightarrow Q, C_{MP}) \Rightarrow Q\end{array}}{Q}$$

Here, let C_{MP} be $(P, P \rightarrow Q, C_{MP}) \Rightarrow Q$. Instances of this schema appear to satisfy both **Finite Premises** and **Consequence among Premises**: All three premises are linked to the conclusion by one amongst themselves that expresses the corresponding entailment, namely C_{MP}. Whereas in the original regress, questioning one of the inferences was not (immediately) to question one of the premises and hence moved Achilles to add new premises, questioning the entailment from P, $P \rightarrow Q$ and the loopy modus ponens premise C_{MP} (i.e. $(P, P \rightarrow Q, C_{MP}) \Rightarrow Q$) to Q *just is* questioning the loopy modus ponens premise. In other words, that the premises of this kind of argument entail its conclusion cannot be questioned without ipso facto questioning one of its premises.

Given our reading of the dialogue, it seems that this is the very kind of argument that should satisfy the Tortoise: As the passages cited above make clear, the Tortoise is willing to grant Achilles his premises, but insists that he make explicit as premises "[whatever] *Logic* is good enough to tell [them]". On our reading these are propositions concerning logical consequence holding between premises and conclusion. An argument of the above schema appear to satisfy this desideratum: One of its premises expresses the relation of logical consequence holding between all of its premises and its conclusion.

Considering the discussion concerning Curry's paradox from above, one way the Tortoise could object is to point out the apparently paradoxical nature of the loopy assumptions. If Achilles' goal was to convince the Tortoise by stating his argument in a particularly explicit form in accordance with **Consequence among Premises**, then offering premises of a form that threatens to lead into paradox may seem less than ideal. Luckily for Achilles, the previous response applies here too: If a solution to the v-Curry is presupposed according to which the relevant premises can be maintained (for example a Curry-incomplete solution or a modification of the logic of '$\Rightarrow$'), then the problem is avoided. Alternatively, Achilles can try to employ a different, Curry-resistant, connective to formulate his linking premises—for example, he could try the aforementioned modal conditional by Nolan or a connective expressing non-factive grounding. A proper discussion of these possibilities is beyond the scope of this paper, but note (1) that non-factive grounding restricts Achilles to arguments whose premises (non-factively) ground their conclusion (like instances of disjunction introduction, but unlike modus ponens), and (2) that these options might raise the task of convincing the Tortoise to accept arguments on the basis of their premises standing in the relation expressed by the Nolan-conditional or *grounding* their conclusion instead of on the basis of their premises *entailing* their conclusion.

Let us now apply the idea to Rosa's (2019, p. 2264) formulation of the infinite regress problem of inference. According to Rosa, the problem can for example be found in Boghossian (2003) and Railton (2004). Since these each involve their own complications, I focus here on Rosa's formulation:

> Assume that no piece of reasoning is doxastically blind: if there is reasoning at all, then the reasoner believes that her premises give support to her conclusion (or the reasoner holds a 'bridge-belief' for short). Now consider a subject S who infers that ψ on the basis of her belief that ϕ. According to our initial assumption, S also believes that ϕ gives support to ψ. Presumably, then, that

> belief was also part of S's pool of premises, and S's inferential belief is actually based on both S's belief that ϕ and S's belief that ϕ gives support to ψ. But then our initial assumption says again that *that* piece of reasoning must not be doxastically blind, and so the reasoner must also believe that premises ψ and ϕ *gives support to* ψ themselves give support to ψ. Therefore, the content of that belief was also part of S's pool of premises, and so on ad infinitum: it turns out that reasoning is impossible (for S would need to draw her conclusion on the basis of infinitely many beliefs). [...]
> There are three ways to respond to this argument: (1) to require the reasoner to have some non-doxastic or non-propositional attitude towards the support relation between premises and conclusion, (2) to still require the reasoner to hold bridge-beliefs, but in such a way that they are not part of the reasoner's pool of premises, or (3) not require any bridge-attitude on the part of the reasoner at all.

Rosa proceeds to discuss several instances of these strategies (due to BonJour, 2014; Fumerton, 2015; Valaris, 2014, 2016; Wright, 2014), but another strategy is possible if we allow for loopy bridge-beliefs like the following:

(B) ϕ, B give support to ψ.

For let ϕ and B be the premises on the basis of which S infers ψ. This piece of reasoning is not doxastically blind: Since S believes B, they believe that ϕ, B give support to ψ. Moreover, the bridge-belief that B is among S's premises. Nevertheless, as before, the regress is avoided in familiar loopy fashion. While developing the idea further is beyond the scope of this paper (e.g. by investigating the notion of epistemic support in B and its Curry potential), we have thus seen how the loopy proposal applies to a contemporary version of Carroll's problem.

8 Conclusion

Let us recap: In this paper I have first motivated the idea that full grounds need to contain propositions (e.g. rules of inference or grounding propositions) linking the respective grounds to the groundee. I have then reconstructed Bolzano's Tortoise argument aimed to refute this idea (and added to it by giving a argument for the viciousness of the regress that Bolzano identified), and I have offered a response based on loopy propositions involving logical consequence or grounding. I have then defended this idea against a number of problems. As I have moreover shown in the last section, such a loopy response can be given to Carroll's Tortoise and at least one of its contemporary relatives as well.

As mentioned above, it is possible to read this paper discussion either as a defense of the loopy proposal, or if ultimately unsympathetic with my responses to the problems, as a completion of Bolzano's argument. Concerning the sympathetic outlook: While I have offered an initial defense of the existence of the required loopy propositions and argued that at least some variants of the proposal can resist Curry-related worries, more work remains to be done here, one being taking a closer look at using self-relating laws instead of loopy grounding propositions. Beyond that, it could be interesting to further develop a theory of grounding based on the loopy idea (for example, to draw out its consequences vis-à-vis the theory of fundamentality), as well as the loopy theory of epistemic support suggested in Sect. 7.

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