



There is overlap in letters: a mereological definition of word types

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Abstract

Classical mereology seems unable to characterise the identity conditions of word types. For example, the same letter types ordered differently result in different word types; but mereological fusions of the same letters are identical, regardless of the order of the letters. We show, however, that by combining classical mereology with plural logic one can give a definition of word types that satisfies the identity criteria of word types. This means that it is not necessary to abandon classical mereology in order to give an analysis of the mereological structure of word types.

Keywords Mereology · Plural logic · Word types · Composition · Parthood

1 Introduction

Classical mereology is a part–whole theory according to which any collection of objects compose exactly one fusion. The two main controversies surrounding the theory concern the ‘any’ and the ‘exactly one’ aspect of the above formulation. Some claim classical mereology is wrong because some collections of objects fail to compose

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a fusion. Others think it is wrong because some collections compose more than one fusion. This latter debate concerns the question of whether parthood is extensional, i.e. whether distinct objects can have the same parts. Those who think no two objects can have the same parts are called ‘extensionalists’, whereas those who think distinct object can be composed of the same parts are called ‘non-extensionalists’. It is this debate that forms the background of our discussion.

Extensionalism is and has been actively defended as the correct position for the mereological structure of entities in various domains. But most extensionalists wave the white flag when faced with word types. Word types need to be distinguished from word tokens. A word token is a concrete instance of its word type. For example, the sentence ‘to be or not to be’ (as currently displayed on your screen or printed on the piece of paper in front of you) is a token of the sentence type TO BE OR NOT TO BE and this latter type contains four word types: TO, BE, OR, and NOT. Note that the first two word types have two instances or occurrences in the above sentence type. An occurrence of a word type is thus not the same thing as a word token because a sentence type does not contain tokens but word types (Wetzel, 1993).

The individuation of word types is a controversial issue (Bromberger, 2011; Hawthorne & Lepore, 2011; Irmak, 2019; Kaplan, 1990, 2011; Lando, 2019; McCulloch, 1991; Wetzel, 1993). For example, David Kaplan (1990, 2011) famously argues that one and the same word type can be spelled and pronounced in different ways. According to him ‘analyse’ and ‘analyze’ are two different spellings of a single word, and *fɛdʒu:l* and *skɛdʒu:l* are two different ways of pronouncing the word SCHEDULE.

However, this conception of word types is not the one usually advanced as an argument against extensional mereologies because it is not obvious what the identity criteria of Kaplanian word types are, nor what the parts of these word types are supposed to be. For example, if the letter S is part of the word ANALYSE, it is at best part of only some instances of this word type since not all correct spellings of that word contain the letter S. This suggests that the relation between a letter type and a Kaplanian word type is maybe not a part–whole relation but, possibly, a relation of (partial) realisation or (partial) instantiation. (For a discussion about the part–whole relation that might hold between letters and word types under a Kaplanian conception of words, see Lando (2019, p. 210ff).)

Instead of this Kaplanian conception of word types, it is a more orthographic conception of linguistic types that is commonly (implicitly) assumed when words are weaponized as counterexamples to extensional mereology (Hempel 1953, p. 110; Rescher 1955, p. 10; Simons 1987, p. 114). This conception individuates word types via their spelling and commonly assumes that we already have, or do not need, clear identity criteria for letter types (these are considered the basic units). A thorough orthographic conception of word types completely ignores semantic and phonological properties of words. Simons and Rescher seem to use a mixed conception of word types which combines semantic and orthographic notions when arguing against extensional mereology. However, their employment of semantic properties also makes their arguments less straightforward. For example, is BANK one word type or two word types under their conception? A thorough orthographic conception takes it to be one word, while under a semantic conception it is two words (for it has two meanings).

A thorough orthographic conception has the advantage of providing a clear identity criterion. And it also suggests that letter types are parts of word types, which are in turn parts of sentence types—thereby making it intuitively plausible that different word types can be composed of the same letter types. (For the remainder of the paper we thus suppose this orthographic conception of linguistic types, which we will make more precise in the Sect. 3.)

A counterexample against extensional mereology is easily devised given the orthographic conception of word types. For if word types are constructed as fusions of letter types, classical mereology fails to provide an adequate account of word types. First, the order in which letters appear matters for the identity of words. But classical mereology is not very sensitive to the order of the parts in a fusion. Second, letters can occur in a word multiple times, but classical mereology cannot have sums that have a part multiple times over. This failure of classical mereology provides fertile ground for alternative mereologies (Bennett, 2013; Cotnoir, 2015). But this brings little solace to the true believer in classical mereology.

An extensionalist who is acutely aware of the problem word types pose to extensional mereology is Achille Varzi. He takes solace in the fact that it is not a problem specific to extensionality since various mereological principles that are independent of extensionality are also violated if we take letter types to be parts of word types (Varzi, 2008, p. 128ff). It is true that a mereological analysis of word types faces problems that are not specific to the extensionality of classical mereology. But that does not make it less of a problem for classical mereology.

We want to investigate an approach to word types that is compatible with classical mereology, and in particular with extensionality. Our reasons for preferring to stay within the land extensionality rather than moving towards a non-extensionalist plane are analogous to the Goodmanian reasons we mention in Sect. 4 below for preferring mereology to set theory. Roughly, we do not accept a distinction in object without a distinction in content.

Obviously, if we drop the idea that word types are *fusions* of letter types, then word types no longer threaten classical mereology. So that is where we will begin. However, as we will show, this does not mean that classical mereology has no role to play in the explication of word types. To the contrary, we argue that if we explicate word types as pluralities of certain fusions, classical mereology provides a fruitful tool in characterising various aspects of word types.

Before we begin, we would like to mention some of our assumptions. First, we assume that word types are abstract because (i) we think this is how word types are commonly understood and (ii) the problem of providing a mereological account of word types seems more difficult when types are abstract. (We briefly consider dropping this assumption in Sect. 5.)

Second, we consider our account successful when it delivers objects that can play the role of word types, and we think some entities can play a certain role if they satisfy the relevant identity criteria associated with that role. This means that our account contains some arbitrary choices. Our approach is thus analogous to certain structuralist positions in mathematics where one points to entities satisfying all the structural conditions that one takes to be necessary and sufficient and then calls it a day.

Third, in our examples we only consider words made from letters. But our account is more general and should work for any writing system in which some basic units make up larger units. So it might have been better to speak of graphemes instead of letters, since this makes it clear our account also covers syllabic and logographic writing systems. However, since every reader of this paper is acquainted with the alphabetical writing system of English we only use examples from this *lingua franca*. (Actually, our account is even more general and is applicable to any entities that are ‘made up’ of some basic entities, such that the identity of the complex entities depends both on the number of occurrences of the basic entities and the order in which they appear. We could thus speak of ‘strings’ rather than ‘word types’. However, we think ‘string’ is more commonly associated—at least in philosophy—with a concrete token rather than a type. To avoid confusion we continue speaking of ‘word types’.)

We also assume that there is no problem individuating letter types. This does not mean that we can determine of every inscription whether it is a token of this or that letter type. It merely means that every letter type is distinct from every other letter type, and of each letter type there is only one. The challenge, we think, is to construct word types of these letter types in line with classical mereology.

Here is the plan. In Sect. 2 we present the basics of classical mereology and plural logic. Then in Sect. 3 we make the structure of word types explicit in terms of identity criteria. Section 4 contains a definition of word types that satisfies the criteria of Sect. 3 using only the machinery of Sect. 2. We then discuss in Sect. 5 how one may understand the type-token relation given our account. We conclude that word types need not keep extensionalists awake at night.

2 Mereology and plural logic

Mereology is the study of the part–whole relation. We present here an axiomatisation of classical mereology (also known as ‘general extensional mereology’, from hereon simply ‘mereology’) that takes overlap (symbolised by ‘ \circ ’) as a primitive notion and defines the following notions.

(Improper Parthood) $x \leq y =_{df} \forall z(z \circ x \rightarrow z \circ y)$

(Proper Parthood) $x < y =_{df} x \leq y \wedge \neg y \leq x$

(Fusion) $Fu(y, \varphi x) =_{df} \forall z(z \circ y \leftrightarrow (\exists x(\varphi x \wedge z \circ x)))$

These definitions ensure that improper parthood is reflexive and transitive, and that proper parthood is irreflexive, transitive, and asymmetric. In the case of a fusion of two objects, x and y , we will write ‘ $x + y$ ’. The following three axioms then provide an adequate basis for mereology.

(Shared Part) $x \circ y \leftrightarrow \exists z(z \leq x \wedge z \leq y)$

(Unrestricted Composition) $\exists x \varphi x \rightarrow \exists y Fu(y, \varphi x)$

(Extensionality) $\forall z(z \circ x \leftrightarrow z \circ y) \rightarrow x = y$

These principles ensure that objects with the same parts are identified. Another consequence is that if x is part of y then the fusion of them is just y . This means that an object can have another object as a part only once. Mereology is thus ‘proudly blind’ to structure (Lando, 2017, p. 83).

Besides mereology we need some basic plural logic. Plural logic studies logical inferences that employ plural referring terms such as ‘they’ and ‘Barcan Marcus, Carnap, and Kripke’. Plural logic thus contains plural terms and a logical constant ‘ \leq ’ (to be read as ‘is one of’). We will use capital letters for plural terms so that ‘ $x \leq X$ ’ expresses that x is one of the X ’s. We take the following three principles to be axiomatic for plural logic. (Plural logic has no complete axiomatisation (Oliver & Smiley, 2016, p. 246ff).)

- (Plural Identity) $X = Y \leftrightarrow \forall z(z \leq X \leftrightarrow z \leq Y)$
 (Not Empty) $\forall X \exists x x \leq X$
 (Comprehension) $\exists x \varphi x \rightarrow \exists Y \forall x (x \leq Y \leftrightarrow \varphi x)$

The axiom of plural identity is, from right-to-left, an extensionality principle to the effect that pluralities with the same members are identical. For our purposes, this is all we need from plural logic. (For more advanced systems of plural logic, see for example Yi (2005, 2006) and Oliver & Smiley (2016).)

Plural logic is expressively rather strong (basic systems are equivalent to monadic second-order logic). However, we think plural logic is acceptable for a classical mereologist who eschews differences between entities that have no difference in parts. First, there is precedent: David Lewis (1991) happily accepted both classical mereology and plural logic, while only grudgingly accepting set theory—and refusing to countenance any other modes of composition beyond mereological composition. Second, plural logic does not introduce novel entities, but only novel ways of speaking about the entities one already accepts. (At least according to the conception of plural logic we favour, see Boolos (1984, 1985) for an elaborate exposition of this conception.)

3 Criteria of identity for word types

We follow orthodoxy in taking word types to be ‘made from’ letter types. Much depends on how this making relation is construed. A mereologically oriented metaphysician might at first consider word types to be mereological fusions of letter types. This solution could remain neutral on the question whether types are concrete or abstract, because fusions can be concrete or abstract. This approach has difficulty capturing the structure of words that are composed of the same letter types but ordered differently. Take, for example, the words LATE and TALE. Since every letter type that overlaps LATE also overlaps with TALE (and *vice versa*), the fusions of these letter types are the same. Hence, if word types are fusions then according to classical mereology, LATE = TALE—this is wrong. A similar challenge concerns the difference between words that are composed of the same letter types but where one of them has more instances of a letter type than the other. Take, for example, the words LATTE and LATE. Here the letter type T has two instances in the first word, but not in the second. However, mereology does not allow for a fusion to have a part multiple times over. (Otherwise put, a fusion of x , y , and x is the same as the fusion of x and y .)

This short discussion about the difficulties for a straightforward mereological approach to word types also clarifies the conditions under which word types should be considered distinct.

- (Kind)** If two words do not have all the same letters, they are distinct.
- (Number)** If the number of letter instances in one word is different from the number of letter instances in another, they are distinct.
- (Order)** If the order of letters in one word is different from the order of letters in another word, they are distinct.

We take the inverses of these three principles to be individually necessary and jointly sufficient for the identity of word types. That is to say, two words are identical if and only if they have all the same letters, the same number of letter instances, and such that the letters are ordered in the same way. As we have seen, mereology fails to capture the last two criteria, **Number** and **Order**.

Unfortunately, pluralities face the same problems as fusions. For if we take words to be pluralities of letter types then **Kind** will be satisfied due to Plural Identity. But there is a problem with **Number** because in our very basic plural logic pluralities cannot have a member multiple times. For example, the plurality of the letters L, A, T, and E is the same as the plurality of the letters L, A, T, T and E. The reason is that every letter of the plurality consisting of L, A, T, and E is also one of the letters of L, A, T, T, and E—and *vice versa*. Moreover, our basic plural logic fails to satisfy **Order** because plural logic does not distinguish between the order of the members of the plurality. In particular, the plurality of L, A, T, and E is the same as that of T, A, L, and E because these pluralities have the same members.

So neither mereology nor basic plural logic can give an adequate characterisation of word types, and both systems fail for the exact same reasons. They both fail to account for multiple instances of a letter type as well as for the order in which the letter types occur in the word type. (For more on the different ways in which a made-from-relation may fail to capture certain structural differences, see Fine (2010, p. 574ff).)

4 Words as functions

Not all is lost, however. Linda Wetzel (1993) constructs word types as n -tuples, thereby providing entities that satisfy the identity criteria given above. However, since n -tuples are functions, and functions are commonly understood as set-theoretical entities, we cannot follow Wetzel directly because this would mean mereology has no role to play in the analysis of word types—which is the goal we set ourselves.

However, if we can construct functions or function-like entities using only plural logic and mereology, we can construct words as functions of a particular kind. More specifically, like Wetzel, we could treat words as n -tuples, where n -tuples are understood as a function from the tuples' index set to the elements of the function. This is the approach we will take here.

We use the following definitions in terms of plural logic and mereology from Carrara & Martino (2011) (with a slight alteration of the last definition):

- (Subplurality) X is a subplurality of Y , $X \sqsubseteq Y$, if for every $x \preceq X$, $x \preceq Y$.
- (Separate) Two pluralities X and Y are *separate* if no $x \preceq X$ and $y \preceq Y$ overlap.

- (Pair) If X and Y are separate, an X - Y pair is the fusion $x + y$ of an $x \preceq X$ and a $y \preceq Y$; x is the X -component and y is the Y -component of the pair.
- (Relation) A relation between two separate pluralities X and Y is any plurality R of X - Y pairs.
- (Function) A (total) function between two separate pluralities X and Y is a relation between X and Y such that for every X -component there is exactly one Y -component. (A function f between X and Y is partial if $X' \sqsubseteq X$ and f is a function from X' to Y .)

(Note that the above definition of function defines an entity that is a function in the conventional sense except for the fact that the domain and co-domain are completely separate—the standard notion of function allows for the domain and co-domain to overlap or indeed be identical.)

With these definitions in place we can define various entities. Let \mathbb{N} be the plurality of natural numbers with 1 being the smallest natural number; and let \mathcal{A} be the plurality consisting of A, B, C, . . . , Z (and possibly punctuation marks). We then define various functions on these pluralities.

- (Letter Combination) A letter combination is any (total or partial) function from \mathbb{N} to \mathcal{A} .
- (Potential Word Type) A potential word type is any letter combination W such that for some $n \preceq \mathbb{N}$, if n is part of a fusion in W then for every $m \preceq \mathbb{N}$ if m is smaller or equal to n , then there is a fusion in W that has m as a part.
- (Word Type) A word type of the alphabet \mathcal{A} is any potential word type that is considered a word at some point in time in some language that uses the alphabet \mathcal{A} .

A few comments about these definitions. Since \mathbb{N} is countably infinite, the first definition gives us uncountably many functions, i.e. pluralities of fusions. We do not need that many objects and one could instead take some finite plurality as the domain of the function. We leave this open and for our definition it does not matter whether the domain is infinite or not. (This seems to be a matter of metaphysical taste.)

The second definition makes an arbitrary choice concerning which of the many letter combinations to use. Once we have given our definition of word types it will become clear that our choice here is indeed arbitrary. So at the end of this section we review this definition.

The third definition is strictly speaking redundant. Everything we will say below already applies to potential word types (and even to letter combinations). But for the sake of fit with reality, we thought it better to make clear which of the potential word types can be regarded as actual word types. However, if one thinks that some potential word types should count as actual word types, or conversely that some word types we include should actually be excluded, then one is free to adjust the last definition accordingly—nothing hinges on this.

So word types are defined as pluralities of fusions consisting of a \mathbb{N} -component and an \mathcal{A} -component. In line with this, we use a plural variable such as ‘ W ’ to symbolize

a function. To illustrate the approach we give our characterisation of the words LATE, TALE, and LATTE.

- LATE is the plurality consisting of the fusions 4+E, 3+T, 2+A, and 1+L.
- TALE is the plurality consisting of the fusions 4+E, 3+L, 2+A, and 1+T.
- LATTE is the plurality consisting of the fusions 5+E, 4+T, 3+T, 2+A, and 1+L.

This approach to word types allows us to define two types of overlap which, in turn, allows us to give the identity conditions for words.

(\mathbb{N} - and \mathcal{A} -Overlap) For $x_1, x_2 \preceq \mathbb{N}$, $y_1, y_2 \preceq \mathcal{A}$, let $z_1 = x_1 + y_1$ and $z_2 = x_2 + y_2$. Then

$$z_1 \mathbb{N}\text{-overlaps with } z_2, \text{ i.e., } z_1 \circ_{\mathbb{N}} z_2, \text{ iff } x_1 = x_2; \text{ and}$$

$$z_1 \mathcal{A}\text{-overlaps with } z_2, \text{ i.e., } z_1 \circ_{\mathcal{A}} z_2, \text{ iff } y_1 = y_2.$$

Note that \mathbb{N} -overlap and \mathcal{A} -overlap are both equivalence relations, because they are defined in terms of identity. Also, any two words \mathbb{N} -overlap because every word has a first letter and thus contains a fusion with 1 as its \mathbb{N} -component. (This follows from our definition of potential word, which we will review below. Bear in mind though that \mathbb{N} is arbitrary: if we use a different plurality, then every word type would overlap a different entity instead.)

We can now say that words W_1 and W_2 are identical if and only if (i) every fusion in W_1 has a fusion in W_2 such that they both \mathbb{N} -overlap and \mathcal{A} -overlap, and (ii) every fusion in W_2 has a fusion in W_1 such that they both \mathbb{N} -overlap and \mathcal{A} -overlap. Formally:

(Identity Theorem) $(\forall z_1 \preceq W_1 \exists z_2 \preceq W_2 (z_1 \circ_{\mathbb{N}} z_2 \wedge z_1 \circ_{\mathcal{A}} z_2) \wedge \forall z_2 \preceq W_2 \exists z_1 \preceq W_1 (z_1 \circ_{\mathbb{N}} z_2 \wedge z_1 \circ_{\mathcal{A}} z_2)) \leftrightarrow W_1 = W_2$

(The proof is in the appendix.) This theorem enables us to show how our definition of word type satisfies the three criteria we identified in Sect. 3. The first was **Kind**, i.e. that if two words do not have the same letters, then they are distinct. To say that two words have distinct letters means, in our definition, that not every fusion in one word \mathcal{A} -overlaps with a fusion in another word. But if that is the case, then, by the Identity Theorem, the words are distinct—as required.

The second criterion, **Number**, stated that words with a different number of letter type instances are distinct. This also follows from the Identity Theorem, so this criterion is satisfied, too.

Finally, **Order** stated that letter types that are ordered differently result in different word types. Since we have already dealt with the case where words do not have the same letters or not the same letter instances, we may suppose that the words have the same letters and the same number of instances of these letters. (As in the case of the word types LATE and TALE.) So let W_1 and W_2 be such that they have the same number of letter instances of the same letters but ordered differently. According to our definition, this means that every fusion of W_1 \mathbb{N} -overlaps with some fusion of W_2 , and *vice versa*. But since the letters are ordered differently, it must be the case that some fusion z_1 of W_1 \mathbb{N} -overlaps with some fusion z_2 of W_2 but that z_1 does not \mathcal{A} -overlap with z_2 (although, of course, it will \mathcal{A} -overlap with some other fusion of W_2). Thus,

by the Identity Theorem, $W_1 \neq W_2$. So each of the three criteria given above are satisfied.

Besides satisfying the three criteria, our approach also makes precise various phrases that we use to describe words. For example, to say that a word has a letter twice is to say, according to the definition of the number of letter instances, that the word contains two numerically distinct fusions that \mathcal{A} -overlap. (There is thus overlap in the word LETTERS.) And that two words are made of the same letters is made precise here by saying that every fusion in the first word \mathcal{A} -overlaps with a fusion in the second word, and *vice versa*. (In the appendix we give an exact formal definition of the first notion, together with some other definitions and theorems. We leave the exact definition of ‘made of the same letters’ as an exercise for the reader.)

Our definition is compatible with classical mereology. Not everyone will think this matters: everything we do using mereology and plural logic can also be done using Wetzel’s analysis of word types that employs set theory. But we think, with Nelson Goodman (1972), that set theoretical entities are deeply problematic and have no place in a final theory. We will not argue the point here and realise that this is a minority view. Still, for those unfamiliar with Goodman’s reasons for eschewing sets, we reiterate his reasons here. (Thanks to a reviewer for asking us to elaborate a bit on this.)

Goodman holds that there can be no distinction between entities without a distinction in ‘content’. What he means by this is that when you ‘break down’ two entities into their basic constituents, you should not get the same resulting objects. Goodman thus objects to set theory because in set theory you *can* get different entities that are ultimately made from the same basic entities. For example, from four objects a , b , c , and d you can get both $\{\{a, b\}\{c, d\}\}$ and $\{\{a, c\}\{b, d\}\}$, and these are two different sets. His reasons for holding that two composite entities cannot be, ultimately, made from the same basic objects are strongly related to his meta-philosophical position according to which a philosopher’s job is ‘to clarify, simplify, [and] explain [...] in understandable terms’ (Goodman, 1972, p. 168). Goodman finds it mysterious how one can get different entities if one starts from the same things and applies the same ‘generating’ or ‘constructing’ relation (be it set formation or mereological composition or any other relation that gives you ‘new’ entities from ‘old’ ones); therefore he does not use set theory. (For more discussion on Goodman’s position, see Lewis’ critical comments in (Lewis, 1991, pp. 38–41) and the response in (Cohnitz & Rossberg, 2006, p. 218ff) with whom we are in agreement on this.)

We realise that not everyone shares Goodman’s outlook and many seem to think that set theoretical results are not at all mysterious. Our result should nonetheless be interesting for everyone: few would suspect that one can construct entities whose identity depends on the order and number of occurrences of its more basic entities, using only plural logic and mereology.

Moreover, we commonly talk about word types as having parts. And our account does *some* justice to this idea for it says that letter types are parts of the fusions that are in word types. This is slightly better than pure n -tuple accounts of word types, for in these cases talk of parthood is not apt: set-membership is not a form of parthood. Moreover, when n -tuples are understood as nested pairs in the Kuratowski style, then it turns out that many letter types appear far more often in a word type than we would intuitively have thought. (If instead n -tuples are understood as functions from an index

set to a codomain, then—if the index set is \mathbb{N} —they ultimately contain the same basic elements as in our account.)

A final point we wish to address here concerns the status of \mathbb{N} . According to our definition, every letter in a word is a fusion of an object in \mathbb{N} and an object in \mathcal{A} . We expect three sorts of worries: one concerns the fact that a word type seems to be made of atomic letters. But our definition says that words are pluralities of composite objects. The second worry concerns the status of \mathbb{N} itself: are we not implicitly helping ourselves to the structure of the natural numbers? And, if we are supposing the natural number structure, does that not show that we are presupposing the functions of successor and addition, and thus use strictly more than merely mereology and plural logic to get our definition of word type? The final worry is whether we could not have done all this without a second plurality in addition to the plurality of letter types.

With respect to the first worry our reply is two-fold. First, and to repeat our structuralist outlook, we consider our definition a success because we can point to entities that can play the role of a word type. The necessary properties for playing the role of a word type were expressed by **Kind**, **Number**, and **Order**. As it turns out, having atomic letter components is simply not part of the job description. Nor is it necessary that words are mereologically composed of letter types.

Of course, this is not going to satisfy those metaphysicians who have less of a structuralist approach. Some might hold that it is simply obvious that word types are composed of atomic elements, in particular letter types. Our second response is a rejoinder to them: our account does do justice to the intuition that the components of a word are *ultimately* atomic, and that half of these components are letter types, because each of the fusions in those words may have atomic parts. That is, it might very well be that I and A are atomic and that the indefinite article I+A thus has two atomic parts. Our opponent should instead argue that there is nothing beyond atomic letters that goes into making word types. But we take it that it is not obvious there is nothing going on in making word types beyond letter types because the order of the letter types and number of times the letter types occur matters to the identity of the word types (see above). So it seems there is *something* going on beyond bringing letter types together when it comes to making word types. However, if they can argue that word types are made of only letter types, then we have to agree that our account fails because we need more than merely letter types. (But note that this argument would also apply to the more standard n -tuple account of, for example, Wetzel. If an n -tuple is understood as a function from an index set into a codomain of letter types, then more is needed than merely letter types. If, instead, an n -tuple is understood as a nested ordered pair, then the word types are made of more than just letters: they also contain the empty set and sets of letters. And in that case there are also ‘ultimately’ more letters in a word type than we intuitively think.)

However, what we need beyond letter types is rather arbitrary because we need not have used *numbers*. This brings us the second worry: are we implicitly supposing something like the Peano axioms by using the plurality \mathbb{N} ? We think not. Instead of using \mathbb{N} , we could have used any plurality that has at least as many objects as the length of the longest possible word type, as long as there is some (possibly conventional) way of imposing a total ordering on the members of the plurality. So instead of \mathbb{N} we could, for example, have used a plurality \mathbb{M} consisting of the minutes in a leap year.

(This plurality consists of 527, 040 distinct objects, which is sufficiently large so that we can define even the (current) longest alleged word, which is the full chemical name for the protein titin and counts 189, 819 letters. If we still need a larger plurality we could instead take the plurality of seconds in a leap year or the minutes from the year 0 CE up to today.) We would then have defined *letter combination* as any (full or partial) function from \mathbb{M} to \mathcal{A} , and defined a *possible word type* as any letter combination such that if it has a fusion that has some minute t as a part, then for every minute s before or simultaneous with t there is a fusion in the plurality that has s as a part. So, for example, the first member in a potential word type would then—by convention—have as a part 1 JANUARY - 00 : 00AM, whereas the second would have 1 JANUARY - 00 : 01AM, etc. (The whole enterprise above would be the same except for some differences in notation.) Since times are not adequately captured by the Peano axioms, or any other axiom system that describes the natural number system, we can be certain that they are not implicitly assumed when we used sub-pluralities of \mathbb{N} .

Indeed, all the theorems in the main text and the appendix apply to the more general notion of letter combinations. Because already there we have many pluralities with the same number of fusions but without these fusions overlapping completely. For example, there is a letter combination of the fusions 1+T, 2+A, 3+L, and 4+E; and another letter combination consisting of the fusions 5635+T, 3964+A, 8917+L, and 8186+E. These are distinct by Extensionality and Plural Identity. One simply has to pick one of these pluralities for the word type LATE. But this is an arbitrary choice, just as it is in set theory when one decides which construction counts as an ordered pair. So all we need is the plurality of letter types and another (sufficiently large) plurality together with an arbitrary rule that says of that second plurality which object in it counts as the first, which counts as the second, etc.

(To be sure, saying that something counts as the first, as the second, etc. does suppose that we can impose on ordinal order on the plurality. But we do not think this means that we end up accepting a non-extensional mereology. Indeed, the arch nominalist/hyperextensionalist Goodman happily used *concatenation* as a primitive while concatenation does suppose that one thing comes after another: ‘we shall write “ $Cxyz$ ” to mean that x and y and z are composed of whole characters of the language, in normal orientation to one another, (...) and that the inscription x consists of y followed by z ’ (Goodman & Quine, 1947, p. 112—our italics). This kind of structuring of entities is unobjectionable to the extensional mereologist for she does not deny that, say, words can be ordered alphabetically. The extensional mereologist merely denies that two wholes can be composed of the same parts. In that respect our definition of word types receives a clean bill of health.)

The final worry is whether this second plurality (be it of natural numbers or of seconds in a leap year or any other sufficiently large plurality) is really necessary. (The following was suggested by an anonymous reviewer). Could we not, for example, take the n -th letter of a word to be a fusion of that letter plus the n letters that come alphabetically after that letter (after skipping one). Thus, LATE is then the plurality consisting of the following four fusions L+N, A+C+D, T+V+W+X, and E+G+H+I+J. The problem with this proposal is that it limits the length of word types to the number of basic symbols in your alphabet minus two because after that you will have to fuse a symbol multiple times—something classical mereology cannot do.

One could remedy this by holding that the alphabet continues after Z with AA, AB, etc. But this only works if we take AA not as a fusion or plurality made from A (in some way) but instead as an atomic letter type. In that case AA has as much in common with A as it has with any other letter type. Indeed, once AA (and AB etc.) is taken to be an atomic letter type there seems to be no point in trying to construct word types out of letter types: most words are then just letters that are alphabetically later than the letter z (and some words may be both a word and a letter, such as A and I in English).

The original problem was that mereology and plural logic did not provide enough distinct objects. For example, TALE and LATTE were identified because we only had T, A, L, and E and their fusions to work with. Once an additional plurality is added (say, of natural numbers or of minutes in a leap year), the problem dissolves as long as we can keep track of their identity criteria. The fact that we ended up with more objects than necessary is not really a problem—and very much expected given that we took mereological composition to be unrestricted.

Before concluding, let us point out that our technique for constructing word types from letter types (plus an additional plurality) can also be used to construct sentence types from word types. (Here too we use an orthographic conception of sentence type; hence the semantic, syntactic, and phonological properties of sentences are ignored.) The most straightforward way to do this is by adding another countable plurality besides \mathcal{A} and \mathbb{N} . Take, for example, the rational numbers between 0 and 1 that can be represented as a fraction with a numerator of 1 and a denominator that is a power of 10. Let's call this plurality ' $\mathbb{Q}^{1/10^n}$ '. We can then let the first word type in a sentence type be a plurality of fusions that have three atomic parts: a letter from \mathcal{A} , a natural number from \mathbb{N} , and the rational number 0.1. The second word type in a sentence type is then a plurality of fusions composed of a letter from \mathcal{A} , a natural number from \mathbb{N} , and the rational number 0.01. Et cetera. For example, the sentence type TO BE OR NOT TO BE is then identified with the following plurality:

$$\begin{aligned} &T + 1 + 0.1, O + 2 + 0.1, B + 1 + 0.01, E + 2 + 0.01, O + 1 + 0.001, R + 2 + 0.001, \\ &N + 1 + 0.0001, O + 2 + 0.0001, T + 3 + 0.0001, T + 1 + 0.00001, O + 2 + 0.00001, \\ &B + 1 + 0.000001, E + 2 + 0.000001. \end{aligned}$$

We can then make sense of multiple occurrences of the same word type in terms of overlap. The two fusions in the subplurality $B+1 + 0.01, E+2 + 0.01$ and the two fusions in the subplurality $B+1 + 0.000001, E+2 + 0.000001$ are such that if a fusion of the first (second) subplurality \mathbb{N} -overlaps with a fusion of the second (first) subplurality, then these fusions also \mathcal{A} -overlap. Hence, they are the same word types. However, they can be said to be different occurrences of that word type because the fusions differ from each other with respect to their $\mathbb{Q}^{1/10^n}$ -parts.

5 On the type-token relation

We have constructed word types as pluralities of binary fusions consisting of a number and a letter type. A natural question is how this construction connects to the type-token relation: does our conception of word types explain how word types are related to their

tokens? (We thank a referee for this journal for asking us to address this question.) Some conceptions of word types go naturally with certain conceptions of the type-token relation. For example, if a type is a set of tokens, the tokens are set theoretical members of their type. And if types are universals, a token may be considered as an instances of its type. Our account does not force us to take a stance on the type-token relation, but there are various answers that are compatible with our account and that could be developed independently of our construction of word types as pluralities of binary fusions.

Before mentioning these answers, it is worth pointing out that the standard n -tuple account of, for example, Wetzel (1993) is also compatible with various answers as to the relation between a type and its tokens. If n -tuples are understood as functions from an index set to a codomain of letters, then one could hold that types are universals and tokens are instances of these universals. After all, functions seem to be abstract and repeatable; much like universals. Moreover, the domain and codomain of these functions seem to consist of universals too: numbers and letter types are commonly thought of as abstract and repeatable entities. Hence, if we follow this line of reasoning, types are universals and tokens are instances of their types.

But under the n -tuple account of word types one might also think of a word type as a program or algorithm telling you how to construct a token of its kind, where every token is an execution of the program. (Though note that there is ultimately little difference between this answer and the previous one if programs are universals.) Note that in both cases there is only one ‘part’ of the type that is instantiated: the objects from the domain only determine the order of instantiation of the letter types (or the order of the execution of the program), while it is the objects from the codomain that get instantiated/executed.

Things get more complex when n -tuples are understood as ordered pairs or nested sets. In the case of ordered pairs (Kuratowski style) the word type AND will be constructed as the set $\{\{A\}, \{A, \{\{N\}, \{N, D\}\}\}$. One can then of course still hold that word types are universals (after all, sets seem to be universals of some sort) and that tokens are instances of a universal, but the instantiation relation will be harder to describe than in the case of n -tuples as functions since some occurrences of letters should not be instantiated if we go the Kuratowski route.

We do not mean this as a criticism of any of these ways of constructing n -tuples. We merely want to illustrate that if one constructs word types as n -tuples, one can treat word types as universals, but one is not forced to do so and one could instead come up with a novel, or *sui generis* relation holding between types and tokens.

Much the same can be said about our construction of word types: if letter types and numbers are universals, then a plurality of fusions of these entities seems to be a universal too—albeit a ‘plural’ universal. It would make sense then to think of word tokens as instances of such universals. And just as the standard n -tuple account could conceive of word types as programs for constructing tokens, so can we.

We could also get very creative and go for a more thorough mereological conception of types. Here is a sketch of what such an account could look like. Letter types could be constructed as fusions of letter tokens. Moreover, since tokens are located in spacetime, a plurality consisting solely of fusions of the positions of the tokens could replace \mathbb{N} in our construction. For example, we may use ‘1’ to denote the first location type, i.e. the

fusion of all positions occupied by the first letter of any word token; and similarly for all other location types. The fusions in this plurality will be very peculiar since word tokens are located at various places: on paper, on screens, in sand, carved in trees, etc. These fusions will partly overlap. For example, in crossword puzzles a single position may both be the second and the fifth position of a certain word token. But the fusions will nonetheless be distinct since the majority of positions occupied by letter tokens do not play such a double role.

If we follow this route, a word type is a plurality consisting of fusions that have a letter type and a position type as parts. (These two parts are fusions of letter tokens and position tokens, respectively). And a word token is related to its type via its parts in the following sense: every letter token of a word token is part of one of the letter types that is in the word type, and every letter token occupies a part of one of the location types that is in the relevant word type. Hence the type-token relation is then completely analysable in terms of the part-whole relation of mereology and the *is one of* relation of plural logic.

6 Conclusion

Both classical mereology and plural logic fail to model certain structural properties. They ignore the order in which objects occur and cannot account for multiple occurrences or instances of an object. When working together, however, they can give an adequate definition of word types, which are entities that exhibit such structural features. So relatively blunt instruments can still bring you highly sophisticated results. It would be interesting to see how far this approach can be pushed. For example, the part-whole relation of structural universals and of states of affairs famously resist a straightforward characterisation consistent with classical mereology—and for much the same reasons as word types were thought to be beyond the pale (Bennett, 2013; Bigelow & Pargetter, 1989; Cotnoir, 2015; Forrest, 2016; Fisher, 2013; Hawley, 2010; Lewis, 1986; Mormann, 2010; Smith, 2009). We hope such entities can receive a similar treatment as word types but leave the details for future work.

It is philosophically interesting that our definition of word types locates, so to speak, structural aspects somewhere between the notion of a fusion and that of a plurality. Neither a fusion nor a plurality is a structured entity. Instead the structural aspects result from a certain interplay between them. This may provide evidence that some kinds of structure can be regarded as a supervenient phenomenon, a byproduct of something lacking order. Such deep metaphysical questions about the nature of order are beyond the scope of this paper, but we hope our result provides some guidance to this enterprise.

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Declaration

Conflict of interest The authors declare that they have no conflict of interest.

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Appendix

In this appendix we give a proof of the Identity Theorem, as well as some related theorems.

We start with a theorem to the effect that if two words have different letters at some point in the sequence, they are distinct:

(Order Theorem) $\exists z_1 \preceq W_1 \exists z_2 \preceq W_2 (z_1 \circ_{\mathbb{N}} z_2 \wedge \neg z_1 \circ_{\mathcal{A}} z_2) \rightarrow W_1 \neq W_2$

Proof Suppose that $z_1 \preceq W_1, z_2 \preceq W_2$ and that $z_1 \circ_{\mathbb{N}} z_2$ but $\neg z_1 \circ_{\mathcal{A}} z_2$. Let z_1 \mathbb{N} -overlap with z_2 at x . So $x \leq z_1$ and $x \leq z_2$. Since W_1 and W_2 are functions, we have that if $x \leq z_3$ and $z_3 \preceq W_1$, then $z_1 = z_3$. (And, similarly, if $x \leq z_3$ and $z_3 \preceq W_2$, then $z_2 = z_3$.) Since they do not \mathcal{A} -overlap, let us suppose that $y_1 \leq z_1$ and $y_2 \leq z_2$, where $y_1 \neq y_2$. So we have by Extensionality that $z_1 \neq z_2$. By our assumption, $z_1 \preceq W_1$; but $z_1 \not\preceq W_2$ since W_1 and W_2 are functions. Hence, by Plural Identity, $W_1 \neq W_2$ \square

We can define the number of letter types in a word W as the number of fusions in W that do not \mathcal{A} -overlap:

(Number of Letter Types) If $\exists z_1, \dots, \exists z_n \preceq W (\neg z_1 \circ_{\mathcal{A}} z_2 \wedge \neg z_1 \circ_{\mathcal{A}} z_3 \wedge \dots \wedge \neg z_2 \circ_{\mathcal{A}} z_3 \wedge \dots \wedge \neg z_{n-1} \circ_{\mathcal{A}} z_n) \wedge \forall v \preceq W ((v \neq z_1 \wedge \dots \wedge v \neq z_n) \rightarrow (v \circ_{\mathcal{A}} z_1 \vee \dots \vee v \circ_{\mathcal{A}} z_n))$, then W consists of n letter types, i.e. $\#W^{\mathcal{A}} = n$.

So the number of letter types in a word is determined by counting non- \mathcal{A} -overlapping fusions, i.e. fusions that \mathcal{A} -overlap count as one letter type. For example, in the word LATTE we count four letter types because the fusions 3+T and 4+T \mathcal{A} -overlap at T whereas all the other fusions are disjoint. From this definition it follows that if all members of a word \mathcal{A} -overlap with members of another word, then the second word has at least as many letter types as the first.

(Types Theorem) $\forall z_1 \preceq W_1 \exists z_2 \preceq W_2 (z_1 \circ_{\mathcal{A}} z_2) \rightarrow \#W_1^{\mathcal{A}} \leq \#W_2^{\mathcal{A}}$

Proof Suppose that for every fusion z_1 in W_1 there is a fusion z_2 in W_2 s.t. z_1 \mathcal{A} -overlaps with z_2 . Let $\#W_1^{\mathcal{A}} = m$. Thus by the definition of number of letter types, there are m fusions in W_1 that are not \mathcal{A} -overlapping. By our supposition, for every such fusion z_1 in W_1 , there is at least one fusion z_2 in W_2 such that $z_1 \circ_{\mathcal{A}} z_2$. Thus W_2 must have at least m fusions that are not \mathcal{A} -overlapping. Hence, $W_1^{\mathcal{A}} = m \leq W_2^{\mathcal{A}}$ \square

(From hereon, the symbols ‘ \preceq ’ and ‘ $<$ ’ are ambiguous: they can mean ‘is an improper part of’ and ‘is a proper part of’ or instead mean ‘is not greater than’ and ‘is strictly

smaller than'. Confusion will not arise, however, because the mereological relations are meant only when '≤' and '<' are flanked by (possibly indexed) variables $x, y, z,$ and $v.$)

We can also count the number of letter type *instances* in a word. This is done by counting the number of distinct fusions that overlap. We will use '#W^y' to symbolize the number of numerically distinct fusions in W that overlap at $y.$ We can make this precise thus:

(Number of Letter Instances) If $\exists z_1, \dots, \exists z_n \leq W((z_1 \neq z_2 \wedge z_1 \neq z_3 \wedge \dots \wedge z_{n-1} \neq z_n) \wedge y < z_1 \wedge z_1 \circ_{\mathcal{A}} z_2 \wedge z_2 \circ_{\mathcal{A}} z_3 \wedge \dots \wedge z_{n-1} \circ_{\mathcal{A}} z_n \wedge \forall v \leq W(y < v \rightarrow v = z_1 \vee \dots \vee v = z_n)),$

then n is the number of instances of the letter y in $W,$ i.e. $\#W^y = n.$

This definition states that the number of instances of a letter y in W is the number of numerically distinct fusions in W that have y as a part. We can show that words with a different number of letter instances of a specific letter type are distinct:

(Instances Theorem) $\#W_1^y \neq \#W_2^y \rightarrow W_1 \neq W_2$

Proof We show the contrapositive. Suppose $W_1 = W_2.$ Then by Plural Identity every fusion that is one of W_1 is also one of W_2 and *vice versa.* So W_1 has n fusions that overlap at y if and only if W_2 does, too. Hence $\#W_1^y = \#W_2^y.$ □

Finally, the main theorem:

(Identity Theorem) $(\forall z_1 \leq W_1 \exists z_2 \leq W_2 (z_1 \circ_{\mathbb{N}} z_2 \wedge z_1 \circ_{\mathcal{A}} z_2) \wedge \forall z_2 \leq W_2 \exists z_1 \leq W_1 (z_1 \circ_{\mathbb{N}} z_2 \wedge z_1 \circ_{\mathcal{A}} z_2)) \leftrightarrow W_1 = W_2$

Proof (\Rightarrow) Suppose the first conjunct of the antecedent, i.e. that for every fusion z in W_1 there is a fusion z_2 in W_2 s.t. z \mathbb{N} -overlaps and \mathcal{A} -overlaps with $z_2.$ Let z_1 be any fusion in $W_1.$ We show that z_1 is in $W_2,$ too. By our supposition, there is a z_2 in W_2 s.t. z_1 \mathbb{N} -overlaps and \mathcal{A} -overlaps with $z_2.$ Thus there is an $x \leq \mathbb{N}$ s.t. $x \leq z_1$ and $x \leq z_2;$ and there is a $y \leq \mathcal{A}$ s.t. $y \leq z_1$ and $y \leq z_2.$ By the definition of fusion: $z_1 = x + y = z_2.$ Hence, z_1 is in W_2

If we suppose the second conjunct of the antecedent, then similar reasoning shows that any fusion z_2 in W_2 is also in $W_1.$

Hence W_1 and W_2 have all the same fusions, thus by Plural Identity, $W_1 = W_2$

(\Leftarrow) We prove the contrapositive. Suppose that *either* (i) there is some $z_1 \leq W_1$ s.t. for all $z_2 \leq W_2$ if z_1 \mathbb{N} -overlaps with z_2 then it is not the case that z_1 \mathcal{A} -overlaps with z_2 *or* (ii) there is some $z_2 \leq W_2$ s.t. for all $z_1 \leq W_1$ if z_2 \mathbb{N} -overlaps with z_1 then it is not the case that z_2 \mathcal{A} -overlaps with $z_1.$

(Case 1) Let $z_1 = x_1 + y_1$ be a fusion in $W_1.$ Then by our supposition, there is no z_2 in W_2 s.t. $x_1 \leq z_2$ and $y_1 \leq z_2.$ So for every fusion z_2 in $W_2, z_2 \neq z_1.$ Hence z_1 is not in $W_2.$ Thus by Plural Identity, $W_1 \neq W_2.$

(Case 2) This case is analogous to the previous case. So in either case $W_1 \neq W_2.$ □

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