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ORIGINAL PAPER

Relocating mathematics: a case of moving texts between the front and back of mathematics

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Abstract

As mathematics departments in the United States began to shift toward standards of original research at the end of the nineteenth century, many adopted journal clubs as forums to engage with new periodical literature. The Bryn Mawr Mathematics Journal Club, maintained episodically between 1896 and 1924, began as a supplement to the graduate course offerings. Each semester student and professor participants focused on a single disciplinary area or surveyed what had been published lately. The Notebooks containing these reports were stored on the open shelves of the college library. These collectively composed documents record ways in which graduate students transcribed and interpreted contemporary literature from the front to the back of mathematics. This article will consider the entries of a single student in which published mathematics was rewritten for a local audience and how the process of relocation animated research at Bryn Mawr.

Keywords The front and back of mathematics · Local and universal · Circulation of mathematical texts · Tacit knowledge · Graduate education · Early-twentieth-century topology

The year 1896 was auspicious for the Bryn Mawr Mathematics Department.[1] The first two PhDs published their dissertations. A generous gift enabled the library to purchase much desired Italian journals (supplementing the French, German, and English periodicals). Bertrand Russell proposed to give a series of lectures on his upcoming book in the foundations of geometry.[2] Many of the senior undergraduates that spring

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planned to "continue their studies" as graduate students (Thomas, 1897, p. 67).

In her departmental report to the president of Bryn Mawr, Professor Charlotte Angas Scott announced the "formation of a Journal Club, to receive reports on special topics and listen to outline accounts of interesting theories that do not naturally present themselves in the regular graduate work."[3] She noted that such a club "has often been considered, but always with the result that it was not thought advisable." However, the number of possible participants "points to the present time as most suitable for the experiment." She suggested as a first topic "the true foundations of geometrical science, these being discussed in their philosophical as well as their mathematical aspect" in complement to Russell's anticipated lectures (Thomas, 1897, pp. 67–68).

During its first year the Mathematics Journal Club met for one hour "once a fortnight to listen to reports on subjects which were not included in the graduate instruction for the year" (Thomas, 1898, p. 71). The talks from November 1896 through May 1897 ranged widely over contemporary mathematics.[4] Regular participants included four graduate students, Professors Scott and James Harkness, and recent doctoral student Isabel Maddison.[5] Scott also expressed thanks to "Professors F. Morley and E. W. Brown, of Haverford College, for their kindness in presenting communications" though it is unclear whether they attended other sessions as well.[6]

The Journal Club remained variably active over almost thirty years. Entries were recorded by hand in the Mathematical Journal Club Notebooks stored on the open shelves of the college library for student perusal.[7] Scott reported that "meetings are a most important adjunct to the provision for formal instruction made by the department, both in the encouragement they offer to students to recast the results of investigations into a form adapted to an assigned purpose, and in the opportunities thus presented of

2 On the organization, contents, and reception of Russell's lectures, see Dunham (2016).

3 On Charlotte Angas Scott's biography and contributions to mathematics see Kenschaft (1982a), Kenschaft and Katz (1982), Kenschaft (1987), and Lorenat (2020b). On Scott's students in particular, see Kenschaft (1982b) and Lorenat (2020a), A history of Bryn Mawr's mathematics department over the first fifty years can be found in Parshall (2015).

4 Between November and May these talks were as follows Scott (1896): 1. On Non-Euclidean Geometry. Professor Scott; 2. Modern Researches on the Number System. Professor Harkness; 3. Theory of Symmetric Figures. F. C. Gates; 4. Curves which cover an area of the plane. Dr. Maddison; 5. Apolarity. Professor Morley; 6. The problem of map colouring. H. S. Pearson; 7. Representation of Regular Groups by Colour Diagrams. E. N. Martin; 8. Infinite Determinants. Professor Brown; 9. The Transcendency of e and π. V. Ragsdale; 10. Regular Reticulations and Regular Branches upon a Riemann Surface. F. C. Gates; 11. Numbers and Functionals of an algebraic corpus. Professor Harkness; 12. Circuits. Professor Scott.

5 James Harkness graduated from Cambridge University as eighth wrangler in 1885. He taught at Bryn Mawr College until 1903 when he accepted a position at McGill University in Canada. Isabel Maddison studied at Cambridge University, Bryn Mawr College, and the University of Göttingen before obtaining her Ph.D. under Scott in 1896 "On Singular Solutions" (Maddison, 1896). She spent the remainder of her career in administration at Bryn Mawr, beginning as the president's secretary and eventually becoming dean.

6 Frank Morley graduated from Cambridge University in 1884 (four years after Scott), began at Haverford College in 1887, and moved to Johns Hopkins University in 1900 where he supervised 48 doctoral students and remained for the rest of his career. Ernest William Brown graduated from Cambridge in 1887 where he also received his master's degree. He worked at Haverford from 1891 to 1907 and then at Yale University from 1907 to 1932.

7 In what follows, the notebook entries will be cited by author, entry title, date, and page number. Though the Notebooks span several books, there are no titles to the separate volumes and a year's worth of entries could span one or more.

becoming acquainted with topics that cannot well be included in the regular courses" (Thomas, 1899, p. 97). Scattered references in these administrative documents show the central role the Journal Club held in graduate student development, both supplementing the course offerings and promoting fields of potential research. A brief summary of the mathematics department for the academic year 1904/05 noted the direct causal relationship between the Journal Club reports and research: "a main subject was selected in Differential Geometry" with reports "presented in special topics arising out of this" resulting in some students "now carrying on investigations in connection with the special topics thus discussed."[8]

Readers of *Synthese* are probably familiar with Reuben Hersh's 1991 call for a better awareness of the front and back of mathematics (Hersh, 1991). Following Erving Goffman, Hersh defines "the 'front' of mathematics" as "mathematics in 'finished' form, as it is presented to the public in classrooms, textbooks, and journals." By contrast the "'back' would be mathematics as it appears among working mathematicians, in informal settings, told to one another in an office behind closed doors" (128).

Historians of mathematics, particularly those who work with unpublished archival sources, have long been attuned to mathematics as it moves from the back to the front. Though unable to reconstruct what happened in spoken conversations "behind closed doors,"[9] extant textual evidence can contain "fragmentary, informal, intuitive, tentative" mathematics, characteristic of the "back" (Hersh, 1991).[10] While the distinction falls apart when considering mathematics before modern standards of publication developed through the nineteenth-century, historical studies of modern mathematics support Hersh's claims that the drafts, notebooks, and private correspondences toward the back of mathematics are different from what appears as published mathematics in the front.

Hersh notes that an "important part of becoming a professional, in mathematics or anywhere else, is to move from the 'front' to the 'back'" (Hersh, 1991, p. 132). Refining Hersh's distinction, Dirk Schlimm characterizes learning mathematics as "appropriating public knowledge and turning it into something private" (Schlimm, 2013, p. 285). The Journal Club Notebooks document part of this process by recording ways in which graduate students engaged with contemporary literature, formulated research questions, and assessed tools for potential solutions.

To witness the process of bringing mathematics to the back, consider Virginia Ragsdale's report "On the Arrangement of the Real Branches of Plane Algebraic Curves" written in the Journal Club Notebook on December 16, 1901. The report brings together half a century of research on her title subject with an emphasis on two recent articles by L. S. Hulburt for the *American Journal of Mathematics*, which were in turn dedicated to summarizing David Hilbert's "Ueber die reellen Züge algebraischer Curven" published in the *Mathematische Annalen* in 1890. The report is a literature review, Ragsdale does not purport to offer any new mathematics. But mathematics does not

[8] The Annual Report for this year is not at the Bryn Mawr College Archives, but some information can be found in the archival source material for these reports among the M. Carey Thomas Official Papers, Scott (1902a).

[9] The challenge of spoken conversations is addressed explicitly in Rowe (2004).

[10] To take three examples that convey the temporal range consider (Knobloch, 2004; Brechenmacher, 2007; MacKenzie, 1999).

move from the front to the back unchanged. Exactly what changes depends on the local process. Each relocation concerns the subject-matter, the individual writer, the mathematics department at Bryn Mawr College, the nebulous geography of Anglo-American mathematics circa 1900, among other variable factors. These interlocking sites of production imprint Ragsdale's report. Moreover, as Catherine Goldstein has emphasized, "the concrete manner of transmission of knowledge through personal or institutional links" is one of several factors that provide "impulse to a mathematical investigation" (Goldstein, 2019, p. 3). Indeed, this impulse can be observed as the selected texts filtered through the expectations and affordances of mathematics research at Bryn Mawr sparked what would become Ragsdale's dissertation. The first aim of this article then is to investigate a case of mathematics moving from the front toward the back and how local features shaped this move.

Ragsdale not only brought mathematics from the front to the back, but also developed a dissertation from the topics and questions first raised in the Journal Club Notebooks. Whereas Hersh described the transition from the front to the back as generative, he likened the opposite direction to a violent purge "of the personal, the controversial, and the tentative, producing a work that acknowledges little trace of humanity, either in the creators or the consumers"' (Hersh, 1991, p. 131). In comparing Ragsdale's Notebook entries with the article she published in the *American Journal of Mathematics* in 1906, we can see precisely what was "purged" or put aside.

Hersh classified the front of mathematics as it appears before "the public," but there are many kinds of publics for mathematics between the individual and absolutely everyone: a colleague is a public, the participants in a departmental club are a public, and the readers of a published article are (hopefully!) a public. Returning to Goldstein, "the way mathematics is made public and circulates has been proved to be both a serious constraint on its form and an essential factor in its transmission" (Goldstein, 2019, p. 5). When rewriting for the Journal Club, Ragsdale changed certain key words that determined her choice of cited references as well as how she formulated and pursued her research objectives. In his study of technical mathematical words, François Lê explains how they "are intertwined in networks of meanings linked to the personal knowledge of the people who employ or read them, and to the collective representations which are made of them, and which include genealogies of (groups of) authors, objects and domains of mathematics that may be naturally connected to them, as well as advocated methods and values for instance—meanings coming from outside mathematics could certainly be added to this list" (François, 2020, p. 102). Accordingly, it is not just that Ragsdale decided to retain "the personal" in her published article, but that her results depended on the local format.

More generally, it would only be possible to purge the personal from mathematics if the universality of mathematics that Hersh describes as a myth was actually true (which he acknowledges is possible). But mathematics is local even when the back of mathematics is hidden, as has been demonstrated many times over. To take one example, when introducing his comparative study of published work on knot invariants in Vienna and Princeton, Moritz Epple focuses on "local knowledge traditions" that inform "the specifics of the mathematical language used in a particular period and region, the possibilities offered and the limits imposed by particular conceptual frameworks or ways of imagination, the differences in proof strategies and standards of rigor, the mathematical and scientific contexts of particular problems, or the social and cultural setting of particular episodes of mathematical work" (Epple, 2004, p. 133).

A close study of Ragsdale's work in the Journal Club and in the *American Journal of Mathematics* is not needed to dismantle the myth of universality, and I will side-step the question of what, if anything, is universal in mathematics. Perhaps, following the "mathematician's conviction" as described by Goldstein, even as the form and uses of a text change, "the sense of true theorems is eternal and universal" (Goldstein, 1995, p. 7). Instead, the second aim of this article is to tease apart a more pernicious myth. If we only witness the front of mathematics, then mathematics appears to be transmitted through published texts. This is true, but an impoverished narrative. In her study of divergent historical and mathematical readings of Fermat's little theorem, Goldstein documents how "*all* reading is contextualized, even if only implicitly, by the prior knowledge of the reader" who determines "on what body of knowledge [a reading] is based; to what questions, for what public, it intends to answer; what contexts it requires to interpret a text; on what criteria it bases its answers" (ibid, 9).[11] Between the published texts are the translations and interpretations that determine what each mathematician reads as familiar or strange, valuable or irrelevant. The existence and persistence of the Journal Club attests to the value of relocalizing published mathematics for collective understanding and use. For those who completed graduate studies, Ragsdale's entries capture a more general pattern of appropriation, complete with false starts, unfinished questions, and promising ideas for future work.

Ragsdale's initial contributions are subtle and can only be recognized up close. But too much detail here would lose the object of the paper in the work of other mathematicians. François Lê and Sebastian Gauthier offer an explicit methodological discussion of this delicate balance in their historical analysis of Louis Mordell's "youthful mathematics" (Gauthier and Lê, 2019). In presenting their research, Lê and Gauthier sought "for an adequate grain for our description of Mordell's mathematics, a grain sufficiently fine not to erase the details and particularities of Mordell, and at the same time not too fine, which would lead to an overly fastidious picture" (435). Their choice of representative texts and details capture "the form of the writing, the way Mordell either expresses arguments and statements, or skips computations and bibliographic references [...] which are historically meaningful, just as the results, the objects, the proofs, and the techniques themselves" (463). A similar attention to the text as material object, as literature, and as mathematics exposes how Ragsdale words her research question in its several incarnations, how she translated published results for her audience, how she exploited the potential of a handwritten document, how she colored illustrations and examples.[12] In a volume on language and textual representa-

[11] "*Toute* lecture est contextualisée, ne serait-ce que, de manière implicite, par les acquis préalables du lecteur ou de la lectrice. Mais une certaine latitude existe dans le contexte mobilisable autour d'un théorème pour lui donner une signification, l'inscrire dans une évolution historique. Situer une lecture, qu'elle soit historienne ou mathématicienne, c'est déterminer sur quel ensemble de connaissances elle s'appuie; à quelles questions, pour quel public elle entend répondre; quel contexte elle sollicite pour interpréter un texte; sur quels critères elle fonde ses réponses."

[12] Because the text is handwritten, underlining is used to indicate emphasis. I have converted these to italics to better accord with the readability of typed documents. However, it should be noted that the practice of emphasizing when underlining is not identical to that of using italics with respect to frequency and volume.

tion in mathematics, this microscopic focus suggests one end of a spectrum of textual proximity for which the other might be the telescopic lens of "big data approaches and natural language processing."

I will begin by situating the Bryn Mawr Journal Club as one iteration of a mathematics seminar, popular in graduate programs during the late nineteenth century. Mathematics professors and administrators formed these spaces in part to bring mathematical students to the edge of their disciplines. Ragsdale's own research continues that of Scott, but can also be seen as a response to Hilbert's 16th Paris Problem. In the following section, I will turn to this problem, and Ragsdale's first engagement with Hilbert in the Journal Club Notebooks. Having identified an open question, she began to assemble tools. Her subsequent two entries, treated in Sect. 3 attest to Ragsdale's efforts and dead-ends. After some delays, in 1906 the *American Journal of Mathematics* published Ragsdale's dissertation—a conjecture on the arrangement of branches of plane curves, the subject of Sect. 4. Shortly afterward Ragsdale presented a very concise version to the Journal Club, affording a comparison between the published and unpublished texts—a final "oscillation between back and front" (Schlimm, 2013, p. 286).

1 The mathematics seminar in late-nineteenth century United States

While the Bryn Mawr Mathematical Journal Club Notebooks are unusual documents given their longevity and preservation, the Club itself can be considered as one form of a graduate seminar or seminary, which were common across many disciplines and institutions by the late nineteenth century. In *The Emergence of the American University* Laurence Veysey has described how American educators looked to graduate seminars at German Universities as the epitome of academic exchange and "intellectual incentive" but in "terms of these goals, the American seminar proved no more automatic in its success than had the German." Nevertheless, whether students "reported on the progress of their researches," "gave book reports," discussed "articles and monographs" or spent the entire class on "a minute study of documents" these seminars became the hallmark of serious graduate programs across most disciplines in Progressive Era United States (Veysey, 1965, p. 155). Evidence of such semi-formal meetings in which students and faculty could discuss recent literature and research within mathematics departments can be seen in Florian Cajori's 1890 report on *The Teaching and History of Mathematics in the United States* (Cajori, 1890)

This report, published under the auspices of the Bureau of Education in the Department of the Interior, drew on "a large correspondence with alumni, and past and present instructors in the higher educational institutions." The anecdotal evidence supplemented responses to "1,000 circulars of inquiry" sent from the Department of the Interior to a wide range of schools, colleges, and universities (ibid, 3). Clubs and seminaries were identified as a means to bridge the gap between learning mathematics and making mathematics, between students and professors. Those at Johns Hopkins University, Cornell University, and the University of Michigan show the wide range of possible approaches at the time.

As students fondly reminisced, from the beginning, there “has always been a mathematical seminary” at Johns Hopkins. In “the time of Sylvester” [...] instructors and more advanced students would present and discuss their original researches.” James Joseph Sylvester dominated the meetings with an authoritarian approach that resembled the competitive atmosphere at Göttingen, where some students similarly struggled to live up to the standards their professors set.[13] When Simon Newcomb succeeded Sylvester in 1884 the seminary became the “Mathematical Society” held weekly with each term “conducted” by a lead professor who “selects for his seminary topics from his special studies.” Similarly, Professor William Story expected students “if possible, to begin where he had left off and carry on investigations along lines pointed out by him” (276).

At Cornell the seminary focused on education. During meetings, the professor raised questions on pedagogy (“What is the place of memory in mathematical teaching?” “What are the relative advantages of lecturing and book work, and how are they best combined?” “How can we best teach geometry?”) and then called for discussion (187). Finally, Michigan offered a much more casual mathematical club “under the control of the students, but an active interest is continually shown by the various instructors.” At meetings “papers of some length are presented, problems discussed, etc.” (253).[14]

Cajori’s circular inquired whether there were “any mathematical seminaries or clubs, and how are they conducted.” Most of the 168 recorded respondents did not have graduate programs and “answered in the negative.” Charlotte Angas Scott reported on behalf of Bryn Mawr in 1889 writing,“No clubs, but seminaries, through part of regular course, but not very formal; they are intended to afford students opportunity of working problems under guidance” (303). At this time, Scott viewed clubs as extracurricular and seminaries as a type of advanced graduate coursework alongside topical lectures.

The Bryn Mawr Mathematical Journal Club primarily served as a site for literature reviews based in publications from the past several decades. Scott advocated for this kind of preliminary study to original research. In an 1893 talk before the Mathematics Club at Girton College—a club she had co-founded as an undergraduate—she urged her listeners to “study periodical literature” (Scott, 1894). Student contributions ranged from a summary of a single paper, to a comprehensive study of a topic, to a report on their own recent findings. Though the Journal Club Notebooks sat in the Bryn Mawr library for student perusal, students occasionally revisited topics or texts that had been treated in a prior year. The public of the Journal Club changed every year with new admissions and graduations. For some students, Scott selected the subject for reports based on a student’s age, experience, or inclinations. Others seem to have chosen their own topics, sometimes against the better judgment of their professors.[15]

[13] For instance, see Parshall and Rowe (1994), chapter 5.

[14] Notably, in 1890 (the year Cajori’s report appeared) professors W. W. Beman, F. N. Cole, and Alexander Ziwet organized a Mathematical Society “to bring together those who may be interested for the presentation and discussion of Mathematical topics embracing the range of School and University work and of Advanced Research.” This was fairly formal, including a constitution, by-laws, and around 20 members (Club, 1890).

[15] The graduate student files for Mary Gertrude Haseman and Monica Healea during the 1920s include brief faculty reports on their Journal Club subjects and participation. See Bryn Mawr (undated).

2 Hilbert's 16th problem

In the summer of 1900 Scott went to Paris for the second International Congress of Mathematicians. That November her general report was published in the *Bulletin of the American Mathematical Society*.[16] Communicating Hilbert's address on the future problems of mathematics, Scott recounted that the "lines along which we may expect the development of any science which is progressing in a continuous manner can be detected by an examination of the problems to which attention is specially paid" (Scott, 1900a, p. 67).[17] She summarized a few of the "problems that M. Hilbert specified in particular as fitted to advance mathematics" including investigating "the relative situation of the circuits that a plane curve of assigned order can possess" (68).[18]

In fact, Scott was in the midst of this investigation. At the twelfth meeting of the Journal Club, on May 17th, 1897, her topic was "Circuits" (Scott, 1897). Scott defined the Theory of Circuits or "the theory of the real branches of Algebraic Curves" as a part of "*Topology* or *Analysis Situs*."[19] Topology "deals with general questions of appearance, that is, of form, number and arrangement of real singularities, +c." She identified one "extensive theory of Topology" as that of "'Knots,' which relates to the determination of the possible ways of connecting double points that are given in number and general arrangement" and another as Circuits, that "which discusses

[16] For a detailed account of the Paris Congress, see Duporcq (1902).

[17] Hilbert's address was first published as Hilbert (1900). The address was later translated into English for the *Bulletin of the American Mathematical Society* by Mary Winston Newson, who had studied at Bryn Mawr University and the University of Chicago before receiving her PhD from the University of Göttingen (Hilbert, 1902). On the significance of his problems for the next century of mathematics see Gray (2000) and Yandell (2002). Yandell mentions Ragsdale along with Karl Rohn as early contributors to Hilbert's 16th problem. In particular he notes that Ragsdale "understood that there should be a relationship between her conjectures and the Euler characteristic, but had not proved this." (277)

[18] In Winston Newson's 1902 translation, Hilbert's 16th problem—"Problem of the Topology of Algebraic Curves and Surfaces"—begins with branches.

> The maximum number of closed and separate branches which a plane algebraic curve of the nth order can have has been determined by Harnack. There arises the further question as to the relative position of the branches in the plane. As to curves of the 6th order, I have satisfied myself—by a complicated process, it is true—that of the eleven branches which they can have according to Harnack, by no means all can lie external to one another, but that one branch must exist in whose interior one branch and in whose exterior nine branches lie, or inversely. *A thorough investigation of the relative position of the separate branches when their number is the maximum seems to me to be of very great interest, and not less so the corresponding investigation as to the number, form and position of the sheets of an algebraic surface in space* (Winston, 1902, p. 465)

[19] For most of the texts considered here "branches" are left undefined as a descriptive property of curves. In Arthur Cayley's *Encyclopaedia Britannica* article on "Curves" he defined a branch of a real curve as "any portion capable of description by the continuous motion of a point" so that "a curve consists of one or more branches." (Cayley, 1877, p. 785).

all possible varieties of complete branches, and their proper classification; or else investigates the possibilities as to circuits on a curve of given order, class, or deficiency" (96),[20]

Scott's entry included a list of the "most important analytical principles used" and a "brief bibliography." She noted that "a fairly complete account of all that has been done in the subject" could be accomplished in a few pages. As she introduced students to a field of emerging research, Scott warned of the "often very difficult" application of analytical expressions in topological investigations "for algebraic analysis concerns itself with *all* roots of an equation, *all* intersections of a line with a curve, while Analysis Situs concerns itself only with real solutions" (97). She then outlined her own progress in the subject, defining terms and illustrating some possible theorems that still lacked formal proofs. Scott's report on "Circuits" is a teaching document full of the "fragmentary, informal, intuitive, tentative" mathematics described by Hersh. Before an audience of students and colleagues, Scott conceded that the term "circulation" for the "smallest number of odd circuits into which the given circuit can be transformed" was not quite right. She added parenthetically "[? should be a different word, however]" (103). This preliminary outline of circuits as a subject where much work was needed, bolstered by the motivation provided by Hilbert's Paris problems, served as an impetus to the research of Scott's student, Virginia Ragsdale.

Ragsdale had earned her bachelor's degree from Guilford College, a small co-educational Quaker college in North Carolina, in 1892. As valedictorian she won a scholarship for $400 to attend Bryn Mawr College, where she secured a second bachelor's degree in physics and mathematics in 1896. Her exemplary performance continued. As a mark of her standing, she was the European Fellow of her graduating class, but elected to stay at Bryn Mawr for one more year, taking post-major mathematics courses and participating in the inaugural Journal Club.[21] In Ragsdale's academic record, Scott noted, "Miss Ragsdale took part also in the proceedings of the Journal Club + contributed a careful and lucid report on the transcendency of e and π" (Scott, 1902b). The following year, Ragsdale used her fellowship funds to study in Göttingen along with two other Bryn Mawr graduates.[22] She then taught at the Bryn Mawr School in Baltimore until receiving a fellowship from the Baltimore Association for the Promotion of University Educated Women that enabled her to return to Bryn Mawr College for her master's degree in 1901. The fluidity between undergraduate and graduate study, interrupted by periods of teaching at private schools, was typical

20 Following Cayley, British and American geometers of the nineteenth century defined the deficiency of a curve of order m with δ nodes and κ cusps as $\frac{1}{2}(m-1)(m-2)-\delta-\kappa$. That is, "the actual number of nodes and cusps" below the maximum $\frac{1}{2}(m-1)(m-2)$ (Cayley, 1877, p. 784). By around 1900, what had been considered the "deficiency" of a curve came to be called the genus, as shown in Lê (2020) and to be discussed below. In 1918, Scott's graduate student Mary Haseman published her dissertation on knots in Haseman (1918).

21 Ragsdale's biography up to 1903 is summarized at the end of her dissertation in a section entitled "Life" (Ragsdale, 1906b).

22 In a chapter on "Internationality: Women in Felix Klein's Courses at the Unviersity of Göttingen," Renate Tobies provides a table of the women enrolled in Felix Klein's courses between 1893 and 1912 (Tobies, 2020). Ragsdale is cited as having studied Mechanics I (WS 1897/1898), Mechanics II and Exercises in mechanics (SS 1898). In her dissertation, Ragsdale wrote that she "studied Mathematics at the University of Göttingen under Professor Felix Klein and Professor David Hilbert." Ragsdale (1906b).

of Scott's graduate students, many of whom had limited financial means. There was little evidence that earning higher degrees would advance professional prospects since most colleges and universities only hired men.[23]

In 1901, when the Mathematical Journal Club reconvened for its third year, Ragsdale presented "On the Arrangement of the Real Branches of Plane Algebraic Curves" (Ragsdale, 1901). The report is handwritten with medium-sized, very readable script and runs ten pages including references. Ragsdale employed two kinds of citations: names, sometimes with year and title, are mentioned in the text and there are also four full citations at the end of the text under *References*.[24] All of these texts are also in Scott's "Circuits" bibliography from 1898. Ragsdale's report is considerately illustrated. There are five figures colored in black, red, and blue. The first figure is integrated with the text, while the latter four are enumerated and separated on a more transparent paper placed adjacent to where they are invoked between pages 120 and 121.

In the first three pages Ragsdale set out the main themes, results, and analytic principles in the past half-century of her topic. She began with Georg Karl Christian von Staudt, who "divides the branches, or circuits, of a curve into the two classes, odd and even, according as the number of points in which they can be cut by a straight line is odd or even." She then introduced Hieronymous Zeuthen's statements on the number of even or odd branches for a non-singular curve. In 1873 Zeuthen had determined that "a non-singular curve of even order must be composed entirely of even circuits; and a non-singular curve of odd order must have one circuit odd and the remaining even." He further proved the existence of a quartic circuit that cuts itself twice and "cannot be projected into the finite." However, Ragsdale restricted herself, "chiefly to the consideration of non-singular curves" (116).

Proceeding chronologically, Ragsdale presented Axel Harnack's proof that the "*maximum number of circuits* on a curve of deficiency p" is $p+1$ and that there exists a curve having this maximum number of real branches for every value of p. Ragsdale then synthesized in two steps what she considered as the "analytic principles underlying Harnack's proof" (117). She accompanied the two principles with an "Illustration" consisting of the algebraic equation $\left(\frac{x^2}{25}+\frac{y^2}{4}-1\right)\left(\frac{x^2}{4}+\frac{y^2}{25}-1\right)=\pm\delta(x-y+5)$ and a colored figure of two ellipses (Fig. 2).

Having illuminated Harnack's principles, Ragsdale defined "the only two possibilities" in the relative position of branches of a non-singular curve—"i. the branches may be *external* to one another; ii. the branches may be *nested*; i.e., so situated that the first lies wholly within the second, the second within the third, and so on." Ragsdale added that it is "obvious that only even circuits, or ovals, can be nested." As "an example of

[23] On professional opportunities for women in mathematics at this time, see Green and LaDuke (2009), Fenster and Parshall (1994), and Rossiter (1982).

[24] All together there are five sources mentioned: von Staudt, *Geometrie der Lage* (1847); Zeuthen, "Sur les différentes formes des courbes planes du quatriéme ordre, *Mathematische Annalen* (1873); Harnack, "Ueber die Vieltheiligkeit der ebenen algebraischen Curven," *Mathematische Annalen* (1876); Hilbert, "Ueber die reellen Züge algebraischer Curven" *Mathematische Annalen* (1890); Hulburt, "Topology of Algebraic Curves," *Bulletin of the New York Mathematical Society* (1892); and Hulburt "A Class of New Theorems on the Number and Arrangement of the Real Branches of Plane Algebraic Curves" *American Journal of Mathematics* (1892).

a curve having the maximum number of real branches all of which are *external* to one another" Ragsdale gave the equation $(x^2 - 9)^2 + (y^2 - 4)^2 - 1 = 0$.

With these definitions and examples, Ragsdale could introduce Hilbert's claim for the number and arrangement of branches in the sextic curve and the potential generalization for curves of higher order. This topic would constitute the subject of Ragsdale's eventual dissertation. In this preliminary Journal Club paper, she only observed that the question required investigation before turning to the complementary subject of nested branches treated in L. S. Hulburt's two recent papers. Writing for American publications two years after Hilbert, Hulburt first provided an exposition and critique of Hilbert's result that for a non-singular curve of even order n no more than $\frac{1}{2}(n - 2)$ of the $p + 1$ ovals can be nested and for odd order n no more than $\frac{1}{2}(n - 3)$ can be nested. In his second paper, Hulburt extended Hilbert's results to certain curves with singularities.

Ragsdale's text moves mathematics from the front to the back, both for her own understanding *and*, because the Journal Club was not private, for the understanding of her audience. In this process of acculturation, Ragsdale's review of branch arrangement responds to a combination of geographically local and temporally ephemeral factors. By writing in English for a British and American population, Ragsdale translated French and German vocabulary. Bryn Mawr as a physical space informed the choice of sources based on what texts were available—periodicals deemed essential to any mathematics department as well as material for Scott's own geometrical research. Delivering a talk before a group of students with mixed mathematical backgrounds promoted the inclusion of concrete examples. Finally, Ragsdale was not only communicating recent publications, but also signaling interest in a line of future study. Accordingly, she reframed past results in terms that could be fruitful toward this end.

Graduate students in mathematics at Bryn Mawr were expected to read French and German. Nevertheless, with some minor exceptions, Journal Club entries were written entirely in English. Accordingly, Ragsdale's report involved a substantial amount of translation. One word is particularly telling. In summarizing Harnack, Ragsdale rendered his "Geschlecht" as "deficiency." As historian François Lê has demonstrated, this use of deficiency was idiosyncratic to the Anglo-Saxon context and contrasted in scope—not just name—with the French and German uses of genus (Geschlechte, genre) at the same time. Lê attributes this national pattern in part to the prominence of Arthur Cayley among British geometers and determines that 'genus' only began to replace 'deficiency' in English texts beginning around the turn of the century François (2020). For instance, in 1900 Scott wrote "the nominal genus (deficiency) of a curve is greater than or equal to the actual genus, inasmuch as the curve may have multiple points other than the stated ones, and their presence diminishes the genus" (Scott, 1900b, p. 223). Ragsdale's report is at the cusp of this change. What was the deficiency of the curve p in 1901 is the genus p by the time her dissertation was published in 1906 (Ragsdale, 1906a).

The choice of two names for the objects under discussion—branches or circuits—suggests a narrower regional source. Throughout the text, Ragsdale used both terms without any clear systematic explanation and approximately a three to one preference to *branches* over *circuits*. None of the authors cited by Ragsdale write of circuits, and though von Staudt's "geschlossene Linie" is "Zug" in Harnack and Hilbert, this seems to be an evolution in specificity rather than two co-existing competing terms in German. While neither of these terms has the same extramathematical connotations as "branch," this appears to be the common French and English translation.

In Scott's Journal Club entry on "Circuits" she determined "the theory of the real branches of Algebraic Curves" as "more restricted" than that of circuits (Scott, 1897, p. 96). However, she did not elaborate what this restriction entailed, and elsewhere the terms are interchanged. For instance, in Scott's 1902 "On the Circuits of Plane Curves" she called attention to the "nature of the individual circuits (or complete branches) that make up a curve of order *n* (Scott, 1902c, p. 388). In Ruth Gentry's Bryn Mawr doctoral dissertation, she described the appearance of a curve as depending "mainly upon the number and nature of its multiple points, multiple tangents, inflexions, and circuits" (Gentry, 1896, p. 1). Though unusual, this usage is not completely idiosyncratic to Scott and her students. Peter Field, an American mathematician who obtained his PhD at Cornell University under Virgil Snyder and then became a faculty member at the University of Michigan, also wrote circuit and branch interchangeably in his dissertation (Field, 1902). In addition, Cayley, with whom Scott studied while in Cambridge, used "circuit" in numerous papers. However, in his article on "Curve" for the ninth edition of the *Encyclopaedia Britannica* (1877), he introduced circuits as "complete branches," that is, the different branches of a curve "connected together at infinity" (Cayley, 1877, p. 477).

By including both words, Ragsdale effectively connected the title of Scott's prior Journal Club report on circuits to the title of Hulburt's article on branches [(which is also cited in Scott (1897)). Though the nonstandard usage was potentially confusing,[25] her repetition reinforced the interchangeability and normalized "circuits."

The use of "deficiency" and "circuits" demonstrates Ragsdale's attention to her professor's standards, but the Journal Club served a student audience. That year included two new graduate students, Harriet Ella Wigg and Carrie Alice Mann, both of whom had just begun at Bryn Mawr. The Journal Club aimed to be a medium in which to explore new texts and research questions, but most Bryn Mawr College graduates would work as secondary school teachers. For them the Journal Club may have been as much an exercise in exposition. Ragsdale had already taught mathematics at private school for several years. This experience perhaps informed her exposition style and decision to accompany abstract principles with concrete examples. In addition to these practical considerations, Ragsdale's appreciation of numerical equations and colored figures resonate with Scott's commitment to "fully worked concrete examples" through which "the true significance of the matter can be generally appreciated" (Scott, 1899, pp. 330–331).

Because Ragsdale's report is handwritten, she had more flexibility in the depiction of figures. Texts situated in the front of mathematics encountered various restrictions depending on a journal's budget and publication equipment. There are no figures in Hilbert's and Hulburt's published texts, and without access to earlier drafts one can only speculate whether figures were ever employed. Harnack included two figures

[25] For instance, in Henri Poincaré's "Sur les courbes tracées sur les surfaces algèbriques" (Poincaré, 1910), he wrote "circuit" to describe the path of a complex variable and "branch" to describe a feature of a, given curve with double points.

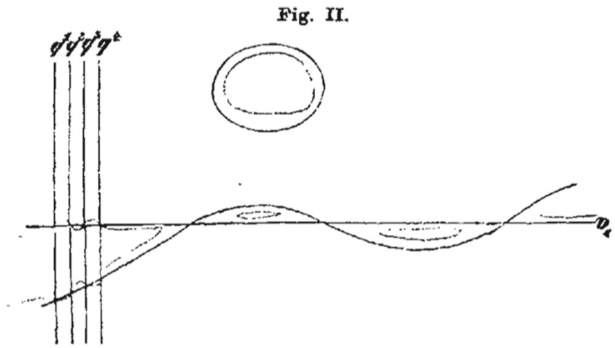


Fig. 1 Harnack's Figure II Harnack (1876)

showing his process and not based on specific given equations. The idea of building the curve through desired intersections is shown in Harnack's Figure II (Fig. 1), in which the path of the newly constructed curve through the desired intersections is suggested in a fainter line (Harnack, 1876).

Ragsdale's first illustration (Fig. 2) echoed Harnack's in a more concrete example. She presented two ellipses and the intersecting line, using pink and blue in complement with plus and minus signs to show positive and negative regions. Her later added figures document a step-by-step construction process, replacing Harnack's straight line with an ellipse.

In her "Fig. i" (Fig. 3) Ragsdale showed the case of $n = 4$ where $C_2 = 0$ in faint black and $E_2 = 0$ in bold black "represent two ellipses cutting each other in four points." She selected eight points on any segment of "E_2 bounded by two consecutive intersections of the curves," which can be seen on the left side of the ellipse in the figure (120). Joining "these in pairs by straight lines" produced the four lines shown in dark blue and "the product of the equations of the lines thus formed is $L_4 = 0$." Finally, in red, C_4 is the quartic "having the maximum number of branches, $\frac{1}{2}(n-2)$ nested branches, lying within the ellipse E_2, and a non-nested oval, b, which cuts E_2 in 8 points following in the same order on the two curves." Further figures showed the "analogous" construction for the cubic and the quintic.

Below the figures Ragsdale warned that "the intersecting arcs of C_n and E_2 cut without the inflexions drawn in the figures. Distortion was necessary to bring the figures into the scope of the paper." This addition of inflexions for visual clarity and compactness can be found in earlier research publications by Scott and her students. For instance, in "On the Higher Singularities of Plane Curves" Scott added a footnote explaining that two figures have been "drawn with inflexions, as the scale is too small to keep them distinct in any other way" (Scott, 1892, p. 320).

In addition to the immediate audience, Ragsdale's report was a product of the mathematical resources available at Bryn Mawr. One of Scott's first decisions in stocking the mathematical library was subscribing to *Mathematische Annalen* and Zeuthen's

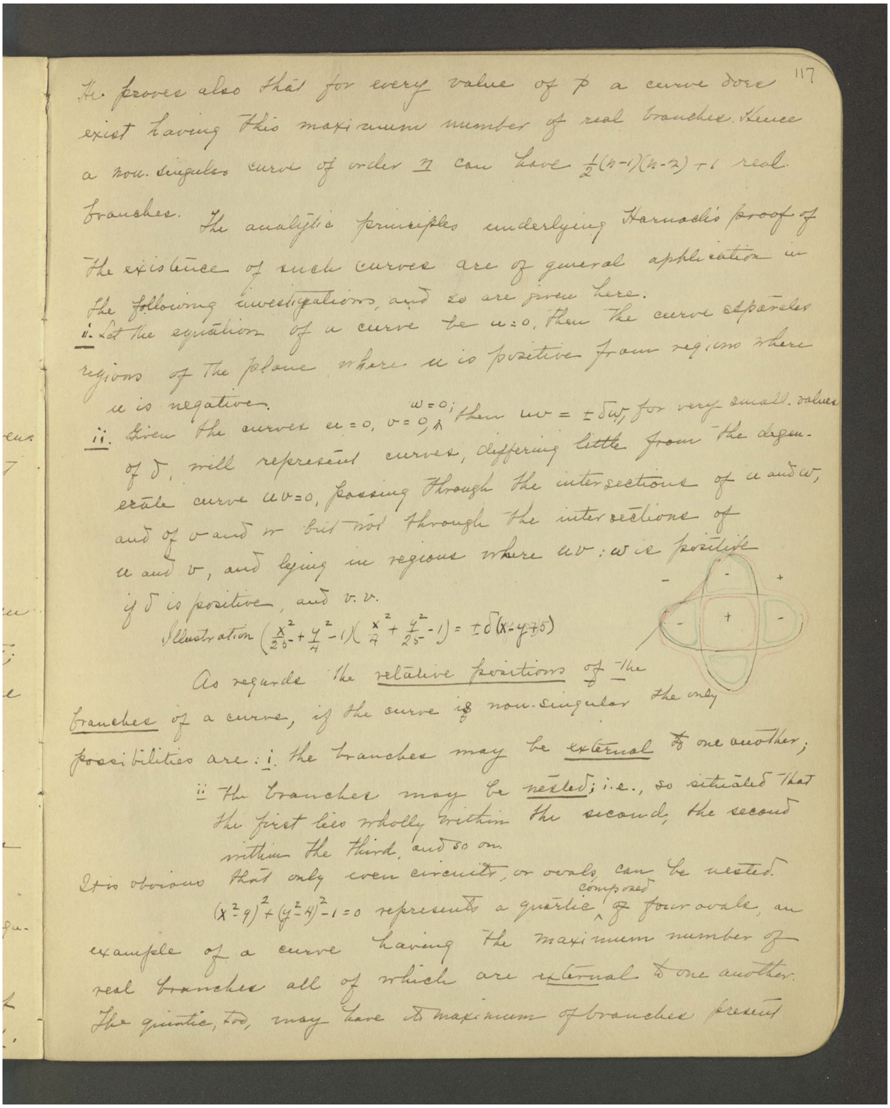


Fig. 2 Ragsdale's illustration (Ragsdale, 1901)

1873 paper there had been a central text for mathematics research at Bryn Mawr College since the mid-1890s.[26]

The significance of Zeuthen for Ragsdale becomes apparent in comparing her text to Harnack, Hilbert, and Hulburt, where Zeuthen was not referenced. In his comprehensive classification of quartics, the results cited by Ragsdale are fairly minor. Zeuthen simply stated the distinction between even or odd circuits on the second page of his text. Similarly, Zeuthen introduced the existence of the non-singular quartic that

[26] On Zeuthen's contributions to analytical geometry, see Michel (2020).

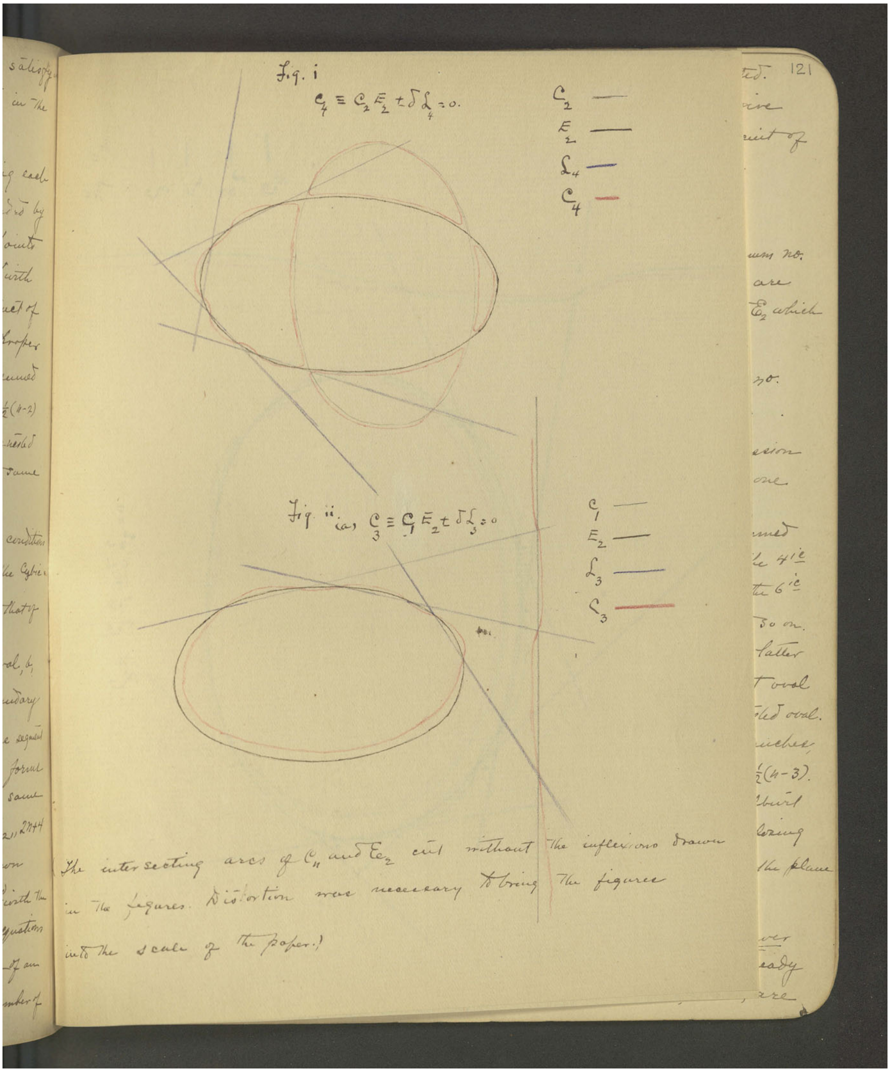


Fig. 3 Ragsdale's Figures i and ii(a) Ragsdale (1901)

is met by every straight line in at least two real points as one example of a quartic with a common point between two external branches. As Ragsdale primarily followed Harnack and Hilbert—who only consider non-singular curves—it is striking that she justified excluding singular curves when they did not.

Zeuthen's article was well-known at Bryn Mawr and its invocation in the study of non-singular circuits familiarized the new objects of study in topology. Scott had cautiously looked to algebraic analysis for principles for investigating topology. Similarly, a careful selection of texts from analytical geometry concerned "form, number, and arrangement" of curves even if not directly applicable to the problems addressed by Harnack and Hilbert (Scott, 1897).

Ragsdale, too, invoked guiding "principles." In two steps Ragsdale provided a synthesis of Harnack's procedure for constructing a non-singular curve of order $n+1$ with the maximum number of branches from a curve of order n with the maximum number of branches.

> i. Let the equation of a curve be $u = 0$. Then the curve separates regions of the plane where u is positive from regions where u is negative.
> ii. Given the curves $u = 0$, $v = 0$, $w = 0$; then $uv = \pm\delta w$, for very small values of δ, will represent curves, differing little from the degenerate curve $uv = 0$, passing through the intersections of u and w, and of v and w but not through the intersections of u and v; and lying in regions where $uv : w$ is positive if δ is positive, and v. v (Ragsdale, 1901, p. 117).

Ragsdale's principle (ii) paraphrased what Harnack had described in greater length through specific examples in the second half of his paper. The main difference between Ragsdale and her predecessors is in the use of positive and negative regions in principle (i) and the conclusion of her principle (ii) also with respect to positive regions.

None of Ragsdale's cited references used the language of positive and negative in describing how the curve separates the plane into regions. Instead they adopted the more evocative language of inside (innerhalb/innere) and outside (ausserhalb/aussere) following von Staudt. However, in Scott's 1897 "Circuits," the first of the "most important analytical principles" in topological investigations was "(i) the curve $f = 0$ divides the plane into regions, in some of which the expression f has a positive value, in others a negative value" (Scott, 1897, p. 97). Further precedent for this designation can be found in Scott's bibliography: "1852. Möbius, II. pp. 92–106." This somewhat oblique reference corresponds to the republication of Möbius's "Über die Grundformen der Linien der dritten Ordnung" in the second volume of his complete works, suggesting this is how the text, originally published in the *Berichte der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig*, would have been available to the Bryn Mawr community (Möbius, 1852).

In this paper, Möbius drew a connection between the sign of coefficients a, b, c and their position on the side of the circles illustrated in his Figure 16 (Fig. 4). There is a striking similarity between Möbius' Figures 16 and 18 and Ragsdale's ellipses in her introductory figure (Fig. 2). With the use of color, Ragsdale created another expression of positive/negative or inside/outside in pink/blue.

Of course, the correspondence between inside/outside and positive/negative (or negative/positive) predates these nineteenth-century uses as a consequence of a coordinate approach to geometry. The practice of dividing the plane into alternating positive and negative regions was also taught in Scott's course on curve tracing. As one of several tools for drawing algebraic curves, Scott included "the method of exclusions" based on the fact that a "curve, represented by a function $F(x, y) = 0$, divides the plane into regions, in which the $F(x, y)$ will be either positive or negative."[27]

[27] The description of Scott's method of exclusions can be found in Haseman (1927, p. 17). Mary Haseman had studied curve tracing under Scott at Bryn Mawr and believed this method to be original to Scott.

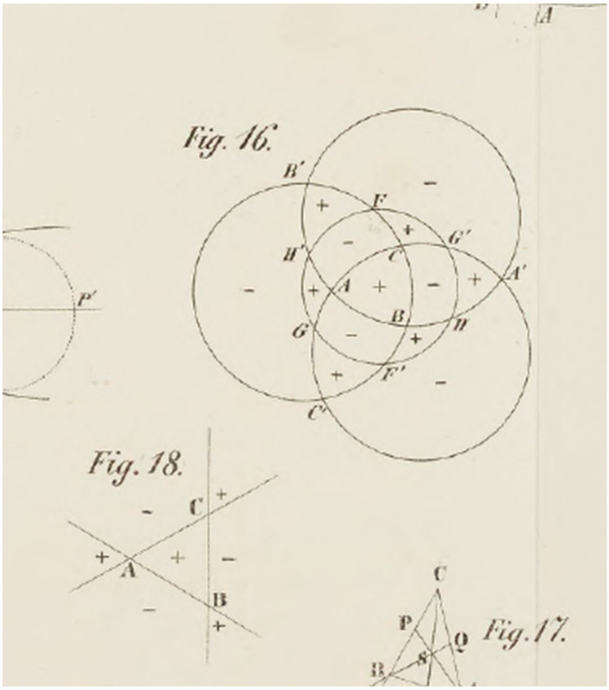


Fig. 4 Möbius's figures 16 and 18 (Mobius, 1852)

Nevertheless, by bringing the positive/negative distinction to the study of branches, Ragsdale struck out on a different path from her cited sources, who had also read Möbius's text. Here Ragsdale did not uniformly adopt the language of her first principle, but instead first referenced Hilbert's research with the language of "external branches":

> Hilbert in a paper, "Ueber die reellen Züge algebraischer Curven" Math. Annal. XXXVIII p. 115, states, without proof, that if the maximum number of branches of the sextic be present then all can not be external to one another. From this it follows that if all the circuits are external the maximum number of branches can not be present. Whether these limitations apply to curves of higher order is still to be investigated (Ragsdale, 1901, p. 118)

Hilbert had placed the claim in a footnote to a discussion of sextic curves.

> I subjected this case $n = 6$ to a further detailed investigation, whereby I found—admittedly in an extremely complicated way—that the 11 branches of a curve of 6^{th} order cannot all be external to one another. This result appears to me of interest because it shows that the topologically simplest case is not always possible for curves with the maximum number of branches (Hilbert, 1890, pp. 118–119) [28]

[28] *) Diesen Fall $n = 6$ habe ich einer weiteren eingehenden Untersuchung unterworfen, wobei ich—freilich auf einem ausserordentlich umständlichen Wege — fand, dass die 11 Züge einer Curve 6^{ter} Ordnung

He professed interest in this result, but did not treat the subject further, perhaps because the discovery was extremely complicated. Hulburt employed Hilbert's results and methods to extend the study of nested branches in certain non-singular curves, but did not mention Hilbert's footnote.

Ragsdale had a question "still to be investigated" and understood how prior mathematicians had addressed the "similar case of nested branches." Could a comparable approach show the arrangement of external branches? Scott sought further resources for her student. Writing on January 14th, 1902, Scott directed Hilbert's attention to his ten-year-old footnote.

> As I am very much interested in topological investigations myself, and have a graduate student who is taking up this line of research, I am anxious to know by what process you have proved this, and whether the process has been applied to other curves. I shall be much indebted to you for information on this point. If the proof has appeared in print, a reference to the periodical and volume will give me what I need (Scott, 1902d).

Scott ended by apologizing "for this trespassing on your time and attention" and pleaded her excuse as "the interest I take in the subject." There is no evidence of a response. In any case, the relationship between Bryn Mawr and Göttingen appears to have been cordial and professional, making it unlikely that her letter would have been ignored. Probably Hilbert did not have a proof, as the subject would continue to be investigated by his doctoral students over the next decade.[29]

While Hilbert himself proved a dead-end, Ragsdale had not yet exhausted Scott's 1897 "Circuit" bibliography, which concluded with a set of texts having "no direct bearing on the subject at present, though there appears to be an underlying connection" (Scott, 1897, p. 101). These included four papers by Walther Dyck on "Analysis Situs" which served as the primary source in Ragsdale's next Journal Club entry on "The Theory of the Characteristic and its Use in Topological Investigations" dated April 21, 1902. A result from Dyck on the number of singularities inside a bounded non-singular curve reminded Ragsdale of a more particular finding in James Clerk Maxwell's "On Hills and Dales." Her next Journal Club entry took up this text and its potential for the study of branches. While valuable as expositions in the back of mathematics these Journal Club entries were unsuccessful in the context of Ragsdale's research question. The usability of these texts could only be determined after they were brought to the back. In particular, Ragsdale's entry "On Hills and Dales," though clearly directed toward resolving Hilbert's footnote, left no apparent trace in her published dissertation.

Footnote 28 continued
keinesfalls sämmtlich ausserhalb und von einander getrennt verlaufen können. Dieses Resultat erscheint mir desshalb von Interesse, weil es zeigt, dass für Curven mit der Maximalzahl von Zügen der topologisch einfachste Fall nicht immer möglich ist. (my translation)

[29] These are Löbenstein (1910) and Kahn (1909). Klara Löbenstein and Grete Kahn were among the first German women to obtain their doctorates in mathematics.

3 Hills and Dales

Unlike the short articles of Hulburt that form the core of Ragsdale's exposition in her entry on branches, the Dyck texts taken together add up to almost 150 pages.[30] In addition, Ragsdale's bibiography references particular passages from Leopold Kronecker, Eugen Netto, and Heinrich Weber totaling an additional 50 pages.[31] Consequently, Ragsdale provides a much more targeted summary of Dyck's references to curves, leaving out or only superficially treating a substantial portion of the cited texts.

Ragsdale divided her entry on the theory of the characteristic into two parts, respectively treating the algebraic and the geometrical definitions. She began with Kronecker.

> In papers published in 1869, 1878 Kronecker shows that with any system of algebraic functions $f_0, f_1, f_2, \cdots f_n$, satisfying certain assumed conditions, there is associated a number, derived algebraically, which is invariant for that system (Ragsdale, 1902b, p. 63)

In the geometrical part, she turned to Dyck's "Beiträge zur Analysis Situs," in which he "discusses the significance of the Characteristic in the consideration of Topological problems." Dyck defined "*geometrically* for a manifold of n dimensions a *characteristic* number which is built up as the manifold itself is developed" and determined "that this number is identical with the characteristic as defined by Kronecker" (70). Ragsdale noted that if "we start with the figure as actually non-existent, the characteristic is zero," then as "elements appear or separate the characteristic increases by unity" and conversely "as elements disappear or unite the characteristic decreases by unity." Ragsdale underlined her conclusion: "*Therefore the characteristic of the manifold, or figure, as now defined, is equal to the sum of the characteristics of the separate elements and is wholly independent of the method of generation*" (71).

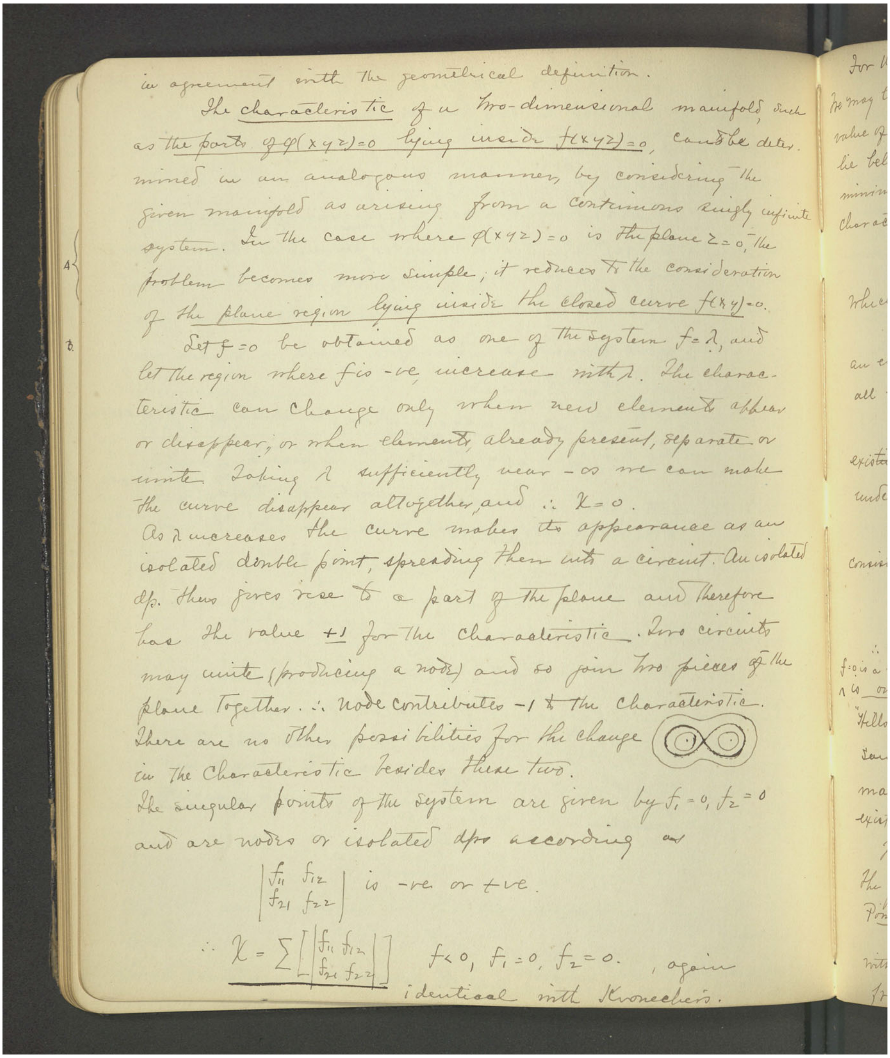
After defining the geometric characteristic in three-dimensions, Ragsdale returned to the subject of curves closely drawing on Dyck's treatment of "Zweidimensionale Mannigfaltigkeiten M_2."[32] Ragsdale explained that there were two possibilities for the change in the characteristic from $\chi = 0$. In the first possibility an "isolated dp. [double point] thus gives rise to a part of the plane and therefore has the value ± 1 for the characteristic." In the second, "circuits may unite (producing a node) and so join two pieces of the plane together" with the node contributing -1 to the characteristic. On the right of the page, Ragsdale included a rough sketch of the second possibility (Fig. 5).

Ragsdale then confirmed the value of the characteristic for the manifold could be calculated based on the number of nodes and isolated double points in the system and corresponded to Kronecker's definition "for all *modes* of generation *of the* manifold." From the properties relating "singular points of any system of curves" Ragsdale determined that inside the bounded non-singular curve $f = 0$ "*the number of isolated dps.* of the system of which $f = 0$ is a member is *one more than the number of nodes*."

[30] Ragsdale's bibliography includes (Aufsatz, 1888; Dyck, 1886, 1887, 1890; Mittheilung, 1885) On the life and work of Walther von Dyck, see Hashagen (2003).

[31] These passages are drawn from Kronecker (1895), Kronecker (1897), Netto (1896), and Weber (1896).

[32] For comparison, Dyck's work on this subject can be found in Dyck (1886, part 3) and (1888 pp. 467–468).



in agreement with the geometrical definition.

The characteristic of a two-dimensional manifold such as the parts of $\varphi(xyz)=0$ lying inside $f(xyz)=0$, can be determined in an analogous manner, by considering the given manifold as arising from a continuous singly infinite system. In the case where $\varphi(xyz)=0$ is the plane $z=0$, the problem becomes more simple; it reduces to the consideration of the plane region lying inside the closed curve $f(xy)=0$.

Let $f=0$ be obtained as one of the system $f=\lambda$, and let the region where f is -ve increase with λ. The characteristic can change only when new elements appear or disappear, or when elements already present, separate or unite. Taking λ sufficiently near $-\infty$ we can make the curve disappear altogether, and $\therefore \chi=0$.

As λ increases the curve makes its appearance as an isolated double point, spreading then into a circuit. An isolated d.p. thus gives rise to a part of the plane and therefore has the value +1 for the characteristic. Two circuits may unite (producing a node) and so join two pieces of the plane together. ∴ node contributes −1 to the characteristic. There are no other possibilities for the change in the characteristic besides these two.

The singular points of the system are given by $f_1=0$, $f_2=0$ and are nodes or isolated dps according as

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}$$ is -ve or +ve.

$$\therefore \chi = \sum \left[\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} \right] \quad f<0,\ f_1=0,\ f_2=0.$$

, again identical with Kronecker's.

Fig. 5 Ragsdale's figure of a node produced by combining circuits (Ragsdale, 1902b)

She added that "Maxwell in a paper 'Hills and Dales', (Phil. Mag. Series IV. Vol. 11. p. 44) enunciates this same theorem." Loosely quoting Maxwell: "If a function of two variables becomes a maximum p times within a certain boundary, then there exist also within that boundary $p-1$ false maxima" (75). This reference was not in Dyck.[33]

Ragsdale remarked that though "the geometrical conception of the characteristic leads to this remarkable information concerning the figure, the latter is by no means uniquely determined by its characteristic." In particular, "the closed branches of a one-dimensional manifold, or figure of lines, contribute nothing to the characteristic

33 Dyck does cite Maxwell's "Treatise on Electricity and Magnetism" (2nd ed) 1881.

and hence the characteristic can give no clue to the number of branches of the curve." She would later doubt whether the theory of characteristic was "the most promising instrument of proof" (Ragsdale, 1906a, p. 403).

Ragsdale closely, but selectively, followed Dyck. Her additions are like those made in 1901: the language of circuits to accompany branches, concrete examples coupled to drawn and colored figures, and further sources, including Maxwell's "On Hills and Dales." In recognizing Maxwell as offering a more particular version of Dyck's result, Ragsdale landed upon the next topic for her Journal Club contribution.

For the fall semester in the Club's fourth year, most entries centered on reviews of single texts in geometry and analysis published over the past half century. Maxwell's "On Hills and Dales" is Ragsdale's first step beyond Scott's 1897 "Circuits" bibliography.

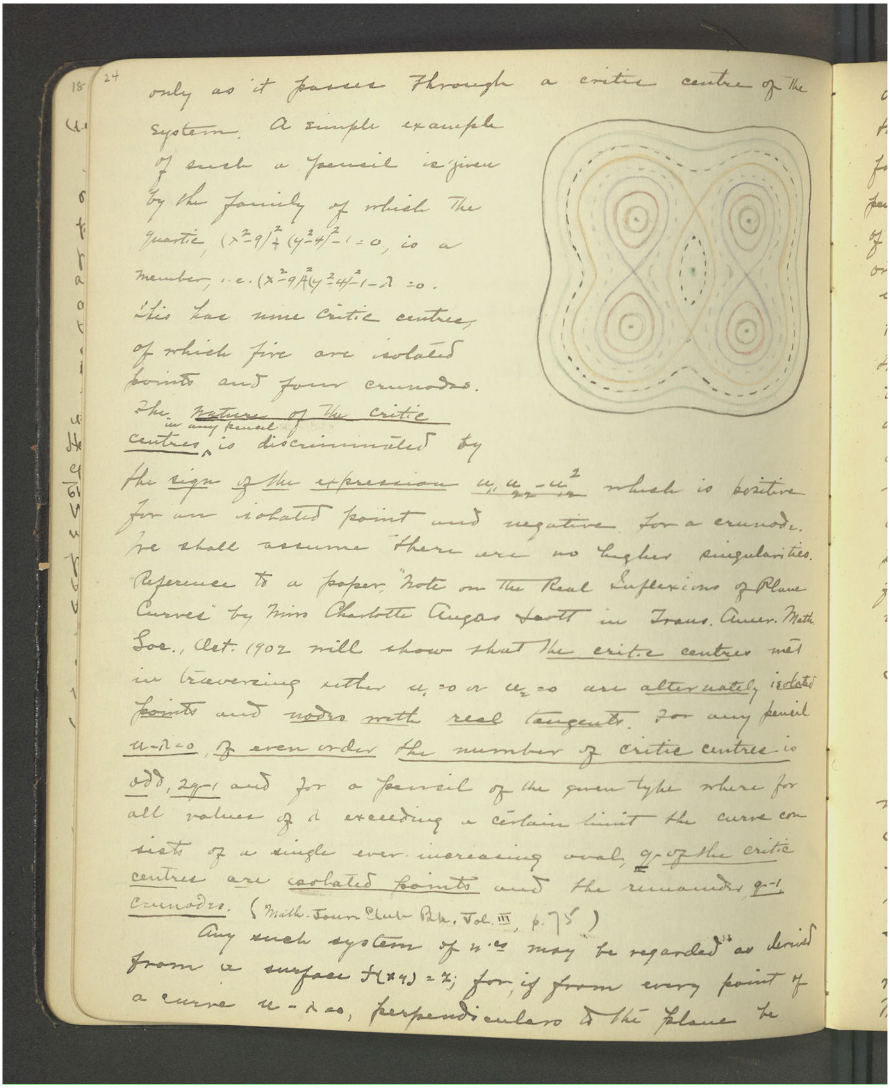
Maxwell published "On Hills and Dales" in the *Philosophical Magazine* in 1870 Maxwell (1870). The article extended an exposition on "the first elements of physical geography" initiated by Cayley in "On Contour and Slope Lines" about a decade before. The paper is dedicated to the problem of defining contour-lines "representing the intersection of a level surface with the surface of the earth" so that "the height of a place" is "mathematically accurate." He began "with a level surface entirely within the solid part of the earth." Maxwell narrated, "let us suppose it to ascend till it reaches the bottom of the deepest sea. At that point it will touch the surface of the earth; and if it continues to ascend, a contour-line will be formed surrounding this bottom (or Immit, as it is called by Professor Cayley) and enclosing a region of depression" (235).[34] This ascent continued and as "the level surface rises these regions of depression will continually expand, and new ones will be formed corresponding to the different lowest points of the earth's surface."

Despite his pastoral title and geological scope, Ragsdale recognized Maxwell's text as treating pencils, critic centres, isolated points, crunodes, and other precise geometrical objects that Maxwell had not invoked by name. As she judged, an "understanding of Maxwell's paper presupposes some knowledge of families of curves" (Ragsdale, 1902c, p. 23). By adding an introduction to curves, Ragsdale was not only endeavoring to help her audience understand Maxwell's paper, but also assaying whether the alignment of summits with maxima and bottoms with minima held interpretative power for the study of branches.

Ragsdale thus began in the plane rather than the center of the earth. She denoted $u = 0$ as a curve of order n or n^{ic}. Then $u - \lambda = 0$ "represents an n^{ic} which has no points in common with the given curve." Letting λ "assume all values from $-\infty$ to $+\infty$" the n^{ic} then "represents a singly infinite system of non-intersecting curves completely covering the plane. There is "one and only one curve of the pencil" for all but exceptional points in the plane.

Ragsdale showed that the number of double points or "critic centres" is $(n-1)^2$. As a "simple example" she included a very colorful picture of the pencil $(x^2 - 0) + (y^2 - 4)^2 - 1 - \lambda = 0$ (Fig. 6) of which the quartic $(x^2 - 9)^2 + (y^2 - 4)^2 - 1 = 0$ is a member (shown in red). This pencil has nine critic centres, five are isolated points

[34] The reference is to Cayley's "On contour and Slope Lines" also published in *Philosophical Magazine* and included as the other text in Ragsdale's bibliography (Cayley, 1859).



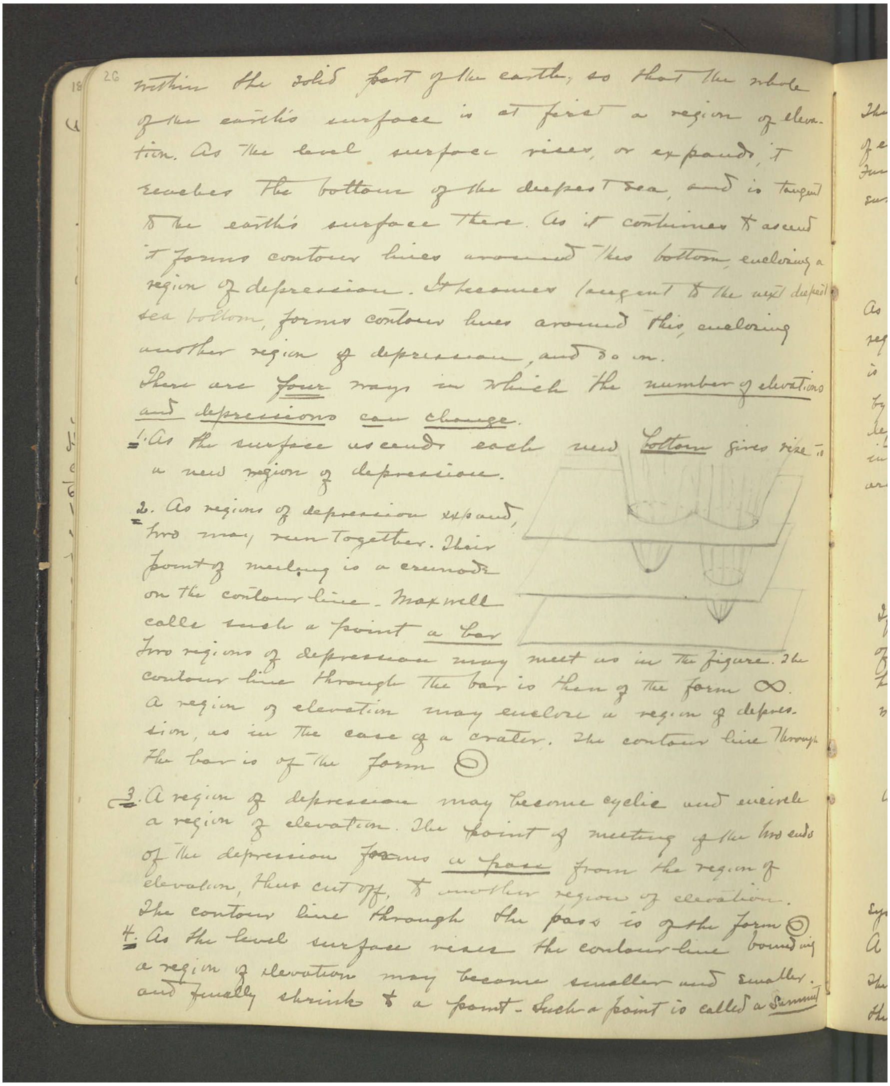
18 24 only as it passes through a critic centre of the system. A simple example of such a pencil is given by the family of which the quartic, $(x^2-9)^2+(y^2-4)^2-1=0$, is a member, i.e. $(x^2-9)^2+(y^2-4)^2-1-\lambda=0$. This has nine critic centres, of which five are isolated points and four crunodes. The nature of the critic centres in any pencil is discriminated by the sign of the expression $u_{11}u_{22}-u_{12}^2$ which is positive for an isolated point and negative for a crunode. We shall assume there are no higher singularities. Reference to a paper "Note on the Real Inflexions of Plane Curves" by Miss Charlotte Angas Scott in Trans. Amer. Math. Soc., Oct. 1902 will show that the critic centres met in traversing either $u_1=0$ or $u_2=0$ are alternately isolated points and nodes with real tangents. For any pencil $u-\lambda=0$, of even order the number of critic centres is odd, $2q-1$ and for a pencil of the given type where for all values of λ exceeding a certain limit the curve consists of a single ever-increasing oval, q of the critic centres are isolated points and the remainder, $q-1$ crunodes. (Math. Journ. Club Bk. Vol. III, p. 75)

Any such system of n^{ics} may be regarded as derived from a surface $u(xy)=z$; for, if from every point of a curve $u-\lambda=0$, perpendiculars to the plane be

Fig. 6 Figure of the pencil with nine critic centres in Ragsdale (1902c)

(shown as such) and four are crunodes (where the orange curves and purple curves intersect).

Ragsdale invoked Scott's recently published "Note on the Real Inflexions of Plane Curves" and her own own recent Journal Club entry on "The Theory of the Characteristic" for further results in obtaining the number and kind of critic centres from the equation of the pencil. Having made the connection to her ongoing research on curves, Ragsdale explained how these pencils relate to Maxwell's study of the earth because any "such system of n^{ics} may be regarded as derived from a surface." For a surface $u(xy) = z$, as the plane $z = \lambda$ moves "upward from $z = -\infty$" it will be "tangent



26 within the solid part of the earth, so that the whole of the earth's surface is at first a region of elevation. As the level surface rises, or expands, it reaches the bottom of the deepest sea, and is tangent to the earth's surface there. As it continues to ascend it forms contour lines around this bottom, enclosing a region of depression. It becomes tangent to the next deepest sea bottom, forms contour lines around this, enclosing another region of depression, and so on.

There are four ways in which the number of elevations and depressions can change.

1. As the surface ascends each new bottom gives rise to a new region of depression.

2. As regions of depression expand, two may run together. Their point of meeting is a crunode on the contour line. Maxwell calls such a point a bar. Two regions of depression may meet as in the figure. The contour line through the bar is then of the form ∞. A region of elevation may enclose a region of depression, as in the case of a crater. The contour line through the bar is of the form [illegible]

3. A region of depression may become cyclic and encircle a region of elevation. The point of meeting of the two ends of the depression forms a pass from the region of elevation, thus cut off, to another region of elevation. The contour line through the pass is of the form [illegible]

4. As the level surface rises the contour-line bounding a region of elevation may become smaller and smaller. and finally shrink to a point. Such a point is called a Summit

Fig. 7 The figure of two regions colliding as a surface ascends in Ragsdale (1902c)

to the surface at every minimum and maximum, and the representations of these in the system of curves will be isolated points." This prefatory exposition of curves ends with a horizontal line dividing page 25 and Ragsdale turned to the specific content of Maxwell's paper.

Synthesizing Maxwell, Ragsdale listed and illustrated "*four* ways in which which the *number of elevations and depressions can change*" and in doing so defined bars, bottoms, passes, and summits (26). A figure on the right shows at once an ascending surface and two formerly separate regions colliding (Fig. 7).

Because the "appearance of each *new* region of elevation is accompanied by a *pass*" and finally "every region of elevation is reduced" to a summit, the number of summits is one more than the number of passes. Similarly, the number of bottoms is one more than the number of bars.

The alterations from Maxwell's original text here are slight. In the second "way," Ragsdale substituted the more precise "crunode" here for Maxwell's "double point." She also compared a region of elevation that encloses a region of depression with the geographic example of a crater—perhaps a consequence of mid-Atlantic American geography as opposed to that of Britain. Tiny sketches of forms illustrated passes and summits.

Ragsdale further interpreted these results with respect to singularities. As Maxwell had written, the "whole of this theory applies to the case of the maxima and minima of a function of two variables which is everywhere finite, determinate, and continuous. The summits correspond to maxima and the bottoms to minima" (Maxwell, 1870, p. 236) Ragsdale added that the "*passes* and *bars*, represented by *crunodes*, are points where the surface is horizontal but where the elevation is *neither* a maximum nor a minimum" (Ragsdale, 1902c, p. 29). Like in the revision of inner/outer to positive/negative, Ragsdale's more precise classifications of singularities unlocked new applications.

Passing over Maxwell's discussion of functions of three variables, Ragsdale almost identically reproduced Maxwell's language to define "lines that are everywhere perpendicular to the contour" as *lines of slope*. Though lines of slope served to form basins or dales, hills, watersheds, and water courses, they were a dead end for the study of curves. In "the system of plane curves $u - \lambda = 0$, there is nothing that corresponds exactly to the *lines of slope*." The tangent lines $u_1 = 0$, $u_2 = 0$ "do pass through all the maxima and minima and other stationary points, and cut across all curves of the system, but the analogy ends there" (30).

Whether or not Ragsdale put aside "On Hills and Dales" after her Journal Club paper, she made no use of the work in her published dissertation. In this respect, the entry is a false start. From another perspective, Ragsdale's reinterpretation of Maxwell was fruitful. Without surfacing in the front of mathematics, "On Hills and Dales" became part of the Bryn Mawr knowledge tradition for the next two decades. The paper formed the subject of Virginia Stoddard's 1908 entry and Elizabeth Cooper's 1922 entry. Moreover, following Ragsdale, Maxwell's text remained a part of topology. As Cooper concluded,

> It is fascinating to find this definite application of Geometry Situs to the familiar aspects of the very ground we walk upon. Cayley and Maxwell have, to use the colloquial phrase, brought Pure Mathematics "down to earth" in a most interesting manner (Cooper, 1922, pp. 74–75)

Though the Bryn Mawr students only pursued Hills and Dales within the Journal Club, the potential of approaching Maxwell as a topological text was eventually realized, albeit obliquely. Maxwell's language of mapping bears strong resemblance to Marston Morse's 1966 lectures on "Pits, Peaks, and Passes" as a fairly elementary means of illustrating critical point theory. Though Morse cited Maxwell, it is in the context of electrostatics and not physical geography.[35] Ragsdale's identification of Maxwell with topology was insightful, but this insight would not enter the front of mathematics through Bryn Mawr.

4 Ragsdale's dissertation and return to the Journal Club

Ragsdale applied to enter doctoral studies in 1902 writing "Subject of Thesis is to be one in Topology. The field of all my seminar work during the present year" (Ragsdale, 1902a). Her application was successful and Ragsdale completed her PhD exams in September 1903. That year her fellowship ended and she took a position teaching mathematics in New York City at Dr. Sachs's School, a private all-girls' school. Between 1902 and 1906 Ragsdale did not contribute to the Mathematical Journal Club. In 1906 Ragsdale moved back to the town of Bryn Mawr as the head of mathematics at Baldwin School, another private all-girls' school. This proximity enabled her to participate in the mathematics department at Bryn Mawr College, where she would also serve as a Reader in Mathematics between 1908 and 1910.

On April 29th, 1905 "Miss Virginia Ragsdale" read a paper "On the arrangement of the real branches of algebraic curves" at the meeting of the American Mathematical Society in New York.[36] Her article of the same title was published in the October, 1906 issue of the *American Journal of Mathematics* and then was reprinted by Bryn Mawr College as her dissertation. In December of that year she reported on the subject for the Bryn Mawr Mathematical Journal Club.

The dissertation centers on the number and arrangement of external branches for higher order curves, a generalization of the investigation prompted by Hilbert's footnote. Though unable to find a proof, Ragsdale presented significant evidence toward a conjecture. The provability of the conjecture rested on whether "all non-singular curves with the maximum number of branches are obtainable" by the Harnack and Hilbert processes "by which curves with the maximum number of branches have been derived" so far Ragsdale (1906a, p. 380). Acknowledging these limitations, Ragsdale assured the reader that regardless of "whether the law is of perfect generality or not, it is of interest to investigate more fully the various types of curves that can be derived by these different methods."

The majority of Ragsdale's paper elaborated procedural steps by which external branches could be constructed and counted as the order of the curve increased. Though Ragsdale followed the methods established by Harnack and Hilbert, she shifted the question from finding an upper bound on the number of nested ovals to finding an upper bound on the number of external branches. For nested ovals, exceeding the maximum would quickly result in a contradiction—the number of intersections with a straight line can exceed the order of the curve. For external branches, careful bookkeeping was required. As Ragsdale showed, there are different "modes of generation" dependent on the positions of the straight lines with respect to the n^{ic} and choice of generating

[35] John Stillwell draws this connection between Cayley, Maxwell, and Morse in his introduction to Morse (2007).

[36] Also in attendance from Bryn Mawr at this meeting were Miss L. D. Cummings, Prof. Charlotte A. Scott, and Mr. J. E. Wright (Cole, 1905).

oval (nested or not). These can lead to multiple "types" of curves generated from the same n^{ic}.

In her Fig. 3 (Fig. 8), Ragsdale provided a schematic diagram of how the internal and external ovals of a 6^{ic}, 8^{ic}, and 10^{ic} may be arranged following Harnack's use of an auxiliary line or Hilbert's use of an auxiliary conic, both described as "small variation from a special degenerate curve." The number of types doubles for each increase of even n, suggesting why Ragsdale stopped with 12. She had already done a great deal of work.

With the "conspicuous label" of *conjecture*, Ragsdale's dissertation adheres to Hersh's portrait of the front of mathematics. But the move from the back to the front was informed by Ragsdale's training at Bryn Mawr and was not aimed at emulating some "universal" form of published mathematics. Some of the examples are excised, but the Bryn Mawr sources and terminology persisted, color is lost but many illustrations are elaborated, and Ragsdale's positive/negative distinction enabled a careful generalization in counting external ovals.

Ragsdale's 1906 publication has the same title as her 1901 Notebook entry, and she likewise continued to interchange circuits and branches without any systematic difference. External validation of the publishing process in the *American Journal of Mathematics* further confirms the legitimacy of this minority usage. On the other hand, the editors in 1906 were Scott, Frank Morley (who had participated in the 1896 Journal Club as faculty at Haverford College), Simon Newcomb, A. Cohen, and "other mathematicians." Without further information of the article selection process, it is not possible to know how Ragsdale's contribution was vetted.

There are some slight changes in terminology between the Journal entry and the published paper. Adopting the new English standard, what was the deficiency of the curve p in 1901 is the genus p in 1906. Of more relevance to Ragsdale's research on non-singular curves, in her article she worded her restriction to only circuits that "can be projected into the finite, and for these the term oval is here employed" (377). Ragsdale had introduced ovals as equivalent to "even circuits" in 1901. Yet, because the results of her publication are limited to even circuits, there is no practical difference in these two designations.

Likewise, the references from Ragsdale's 1901 introduction remain intact, rewritten with a higher standard of formality. Whereas Ragsdale's initial citation to Zeuthen's non-finite curves seemed exceptional, here she dedicated an entire footnote to the subject of curves that cannot be projected into the plane. Ragsdale supplemented the example from Zeuthen with references to Cayley's 1865 *On Quartic Curves* and Scott's 1902 *On the Circuits of Plane Curves*—further cementing the importance of Bryn Mawr College in determining the relevance and accessibility of literature (377). The footnote also established a lineage ending with Scott, potentially bolstering the reputation of Ragsdale's advisor and training for the front of mathematics. Such an advertisement would have been superfluous for the Journal Club.

Besides Cayley and Scott there are no other sources in Ragsdale's publication that are not also in her Journal Club entries. While her study of Dyck and Maxwell indicate that Ragsdale sought further means of treating Hilbert's footnote, she eventually settled on the resources already at hand in 1901.

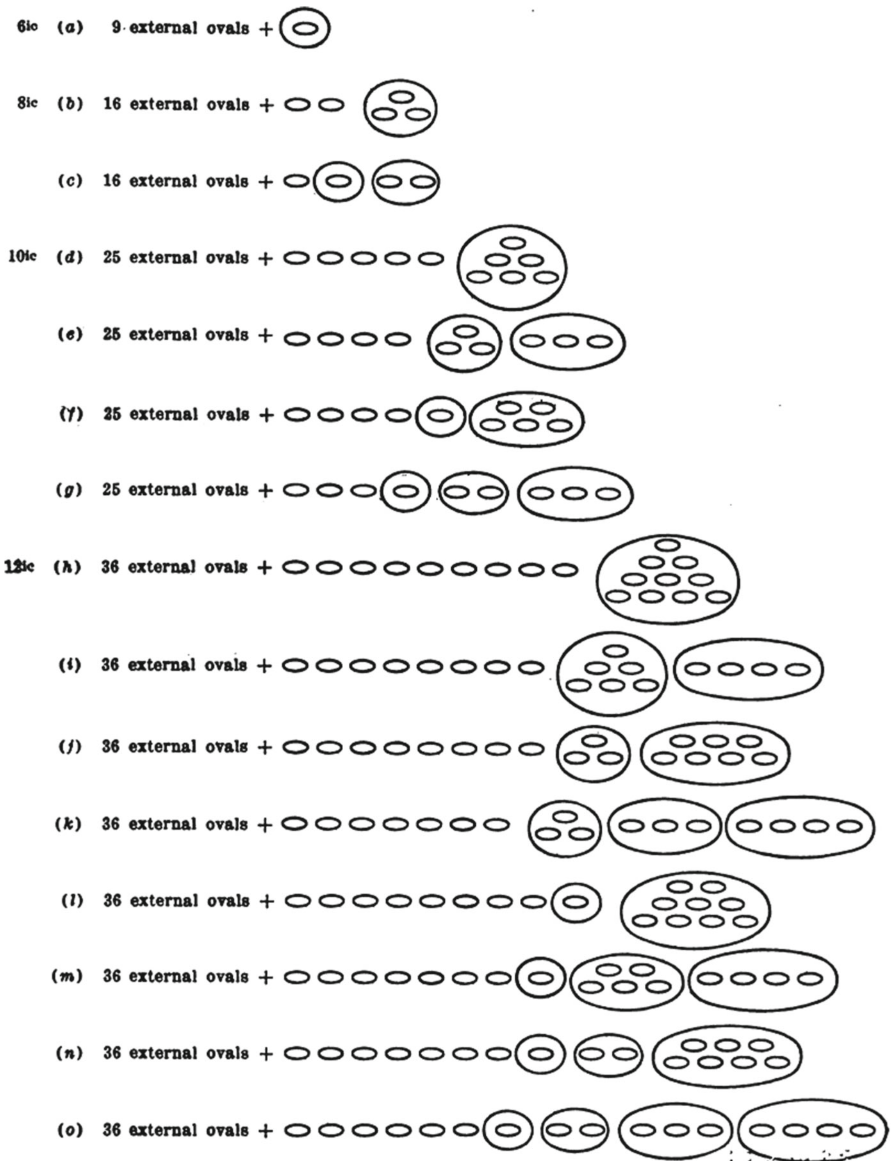


FIG. 3.

Fig. 8 Ragsdale's Fig. 3 from (Ragsdale, 1906a)

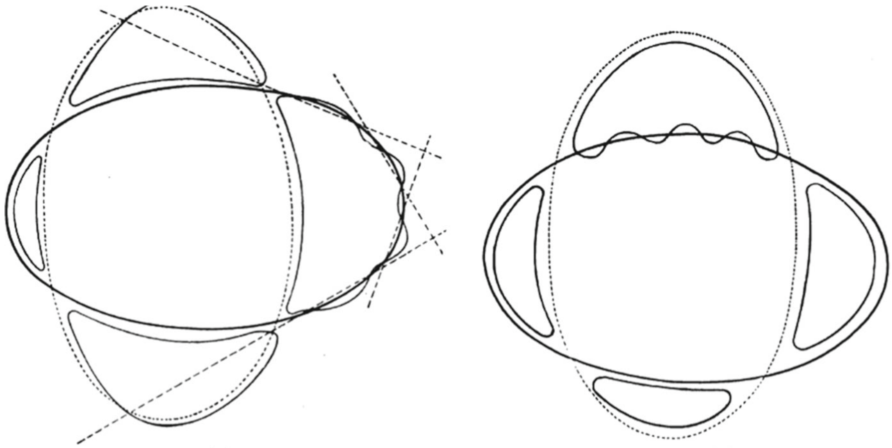
The theory of the characteristic is addressed only in the conclusion where Ragsdale considered "several other forms in which the theorem can be stated that are of interest, either as facts resulting from the theorem if established in its preceding form, or as statements which may afford a better starting point for the proof of the theorem" (400). However, the approach is no longer optimistic. Though Ragsdale could determine bounds on the value of the characteristic with respect to the number of isolated points and nodes, these relations were "interesting, but afford no clue to the solution" of her conjecture. Further, the results from studying the characteristic were "applicable to curves of even order only." After putting aside the theory of characteristic, Ragsdale proposed "the most interesting form" in which the theorem could be stated relates to "the number of regions into which the plane is divided by the curve." In this setting the conjecture can be read as an upper and lower bound on the number of regions in which the curve of even order C_{2n} is respectively positive and negative, suggesting "some underlying relation to the theory of Multiply-connected Surfaces."

Was this hesitancy atypical in a published work? Was it also a feature of the training at Bryn Mawr? In her obituary of Cayley in 1895, Scott had marveled that "not every mathematician [...] will lecture to a class of specialists on the incomplete investigation of the night before, and end up with the remark, obviously genuine, 'Perhaps some of you may find this out before I do"' (Scott, 1895, p. 139). Scott similarly laid bare unfinished work in progress through the Journal Club. Scott had annotated her definition of "circulation" with question marks and dissatisfaction and openly criticized it as "neither compact nor convincing" in 1897 (Scott, 1897, p. 104). When she published the definition, her argument was only tentative, suggesting "it may be necessary to introduce some term as *circulation*" and hoping to consider the case "at some future time" (Scott, 1902c, p. 398). Both Scott and Ragsdale were writing for the *American Journal of Mathematics* and the venue may have been unusually open to more speculative writing. In any case, in contrast to Hersh's depiction, some of what made it to the front of mathematics was uncertain.[37]

Ragsdale maintained and expanded the use of figures introduced in 1901. Her first and second figures (Fig. 10) demonstrate the value of the positive/negative distinction. The evidence for Ragsdale's conjecture is neatly synthesized in her third figure (Fig. 8), in which all possible maximal arrangements of external ovals are presented in succession. Six other schematic figures through the text indicate arrangements of ovals produced by the multiple modes of generation.

Along with the in-line figures, Ragsdale's text concluded with two plates, respectively showing the constructive results of the Harnack and Hilbert processes (Fig. 9). These plates are very similar to Harnack's Figure II (and thus Ragsdale's 1901 figures) with the addition of a graphical innovation. In her Journal Club entries, Ragsdale could use multiple colors to distinguish between the different components of the equation. In print, Ragsdale accomplished the same effect through solid, dotted, and dashed lines that are cleverly communicated in the equations accompanying each figure. This key is a marked improvement from the much more subtle faint versus heavy line employed by Harnack. Ragsdale applied this palette to her figures of intersecting ellipses from her Journal Club entry. The orientation has shifted and the colors are gone, but the first two figures convey the same constructive process. By 1906 some mathematical texts did contain color, but this would have been an added expense. By designing a correspondence between colors and line types, Ragsdale overcame this potential challenge without sacrificing the visual clarity she had first displayed for the Bryn Mawr Journal Club.

[37] For a more contemporary debate of the value of uncertainty at the front of mathematics, see Jaffe and Quinn (1993) and the associated responses.



$C_6 \equiv C_2 \cdot E_4 + \delta \prod_{i=1}^{i=4} l_i = 0.$

FIG. 1.

$C_6 \equiv C_2 \cdot E_4 + \delta \prod_{i=1}^{i=4} l_i = 0.$

FIG. 2.

Fig. 9 Ragsdale's Plate 2, Figs. 1 and 2 (Ragsdale, 1906a)

Once Ragsdale established the preliminary background on curves and branches, she pivoted to Hilbert's provocative footnote and the intended purpose of her research: "if the non-singular sextic have its maximum number of branches, eleven, these cannot all lie external to one another." She noted that "Hilbert speaks of the process by which he arrived at this conclusion as 'ausserordentlich umständlich,' but no hint as to the character of the argument is given, and no proof of the statement has ever been published." Further, "if such a limitation on the arrangement of the ovals does exist for the 6^{ic}, there arises at once the question as to the existence of a similar limitation for all non-singular curves with the maximum number of branches." Since "no such restriction holds" for curves of odd order, Ragsdale pursued "the discussion of the question for curves of even order." But first, she proposed, "it is convenient to cast Hilbert's statement into a slightly different form" (378).

There are two types of the 6^{ic} "derived from his [Hilbert's] method of generation" leading to an oval O. In the first, there is "1 oval inside O, and 9 outside" and in the second there are "9 ovals inside O, and 1 outside." Because "the number of ovals 'inside' and 'outside' are interchanged in the two cases" Ragsdale made the "natural inference" that the "distinction between the 'inside' and 'outside' of a closed circuit [...] is based on no distinctive or permanent property of any one region of the plane."

Since the property itself was not permanent, the language should be similarly malleable. So "the division of the plane by the curve $u = 0$ into regions where u is positive and regions where u is negative offers a more promising basis for investigation of the problem, because of the element of arbitrariness introduced in ascribing to a certain region the positive rather than the negative sign." With positive/negative Ragsdale stepped away from a visually pre-determined designation to one that is based on arbitrary conventions.

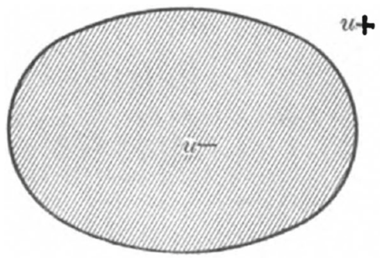


FIG. 1.

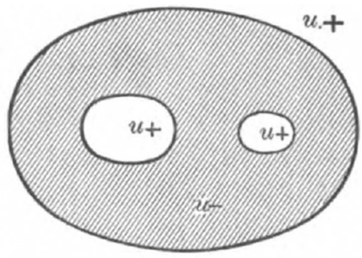


FIG. 2.

Fig. 10 Ragsdale's Figs. 1 and 2 (Ragsdale 1906a)

For the non-singular, even curves under consideration, Ragsdale invoked "the usual convention" letting "the sign be so determined so that the expression *u* is positive at infinity."[38] Then a "region where *u* is negative may be a region bounded by a single circuit as in Fig. 1, or a region bounded by two or more circuits as in Fig. 2" (379). Ragsdale replaced the colors of the Notebook with shading to further distinguish the two regions (Fig. 10).

Following these illustrations, Ragsdale defined "an *internal* oval" as "an oval which cuts off in the midst of a region where *u* is negative a region in which *u* is positive" and conversely "an *external* oval" as "an oval which cuts off in the midst of a region where *u* is positive a region in which *u* is negative." By comparison with Zeuthen's definition,,[39] Ragsdale's external and internal distinction based on positive and negative values of *u* is designed for the context of nested ovals. This is particularly useful in her treatment of annular regions where the external/internal no longer corresponds respectively to outside/inside (see Figs. 9 and 11). As Ragsdale explained, the "circuits lying in the 2nd, 4th, 6th,.... regions lie 'inside' certain ovals but are themselves external ovals" (386).

With the definitions of internal and external, "Hilbert's statement can be expressed as follows: *If the non-singular sextic have its maximum number of branches, at least one of the eleven ovals must be internal*;—that is, not more than ten of the eleven ovals can be external."

This "slightly different form" lends itself to Ragsdale's conjectured generalization (pp. 378–379). She elaborated "as curves of higher order are investigated a most interesting law governing the arrangement of the ovals presents itself so persistently, and in curves of such widely different types, as to give strong reasons for belief in the existence of a general theorem" that is, "*if the non-singular* $2n^{ic}$ *have the maximum number of branches, at least* $\frac{1}{2}(n-1)(n-2)$ *of the* $p+1$ *ovals must be internal; or not more than* $n^2 + \frac{1}{2}(n-1)(n-2)$ *can be external.*"

At the end of her paper, Ragsdale extended the internal/external division to the broader context of odd circuits that divide "the plane into two regions both infinite,

[38] At the end of this paper, Ragsdale noted that "no modification of these statements is necessary, if the sign of C_{2n} be so chosen that it is negative at infinity."

[39] "Une branche d'ordre pair sans points multiples, divise le plan en deux parties, dont l'une qui renferme des branches d'ordre impair s'appelle *l'externe* l'autre qui n'en renferme aucune s'appelle *l'interne*." (Zeuthen, 1874, p. 410)

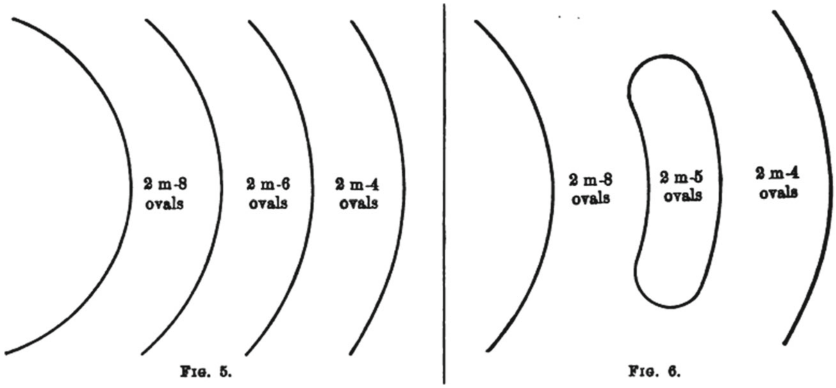


Fig. 11 Ragsdale's Figs. 5 and 6 (Ragsdale 1906a)

in one of which C_{2n+1} is positive, and in the other, negative"—another situation in which "ovals may lie each outside the others" yet still be "internal" (404).

Comparing Ragsdale's positive/negative distinction in 1901 with that in 1906 there are a few remarkable additions. First, Ragsdale justified her decision explicitly with respect to the value of arbitrariness. Such a change, though slight, raises the inscrutable question of what need not be said in the semi-private of a Journal Club. In her writings on duality, Scott claimed "a judicious misuse of conventions is frequently essential to progress" (Scott, 1897, p. 125). Ragsdale's added justification enabled exporting her mathematics to a new audience with different standards and expectations.

Secondly, Ragsdale applied the positive/negative distinction to create new categories and generalize her findings. The feature becomes essential not only to the form of Ragsdale's dissertation but also the contents. While Ragsdale's conjecture was eventually proved false, her definitions persisted.[40]

In contrast to the continuity in the sources, terminology, and figures Ragsdale employed in 1901 and 1906, the published version contained no equations of specific curves grounding the general results. This absence may speak to a difference of audience or simply a concession to affording more space to new results. Regardless of the motivation, the elimination of concrete numerical examples is systematic. Perhaps such a "purge" advances a myth about the generality or abstraction of early-twentieth-century mathematics.[41]

Indeed, two months after publication, when Ragsdale relocated her results back to the Journal Club, she replaced the detailed accounting and procedural steps with a representative example. To show the results of Harnack's process, Ragsdale inserted her Plate I, changing by hand the figure enumeration. For the "less simple" Hilbert process, Ragsdale explained that if "the 1st mode of generation be used throughout for the derivation of the $2n^{ic}$, the nest of $n-1$ ovals is built up from the inside outward, and the ovals [that] sit nested in the $n-2$ annular regions formed by successive ovals of the

40 For instance, in Kahn (1909). Later in the twentieth century, Ragsdale's internal/external division was revised as even/odd, so that "An oval of a curve is called *even* (resp. *odd*) if it lies inside of an even (resp. odd) number of other ovals of this curve" (Itenberg & Viro, 1996, p. 23).

41 On the meanings and values of generality for mathematics, see Chemla and Chorlay (2017).

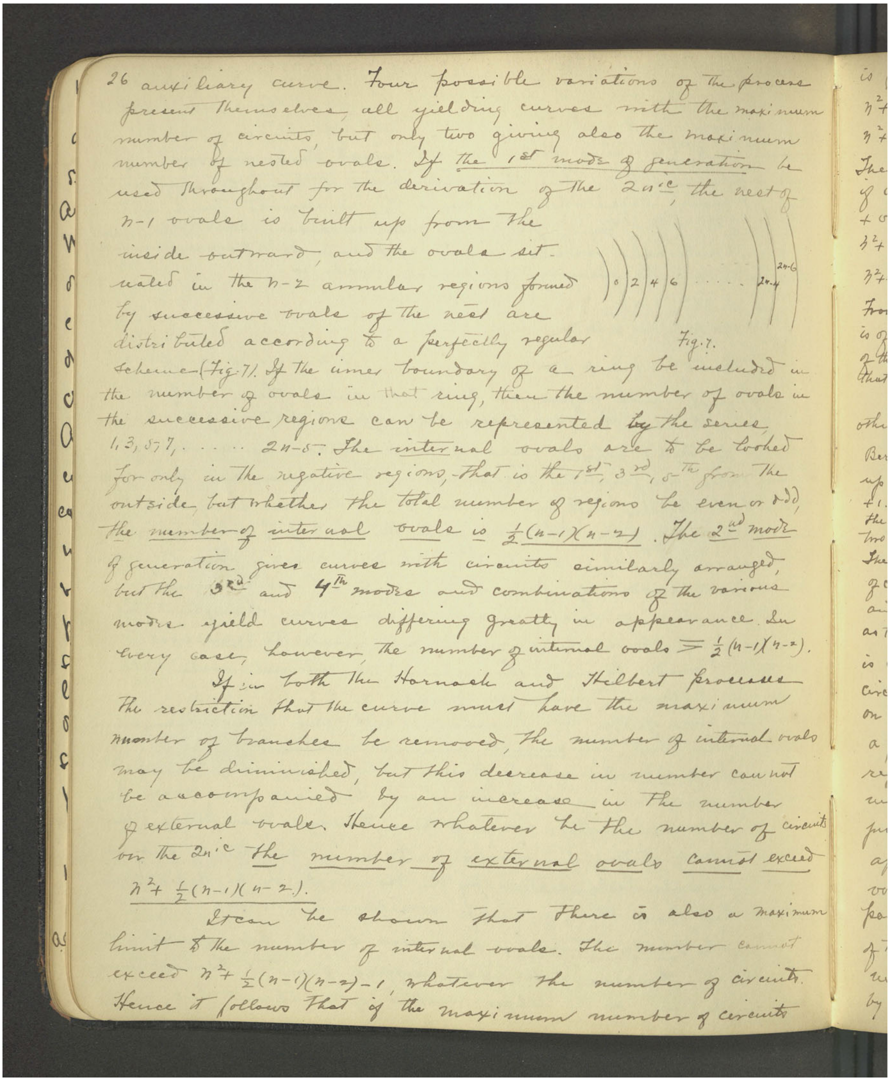

26 auxiliary curve. Four possible variations of the process
present themselves, all yielding curves with the maximum
number of circuits, but only two giving also the maximum
number of nested ovals. If the 1st mode of generation be
used throughout for the derivation of the $2n^{ic}$ the nest of
$n-1$ ovals is built up from the
inside outward, and the ovals sit-
uated in the $n-2$ annular regions formed
by successive ovals of the nest are
distributed according to a perfectly regular
scheme (Fig. 7). If the inner boundary of a ring be included in
the number of ovals in that ring, then the number of ovals in
the successive regions can be represented by the series
1, 3, 5, 7, ... $2n-5$. The internal ovals are to be looked
for only in the negative regions, that is the 1st, 3rd, 5th from the
outside, but whether the total number of regions be even or odd,
the number of internal ovals is $\frac{1}{2}(n-1)(n-2)$. The 2nd mode
of generation gives curves with circuits similarly arranged,
but the 3rd and 4th modes and combinations of the various
modes yield curves differing greatly in appearance. In
every case, however, the number of internal ovals $= \frac{1}{2}(n-1)(n-2)$.
If in both the Harnack and Hilbert processes
the restriction that the curve must have the maximum
number of branches be removed, the number of internal ovals
may be diminished, but this decrease in number cannot
be accompanied by an increase in the number
of external ovals. Hence whatever be the number of circuits
on the $2n^{ic}$ the number of external ovals cannot exceed
$n^2+\frac{1}{2}(n-1)(n-2)$.
It can be shown that there is also a maximum
limit to the number of internal ovals. The number cannot
exceed $n^2+\frac{1}{2}(n-1)(n-2)-1$, whatever the number of circuits.
Hence it follows that if the maximum number of circuits

Fig. 12 Ragsdale's Fig. 7 (Ragsdale 1906c)

nest are distributed according to a perfectly regular scheme (Fig. 7)" (Ragsdale 1906c, p. 26). Figure 7 (Fig. 12) portrayed how each annular region contains a successive even number of ovals. Ragsdale was thus able to abbreviate the general structure of her paper through the specific case of the first mode of generation that illustrated the process and challenges of her method for all four modes and their combinations.

Ragsdale began this final exposition of her research with Hilbert's 1900 "address before the Mathematical Congress in Paris"—thus making explicit the connection suggested by Scott's attendance and report. By 1906 the significance of Hilbert's Paris Problems and their solvers was already well-known to the American mathematical community. Such an impressive historical link was at once celebratory and potentially inspirational for a graduate student audience.

Rather than simply referring the Bryn Mawr mathematics department to the full article as published in the *American Journal of Mathematics*, Ragsdale re-presented her article before the Journal Club. Scott had been present in 1901, but the rest of the ephemeral Journal Club audience was new. As with her first report of the same name, Ragsdale could not rely on prior exposure to her subject matter among the graduate students present. The much briefer report, fit to be presented within an hour, may be read as an advertisement for the publication. Even though Ragsdale's published article is distinctly a product of her situation at Bryn Mawr it did not pass unchanged—or only compressed—from the front to the back.

Similarly, each review of Ragsdale's article betrayed personal readings of the piece's value. In the *Bulletin des sciences mathématiques*, Jules Tannery described her "intéressante contribution à l'étude du nombre des branches réelles" explaining how Ragsdale applied the method of Harnack and Hilbert "de maniére à avoir des renseignements précis sur les différents types de courbes qui dérivent de cette méthode" (Tannery, 1906). In the *Revue semestrielle*, Pieter Hendrik Schoute admired the "several diagrams and two beautiful plates" (Schoute, 1907). Most of the two-page review by E. Meyer in *Jahrbuch über die Fortschritte der Mathematik* is dedicated to explaining Ragsdale's "internal" and "external" designation, suggesting the potential utility of this division beyond the immediate setting (Meyer, 1909).

In 1907, Edmund Wright—who had become an associate professor of mathematics at Bryn Mawr in 1903—published "The Ovals of the Plane Sextic" in the *American Journal of Mathematics*, in which he referenced Ragsdale's recent contribution and provided a proof of Hilbert's footnote adopting a different approach tailored to this narrower topic (Wright, 1907). Ragsdale's results also circulated back to Göttingen University. Hilbert's student Grete Kahn invoked the "Amerikanerin V. Ragsdale," and cited Ragsdale's definitions of "ein Oval intern" and "ein Oval extern" (Kahn, 1909, p. 6).

Publishing Ragsdale's dissertation did not make it universal, still it was usable beyond its local environment. An important part of becoming a mathematician is not only moving "from the 'front' to the 'back,'" but also learning to recognize published mathematics as something relocatable. Perhaps this is part of what Hersh meant by developing "a less naive, more sophisticated attitude toward the myths of the profession" (Hersh, 1991, p. 132). Because when mathematics is not universal, it requires frequent reinterpretation.

5 Conclusion

In describing the Mathematical Journal Club at Bryn Mawr college as roughly equivalent to graduate seminars in mathematics, I glossed over the difference between seminars at mathematical centers like the Universities of Göttingen or Chicago, in which students could learn a wide range of mathematics directly from the faculty and other department members who were producing it. Though Scott and her colleagues were active researchers and among the most prolific writers in American mathematics, there were never more than two professors of mathematics at Bryn Mawr during the entirety of Scott's forty-year tenure. Scott built up an exceptionally robust college library given her budgetary constraints and made her personal collection available to students upon request. Yet, the availability of published texts was insufficient. The creation and perseverance of the Journal Club confirms the value of local intervention in communicating mathematics, especially for emerging mathematicians. Students learned to bring mathematics to the back where they could share their collective interpretations. Scott had first begun a mathematics club at Girton College and believed from experience that it "may do much" in creating a "mathematical culture" for students (Scott, 1894, p. 4).

The mathematical culture at Bryn Mawr is evident in Ragsdale's conflation of branches and circuits, her sustained citation to Zeuthen's work, and her interpretation of a text on "physical geography" as one on "families of curves." By identifying Hilbert's footnote as an important unanswered question in the study of circuits, Ragsdale could marshal Scott's bibliography for further sources, sustaining the initial "impulse to a mathematical investigation" initiated in rewriting Hulburt (Goldstein, 2019, p. 3). This case study demonstrates the local idiosyncrasies in moving mathematics from the front to the back and from the back to the front. The latter process is not simply getting "rid of all the loose ends" or adopting a "standard style" and the front/back axis should not be conflated with a universal/local axis in mathematics (Hersh, 1991, p. 131). Still Ragsdale's publications for distinctly different audiences attest to the difference between the front and the back of mathematics. She did not simply submit her Journal Club entry for publication nor did she paste all of her published article into the Journal Club Notebooks. The most ephemeral features of her 1901 text were those particularly adapted to the medium of a document handwritten on paper: multicolored figures, informal citations, underlined words, traces of erased text or arrows indicating late additions of certain words, the individuality of handwriting itself. Such aspects do not usually survive the constrained form of a typeset journal. By contrast, the removal of concrete numerical examples between the Notebooks and the *American Journal of Mathematics* was a decision by the author and/or editors about what mathematical content should be published. Ragsdale may have added such "illustrations" for her captive student audience, whereas the audience for her published article would be mostly professional and voluntary. Multiple local levels overlaid in rewriting a text.

Additional historical research on journal clubs and their equivalents in training mathematicians could help to clarify the extent to which literature reviews—recasting "the results of investigations into a form adapted to an assigned purpose" (Thomas, 1899, p. 97)—characterized the transition from students to researchers beyond the Mathematical Journal Club. To what extent did this training persist across other disciplines at Bryn Mawr College, American graduate training at universities, women's colleges elsewhere in the world, etc.? Such local historical practices might be compared to those in educating mathematicians today, for instance, the "list of normative dictates" Jeremy Avigad presents as "strategies that one might urge upon an aspiring young mathematician" (Avigad, 2021, p. 7388).

The Journal Club Notebooks pull back the curtain to show the back of mathematics. The diverse, subjective, local, and tentative entries in their pages dispel the "more general myths" enumerated by Hersh. But so does Ragsdale's published dissertation as well as the publications that she cites. Consider Hilbert's footnote, where he confesses to an extremely complicated investigation and leaves an interesting question unanswered. If the standard style of published mathematics today suggests the unity, objectivity, universality, and certainty of mathematics, then these are late-twentieth century (and not universal) values.

The Notebooks also demystify the process of becoming a mathematician. Scott urged her students "to advance Mathematics." In the talk Scott gave to the Mathematics Club at Girton College, she explained how following up "some special passage in your reading" could develop into research.

> Never mind if your first steps along the line thus marked out lead you only to known results; that is no reason for stopping; your next steps may take you into the unknown, and at any rate your investigations will be of value to yourselves, giving continuity and coherence to your work, and substituting for your mosaic of information an organic body of knowledge (Scott, 1894, p. 4)

Scott's description of the personal value of taking up mathematics also justified the local value of the Journal Club. Students could teach each other the process and outcomes of relocating published work. Scott knew that making mathematics could be difficult. Describing a student's progress to the Bryn Mawr College President, she acknowledged that "it is absolutely impossible to say that work, however good, on a given subject for a given time will produce *any* result" (Scott, 1906). Providing a more stable record in which such struggles could be seen is perhaps another motivation for why Scott placed the Notebooks on the library shelf,[42] Any student could glimpse texts moving between the front and back of mathematics.

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[42] The value of this strategy is supported by the National Science Foundation funded *Women Achieving Through Community Hubs in the United States (WATCH US)* which lists among their "Best Practices for Graduate Programs": "Faculty and speakers should give examples of how they struggled through difficulty. Giving students intsight into how successful matheamticians have struggled will allow them to imagine themselves succeeding through their struggles as well." (Watchus, 2019).

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