Synthese (2023) 201:28
https://doi.org/10.1007/s11229-022-04028-8

ORIGINAL RESEARCH

Ground first: against the proof-theoretic definition of ground

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Received: 3 June 2022 / Accepted: 22 December 2022 / Published online: 11 January 2023

Abstract
This paper evaluates the proof-theoretic definition of ground developed by Poggiolesi in a range of recent publications and argues that her proposed definition fails. The paper then outlines an alternative approach where logical consequence relations and the logical operations are defined in terms of ground.

Keywords Ground · Logic · Proof-theory · Definition

1 Introduction

What is the relationship between ground, proof, and consequence? In a series of recent publications[1] Francesca Poggiolesi and her collaborators have proposed that ground should be defined in terms of logical consequence and complexity, the idea being that the grounded is both a consequence of and is more complex than its grounds. Most of this paper is taken up by the negative task of refuting this account of ground. But the paper also makes a positive proposal: instead of defining ground in terms of consequence we should rather define both consequence (in general) and the logical operations (in particular) in terms of ground.

For the reader's benefit here is an overview of the paper.

Section 2 introduces notation and terminology. Section 3 then presents Poggiolesi's proposed definition of complete immediate (formal) ground. Section 4 develops the main argument against her account. It begins by observing that she has, at best, defined the grounding relation for conjunctive, disjunctive, and negated propositions. Section 4.1 then raises a general worry about how the account can be extended to accommodate

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[1] See Poggiolesi (2016a, 2016b, 2018, 2020a, 2020b, 2020c, 2020d), Poggiolesi and Francez (2021) and Rossi et al. (2021).

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other types of logically complex propositions. Supposing that this in fact can be done, Sect. 4.2 raises the philosophically fundamental objection that the enumerative nature of the resulting definition means both that it cannot capture what is *common* to distinct cases of ground and that it makes the topic neutral notion of ground be about each particular logical operation. Section 4.3 argues that certain widely accepted cases of non-logical grounding cannot be accommodated by accounts like Poggiolesi's, showing that definitions like hers have limited scope. Finally, Sect. 4.4 argues that phenomena related to the puzzles of ground show that the account cannot work even for logical ground. Section 5 sketches an alternative view about the relationship between ground and consequence. Instead of defining ground in terms of consequence and complexity, one should define consequence in terms of ground (Sect. 5.1) and one should define the logical operations in terms of their grounding profile (Sect. 5.2); this throws new light on the distinction between the various types of ground (Sect. 5.3). Finally, I indicate how one can develop a natural deduction system for explanatory inferences (Sect. 5.4). After concluding in Sect. 6, the brief Sect. 1 establishes some results about ground and consequence that are baldly stated in the main text.

2 Preliminaries

2.1 Ground as a relation

If it is the case that ϕ one may ask what *makes* this the case; or one may ask *in virtue of* what it is the case; or one may ask in what its being the case *consists*. The answer(s) to these questions gives the grounds for ϕ.

How should claims about ground be expressed? Some—for instance Fine (2012) and Dasgupta (2017)—employ a sentential operator; others take ground to be a relation between facts or propositions (see, e.g., Audi, 2012; 2010). Poggiolesi often speaks of grounding as a *metalinguistic* relation between sentences.[2] Taken literally, this view is a non-starter. It is important that one and the same grounding claim can be expressed in Greek, German, and English: otherwise how could one say that Bolzano agreed with Aristotle that it is *because* you are pale that we think truly that you are pale, and not the other way around?

If grounding is a relation at all it has to be a relation between what sentences express. Some philosophers draw a distinction between *representational* and *worldly* grounding. Here, e.g., is Correia (2017, p. 508):

> [a]suming grounding to be a relation, on a worldly conception it is natural to take the items related to be worldly items, say states of affairs or situations, whereas on a representational conception it is natural to take them to be representations, say propositions of some kind.

The difference is illustrated by self-disjunctions. Those who adopt a worldly conception of ground identify the fact that p and the fact that $p \vee p$ and thus deny that the fact that p grounds the fact that $p \vee p$; those who adopt a representational conception,

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[2] See e.g., Poggiolesi (2018, p. 1234; 2020d, pp. 29, 34–35; 2020b)

on the other hand, hold that the proposition that p grounds the proposition that $p \vee p$. Since Poggiolesi adopts a representational conception of ground[3] I therefore propose to treat grounding as a relation between propositions.[4]

Adopting this view does not tilt the playing field against Poggiolesi as long as the following claim correlating sentential and propositional grounding holds:[5]

(Correlation) The sentence S_0 grounds$_{\text{sent}}$ the sentence S_1 iff the proposition expressed by S_0 grounds$_{\text{prop}}$ the proposition expressed by S_1

I will not offer a fully worked out theory of propositions here, but given the hyperintensional nature of ground they have to be fine-grained. Moreover, they must have a quasi-syntactic structure; in particular, I assume that the notion of *substituting* an item for a constituent in a proposition makes sense. While these are substantive assumptions[6] making them does not put Poggiolesi at a disadvantage: she requires these assumptions to state her proposed definition.

To facilitate the discussion I introduce the following notation. If ϕ is a sentence, $[\phi]$ stands for the proposition expressed by ϕ. I use $p, q, r, \ldots$ as variables for propositions. I write $p \wedge q$ to stand for the proposition that results from applying $\wedge$ to p, q (and similarly for $p \vee q, \neg p, \ldots$). I use $\Gamma, \Delta, \ldots$ as variables for multisets of propositions.[7]

2.2 Full vs. complete ground

The number of distinct notions of ground that have been distinguished in the literature verges on the embarrassing. In this paper, the main notions are *full immediate ground* and *complete immediate ground*—both in their factive variety. (Non-factive notions of ground make an appearance in Sect. 5.) I write $\{p_0, p_1, \ldots\} \ll q$ to mean that $p_0, p_1, \ldots$ fully and immediately ground q. Following proof-theoretic practice, when no confusion results, I will drop the set-brackets and simply write $p_0, p_1, \ldots \ll q$. To say that $\ll$ is *factive* is to say that if $p_0, p_1, \ldots \ll q$ then q as well as each of $p_0, p_1, \ldots$ is the case. (Working with factive ground, one has to take ground to be a relation between *true* propositions.) To say that the propositions $p_0, p_1, \ldots$ *fully* ground the proposition q is to say that nothing need be added to the propositions $p_0, p_1, \ldots$ in order to have a full explanation of q. It is *immediate* in the sense that if $p_0, p_1, \ldots$ ground q then their grounding q need not be seen as mediated through their grounding some $r_0, r_1, \ldots$ that in turn ground q.

A proposition may have many distinct full immediate grounds. For instance, if both p and q are true orthodoxy has it that the proposition $p \vee q$ is grounded in each of p

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[3] This is clear from the grounding principles she endorses, but for an explicit endorsement see Poggiolesi (2020c, pp. 78–79.

[4] While I will go along with talking about representational ground, I find the distinction unhelpful. In my view, those who accept that $p \vee p$ is grounded in (and thus distinct from) p are best seen as holding that the world itself is very fine-grained (cf. Dorr, 2016, p. 77.)

[5] Schnieder (2016, p. s1343) makes a similar claim about logical consequence.

[6] For some worries see Goodman (2017) and especially Fritz Forthcoming.

[7] I follow Poggiolesi in using multisets. For present purposes nothing hinges on this; but using multisets is required if one wants—as one should—to claim that the grounds for $p \wedge p$ are p, p and not p by itself.

of and q. Moreover, immediate and mediate ground are not exclusive. For instance, p is both an immediate and a properly mediate ground of $p \vee (p \vee s)$.[8]

Poggiolesi—following Bolzano—targets rather the notion of *complete* and immediate ground. She informally elucidates this notion as follows: "those truths each of which" contributes to ground the truth C is a complete ground of C. (Poggiolesi, 2018, p. 3150). Disjunction provides a good illustration. If both p, q are true then neither p nor q are complete grounds for $p \vee q$; rather, the sole complete ground for $p \vee q$ is p, q (taken together).[9]

3 Poggiolesi's definition of ground

The most common view in the literature takes ground as a primitive. But given that many find the notion obscure[10] having a definition of the relation in unproblematic terms would be beneficial. But what sense of definition is the relevant one? Since it is the grounding relation itself that is alleged to do important work in metaphysics, one needs a definition of the relation itself; in other words, one needs a *real* definition of the grounding relation. I will thus evaluate Poggiolesi's definition as a real definition of the grounding relation.[11]

3.1 Ground and explanatory arguments

Poggiolesi's core idea is to connect ground to *explanatory arguments*, the idea being (roughly) that Γ grounds p iff there is an explanatory argument from premisses (exactly) Γ to p. Here is how Poggiolesi puts the idea[12]

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[8] The mediate grounding relation $<$ is defined from the immediate grounding relation in the natural way: it is the least relation $<$ that contains $\ll$ and is closed under the principle of Cut. That is, one requires that if $\gamma_0, \gamma_1, \ldots, \Delta \ll p$ and $\Sigma_0 < \gamma_0$, $\Sigma_1 < \gamma_1, \ldots$ then $\Sigma_0, \Sigma_1, \ldots, \Delta < p$. The partial grounding relation $\prec$ is defined as follows: $p \prec q$ iff for some Γ we have $\Gamma, p < q$.

[9] Poggiolesi is, of course, free to focus on whichever notion of ground she is interested in, but here is one reason for thinking that full ground is the more important notion. There might be cases where one can assert that a proposition has a full ground, but one cannot assert that it has a complete ground. This situation arises for those who accept intuitionistic but not classical logic. For consider the disjunction $[0 = 0] \vee p$ where p is some as of yet undecided proposition. Then $[0 = 0] \vee p$ has a full ground—*viz.* $[0 = 0]$—but one has no reason to think that it has a complete ground. For in order to determine whether it is $[0 = 0]$ or rather $\{[0 = 0], p\}$ that is the complete ground we have to be able to decide whether p. But p was chosen to be an undecided proposition. (In fact, one can show that if every true proposition has a complete ground, then every proposition is either true or false.)

[10] See e.g., Hofweber (2009), Daly (2012), Koslicki (2015), Wilson (2014), and Wilson (2016).

[11] Maybe Poggiolesi just wants to give a definition of the word "ground" or give a definition of the concept *ground*? It is unclear what metaphysical significance such definitions would have, but as will become clear her definition also fails as a nominal or a conceptual definition (see footnote 23).

[12] Such "argumentative" approaches to ground have been developed by several authors independently. Litland (2012, 2017, 2018b): develops a version of this view to account for iterated ground; Wilsch (2015a; 2015b) uses a deductive-nomological account of explanation to give a reductive account of ground. More recently, Kovacs (2022) has tried to use a "unificationist" account of explanation to account for ground. The earliest version of the view of which I am aware is Barker (2013), where the view is developed in an expressivist manner.

> [G]rounding is a special sort of inference relation. Indeed, just as for a logic L there are *inference* rules of the form $\frac{A_1, \ldots, A_n}{B}$ that we can read as "from the premisses $A_1, \ldots, A_n$ we can infer that a certain conclusion B is true", there are also *grounding rules* of the form $\begin{array}{c} C_1, \ldots, C_m \\ \hline\hline D \end{array}$ read as: "the premisses $C_1, \ldots, C_m$ are the grounds for or the reasons why the conclusion D is true." (Poggiolesi, 2020c, p. 71)

Some terminology will be helpful. An explanatory *inference* from Γ to p corresponds to Γ's immediately grounding p; an explanatory *argument* from Γ to p corresponds to Γ's mediately grounding p.

As noted above p only *completely* grounds $p \vee q$ if q is not true. Since Poggiolesi wants to capture complete ground, she is interested in characterizing what it is to be an explanatory inference from Γ to p *on condition* C. (Example: the inference from p to $p \vee q$ is explanatory only on condition that $\neg q$.) I will write $\Gamma[C] \Vdash q$ to mean that q can be explanatorily inferred from Γ on condition C. Since the phrase "q may be explanatorily inferred from Γ on condition C" is unwieldy, I will typically write "Γ immediately and completely grounds p on condition C" for $\Gamma[C] \Vdash q$.[13]

3.2 Defining complete ground

I use $\vdash$ for the derivability relation of classical logic; when Γ is a (multi)set of propositions I write $\neg(\Gamma)$ for $\{\neg\gamma : \gamma \in \Gamma\}$.[14] Poggiolesi then proposes the following definition of complete immediate ground.

Definition 3.1 Γ completely and fully grounds p on condition C (in the present notation: $\Gamma[C] \Vdash p$) iff

Positive Derivability $\Gamma \vdash p$;

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[13] In saying that $\Gamma[C] \Vdash p$ one has not yet introduced a *proposition* the obtaining of which ensures that Γ grounds p on condition C. (Compare: in defining what it is for the proposition q to follow from the proposition p one has not yet made sense of the strict conditional proposition $p \Rightarrow q$. Like Litland (but unlike Wilsch) Poggiolesi (2018, p. 1234) goes on to introduce an operator $\rhd$ such that if $\Gamma \Vdash p$ and Γ is the case, then one can infer $\Gamma \rhd p$. This is similar to the proposal of Litland (2017; 2018b). According to Litland if $\mathcal{E}$ is an explanatory argument from premisses exactly Γ to conclusion p, then the argument that continues by discharging all of Γ and concluding $\Gamma \rhd p$ is itself explanatory. One can then apply the $\rhd$-introduction rule again to conclude that $\Gamma \rhd p$ is *zero-grounded* (in the sense of Fine, 2012, pp. 47–48) thus providing an answer to the question what grounds ground. (It is worth pointing out that Scott (1971) deployed a similar idea to introduce an object-language strict conditional assuming an understanding of conditional assertion.) Unlike Litland, Poggiolesi does not take arguments that end with $\Gamma \rhd \phi$ to be themselves explanatory. This means that Poggiolesi's view is incomplete in a way Litland's is not. For by saying how $\Gamma \rhd p$ is grounded Litland purports to give a complete account of the nature of the operation $\rhd$. (It is that operation such that the propositions formed by applying it to Γ, p are zero-grounded iff there is an explanatory argument from Γ to p.) Poggiolesi, on the hand, does not say enough about $\rhd$ to settle its nature. Of course, if Litland's view is wrong this is advantage Poggiolesi. (Litland's view is subject to many of the same objections as the views of Bennett (2011, 2017) and deRosset (2013); for objections to those views see, e.g., Dasgupta (2014, 2019), Sider (2020), Thompson (2019), Carnino (2016).) In any case Poggiolesi is left with a challenge: without an account of iterated ground she has not defined the grounding operation.

[14] Throughout I will take logical consequence to be a relation between propositions and explanatory inferences to be inferences from propositions to propositions. (After all, both Aristotle and the Bolzano can explanatorily infer that it is true that Socrates is pale from the premiss that Socrates is pale.)

Negative Derivability $\neg(\Gamma), C \vdash \neg p$; and
Complexity Γ, C is immediately less g-complex than p.

The requirement of Negative Derivability captures *complete* grounding. Even if p is true it only grounds $p \vee q$ *on condition* that $\neg q$. This is captured by the fact that conditional on $\neg q$, $\neg(p \vee q)$ is a consequence of $\neg p$. [15]

The definition of ground thus relies both on the (a?) notion of derivability, and the notion of g-complexity. Both are problematic, but especially the latter.

3.3 Defining g-complexity

In many cases the grounds seem "less complex" than what they ground. But—as Poggiolesi (2016b, pp. 3152–3153) conclusively demonstrates—standard logical measures of complexity will not yield the right results. She therefore develops the new notion of *g-complexity*.

In Poggiolesi (2016b, 2018) this notion is defined for propositions formed using the logical operations $\neg, \wedge, \vee$.[16] First some preliminaries: if ϕ is a proposition, the *converse* of ϕ is $\neg^{n-1}\psi$ if ϕ is $\neg^{n}\psi$ and n is odd and it is $\neg^{n+1}\psi$ if n is even. Following Poggiolesi write ϕ^* for the converse of ϕ. Some examples: $(\neg\neg\neg p)^*$ is $\neg\neg p$. $(\neg p)^*$ is p, and p^* is $\neg p$.

According to Poggiolesi the grounds for $p \wedge (q \wedge r)$ are the same as the grounds for $(q \wedge p) \wedge r$. (And similarly, for disjunction.)[17] To ensure this she introduces the relation of associative-commutative equivalence. This is the relation $\cong$ that holds between two propositions p, q if q can be obtained from p by repeated application of the commutativity and associativity of conjunction and disjunction.

The definition of g-complexity is then:

> [A] multiset M of propositions is *completely and immediately less g-complex* than a proposition C, if, and only if:
>
> - $C \cong \neg\neg B$ and, $M = \{B\}$ or $M = \{B^*\}$; or
> - $C \cong (B \circ D)$ and, $M = \{B, D\}$ or $M = \{B^*, D\}$ or $M = \{B, D^*\}$ or $M = \{B^*, D^*\}$.
>
> (Poggiolesi, 2016b, p. 3158, Definition 4.8); similarly Poggiolesi, 2018, p. 1238, Definition 3.6)

An example might be helpful. (In what follows p, q are assumed to be atomic.) The (multi)sets that are immediately less g-complex than $\neg p \wedge \neg\neg q$ are $\{\neg p, \neg\neg q\}$, $\{\neg p, \neg\neg\neg q\}$, $\{p, \neg\neg q\}$, $\{p, \neg\neg\neg q\}$. Note how $\neg p$ and $\neg\neg q$ are treated differently

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[15] I should point out that none of the criticisms in the paper turn on Negative Derivability—though see footnote 9.

[16] In Poggiolesi (2020b, d) the definition is extended to relevant implication; in Poggiolesi and Francez (2021) the definition is extended to exclusive as well as to ternary disjunction; and in Rossi et al. (2021) it is extended to the quantifiers.

[17] Poggiolesi (2016b, pp. 3156–3157) justifies this on the grounds that $p \wedge (q \wedge r)$ and $(q \wedge p) \wedge r$ concern the same issue or have the same subject matter. For reasons given by Krämer (2019, pp. 1665–1667) I am not persuaded by Poggiolesi's argument, but my criticism of Poggiolesi will not turn on this.

by the converse operator *. (For further examples, especially ones involving associate-commutative equivalence, see Poggiolesi, 2016b, p. 3158.)

4 Against definitional adequacy

It might be useful to begin with a poor objection. Poggiolesi thinks that the grounds for $p \wedge q$ are exactly p, q and the grounds for $\neg(p \wedge q)$ is whichever one of $\neg p$, $\neg q$ is true if only one is true, and $\neg p$, $\neg q$ together if both are true. Clearly, the definition of g-complexity is just engineered to get these results about complete immediate ground. It would, however, be a mistake to object that this renders the definition circular. The definition of g-complexity is given purely in terms of the structure of the propositions, with no mention of ground. It is of course true that one's inchoate understanding of ground helps one single out one of the many possible definitions of complexity; but this does not render the definition itself circular.

But while the structural nature of the definition of g-complexity saves it from circularity, it is this structural nature that dooms it as a definition. For this means that the definition of "being immediately less g-complex than"—and thus the definition of immediate complete ground—is given just for the propositions formed using the operations $\wedge$, $\vee$, $\neg$. One thus—*at best*—gets a definition of ground for propositions formed using $\wedge$, $\vee$ and $\neg$, and not a definition of ground for arbitrary propositions.

This basic observation can be developed into a technical challenge and a philosophical objection.

4.1 The conservativity challenge

One may worry about the *conservativity* of the definition of immediate complete ground. This worry starts with the observation (see footnote 16) that once one considers further logical operations the definition of g-complexity has to be given anew. Consider collections of logical operations $\mathcal{C}_0 \subseteq \mathcal{C}_1$ and suppose one has defined complete immediate ground for $\mathcal{C}_0$ and for $\mathcal{C}_1$. Call these grounding relations $\Vdash_0$ and $\Vdash_1$. Suppose Γ, p are formed using just the operations in $\mathcal{C}_0$. A failure of conservativity would take the form of having $\Gamma \Vdash_1 p$ but not $\Gamma \Vdash_0 p$.

Poggiolesi (2020b, Theorem, 3. 12) address this worry when she proves that if one extends $\wedge$, $\vee$, $\neg$ with a (relevant) conditional, then one can establish the relevant conservativity result. However, this just shows that things work out in this case: one would like some evidence that every case works out. Ideally, one would like to find some properties such that adding any operations with those properties yield a conservative extension of the grounding relation.

Stating the relevant conditions and establishing such a general conservativity result seems to me to be the most pressing technical challenge for Poggiolesi's program.

4.2 The commonality and specificity objections

Modulo having the conservativity result one could say that what Poggiolesi has defined is just the fragment of the grounding relation that concerns propositions formed using just $\neg$, $\vee$, $\wedge$; the grounding relation itself is what one obtains when one has defined g-complexity for all logical operations. The grounding relation $\Vdash$ is, as it were, the "limit" of $\Vdash_{\mathcal{C}}$ as the collection of logical operations is expanded. However, while this might define the *extension* of the grounding relation, it would not be a definition of ground itself. The problem is that the resulting definition would be a mere list. To appreciate the problem the diagnosis of list-like definitions in Rosen (2015) will be helpful.[18]

The prime numbers are 2, 3, 5, 7, 11, ... But as Rosen points out it would be absurd to *define* the property of being a prime number as the property of being either 2 or 3 or 5 or 7 or 11, ... And this is for two reasons. First, the definition itself would not show what the primes have in common, what makes them prime. (Of course, the primes all have the property of either being 2 or 3 or 5 or ...; but clearly such "Cambridge-commonalities" are not enough to show that the primes have something interesting in common.) Second, the definition is "overly specific": while the property of being a prime is had by specific numbers, the property of being a prime is ontologically independent of any particular number.[19]

Poggiolesi's definition of ground suffers from the same two flaws. First, it fails to reveal what is common to all the cases of ground. Second, it is overly specific in that it makes the topic neutral, wholly general notion of *because* be about particular logical operations like conjunction, disjunction, and negation.

Does the objection prove too much? Does it rule out all reductive accounts of ground? It does not. It might be instructive to consider three existing views that are left unscathed by the objection.

Correia (2018) has developed a reductive account of (mediate) ground that has some similarities with Poggiolesi's. According to Correia, Γ grounds p iff Γ metaphysically entails p and Γ is relatively more fundamental than p. The relation of relative fundamentality is, however, not defined, but is rather taken as a primitive.[20]

Wilsch (2015a, 2015b) has developed a reductive deductive-nomological account of ground where Γ grounds p iff p can be derived* from Γ together with just the metaphysical laws, where the metaphysical laws are (expressed by) certain universally quantified conditionals.[21]

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[18] For the seminal criticism of list-like definitions, see Field (1972, pp. 362–363).

[19] Contrast: a haecceitistic property like *being identical to Socrates* is not just had by the man himself, it depends for its nature—and arguably: existence—on him.

[20] Poggiolesi could do something similar, taking the notion g-complexity as primitive, holding that we have an intuitive grip on the notion. If one takes that view, one no longer has a *definition* of g-complexity; rather, what Poggiolesi offers is a substantive thesis about what is less complex than what. There is, however, no textual evidence that this is how Poggiolesi views the matter. In any case, as argued in Sects. 4.3 and 4.4, complexity-based accounts cannot work.

[21] In order to avoid triviality, in a derivation* one can only use the rules of universal instantiation and *modus ponens*.

Finally, the account proposed by Litland (2017, 2018b) takes as basic certain (rules of) explanatory inference; explanatory arguments are the arguments that result from composing such explanatory inferences. While this view is non-reductive about explanatory inference it is reductive about *ground*: for Litland Γ grounds p iff Γ is the case and there is an explanatory argument from Γ to p.

If we take the relative fundamentality relation (Correia), the laws (Wilsch), or the explanatory inferences (Litland) to be defined by *enumeration* the commonality and specificity objections apply. But as long as Correia holds that the relative fundamentality relation is not given by enumeration, but rather is general or qualitative the objection fails.[22] Similarly, Wilsch and Litland avoid the objection as long as they do not take the laws (explanatory inferences) to be given by enumeration. Litland could, e.g., say that there is a general property $\mathcal{E}$ of being an explanatory inference. For Γ to immediately ground p is for there to exist some inferences $I_0, I_1, \ldots$ such that each I_i falls under $\mathcal{E}$ and one can derive p from premisses Γ using just the inferences $I_0, I_1, \ldots$. Wilsch could, e.g., say that there is a general property $\mathcal{L}$ of being a law governing ground and for Γ to ground p is for there to be some $L_0, L_1, \ldots$ such that each of the L_i fall under $\mathcal{L}$ and such that p is derivable* from Γ using just $L_0, L_1, \ldots$ as auxiliary premisses.

This suggests a way forward for the proof-theoretic definition of ground: one must be able to define g-complexity in general terms, without mentioning any particular logical operations. Future efforts should be directed at giving such a general definition rather than dealing with the logical operations one by one.[23]

However, there are strong reasons for thinking that no such account can be developed. Many standard cases of ground involve no increase in complexity from the grounds to the grounded. I first discuss cases of "non-logical" ground (Sect. 4.3) before I take up cases turning on the so-called "puzzles of ground" (Sect. 4.4).

4.3 Non-logical ground

It is widely assumed that facts about determinates ground facts about determinables. For instance, that the vase is crimson grounds that it is red. Poggiolesi and Genco (forthcoming) attempts to treat this as a case of *conceptual ground*. Their idea is that in cases of conceptual ground the transition between the grounds and the grounded is underwritten by a *definition* of the grounded. Adopting an inferential understanding of definition, they hold that definitions are given by introduction rules. For instance, they propose that the definition of *being a bachelor* should be understood as being given by the following introduction rule:

$$\frac{x \text{ is unmarried} \qquad x \text{ is a man}}{x \text{ is a bachelor}}\ \text{bachelor-introduction}$$

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[22] What does it mean to say that the relative fundamentality relation is general or qualitative? We can spell this out in essentialist terms as follows: there are no propositions Γ and no proposition p such that it is essential to the relative fundamentality relation that it holds between Γ and p.

[23] I have taken Poggiolesi to have attempted to give a real definition of the grounding relation. The commonality and specificity objections are even stronger when directed against nominal or conceptual definitions: after all, somebody can understand the word "ground" or grasp the concept *ground* without having a word (or concept) for any particular logical operation.

This looks quite similar to Litland’s view (which they do not discuss) but a crucial difference is that they do not take the rules to be *given as explanatory*; rather, it is only when the rules give us an increase in“conceptual complexity” from the premisses to the conclusion that we have a case of conceptual grounding.

But how is this supposed to work? *Being unmarried*, *being a man*, and *being a bachelor* all look like atomic properties. Their idea is that *being a bachelor* is covertly complex, being defined in terms of *being a man* and *being unmarried*. Maybe this works in the case of *being a bachelor*, but how is *red* definable in terms of *crimson*? Here is what Genco and Poggiolesi say.

> the color red can be defined as the set of all types of red—crimson, scarlet, . . . — and hence can be seen as composed of them. In this case, the color red will count as more complex than the color crimson. (Poggiolesi and Genco forthcoming)

This is not promising for a number of reasons.

First, and pedantically, colors are not *sets* of colors; presumably what is meant is that *being red* is defined as the disjunctive property *being either crimson, or scarlet or burgundy or* . . . .

Second, this is a controversial view of color As Rosen (2010, pp. 128–129) points out, it is arguable that someone can know the nature of the color red without knowing each of the infinitely many determinate shades of red. This speaks against any disjunctivist account of the colors.[24]

Third, the structure of determinates and determinables gives rise to a special problem for complexity-based accounts. These are arguably cases of *dense* grounding, where if p grounds q there is always an r such that p grounds r and r grounds q. (Between *crimson* and *red* there are intermediate shades of red that are less specific than crimson but more specific than red.)[25] The proof-theoretic complexity measures developed so far all yield non-dense measures of complexity; it is not clear that any proof-theoretic account can be generalized to such cases.

Fourth, the determinate/determinable case illustrates a wider phenomenon. There are arguably many cases where we have conceptual grounding but where the grounded and the grounds are not definable in terms of each other. (For further examples and discussion see Chalmers (2012, pp. 452–460).) That we cannot *define* the grounded in terms of the grounds is not say that the connection between the grounds and the grounded is a mystery. Following Chalmers one might think it is a priori scrutable that the relationship of ground holds or one might think it lies in the essence of the grounded and the grounds that the relationship holds, without accepting that these essences can be expanded to (real) definitions.[26]

The relationship between determinates and determinables is not the only case of non-logical ground; another standard example is that the existence of a set is grounded in the existence of its members. Complexity-based views of ground run into serious problems accounting for this case.

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[24] This point creates a problem for Litland too. The explanatory rules cannot take the form $\frac{x\text{is crimson}}{x\text{is red}}$. Rather, the rules have to take a more general form like

[25] For more about dense grounding see Werner (2020) and Clark (2018).

[26] For more on such partial essences, see Dasgupta (2015, pp. 460–463).

Let x be Socrates and y be his singleton, and consider the propositions $[Ex]$ and $[Ey]$. How can one ensure that $[Ex]$ grounds $[Ey]$? Suppose one grants that $[Ey]$ is a consequence of $[Ex]$; in what sense is $[Ey]$ more complex that $[Ex]$? *Qua* propositions they look equally complicated: they are both simply attributions of existence to an object. (Of course $[Ey]$ is more complex than $[Ex]$ precisely in that the former is grounded in the latter, but someone who wants to define ground cannot leave it at that.)

It might be instructive to consider a failed attempt at accounting for the complexity. Consider the *sentences* "Socrates exists" and "{Socrates} exists". The sentences plausibly differ in complexity: "{Socrates}" is a complex *term*, while "Socrates" is a simple one. Authors like Glazier (2016, pp. 21, 28-31) and Donaldson (2017, pp. 783-784) have suggested that there is an analogous distinction at the propositional level. Whereas the proposition $[Ey]$ contains just the object {Socrates}, the proposition $[E\{x\}]$ contains a *complex* built up out of Socrates and the operation of set-formation, but it does not contain {Socrates}; rather, {Socrates} is the *value* of this complex.[27]

If such a distinction can be made out, there is a fairly clear sense in which $[E\{x\}]$ is more complex $[Ex]$. One would then have a complexity-based explanation of how $[Ex]$ grounds $[E\{x\}]$. Can one turn this into an account of how $[Ex]$ grounds $[Ey]$? The natural idea is to say that $[Ex]$ grounds $[Ey]$ because $[Ex]$ grounds $[E\{x\}]$ and y is the value of $\{x\}$.

However, this will not work. For Socrates is the value of the complex expressed by "the member of the member of {{Socrates}}"; and since the complex expressed by "the member of the member of {{Socrates}}" is more complex than Socrates, the proposal has the absurd consequence that the existence of Socrates is grounded in the existence of {Socrates}!

The problem is, of course, reminiscent of Quine's objections to the possibility of quantifying into modal contexts. The solution to the problem has to be to allow only certain complexes whose values are x and y. But on what basis should one let some complexes in and keep others out? Complexity appears to provide no guide.[28]

4.4 Internality and the puzzles of ground

The above objections turned on cases where we have grounding but we do not have an increase in complexity. Maybe one could set these cases aside as being outside

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[27] There is an underexplored connection here to the early Russell's views on denoting concepts (Russell, 1982).

[28] An anonymous referee suggested that a possible solution would be to insist that the complexes be *super-rigid* in a sense analogous to how an expression is super-rigid (Chalmers, 2012). For Chalmers an expression is *super-rigid* if it has the same referent in every epistemically possible scenario and every metaphysically possible world. But this will not work: for replace Socrates in the above example with ∅. Then both the complex denoted by "{∅}" and the complex denoted by "the member of the member of {{∅}}" are super-rigid, and one has the same problem as above.

the scope of the proposal.[29] However, the complexity based accounts also runs into problems when there is an increase in complexity.

Let T be the propositional truth-predicate. The following is a natural principle about propositional truth.

(Aristotle's Principle) If Tp, then $p \ll [Tp]$

Poggiolesi wants ground to work as follows.

(Irreflexivity) The relation of mediate partial ground is irreflexive

($\vee$-grounding) True disjunctions are grounded in the true disjunct or in both disjuncts together

A complexity-based account of ground will hold that whether p, q grounds $p \vee q$ turns only on the complexity of p, q and $p \vee q$ and whether p, q are both true. Thus the grounding of disjunctions has to satisfy the following restricted Internality principle:

($\vee$-Internality) $p, q < p \vee q \rightarrow \Box(p \wedge q \wedge p \vee q \rightarrow p, q < p \vee q)$

However, a straightforward variation of the counterexample to Internality given in Litland (2015, pp. 489–491) shows that (Irreflexivity), ($\vee$-grounding), ($\vee$-Internality), and (Aristotle's Principle) are jointly inconsistent.[30] There is, of course, no consensus about how to respond to inconsistencies like this and I do not wish to promote a particular resolution here. However, I wish to argue that complexity-based accounts like Poggiolesi's face particularly severe problems.

Since giving up ($\vee$-Internality) amounts to giving up a complexity-based account set that option aside.[31] The options are thus to reject either (Irreflexivity), ($\vee$-grounding), or (Aristotle's Principle). There are principled ways rejecting ($\vee$-grounding). deRosset (2021) does so by rejecting the (standard) notion of immediate ground; Lovett (2019, 2020) does so by adopting a more coarse-grained conception of proposition. Woods (2018) and Correia (2014) develop accounts that reject (Irreflexivity). But given the structure of her view this is not an option for Poggiolesi, and so she has to reject (Aristotle's Principle).

A principled way of doing this is to reject that there is an untyped notion of propositional truth and rather adopt the predicativist line developed by Korbmacher (2018b, 2018a) where propositions and truth-properties come in orders where a truth-property of a given order can only be applied to a propositions of lower orders. However, such a predicativist line is unlikely to sit well with ($\vee$-Internality). The problem is that the prevalence of empirical and contingent self-reference—one of the main morals of

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[29] It is worth noting that other argumentative approaches have no problems with these cases: Wilsch's account will simply posit laws governing determinates, determinables, and set existence and Litland will simply take the relevant rules to be *given* as explanatory. If Poggiolesi's proposal cannot deal with non-logical ground this is a significant drawback of her account.

[30] In fact, the inconsistency persists if we drop (Aristotle's Principle) and hold that that Tp is identical to p.

[31] Litland adopts this option and develops a view where the hierarchy of ground is not fixed independently of how things contingently are; rather, the grounds have to find their own levels depending on how things contingently stand (for details see Litland, 2020). (For even more radical rejections of Internality, see Skiles (2015), Leuenberger (2013), and Baron-Schmitt (2021).)

Kripke's seminal 1975—makes it hard to see how it can be an internal matter what the order of a given proposition is.

This argument is not decisive; there might be a way of assigning orders that saves the account. However, the existing measures of complexity do not provide any clue about how such an assignment of orders is supposed to work.[32]

5 Ground and consequence

While I have argued that Poggiolesi's attempt at defining ground in terms of logical consequence fails, I should stress that Poggiolesi attempts to answer an important question: when p grounds q and q is a logical consequence of p one would like to have an explanation of how the fact that p grounds q relates to the fact that q is a consequence of p. But rather than using logical consequence to define ground I propose that the correct order of explanation uses ground to define logical consequence.

I should stress that the resulting notion of consequence need not be seen as in conflict with other notions of consequence like *(necessary) truth-preservation in virtue of form*. Such accounts can be seen as special cases of the present account; and for many purposes those accounts will serve perfectly well. But, as I hope to indicate, there are purposes for which something like the ground-theoretic account of consequence and the logical operations is required.[33]

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[32] While the goal of this paper is not to settle the debate about the so-called "puzzles of ground" (Fine, 2010) I should say something about how Rossi et al. (2021) attempt to deal with the puzzles. Some of the puzzles turn on the standard views of grounds for existential and universal generalizations. Rossi et al. (2021) respond to these puzzles by giving a different account of the grounds for existentially and universally quantified propositions. Their approach is a technical success, but philosophically the view is so underdeveloped that it is no solution at all.

To formulate the grounds for existential propositions they employ Hilbert's ϵ-calculus. When $F(x)$ is a formula that is satisfied by some object then $(\epsilon x Fx)$ "is a name for an indeterminate object satisfying $F(x)$" (Rossi et al., 2021, p. 1422). They then propose that the ground for $[\exists x Fx]$ is $[F(\epsilon x Fx)]$. For the case of universally quantified propositions they employ (a simplified version of) Fine's theory of arbitrary objects (Fine, 1985). An arbitrary object $\mathfrak{a}$ is a particular object that has as its *values* all objects (including itself). They then propose that the ground for $[\forall x Fx]$ is $[F(\mathfrak{a})]$.

I have no quarrel with arbitrary objects; and if arbitrary objects are accepted, there is no problem interpreting ϵ-terms: interpret $\epsilon x Fx$ as standing for the restricted arbitrary object that has as its values all the Fs—if there are Fs—and interpret $\epsilon x Fx$ as the universal arbitrary object if there are no Fs. The problem is that the authors say nothing about what grounds propositions involving arbitrary objects (or propositions involving ϵ-terms). If nothing is said about the grounds for $[F(\epsilon x Fx)]$, no wonder that no contradiction can be derived from holding that the grounds of $[\exists x Fx]$ is exactly $[F(\epsilon x Fx)]$. (And similarly, for the claim that $[F(\mathfrak{a})]$ grounds $[\forall x Fx]$.)

The natural view about the grounds for $[F(\mathfrak{a})]$ is that the grounds are $[F(a_0)]$, $[F(a_1), \ldots]$ taken together—where a_0, a_1 are the values of $\mathfrak{a}$. (Possibly, one also wants a totality proposition to the effect that $a_0, a_1, \ldots$ are *all* the values of $\mathfrak{a}$.) But this obviously reinstates the puzzles. It is not an option to hold that $[F(\mathfrak{a})]$ is ungrounded. For $[F(\mathfrak{a})]$ entails each instance $[F(a_0)]$, $[F(a_1)]$, ..., and so there are necessary connections between distinct propositions. Such necessary connections cry out for explanation. One tempting explanation is in terms of having common grounds, but if $[F(\mathfrak{a})]$ is ungrounded, no such explanation can be given.

Unless, more is said about the grounds for propositions involving arbitrary objects and ϵ-terms, the verdict must be that their proposal amounts to no more than simply postulating that true universally and existentially generalized propositions have immediate (complete) grounds that do not lead to paradox.

[33] I should stress that the idea of defining logical consequence in terms of ground is not novel here—related ideas are found in Schnieder (2018) and Correia (2014)—but the idea is developed differently here.

5.1 Defining consequence

In this section ground will mean mediate non-factive ground. Before one can define the notion of logical consequence the notion of material consequence should be defined. A set of propositions is *ground-closed* iff every proposition grounded by some propositions in the set is itself in the set. (Formally, Γ is ground-closed if whenever Γ_0 grounds p for some $\Gamma_0 \subseteq \Gamma$, then p is in Γ.) Call a set of propositions *ground-prime* if whenever some propositions in the set have a common full ground, then they have a common full ground in the set. (Formally, Γ is ground-prime iff for all $\Delta \subseteq \Gamma$ if there is Σ such that Σ grounds δ for each $\delta \in \Delta$, then there is $\Sigma' \subseteq \Gamma$ such that Σ' grounds δ for each $\delta \in \Delta$.)[34] One then defines p to be a *material consequence* of Γ iff for all prime, closed sets of propositions Γ^+ if $\Gamma \subseteq \Gamma^+$, then $p \in \Gamma^+$. It is helpful to think of a closed, prime set of propositions as giving a fully determinate specification of how the propositions in that set obtain; for p to be a material consequence of Γ then is for p to be included in any fully determinate specification of how the propositions Γ obtain.

To illustrate how this works one needs to know how propositions formed using $\wedge, \vee, \neg$ are fully immediately grounded. To state this account it is useful to face up to a constant embarrassment in the theory of ground: what to say about the grounds for negations? It is commonly observed that there seems to be no way of characterizing the grounds for the negation of a proposition p in terms of the grounds for p. (This is particularly clear in the case where p is ungrounded.) Truthmaker theorists face the same problem and typically respond by going "bilateral": propositions are assigned both truthmakers and falsemakers (Fine, 2017a, 2017b).

I believe the grounding theorist should adopt a similar approach. In addition to the notion of ground one must help oneself to the notion of (immediate) *antiground*, where the immediate antigrounds for a proposition p are the immediate grounds for the negation of p (cf. Litland, 2022). Think of the antigrounds for p as those propositions the obtaining of which excludes p's being the case; an illustration from outside of logic might be that an object's having a fully determinate shade of color excludes its having any other fully determinate shade of color.

Using $\ll$ for the relation of immediate full ground and $\gg$ for the relation of immediate full antiground, I propose the following account of how propositions formed using just $\wedge, \vee, \neg$ are (anti)grounded:

Definition 5.1 Immediate grounds for conjunctions, disjunctions, and negations

($\neg_{\ll}$) $\Gamma \ll \neg p$ iff $\Gamma \gg p$;
($\neg_{\gg}$) $\Gamma \gg \neg p$ iff $\Gamma = \{p\}$;
($\wedge_{\ll}$) $\Gamma \ll p \wedge q$ iff $\Gamma = \{p, q\}$;
($\wedge_{\gg}$) $\Gamma \gg p \wedge q$ iff $\Gamma = \{\neg p\}$ or $\Gamma = \{\neg q\}$;
($\vee_{\ll}$) $\Gamma \ll p \vee q$ iff $\Gamma = \{p\}$ or $\Gamma = \{q\}$;
($\vee_{\gg}$) $\Gamma \gg p \vee q$ iff $\Gamma = \{\neg p, \neg q\}$

These clauses are, of course, very similar to the truthmaker theorist's clauses for truth- and falsemaking. Note, however, that in standard truthmaker theory the falsemakers for

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[34] From now on I simply write "prime" and "closed" for "ground-prime" and "ground-closed".

$\neg p$ are the truthmakers for p, and thus the proposition p is identical to the proposition $\neg\neg p$.[35] On the present account, in contrast, the sole antiground for $\neg p$ is p, and thus p and $\neg\neg p$ will be distinct.

To illustrate the definition of material consequence one can show that q is a material consequence of $(p \wedge q) \vee (r \wedge q)$. Any prime, closed set of propositions Γ containing $(p \wedge q) \vee (r \wedge q)$ contains either $p \wedge q$ or $r \wedge q$. In the former case we have $\{p, q\} \subseteq \Gamma$; in the latter case we have $\{r, q\} \subseteq \Gamma$. In either case we have $q \in \Gamma$. This shows that q is a material consequence of $(p \wedge q) \vee (r \wedge q)$.

One does not yet have a *formal* notion of consequence. Consider the propositions [the vase is red] and [Trump lost reelection]. The proposition [something is colored] is a material consequence of [the vase is red] $\wedge$ [Trump lost reelection]. To obtain a formal consequence relation one proceeds in the standard way. Say that p is a formal grounding consequence of the propositions Γ if whenever Γ^+, p^+ are some propositions that result from Γ, p by a uniform substitution that leaves $\wedge, \vee, \neg$ fixed then p^+ is a material consequence of Γ^+.[36]

Does this notion of grounding consequence coincide with a familiar consequence relation? It does. By adopting some ideas due to Correia (2014) one can show that it coincides with the consequence relation of First Degree Entailment. (The details are relegated to Sect. 1.)

However, there are natural variations of grounding consequence that coincide with other consequence relations. Say that a set of propositions Γ is *coherent* if for no proposition p both p and $\neg p$ are in Γ. Say that p is a *coherent material consequence* of Γ iff for any coherent, prime, closed set of propositions Σ, if $\Gamma \subseteq \Sigma$ then $p \in \Sigma$. Obviously, many sets of propositions have incoherent non-factive grounds—consider, e.g $\{p \wedge q, \neg p \vee r\}$. However, for non-dialetheists only the coherent grounds of some propositions can *obtain*. Coherent material consequence is thus tied to *factive* ground as follows: for p to be a coherent material consequence of Γ is for any factive grounds for Γ to have some factive grounds that contain grounds for p. One defines formal consequence in terms of substitutions, and one can then show that p is a coherent formal consequence of Γ iff p is a strong Kleene consequence of Γ.

A set of propositions Γ is *complete* iff for every proposition p, either p or $\neg p$ is in Γ. We then say that q is a complete material consequence of Γ iff for every complete, closed Σ such that $\Gamma \subseteq \Sigma$ we have $p \in \Sigma$. Complete material consequence is the appropriate relation of consequence if one thinks that for every proposition either it or its negation is factively grounded. Defining formal complete consequence in the obvious way, one can then show that p is a complete consequence of Γ iff p follows from Γ in the Logic of Paradox.

Finally, say that p is a classical material consequence of Γ iff whenever Σ is coherent, closed, and complete and $\Gamma \subseteq \Sigma$, then $p \in \Sigma$. Classical material consequence is the appropriate notion of consequence if one thinks that for each proposition p exactly one of p and $\neg p$ is factively grounded. Defining formal classical consequence in the

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[35] For a non-standard truthmaker theory that avoids this consequence see Krämer (2018, 2019).

[36] The attentive reader will not have missed the connection with a broadly Tarskian account of logical consequence. The Tarskian defines p to be a formal consequence of Γ if for any substitution instances Γ^*, p^* that leave $\wedge, \vee, \neg$ fixed: whenever Γ^* is *true*, then p^* is *true*.

obvious way one can show that p is a classical consequence of Γ iff p follows from Γ in classical logic.

5.2 Defining operations

These claims about grounding consequence requires Definition 5.1. However, one does not need to *assume* that this is how disjunctions, conjunctions, and negations are immediately grounded. A natural view is that the logical operations are defined in terms of how propositions formed by applying them are (anti)grounded, the rough idea being that what makes a binary propositional operation R the conjunction operation is that the proposition Rpq is grounded in exactly p, q (taken together), and antigrounded in each of the negations of p and q. In contrast, what makes a binary operation R the disjunction operation is that Rpq is grounded in each of p and q; and antigrounded in $\neg p, \neg q$ (taken together). One might say that logical operations are individuated by their "Grounding Profile"—the contribution they make to how propositions formed using them are (anti)grounded. On this view the principles in Definition 5.1 are not substantive assumptions about conjunction, disjunction, negation, but are rather definitional truths about some operations.

Here are three considerations in favor of this view.

First, one might worry about what makes it the case that the logical operations have the grounding profiles they do. The *cognoscenti* may, for instance, have noted that the grounding profile $\vee_{\ll}$ omits the "amalgamating" case, of p, q together grounding $p \vee q$. What could determine whether disjunction is amalgamating? A reasonable response is to adopt a *plenitudinist* view about the logical operations: any (coherent) grounding profile gives rise to a distinct logical operation. There are thus simply two different disjunction operations. One where the disjunction of p, q is grounded in whichever one of p, and q is the case and another where the disjunction of p, q is grounded also in p, q, if they are both the case.[37]

Second, McSweeney (2020) has recently objected that our intuitions about cases of logical ground—for instance, that a proposition grounds its double negation—-can be explained away as really being intuitions about "meaning-determination" or "truth-determination". She argues that while there is reason to think that the truth value (meaning) of a disjunction is determined by the truth values (meanings) of the disjuncts there is no reason to think that disjunctions are *metaphysically* grounded in their disjuncts.[38] However, on the above plenitudinist line there is *some* operation such that propositions formed using it are metaphysically grounded in each of the propositions from which it is formed.

Third, unlike Poggiolesi's view the present view does not have to give a new definition of the grounding relation once on considers further logical operations. One simply has to specify grounding profiles for those further operations. The possibility of defining logical operations by specifying grounding profiles—as opposed to by specifying truth conditions—opens up a range of novel possibilities. While this is not

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[37] And there are more than *two*: there will also be a Poggiolesi-disjunction which is grounded in whichever of p, q is true if only one of them is true; and otherwise is grounded in both (taken together).

[38] Merlo (2022) makes related arguments.

the place to develop this in any detail, it is worth sketching how this viewpoint can throw new light on conditionals.

Yablo (2016) introduced the idea of an "incremental conditional" in the framework of truthmaker semantics. His idea was that the truthmakers for a conditional $p \to q$ should be a state that when fused with an arbitrary truthmaker for p yields a state containing a truthmaker for q. Fine (2014, 2020) showed how this idea could be made precise and showed that the validities of intuitionistic propositional logic are all verified by the minimal (or null) truthmaker; in contrast, distinctively classical validities have more substantive truthmakers.[39]

It seems possible to do something similar in terms of ground. As a first pass: for some propositions Γ to ground a conditional $p \to q$ is for q to be a material consequence of Γ and p. This view has the pleasing consequence that $p \to p$ will be zero-grounded for all p. I conjecture that once the account is worked out all the validities of intuitionistic logic will be zero-grounded.[40]

5.3 Conceptual and metaphysical ground

Many philosophers distinguish between logical, conceptual, and metaphysical ground; however, the relationship between these notions is rarely made clear. Are these irreducibly distinct notions of ground? Are they species of a common genus? An advantage of the present view is that, on mild essentialist assumptions, one can treat logical and conceptual ground as species of metaphysical ground.[41] Slightly simplified, the mild essentialist assumption is this. If $\Gamma \ll p$ there is some generalization of this grounding claim such that

(i) $\Gamma \ll p$ follows logically from this generalization together with the truth of Γ;
(ii) the generalization is true in virtue of the nature(s) of (some constituents of) the proposition p.[42]

I then propose:[43]

(Logical Ground) Γ *logically grounds* p if the generalization from which $\Gamma \ll p$ follows is true in virtue of the logical operations in p

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[39] Related ideas are explored in Leitgeb (2019).

[40] I should stress that the above is but a first pass. As the attentive reader will have observed, $p \wedge q, r$ will be a ground for $p \to q$, but $p \wedge q, r$ does not seem wholly relevant to $p \to q$. Two remarks on this. First, I believe it is possible to obtain a more "exact" account by defining what it is to "subtract" the grounds for p from the grounds of q. The grounds for $p \to q$ are then the results of such subtractions (cf. Yablo, 2016). Second, for a plenitudinist it is not problematic if there is *some* conditional that permits irrelevant grounds.

[41] I am setting aside the issue of whether there are non-metaphysical notions of ground like "natural" and "normative" ground. For discussion see Fine (2012), Berker (2018), and Litland (2018a).

[42] This statement of the essentialist idea simplifies Fine (2012, p. 75); related ideas are explored in Rosen (2010, pp. 129–133) under the label "Formality". Fine's more complicated formulation is required to deal with the cases like the one discussed in footnote 24. Since the details would detract from the flow of the paper, I refer the interested reader to Fine's statement. (For more on such essentialist claims see Audi (2012, pp. 693–696) and Trogdon (2013).)

[43] Those who are skeptical of essentialist notions could rephrase the proposals in terms of logical, conceptual, and metaphysical modality.

(Conceptual Ground) Γ conceptually grounds p if the generalization from which $\Gamma \ll p$ follows is true in virtue of the nature of the concepts figuring in p

Some illustrations. [It is raining] together with [it is windy] logically grounds [it is raining] $\wedge$ [it is windy]. The reason is that the generalization $\forall p \forall q (p \wedge q \rightarrow (p, q) \ll p \wedge q)$ is true in virtue of the nature of the logical operations. And the grounding claim [It is raining], [it is windy] $\ll$ [it is raining] $\wedge$ [it is windy] follows logically from this generalization (and the fact that it is raining and windy).

That Bob is a bachelor (Bb) is grounded in his being a man (Mb) together with his being unmarried (Ub). The relevant generalization is $\forall x (Ux \wedge Mx \rightarrow ([Ux], [Mx] \ll [Bx]))$. This generalization is true in virtue of the nature of the concept *bachelor*. And $[Ub]$, $[Mb] \ll [Bb]$ follows logically from this generalization and the fact that Bob is a man and is unmarried.

The grounding claim that the liquid in the glass is water because it is made of molecules that are made of H, H, O, on the other, hand is a claim of mere metaphysical ground. For the generalization that anything is water if it is made of molecules made of H, H, O is not true in virtue of the nature of the *concept* (as opposed to the *property*) of being water.[44]

Of course, the distinction between logical ground, on the one hand, and conceptual and metaphysical ground, on the other, is not very informative unless one has an account of what makes something a logical operation (concept). Is there a non-pragmatic way of demarcating the "logical" operations (concepts) from other operations (concepts)? While this is a fair challenge, it is also everyone's—for instance, the Tarskian account of logical consequence faces exactly the same challenge.[45]

5.4 Explanatory arguments and normal form

The above accounts of logical consequence and the logical operations are given in terms of (anti)grounding. It is time to bring this back to explanatory arguments. To motivate what follows it will help to begin with a peculiar feature of Poggiolesi's account.

According to Poggiolesi the ground for $\neg(p \vee q)$ is $\{\neg p, \neg q\}$—if p, q themselves are unnegated. However, the grounds for $\neg(\neg r \vee \neg s)$ is not $\{\neg\neg r, \neg\neg s\}$ but rather $\{r, s\}$. This—as noted by Krämer (2019, p. 1665n50) —means that Poggiolesi's logic of ground is not closed under uniform substitution: what motivates Poggiolesi to adopt this view?[46]

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[44] For what it is worth, I am somewhat skeptical that there is a a clear enough notion of a *concept* to have a stable distinction between conceptual and metaphysical ground. Those who share this skepticism should take the distinction to be conditional on making out the requisite notion of a concept. (For a completely different take on the distinction between metaphysical and conceptual ground see Smithson (2020).)

[45] For what it is worth I believe that it can be met by extending the Tarski-Sher "invariantist" account of the logical operations to the ground-theoretic setting. But this has to await another occasion.

[46] In fact, given what she says elsewhere, Poggiolesi's view commits her to grounding contexts's being *opaque* in the sense that for some Γ, p, r, s we have both $\Gamma \ll p$ and $r \approx s$ while we do not have $\Gamma(s/r) \ll p(s/r)$. (Here I use $p \approx q$ to mean that the propositions p, q are identical and use $p(s/r)$ to mean the proposition that results from p by substituting s for r; similarly, for $\Gamma(s/r)$.) In Poggiolesi

Consider the standard view about negated disjunctions

$$\neg\neg p, \neg\neg q \ll \neg(\neg p \vee \neg q) \tag{1}$$

and the standard rule for the grounding of double negations

$$p \ll \neg\neg p \tag{2}$$

Putting (1) and (2) together one obtains the *mediate* grounding claim

$$p, q < \neg(\neg p \vee \neg q) \tag{3}$$

The argument against (1) turns on the observation that the (classical, intuitionistic, minimal) natural deduction proof that proceeds from p, q via $\neg\neg p, \neg\neg q$ to $\neg(\neg p \vee \neg q)$ is not in *normal form* (Poggiolesi 2016a, pp. 300-303). Here is the proof:

$$\dfrac{\dfrac{\dfrac{}{\neg p \vee \neg q}\,{\scriptstyle 1} \qquad \dfrac{\dfrac{\dfrac{p \qquad \dfrac{}{\neg p}\,{\scriptstyle 2}}{\bot}}{\neg\neg p}\,{\scriptstyle 2} \qquad \dfrac{}{\neg p}\,{\scriptstyle 3}}{\bot} \qquad \dfrac{\dfrac{\dfrac{q \qquad \dfrac{}{\neg q}\,{\scriptstyle 4}}{\bot}}{\neg\neg q}\,{\scriptstyle 4} \qquad \dfrac{}{\neg q}\,{\scriptstyle 5}}{\bot}}{\bot}\,{\scriptstyle 3,5}}{\neg(\neg p \vee \neg q)}\,{\scriptstyle 1}$$

In this proof the formulae $\neg\neg p$ and $\neg\neg q$ are *maximum formulae* in the sense that they are introduced by means of the negation-introduction rule (*Reductio ad Absurdum*) and then serve as major premisses to ¬-elimination. Proofs in normal form do not contain maximum formulae and since Poggiolesi takes claims of mediate (logical) ground to correspond to normal proofs she is forced to reject (1).

This argument should be taken seriously. If an explanatory argument is the result of composing explanatory inferences and the result of applying an elimination rule does not result in an explanatory inference, an argument containing a maximum formula cannot be explanatory. But why think that the natural deduction system for explanatory inference should be based on a natural deduction system for classical logic? I do not see any reason to assume this.

This is not the place to develop a system for explanatory inference in detail, but let me indicate what I take to be a promising approach. One should work with systems where (some) rules of inference are simply *given* as explanatory rules. Such an approach faces the problem of what to do with negation. While this might not be the optimal solution, it is natural to adopt *bilateralism*: just as one might need to introduce a primitive relation of antiground between propositions, one might need to introduce

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Footnote46 continued
(2020c, forthcoming) she seems to argue that $\neg(p \wedge q) \approx (\neg p \vee \neg q)$. According to Poggiolesi, however, the immediate ground of $\neg\neg(p \wedge q)$ is just $p \wedge q$, while the immediate grounds for $\neg(\neg p \vee \neg q)$ is p, q. If one accepts the identification between $\neg(p \wedge q)$ and $\neg p \vee \neg q$ then one either has to deny that ground is a relation between propositions or one has to hold that contexts involving negation are opaque. Both views, but especially the latter, are excessively costly.

the notion of an *explanatory rejection*. While the premisses of an explanatory inference to conclusion p are answers to the question “why p?”, the premisses of an explanatory rejection with conclusion p are answers to the question “why not p?”

Write ”$-$” next to the conclusion of a rule R to indicate that the conclusion of R is rejected. Crucially, “$-$” must not be confused with negation: while negation is an iterable sentential operator “$-$” is a *force*-indicator.[47] I then propose the following rules for explanatory inference to and rejection of negations.

$$\dfrac{\Gamma \qquad \dfrac{\dfrac{}{\Gamma}\,1}{-p}\,R}{\neg p}\,1,\ \neg\text{-explanation} \qquad\qquad \dfrac{p}{-\neg p}\,\neg\text{-rejection}$$

What the ¬-explanation rule says is: if there is an explanatory rule that takes us from Γ to the rejection of p, then the inference from Γ to $\neg p$ is explanatory.[48]

Here are the rules for disjunction.

$$\dfrac{p}{p \vee q}\,\vee\text{-explanation} \qquad \dfrac{q}{p \vee q}\,\vee\text{-explanation} \qquad \dfrac{\neg p \qquad \neg q}{-(p \vee q)}\,\vee\text{-rejection}$$

Using both these rules one has the following explanatory derivation of $\neg(\neg p \vee \neg q)$ from p, q. Note that this derivation uses only explanatory rules.

$$\dfrac{\dfrac{\mathrm{p} \qquad \dfrac{\dfrac{}{p}\,1}{-\neg p}\,\neg\text{-rejection}}{\neg\neg p}\,1,\ \neg\text{-explanation} \qquad \dfrac{q \qquad \dfrac{\dfrac{}{q}\,2}{-\neg q}\,\neg\text{-rejection}}{\neg\neg q}\,2,\ \neg\text{-explanation} \qquad \dfrac{\dfrac{}{\neg\neg p}\,3 \qquad \dfrac{}{\neg\neg q}\,4}{-(\neg p \vee \neg q)}\,\vee\text{-rejection}}{\neg(\neg p \vee \neg q)}\,3,4,\ \neg\text{-explanation}$$

Here I have only given explanatory (i.e., introduction) rules for ¬ and ∨; clearly, elimination rules also have to be provided and the features (normalization, subformula property, ...) of the resulting system has to be investigated.[49]

6 Conclusion

This paper has argued that Poggiolesi’s proof theoretic definition of ground is a failure. But let me end by stressing that it is just the proof-theoretic *definition* of ground that fails. None of the above criticisms should be construed as objections to using proof-theoretic techniques in the study of ground—indeed, Poggiolesi’s own work contains numerous rigorous arguments and ingenious ideas that will prove useful for anyone who explores the logic of ground.

Acknowledgements Thanks to Ray Buchanan for helpful advice on this paper. Thanks also to a number of especially conscientious referees whose meticulous comments have greatly improved the paper both in structure and content.

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[47] For more on the distinction between rejection and negation see Rumfitt (2000).

[48] The format of this rule is slightly awkward. It is written in this way to ensure that what immediately explains $\neg p$ is Γ, and not $-p$.

[49] This is not the place to do this, but one can find elimination rules for ¬, ∨ by using the resources of higher-order natural deduction (Schroeder-Heister, 1984)) and the distinction between plain and explanatory arguments (Litland, 2018b).

Funding Open access funding provided by University of Oslo (incl Oslo University Hospital)

Declarations

Conflicts of interest The author reports no conflicts of interest

A Grounding Consequence

This appendix precisely defines the notion of (formal) grounding consequence and establishes the results mentioned in the main text.

Definition A. 1 An immediate grounding structure is a tuple $\mathcal{G} = \langle \mathbb{P}_\mathcal{G}, \ll_\mathcal{G}, \wedge_\mathcal{G}, \vee_\mathcal{G}, \neg_\mathcal{G} \rangle$ such that:

- $\mathbb{P}_\mathcal{G}$ is a set (intuitively of propositions)
- $\wedge_\mathcal{G}, \vee_\mathcal{G}$ are functions $\mathbb{P} \times \mathbb{P} \to \mathbb{P}$ such that $p \wedge_\mathcal{G} q = r \wedge_\mathcal{G} s$ iff the multiset $\{p, q\}$ is identical to the multiset $\{r, s\}$. (Similarly for $\vee_\mathcal{G}$.)
- $\neg_\mathcal{G} : \mathbb{P} \to \mathbb{P}$ is an injective function.
- $p \wedge_\mathcal{G} q \neq r \vee_\mathcal{G} s \neq \neg_\mathcal{G} t$ for for all $p, q, r, s, t \in \mathbb{P}$.
- $\ll_\mathcal{G}$ is a relation between multisets of propositions and propositions satisfying the following conditions. (From now on the subscripts on $\wedge, \vee, \neg, \mathbb{P}, \ll$ will be dropped when no confusion arises.)
  - $\Delta \ll \neg\neg p$ iff $\Delta = \{p\}$;
  - $\Delta \ll p \wedge q$ iff $\Delta = \{p, q\}$;
  - $\Delta \ll \neg(p \wedge q)$ iff $\Delta = \{\neg p\}$ or $\Delta = \{\neg q\}$;
  - $\Delta \ll p \vee q$ iff $\Delta = \{p\}$ or $\Delta = \{q\}$;
  - $\Delta \ll \neg(p \vee q)$ iff $\Delta = \{\neg p, \neg q\}$

Note that the relation of antiground plays no role in this definition; while I believe it is needed for philosophical purposes it is not needed for the present technical points.

One of the grounding structures $\mathcal{I} = \langle \mathbb{P}_\mathcal{I}, \ll_\mathcal{I}, \neg_\mathcal{I}, \wedge_\mathcal{I}, \vee_\mathcal{I} \rangle$ is the *intended* one; here $\mathbb{P}_\mathcal{I}$ is the set of all propositions and $\ll_\mathcal{I}$ is the real grounding relation.[50]

Given a grounding structure $\mathcal{G}$ the atomic propositions of $\mathcal{G}$ are the elements of $\mathbb{P}_\mathcal{G}$ that are not in the range of $\wedge_\mathcal{G}, \vee_\mathcal{G}, \neg_\mathcal{G}$. A *literal* of $\mathcal{G}$ is an atomic proposition of $\mathcal{G}$ or the negation of an atomic proposition of $\mathcal{G}$.

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[50] I am setting aside some cardinality issues here; they do not matter for present purposes.

The mediate grounding relation $<$ over $\mathcal{G}$ is the smallest relation between multisets of $\mathbb{P}$ and $\mathbb{P}$ such that $<$ contains $\ll$ and is closed under the principle of Cut. That is, if $\gamma_0, \gamma_1, \ldots, \Delta \ll p$ and we have $\Sigma_0 < \gamma_0, \Sigma_1 < \gamma_1, \ldots$ then $\Sigma_0, \Sigma_1, \ldots, \Delta < p$.

Definition A. 2 Let $\mathcal{G}$ be a grounding structure and let $\Gamma \subseteq \mathbb{P}_{\mathcal{G}}$.

(i) Γ is *ground-closed* in $\mathcal{G}$ iff whenever $\Gamma_0 < p$ for some $\Gamma_0 \subseteq \Gamma$, then p is in Γ.
(ii) Γ is *ground-prime* in $\mathcal{G}$ iff for all $\Delta \subseteq \Gamma$ if there is Σ such that $\Sigma < \delta$ for each $\delta \in \Delta$, then there is $\Sigma' \subseteq \Gamma$ such that $\Sigma' < \delta$ for each $\delta \in \Delta$.
(iii) The ground-closure of Γ in $\mathcal{G}$ is the least $\Gamma^+ \supseteq \Gamma$ such that Γ^+ is ground-closed in $\mathcal{G}$.

When $\mathcal{G}$ is clear from context I just write "closed" instead of "ground-closed in $\mathcal{G}$" (similarly for "ground-prime in $\mathcal{G}$" and "ground-closure in $\mathcal{G}$").

Definition A. 3 If $\mathcal{G} = \langle \mathbb{P}_{\mathcal{G}}, \ll_{\mathcal{G}}, \wedge_{\mathcal{G}}, \vee_{\mathcal{G}}, \neg_{\mathcal{G}} \rangle$ is a grounding structure a *substitution* on $\mathcal{G}$ is a function $^*\colon \mathbb{P}_{\mathcal{G}} \to \mathbb{P}_{\mathcal{G}}$ that respects $\wedge, \vee, \neg$, that is:

(i) $(p \wedge q)^* = p^* \wedge q^*$;
(ii) $(p \vee q)^* = p^* \vee q^*$; and
(iii) $(\neg p)^* = \neg p^*$

Definition A. 4 (i) p is a *material consequence* of Γ in $\mathcal{G}$ iff for all closed, prime Σ if $\Gamma \subseteq \Sigma$ then $p \in \Sigma$;
(ii) p is a *coherent material consequence* of Γ in $\mathcal{G}$ iff for all closed, prime, coherent Σ if $\Gamma \subseteq \Sigma$ then $p \in \Sigma$;
(iii) p is a *complete material consequence* of Γ in $\mathcal{G}$ iff for all closed, prime, complete Σ if $\Gamma \subseteq \Sigma$ then $p \in \Sigma$;
(iv) p is a *classical material consequence* of Γ in $\mathcal{G}$ iff for all closed, prime, complete, and coherent Σ if $\Gamma \subseteq \Sigma$ then $p \in \Sigma$;
(v) p is a *relative (coherent, complete, classical) grounding consequence* of Γ iff for all grounding structures $\mathcal{G}$ it is the case that p is a (coherent, complete, classical) material consequence of Γ in $\mathcal{G}$
(vi) p is a formal (coherent, complete, classical) grounding consequence of Γ iff p^* is a (coherent, complete, classical) material consequence of Γ^* in $\mathcal{I}$, for all substitutions * on the intended grounding structure $\mathcal{I}$.

To establish the connection between grounding consequence and familiar notions of consequence we introduce valuations.

A *valuation* is a relation between atomic propositions and the truth-values T, F. A given proposition p might be related to exactly one of the values T, F, both of the values, or neither of the values. We extend v to a valuation v^+ of all propositions formed by applying $\wedge, \vee, \neg$ to the atomic propositions in accordance with the four-valued truth tables for First Degree Entailment. A *situation* is a collection of literals. If v is a valuation we associate with v a situation $S(v)$ as follows: p is in $S(v)$ iff $v(p, T)$, and $\neg p$ is in $S(v)$ iff $v(p, F)$.

One can now establish:

Proposition A. 5 *[(i)] Let $\mathcal{G}$ be a grounding structure and let v be a valuation.*

1. *r is in the ground-closure of $S(v)$ iff $v^+(r, T)$.*
2. *$\neg r$ is in the ground-closure of $S(v)$ iff $v^+(r, F)$.*

From this we get:

Proposition A. 6 *If p is first degree entailed by Γ, then p is a (formal, relative) grounding consequence of Γ.*

Proof Let $\mathcal{G}$ be any grounding structure and suppose that p is first degree entailed by Γ. Let Γ^+ be any ground-closed, prime set of propositions containing Γ. One can show that Γ^+ contains some situation S such that Γ is in the ground-clousure of S. Now let v be the valuation such that $S(v) = S$. By Proposition 5 $v^+(\gamma, T)$, for each $\gamma \in \Gamma$ and thus $v^+(p, T)$ since p is first degree entailed by Γ. Since Γ^+ is ground-closed, $p \in \Gamma^+$ and thus p is a material consequence of Γ in $\mathcal{G}$. Since $\mathcal{G}$ was arbitrary this shows that p is a relative grounding consequence of Γ.

To show that p is a formal grounding consequence of Γ it suffices to observe that First Degree Entailment is preserved under substitutions. □

Proposition A. 7 *If p is not first degree entailed by Γ then there is a grounding structure $\mathcal{G}$ such that p is not a material consequence of Γ in $\mathcal{G}$.*

Proof Consider the grounding structure $\mathcal{G}$ where the literals formed from the atomic propositions occurring in Γ and p have no grounds. Take a valuation v such that for each $\gamma \in \Gamma$, $v^+(\gamma, T)$ but not $v^+(p, T)$. Then by Proposition 5 each γ in Γ is in the closure of $S(v)$ but p is not in the closure of $S(v)$. However, since no proposition in $S(v)$ has any grounds the closure of $S(v)$ is prime. This show that p is not a grounding-consequence of Γ. □

Corollary A. 8 *If p is not first degree entailed by Γ then p is not a relative grounding consequence of Γ.*

Establishing the corresponding result about formal grounding consequence runs into a problem. Consider the intended structure $\mathcal{I}$. Take a situation $S(v)$. The propositions in $S(v)$ do not have any conjunctive, disjunctive, or negated grounds, but that is not to say that they do not have any grounds at all. This leaves open the possibility that every prime set containing $S(v)$ will contain a ground for p even though p is not in the closure of $S(v)$.

A grounding structure $\mathcal{G}$ is *humean* if for every situation $S(v)$ and every p if p is not in the ground-closure of $S(v)$ then there is a substitution * and a prime set Σ such that $S(v)^* \subseteq \Sigma$ but $p^* \notin \Sigma$.[51]

Proposition A. 9 *If $\mathcal{I}$ is humean then, if p is not first degree entailed by Γ, then p is not a formal grounding consequence of Γ.*

Proof Let $S(v)$ be a valuation witnessing that Γ does not first degree entail p. By Proposition 5 each γ in Γ is in the closure of $S(v)$ but p is not. Since $\mathcal{I}$ is humean let * be a substitution and Σ be prime and closed such that $S(v)^* \subseteq \Sigma$ but $p^* \notin \Sigma$. Then we have $\Gamma^* \subseteq \Sigma$, but not $p^* \in \Sigma$. This shows that p is not a formal grounding consequence of Γ. □

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[51] This condition might appear *ad hoc*, but note that it is satisfied by every-grounding structure where there are (sufficiently many) ungrounded literals.

Proposition A. 10 *(i) Coherent relative (formal) grounding consequence coincides with Strong Kleene consequence.*

(ii) Complete relative (formal) grounding consequence coincides with Logic of Paradox consequence.

(iii) Classical relative (formal) grounding consequence coincides with classical consequence.

Proof I only consider the case of Strong Kleene consequence. The other two are more straightforward and are left to the reader.

Suppose p is a Strong Kleene consequence of Γ and let $\mathcal{G}$ be any grounding structure. Let Σ be a coherent, closed, and prime set containing Γ. Let S be the largest situation contained in Σ. Let v be a valuation such that $S(v) = S$. Since Σ is prime each $\gamma \in \Gamma$ is in the closure of $S(v)$; thus, by Proposition 5, $v^+(\gamma, T)$ for each $\gamma \in \Gamma$. Since Σ is coherent, v does not assign both T and F to a single proposition. But then, since p is a Strong Kleene consequence of Γ, it is the case that $v^+(p, T)$. By Proposition 5, p is in the closure of $S(v)$ and so p is in Σ. Since Γ was arbitrary this shows that p is a coherent relative grounding consequence of Γ.

If p is not a Strong Kleene consequence of Γ let v be a valuation witnessing this. Let $\mathcal{G}$ be a grounding structure where each proposition in $S(v)$ is ungrounded. Since v does not assign both T and F to a single proposition, by Proposition 5, $S(v)$ is coherent. Since each proposition in $S(v)$ is ungrounded the closure of $S(v)$ is both closed and prime. By Proposition 5, again, Γ is in the closure of $S(v)$ but p is not. This shows that p is not a coherent relative grounding consequence of Γ.

To account for formal coherent grounding consequence one needs to assume that the intended grounding structure is *coherently humean*, that is, that if $S(v)$ is coherent and p is not in the ground-closure of $S(v)$ then there is a substitution * such that there is a coherent, closed, and prime Σ such that $\Gamma^* \subseteq \Sigma$ but $p^* \notin \Sigma$. □

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