Enculturation and the historical origins of number words and concepts

César Frederico dos Santos^{1,2}

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Abstract

In the literature on enculturation—the thesis according to which higher cognitive capacities result from transformations in the brain driven by culture—numerical cognition is often cited as an example. A consequence of the enculturation account for numerical cognition is that individuals cannot acquire numerical competence if a symbolic system for numbers is not available in their cultural environment. This poses a problem for the explanation of the historical origins of numerical concepts and symbols. When a numeral system had not been created yet, people did not have the opportunity to acquire number concepts. But, if people did not have number concepts, how could they ever create a symbolic system *for numbers*? Here I propose an account of the invention of symbolic systems for numbers by anumeric people in the remote past that is compatible with the enculturation thesis. I suggest that symbols for numbers and number concepts may have emerged at the same time through the re-semantification of words whose meanings were originally non-numerical.

Keywords Enculturation · History of numbers · Number concepts · Numerical cognition · Philosophy of mathematics · Re-semantification

1 Introduction

According to the enculturation thesis, higher cognitive capacities, such as reading, writing, and calculating, result from transformations in the brain driven by cultural practices (Menary 2015; Fabry 2018). In the process of learning cultural artifacts such as symbolic systems, evolutionarily ancient neural circuits are recycled or reused and thus transformed into the new neuronal networks that implement higher cognitive functions. Mathematical cognition is often cited as an example of a higher cognitive

✉ César Frederico dos Santos
c.f.dos.santos@vu.nl ; cesar.frederico@ufma.br

¹ Department of Philosophy, Vrije Universiteit, Amsterdam, Netherlands

² Department of Philosophy, Federal University of Maranhão, São Luís, Brazil

capacity that is acquired through enculturation (Menary 2015; Jones 2020; Pantsar 2020; Fabry 2020). The importance of enculturation to mathematical cognition is suggested by findings and theories in the field of numerical cognition according to which culturally created symbols for numbers—number words and digits—play an indispensable role in the transition from our genetically evolved capacities to deal with discrete quantities, which are imprecise for collections of more than three or four items, to true numerical competence, which is precise no matter the size of the involved collections (for reviews of the main findings and theories in the field of numerical cognition, see Dehaene (2011), Gilmore et al. (2018), Nieder (2019), Knops (2020); for an account of how these findings and theories fit the enculturation framework, see Pantsar (2019)).

The enculturation thesis contrasts with nativistic accounts according to which we are born with pre-determined basic cognitive capacities and mental contents whose maturation leads naturally to the higher cognitive capacities observed in adults. In the field of numerical cognition, nativistic views are supported, for example, by Gelman and Gallistel (1978), and Butterworth (2005). In their accounts, symbols for numbers are seen as cultural creations that merely externalize internal numerical concepts whose origins are due to genetic evolution. In the enculturation account, by contrast, proper numerical concepts are not inborn. Although the proponents of the enculturation thesis do not deny that we are born with “proto-arithmetical” skills, they claim that *proper* arithmetical skills result from the integration between these genetically evolved proto-arithmetical skills, culturally created symbols for numbers, and culturally created numerical practices, such as counting and calculation algorithms (Pantsar 2019; Menary 2015; Fabry 2020). Accordingly, in the enculturation framework, the availability of symbols for numbers and numerical practices in the cultural environment where individuals are raised is believed to be a *precondition* for individuals to acquire proper numerical competence.

The enculturation thesis and, more generally, accounts of numerical cognition in which culturally created symbolic systems for numbers play an indispensable role in the acquisition of numerical competence, have been criticized by Pelland (2018a, b, 2020). He observes that, if the availability of symbols for numbers and numerical practices in the cultural environment where individuals are raised were a precondition for the emergence of numerical competence in individuals, then the very creation of symbols for numbers for the first time would become a mystery. Results from numerical cognition studies have shown that, by relying on our genetically evolved capacity to estimate the size of collections, we are unable to clearly distinguish between collections of, say, ten and eleven items (Gilmore et al. (2018, ch. 2), Knops (2020, p. 16ff.)). In the enculturation account, the only way to become capable of accurately distinguishing between such cardinal sizes is by learning to count, which requires the previous availability of a counting system in one’s cultural environment. As cultural artifacts, counting systems must have been created at some point in the past. But how could anyone have ever invented a counting system for the first time without knowing that there is a difference between, say, ten and eleven? “[H]ow ... can we rely on external symbols for numbers in our explanation of the development of numerical content when the existence of such symbols in turn depends on the existence of number concepts?” (Pelland 2018b, p. 185). According to Pelland, the existence of people in

the remote past who were able to develop numerical content from scratch, without the aid of previously available symbols for numbers,

goes against Menary's claim that mathematical practices "are part of the niche that we inherit—they are part of our cultural inheritance" (2015a, p. 16), since such practices have not been a part of everyone's cultural niche. People nowadays only inherit mathematical practices because they were invented (and thus not inherited) by other people in the past (Pelland 2020, p. 11).

For Pelland, the existence of people in the remote past who were able to develop numerical content from scratch should count as counterevidence against the claim that the availability of symbols for numbers and numerical practices in the cultural environment where individuals are raised is a precondition for individuals to acquire proper numerical competence. Pelland proposes that there must be another way of acquiring number concepts that does not require the aid of preexisting external symbols for numbers:

if we are trying to explain the ontogeny of number concepts, our theory should apply to *everyone* capable of thinking about numbers. But since some people seem to have been able to think about numbers without external aids in the (distant) past, any account that depends on such support will not apply to every case of numerical cognition. At best, such externalist accounts could describe how numerical cognition emerges in a numeral-enriched environment. Even so, the fact that it is possible to develop some basic number concepts without external support seems to suggest that cases that do involve external support might somehow appeal to a more fundamental process, which the externalist framework is leaving out. ... the ontogeny of number concepts in a world where symbols for numbers abound cannot be completely separated from past cases of numeral-free ontogeny since the former depends on the latter in important ways (Pelland 2018b, pp. 185–186).

Pelland is right in requiring that explanations of the ontogenetic development of numerical cognition be compatible with explanations of the historical emergence of number concepts. These stories are not independent of each other, since the pioneers of numerical cognition must have experienced an ontogenetic process at least similar in crucial aspects to the one today's children undergo. In face of this, Pelland suggests that the

externalist approach needs to be replaced by one that focuses more on internal cognitive processes, since any appeal to external symbols for numbers must come *after* we have explained the emergence of numerical cognition *internally*, given that external symbols for numbers depend on the construction of internal representations with numerical content for their existence (Pelland 2018b, p. 179).

One of the major difficulties that Pelland's suggestion faces is that such an internal cognitive capacity to engender number concepts in the absence of external aids has

never been detected.¹ On the contrary, putting aside nativistic accounts, it is a well-established fact that proper numerical competence does not arise without the aid of external symbols and practices. Yet, as Pelland convincingly argues, if the enculturation account is to be considered tenable, then an explanation of the emergence of symbolic systems for numbers “in cultures that lack any proto-numerical artefacts, practices, or experts to learn from” (Pelland 2020, p. 11) has to be provided.

In this paper, I propose an account of how symbolic systems for numbers may have been invented by anumeric people in the remote past that is compatible with the enculturation thesis. I assume Pelland’s desideratum that explanations of the ontogenetic development of numerical cognition in numeral-enriched environments should be compatible with explanations of the historical emergence of number concepts in numeral-free environments. However, I reverse Pelland’s reasoning. Instead of assuming that, in the absence of an external symbolic system for numbers, the pioneers of numerical cognition must have acquired numerical competence by means of internal cognitive processes only, I assume that any explanation of the historical emergence of numerical cognition should be built around the fact that it does not emerge without the aid of external symbolic resources. I propose that the pioneers of numerical cognition must have experienced external cognitive stimuli similar to the ones that produce number concepts in contemporary children. The question is which *non-numerical* symbols and practices available in their cultural environment are likely to have triggered the cognitive processes that produced number concepts in their minds for the first time. I draw on linguistic and cognitive studies of contemporary anumeric or few-number cultures² from the Amazon forest, such as the Pirahã (Everett 2005) and the speakers of Nadahup (aka Maku) languages (Epps 2006, 2013), to answer this question.

This paper is structured as follows. In Sect. 2, I briefly review the way children raised in numeral-enriched environments acquire number concepts according to the available evidence from numerical cognition and developmental psychology. In Sect. 3, I advance hypotheses about how the very first numeral systems may have emerged in anumeric cultures by means similar to the ones that produce number concepts in today’s children. In the remaining sections, I present data from cognitive and linguistic studies with anumeric and few-number cultures to support the suggested hypotheses.

2 How today’s children acquire number concepts

Humans and many non-human animals are born with the ability to perceive *numerosities* (a property of perceived collections approximately related with cardinality) (Agrillo 2015; McCrink and Birdsall 2015). Two regimes of numerosity perception have been identified: *subitizing*, through which the cardinal size of perceived collec-

¹ Unless one assumes a nativistic stance such as the one defended by Gelman and Gallistel (1978), which is not what Pelland does. Pelland acknowledges that there is “a gap between the rudimentary numerical content produced by our evolutionarily ancient brains and the arithmetically-viable numerical content that comes to be associated with numeration systems like Indo-Arabic or Roman numerals” (Pelland 2020, pp. 2–3).

² Few-number cultures are small-scale cultures whose languages have only a small list of number words (De Cruz et al. 2010).

tions of up to three or four items is accurately determined, and *estimation*, through which the cardinal size of perceived collections of more than three or four items is approximately determined, with error increasing as size increases (Gilmore et al. (2018, ch. 2), Knops (2020, p. 16ff.)). These genetically evolved abilities are fairly limited; by relying solely on subitizing and estimation, one cannot accurately discriminate between collections of nine items and collections of ten items, for example. To distinguish these genetically evolved abilities from the proper arithmetical abilities displayed by numerate humans, the former have been described as *quantal* (Núñez 2017) or *proto-arithmetical* (Pantsar 2014) abilities.

Putting aside nativistic accounts such as Gelman and Gallistel's (1978),³ the consensus view in numerical cognition is that the acquisition of number words and the mastery of the counting procedure is what allows children to bridge the gap between their limited quantal abilities and truly numerical competence (Dehaene 2011; Carey 2009; Gilmore et al. 2018; Knops 2020). This is accepted even by more recent nativistic accounts, such as Margolis & Laurence's (Margolis and Laurence 2008; Laurence and Margolis 2007), according to which concepts for numbers smaller than four or five are innate, whereas concepts for larger numbers are acquired through experience with culturally created symbolic systems for numbers.

While several different accounts of the cognitive processes that allow children to do the transition between their limited quantal skills and proper arithmetical competence have been proposed (I present Dehaene's (2011) and Carey's (2009) accounts below), there is a fairly general consensus among researchers about the stages through which children progress during this transition. Le Corre et al. (2006) established the now standard classification of the milestones in this process. In the earliest phase, children start learning the sequence of number words by rote, without understanding what these words mean (Wynn 1990). "Until the age of about 2 years, infants will predominantly sing-song numbers without attaching any meaning to them" (Knops 2020, p. 116). In a second phase, children start associating number words with their meanings progressively. In numerical cognition studies, children's progress is assessed by their ability to pass the so-called Give-N test (Wynn 1990). In this test, a certain number of objects—e.g., toy dinosaurs—is made available in a large bowl placed near the child being tested. Then the experimenter asks: "Could you give me n dinosaur/s?" If the child succeeds in giving exactly n dinosaurs and does not give the same quantity when another number is asked, this shows that she has conceptual understanding of the word ' n .'

Children's performance in the Give-N test has shown that they learn the meaning of the first number words in a piecemeal manner. First, they learn the meaning of the spoken number word for one, i.e., they respond correctly to the Give-One task. Le Corre et al. (2006) call children at this stage "one-knowers." Four or five months later, typically developing children learn the meaning of the number word for two, and then become "two-knowers" (they are able to succeed in the Give-One and Give-Two tasks). Other four or five months elapse until children learn the meaning of the number word for three and become "three-knowers" (they are able to succeed in the Give-One, Give-

³ For nativistic accounts such as Gelman and Gallistel's, there is no difficulty in explaining the historical origins of symbols for numbers, since they are seen as simply externalizing contents previously developed through genetic evolution. This is why I am putting them aside.

Two, and Give-Three tasks). Le Corre et al. (2006) collectively call children at these three initial stages *subset-knowers*, because they know only a subset of the sequence of number words. Other four or five months later, when children finally learn the meaning of the number word for four, their comprehension of number words progresses faster: they become able to pass the Give-N task for arbitrarily larger numbers up to the limit of their already-memorized list of number words. This coincides with the moment when children have finally mastered the counting procedure, and thus can make use of it to respond to the Give-N task no matter the number asked.

Mastering the counting procedure is not a simple task. It is widely accepted that, in order to learn how to count, children have to learn to correctly apply the following five rules:

1. *One-to-one Correspondence*: each item of the counted collection must be paired with one and only one number word.
2. *Stable Order of Counting Words*: the order in which number words are used for tagging items must follow a stable order, i.e., the order must be kept constant across different counting events.
3. *Order Irrelevance of Items*: the order in which items are paired with number words is irrelevant, i.e., the order may change across different counting events.
4. *Abstraction*: counting applies to any collection of all sorts of discrete objects.
5. *Cardinality*: the number word used for tagging the last item of a collection represents the cardinality of the whole collection.

These five rules are known as the *counting principles*, and their first formulation is due to Gelman and Gallistel (1978). The first four principles determine how to count. They summarize the procedural knowledge one must have in order to count. The last principle, known as the cardinality principle (CP), refers to the information one obtains by counting, namely, the cardinality of the counted collection.

Le Corre et al. (2006) call “CP-knowers” children who are able to pass the Give-N test for arbitrary number words in their memorized list. This is because knowledge of the cardinality principle is what enables children to rely on the counting procedure in order to pass the Give-N test for numbers larger than three or four. Although subset-knowers may be able to count satisfactorily, many times applying the first four counting principles correctly, they display poor understanding of the cardinality principle. This is shown by the fact that, just after counting a collection, if asked how many objects they have just counted, subset-knowers do not answer with the last word they have used; they prefer to recount the set, clearly showing that they do not yet understand that the last word used in a counting event represents the cardinality of the whole collection (Wynn 1990; Le Corre et al. 2006).

For typically developing children in industrialized societies, it takes about one and a half years to go from one-knower to CP-knower (Le Corre et al. 2006). During this process, children are believed to construct mental contents that progressively endow number words with meaning. In the literature on numerical cognition, these mental contents are called “number representations” or “number concepts.”

The cognitive processes that underlie the construction of number concepts in children’s minds are a controversial topic in the field of numerical cognition. One point of contention is the role each of the cognitive systems believed to implement in the brain

the abilities to subitize and estimate plays in the construction of number concepts. To date, the prevailing view is that humans and non-humans animals share two basic non-verbal mechanisms to detect numerosities: a precise system for smaller numerosities, called the Object Tracking System (OTS), Object File System (OFS), or Parallel Individuation System (PIS), responsible for the ability to subitize; and an imprecise system for larger numerosities, called the Approximate (or Analog) Number System (ANS), responsible for the ability to estimate (Feigenson et al. 2004).

In Dehaene's account of the cognitive processes underlying the construction of number concepts, children learn the meaning of number words by mapping them onto representations of numerosities provided by the ANS (Dehaene 2011, 2009). The ANS produces fuzzy representations of numerosities (estimates), whose fuzziness increases proportionally to the sizes of the perceived collections. Thus, initially, the mapping of number words onto the ANS provides number words with approximate meaning only. Over time, however, as children become proficient in using number words and counting, this mapping causes ANS fuzzy representations to become exact representations of number. In other words, ANS fuzzy representations are sharpened by their recurring association with symbols for numbers (Verguts and Fias 2004). In this account, even the meanings of the words for numbers from one to three (or four) are provided by the ANS (Dehaene and Cohen 1995), although Dehaene acknowledges that subitizing is implemented by the OTS. "Somehow, around the age of 3 or 4, these two systems [OTS and ANS] snap together" in the construction of number concepts (Dehaene 2011, p. 260). The transformation of fuzzy ANS estimates of numerosities into precise representations of numbers is an instance of what Dehaene calls "neuronal recycling" (Dehaene 2005). The claim is that preexisting brain circuitry that originally served the ANS (by delivering fuzzy estimates) is transformed so that it can implement a new function, namely, ascribing number words with meanings by delivering precise representations of number.

Menary (2014) endorses Dehaene's account, and makes neuronal recycling (embedded in the notion of "learning driven neural plasticity") one of the main tenets of his account of mathematical cognition as a case of enculturation (Menary 2015). In spite of this, there are other accounts of the cognitive processes through which number concepts are constructed in children's minds that do not involve neuronal recycling and that are also compatible with the enculturation framework (at least insofar as neuronal recycling is not viewed as a defining attribute of enculturation). This is the case of Carey's (2009) account.

Carey's account of the cognitive processes underlying the construction of number concepts is based on the observation that subset-knowers cannot rely on counting to pass the Give-One, Give-Two, and Give-Three tests, since "children in the subset-knower period do not know the significance of counting" (Carey et al. 2017, p. 244). The natural hypothesis is that the ability to subitize plays an important role in the acquisition of the meaning of the words for one, two, and three or four, since by subitizing we can grasp the cardinal size of collections of up to three or four items without counting them (Carey 2009; Carey et al. 2017). Thus, Carey proposes that children learn the meaning of the first three or four number words by mapping them onto representations provided by the cognitive system behind subitizing, the PIS. Since the PIS does not contain representations of numbers (it represents cardinal sizes

implicitly by establishing one-to-one correspondences between the objects of the target collection and its memory slots or “object files” (Izard et al. 2008; Simon 1997)), Carey postulates the existence of “enriched parallel individuation.” Enriched PI is the PIS enriched with the capacity to build long-term memory representations of collections of one to four items (Carey et al. 2017). In a first stage of number learning, the recurring association of the words for one, two, and three or four with representations produced by enriched parallel individuation has the power of fixing these representations as the meanings of the first number words. In a second stage, when number concepts for the three or four smallest numbers are in place and children have already mastered the cardinality principle, children are believed to generalize a semantical rule. They realize that the meaning of a number word corresponds to the meaning of the previous word in the counting sequence plus one.⁴ In this manner, they infer the meaning of ‘five’ from the meaning of ‘four’, the meaning of ‘six’ from the meaning of ‘five’, and so on, for all the number words in their already-memorized counting list. However, this semantic generalization does not occur immediately after children become CP-knowers. Recently turned CP-knowers are more likely to have just mastered a purely procedural rule according to which the last number used in a count is the answer to the question ‘how many?’ (Davidson et al. 2012; Spaepen et al. 2018). Becoming a “mechanical” CP-knower is a pre-requisite for making the semantic generalization (Carey and Barner 2019, p. 831).

For Carey, as for Dehaene, culturally created symbolic systems for numbers play an indispensable role in the acquisition of numerical competence. In this sense, both Carey’s and Dehaene’s accounts are in line with Menary’s enculturation thesis. Even Margolis & Laurence’s (Margolis and Laurence 2008; Laurence and Margolis 2007) nativistic account does not need to be seen as fully incompatible with the enculturation thesis. True enough, in their account concepts for one, two, and three (or four) are innate, but they acknowledge that beyond this limit “an external structured symbol system helps children to extend the innate system” (Laurence and Margolis 2007, p. 147). This extension is done by means of a semantical generalization quite similar to the one proposed by Carey, in which the counting sequence plays an indispensable role (Margolis and Laurence 2008). Thus, in Margolis & Laurence’s account, just as in Carey’s and Dehaene’s, concepts for numbers larger than three or four also result from a process of enculturation.

Although Carey’s proposal follows more closely the various stages through which children acquire number concepts, Dehaene’s account can also make sense of children’s transition across these stages. As in Carey’s account, in Dehaene’s account it can be said that children learn the meaning of the words for the three or four smallest numbers first because they can clearly perceive collections with the corresponding cardinalities by subitizing; the estimates that the ANS produces for such collections

⁴ This generalization, known as the bootstrapping step, has been criticized on the basis that children, equipped only with the cognitive capacities Carey ascribes to them, would not be able to make the correct generalization. Beck (2017) gives an overview of the criticisms the bootstrapping notion has received, and proposes a solution which is most in line with the re-semantification hypothesis I advance in the next section. Basically, Beck proposes that “computational constraints” are the external elements that help children make the correct generalization. Beck’s “computational constraints” roughly correspond to what I call, following Krämer (2003), “operational rules.” They constrain the way number words are used in sentences and for counting, endowing number words with conceptual roles from which children can infer new contents.

are already sufficiently precise to provide these words with meanings more easily.⁵ The discontinuity in the learning process observed when children learn the meaning of the number word for four is also explainable. For quantities above three or four, ANS estimates become too fuzzy to allow children clearly distinguish the size of collections of more than three or four items. Thus, children will not accurately notice the difference in size between larger collections before they have learned to count. When children become “mechanical” CP-knowers, i.e., when they learn the mechanical procedure through which a different number word is ascribed to collections with different sizes even if their difference in size is not noticeable through the ANS, the process of sharpening the ANS approximate representations of number can start. Over time, the recurring association between the same number word (the last used in a counting event) and the same cardinal size will sharpen the corresponding ANS estimate until it becomes an exact representation of number.

Thus, in both Dehaene’s and Carey’s accounts, number words and the counting procedure play an indispensable role in the construction of number concepts. In Dehaene’s account, if children do not learn number words and the counting procedure, they cannot obtain concepts for numbers above three or four (although they may have something close to innate concepts for numbers below four provided by the ANS). In Carey’s account, if children do not learn words for one, two, and three, they cannot obtain the corresponding concepts, since it is the use of these words that promotes the fixation of representations provided by enriched parallel individuation in long-term memory. And, if children do not learn to count, they cannot acquire concepts for larger numbers, since they will not be able to generalize the rule according to which the meaning of a number word in the counting sequence corresponds to the meaning of the previous word in the sequence plus one. The indispensability of culturally created external symbols for numbers to the acquisition of numerical content is what makes the historical emergence of numerical content puzzling. In the next section, I propose a solution to this puzzle.

3 The re-semantification hypothesis

We saw above that children initially experience number words as meaningless and counting as a mechanical procedure. Even recently turned CP-knowers, who are able to determine precisely that a collection has, say, ‘five’ or ‘six’ elements, do not yet have a clear idea of the numbers corresponding to these words (Davidson et al. 2012; Spaepen et al. 2018). Conceptual understanding comes only later. As Knops (2020, p. 115) describes it, “over several years during development, there may exist this gap between procedural knowledge (i.e. reciting the count words) and conceptual knowledge (i.e. the understanding of numeral meaning).”

The observation that children can recite number words and execute the counting procedure mechanically, without knowing their meanings, shows that having number

⁵ A consequence of there being two systems responsible for numerosity perception is that numerosities with one to four items may be represented twice in the brain (Dehaene 2011, p. 258). A number of studies have shown that, under certain circumstances, both the ANS and PIS may be recruited for the perception of small collections (see, e.g., Izard et al. (2008), Hyde (2011), Chesney and Haladjian (2011)).

concepts is not a pre-requisite for using number words and executing the counting procedure. In historical terms, this opens up the possibility that number words and the counting routine may have been created in the absence of number concepts. For Pelland, however, this possibility is a non-starter,

since any appeal to external symbols for numbers must come *after* we have explained the emergence of numerical cognition *internally*, given that external symbols for numbers depend on the construction of internal representations with numerical content for their existence (Pelland 2018b, p. 179).

And yet, no such internal cognitive capacity to engender number concepts in the absence of external aids has ever been detected, as mentioned above. How can we get out of this paradoxical situation?

The first thing to take into account is that symbols are not only a means of expression and denotation. If we think of symbols as only a means to express ideas that were previously engendered in someone's mind or to denote things that exist in the world, then it seems impossible that a symbol could come into existence before the emergence of the idea or thing it refers to. But symbols serve also an *operational* function (Krämer 2003). When symbols are used to perform operations, there is no need to refer to something previously available; symbols can be “de-semanticized” and become mere tokens that are manipulated according to certain rules in order to achieve a certain outcome.

Krämer (2003) introduces the concept of *de-semanticization* in the context of what she calls “operative writing.” Some examples of systems of operative writing are the formal languages of logic, the programming languages of computer science, and the language of school arithmetic. As opposed to other systems of writing, the primary function of systems of operative writing is not the composition of texts for communication, but the solution of problems or the fulfilment of cognitive tasks. Systems of operative writing are cognitive tools in that they facilitate, improve or enable the performance of certain operations for which our “bare brains” are limited or are not completely suitable (Dascal 2002; Heersmink 2013). Besides syntactic and semantic rules, these systems also include *operational rules* that specify all or some of the actions that must or can be performed in order to solve a problem. For example, formal systems of logic have inference rules that determine which symbolic transformations are allowed in the system so that theorems can be derived from axioms. Finding a chain of authorized transformations that starts with the axioms and finishes with a target formula is a way of showing that this formula is a theorem of the system. As Krämer puts it, systems of operative writing are at the same time “a medium for representing a realm of cognitive phenomena” and “a tool for operating hands-on with these phenomena in order to solve problems or to prove theories pertaining to this cognitive realm” (Krämer 2003, p. 522).

A remarkable property of operational rules is that they can be operated mechanically, i.e., without the agent needing to pay attention either to the purpose of the rules or to the meanings of the symbols she is operating with. Usually, both the operational rules and the symbols of a system of operative writing have an intended semantics, but the agent can temporally “turn off” their semantical content and just make the symbolic transformations prescribed by the rules. This is where de-semanticization comes

in: when manipulated mechanically, symbols are no longer seen as signs standing for something else, but become self-contained objects, mere links in the chain of steps that are required to fulfil a cognitive task.

Counting can be described as a symbolic system whose purpose is the performance of a cognitive operation, similarly to Krämer's systems of operative writing, with the difference that counting is oral. When counting, we do not write down symbols, but utter words as a means to fulfill a cognitive task, namely, the assessment of the cardinality of a collection. The counting procedure, just as systems of operative writing, can be seen as consisting of two parts: a set of symbols and a set of rules that govern how symbols should be used, the operational rules. The set of symbols consists of an ordered list of words. The operational rules are those codified in Gelman and Galistel's counting principles. As operational rules, the counting principles specify the actions that must be performed so as to determine the cardinality of collections with more than three or four items. This is why counting words can be de-semanticized and the counting procedure can be operated mechanically, without the agent necessarily knowing what she is doing. Provided that the agent applies the operational rules appropriately, the outcome will be correct, regardless of her awareness of this.

Thus, in principle at least, anumeric people could execute the counting procedure by following the five operational rules codified in the counting principles. In historical terms, this opens up the possibility that anumeric people in the remote past started using certain meaningless words in a routine similar to counting, which ended up eliciting in their minds the production of number concepts through cognitive processes similar to the ones contemporary children undergo (this is the so-called “ritual” or “eeny, meeny, miny, mo” hypothesis about the origins of number words which I discuss in Sect. 7). But why would people ever engage in a (for them) meaningless practice? And how could they ever create such a (for them) meaningless practice?

The phenomenon of *re-semanticization* may be the answer. Perhaps the relevant words and practices were not meaningless, but have other meanings before they were re-semanticized as number words. Re-semanticization, as Dutilh Novaes (2012) describes it, refers to the action of giving to a formalism a semantic interpretation that was not the one intended when the formalism was first developed. Simply put, “[t]he idea behind re-semanticization is that a formalism which is developed to characterize a specific phenomenon *A* can then be reinterpreted on another phenomenon *B*” (Dutilh Novaes 2012, p. 204). Reinterpretation of formal systems is a common practice in mathematics and logic, specially in model theory, where the many models that a given axiomatic system has besides its intended one (if any) is a topic of investigation.⁶ But the cases of re-semanticization that I am interested in here are significantly different from mere cases of reinterpretation, in that it is the *de*-semanticization of the operational symbolic system under consideration and the mechanical computations that ensue that make room for the creation of a new interpretation of it (more on this in Sect. 7).

⁶ Whether, in model theory, formalisms are literally reinterpreted or mathematicians just discover models that have existed all along is a controversial topic in the philosophy of mathematics. Be that as it may, the point here is that even the discovery of a preexisting model may be seen as resulting, in cognitive terms, from a process of re-semanticization. See Dutilh Novaes (2012, pp. 204–206).

Such a radical form of re-semanticization opens up the possibility that the symbols that eventually gave rise to number words were not symbols for numbers since the beginning. It may be that originally meaningful but *non-numerical* symbols have been used for the operation of a cognitive task where they were de-semanticized. If this cognitive task produced the right kind of cognitive stimuli, the de-semanticized symbols could end up engendering number concepts, and could be subsequently re-semanticized as number words. But which cognitive practice could generate the right kind of stimuli?

The operation of a mechanical procedure following the five counting principles is what triggers the development of concepts for numbers above three or four in today's children. A cognitive task similar to counting, but one that does not require previous competence with numbers for its invention, might have served the same function for our ancestors. The natural candidates are tallying techniques based on the principle of one-to-one correspondence. It is easy to see that tallies are regulated by at least three of the counting principles: one-to-one correspondence, order irrelevance of pairing acts, and abstraction. The two remaining principles—stable order and cardinality—may have been suggested by a particular kind of tally, viz., body-part tallies, as we will see below (Hurford 1987; Wiese 2007). The hypothesis is that, by using purely mechanical tallying procedures regulated by the five counting principles, our ancestors may have ended up activating the cognitive processes that engender number concepts. Then the symbols they used for tallying were re-semanticized to refer to number concepts. I explain how purely mechanical tallying procedures may have emerged in anumeric cultures in Sect. 5. The emergence of proper number words from body-part tallying systems (when names of body parts or descriptions of gestures are re-semanticized as number words) is the topic of Sects. 6 and 7.

When it comes to the first three or four pairs of number words/concepts, the solution may be a bit simpler. We saw above that the acquisition of the first three or four number concepts takes place before the counting procedure is fully mastered. This means that, in a hypothetical historical picture of the emergence of number words/concepts for the first time, the words/concepts for one, two, and three may have emerged even before the invention of tallying techniques, although they cannot have emerged independently of symbols to refer to the corresponding numerosities (at least in Carey's account). One possibility is that inflections to represent grammatical number may have given rise to the first pairs of number words/concepts (Carey 2009). Other possibility is that words that originally designated particular collections with the relevant numerosities ended up being co-opted to refer to the size of these collections, thus giving rise to the first pairs of number words/concepts (Decock 2008). I consider these two hypotheses in Sect. 4.

If these hypotheses are tenable, the historical emergence of numerical cognition in originally anumeric cultures is made compatible with the ontogenetic processes through which people living in cultures where number words are available acquire number concepts, and therefore compatible with the enculturation framework. These are hypotheses about historical processes and, as such, are in need of empirical support. To provide the required evidence, however, is not a simple task, since the history of today's fully-fledged numeral systems is lost in the past. As Hurford (1987, p. 82) puts it “[t]he remoteness in time and space of the origins of numeral systems, together

with the possible effects of cultural and linguistic mixing and borrowing, make the evolutionary picture hard, if not impossible, to discern by any method resembling direct observation.” Where direct observation fails us, we need to recruit other methods. Linguistic studies of contemporary anumeric or few-number cultures are a valuable source of data for this task. As we will see in the next sections, evidence gathered in these studies lends support to the hypotheses outlined above.

4 The first number words/concepts: one, two, three

It is controversial whether the concepts for one, two, and three (and possibly four) are innate. As mentioned above, in Margolis & Laurence’s account they are, and in Dehaene’s account the ANS representations of small numerosities can be conceived of as being quite close to representations of numbers, since they are much more precise than representations of larger numerosities (although some sharpening is still needed, since in the ANS “number line” the difference between two and three is larger than the difference between one and two). If these concepts are innate, then their creation is due to genetic evolution and we have to explain only the creation of words for designating them. Otherwise, if they are not innate (such as in Carey’s account), then their creation depended on the previous creation of words that could trigger the cognitive processes that give rise to them. Here I assume the non-nativistic scenario, although the story below can be easily simplified and turned into a story of the emergence of words in a scenario where innate concepts were already available.

There are some linguistic clues that indicate that words for the first three or four numbers do not originate in a counting or tallying technique. In many languages, words for the smallest numbers inflect, i.e., agree in gender or case with other words in their syntactic environment. According to Hurford (1987, p. 114),

[i]t is of the essence of a rote-learned sequence of words that each word have a single form ... So numeral words which originate in the recited rote-learned sequence would be expected to be uninflected. The occurrence of variant inflected forms of words for 1, 2, 3 (and 4) suggests that these words originate in ways more closely integrated with their eventual use as modifiers of nouns indicating collections of things.

In addition to languages in which words for the smallest numbers inflect, there are also some languages in which the words used in the counting sequence for small numbers are completely different from the words used for the same numbers in other contexts. For example, in Kombai, a language spoken in New Guinea, the words for one and two are, respectively, *mofenadi* and *molumo* (or *lumo*), whereas in their body-based counting sequence the words for one and two are *raga* and *ragaragu*, respectively (De Vries 1994).

Following Hurford’s suggestion, Carey proposes that words for the smallest numbers may have been derived from grammatical number. Grammatical number differs from the word class of numerals. Grammatical number is a category of inflections that distinguish references to one item (singular) from references to more than one item (plural). Some languages also have inflections to distinguish references to collections

of two items (dual) and three items (trial). In English, grammatical number is marked, e.g., by the use of the plural suffix -s and by the singular article ‘a’/‘an’. Carey proposes that

historically, the initial meaning of “one” overlapped substantially that of the singular determiner “a,” and ... the initial meaning of “two” overlapped substantially that for dual markers in languages that have them, and the initial meaning of “three” overlapped substantially that for a trial marker (Carey 2009, p. 323).

Carey’s suggestion is viable in a non-nativistic scenario, since the use of singular/dual/trial/plural markers does not require previous familiarity with numbers. To see why, consider how duals are used in Arabic. In this language, the dual form is obtained by the addition of a suffix to the singular form. Thus, in Arabic it is possible to say “I read two books” without using the Arabic number word for two, by using an expression roughly translated as “I read book.dual” (FCLangMedia 2014). Numerical competence is not required in order to understand the use of the dual suffix because the ability to subitize suffices here. In fact, the same goes for the singular/plural distinction in English: because subitizing/estimation allows us to clearly distinguish a single object from collections of two or more, plural and singular nouns can be used independently of numerical competence. Even so, users of grammatical number have already met one of the conditions for the development of number concepts, since they are already paying attention and referring to cardinalities. In Carey’s account, this means that the fixation of representations produced by enriched parallel individuation can start to take place. In Dehaene’s account, the use of grammatical number to refer to small numerosities may be what starts the process of sharpening numerosity representations of approximately one, two or three items, resulting in exact representations of these numbers (by equalizing the differences between one and two, and between two and three in the ANS “number line”).

Another process of creation for words/concepts for the smallest numbers is suggested by Epps’s (2006) studies of the development of numeral systems in Nadahup languages. The Nadahup family includes four documented languages—Nadëb, Dâw, Hup, and Yuhup—whose numeral systems’ upper limits range from three to 20. Interestingly, Nadahup numerals still preserve transparent etymologies, “a cross-linguistically unusual feature suggestive of their relatively recent development” (Epps 2006, p. 259). Nadahup numerals for the smallest numbers are not derived from grammatical number markers, but from words with other, non-numerical, meanings. In Hup, the number word for one—*ʔayǔp*—seems to be derived from the demonstrative pronoun ‘*yúp*’ (meaning “that”), which is used for abstract, absent or intangible entities. In Yuhup, the number word for one—*câh*—also means “other,” and a variant of it—*câhyápã*—means “other individual.” The words for two and three reveal even more interesting etymologies. In Hup, one of the variants of the word for two—*kəwəg-ʔap*—literally means “eye quantity,” and one of the variants of the word for three—*mót-wig-ʔap*—literally means “rubber tree seed quantity.” Eyes are a universal paradigmatic case of a collection that always comes in pairs, whereas rubber tree seeds are familiar triplets in Hup speakers’ environment.

The rubber tree (*hevea* sp.) has a large, distinctive, three-lobed seed or nut (in Hup, *mót-wig*) which is culturally highly salient, being used among the Hupd'əh and other peoples of the region to make a popular children's toy, and associated with an edible fruit (Epps 2006, p. 264).

The Hupd'əh are the people who speak Hup. But how could rubber tree seeds and pairs of eyes give rise to number concepts? Decock (2008, p. 464) suggests that anumeric societies could use immutable collections of permanent objects—which he calls “canonical collections”—as standards against which other collections can be compared by one-to-one correspondence. This seems to be exactly what Hup speakers have done. The distinct lobes *a*, *b*, and *c* of a rubber tree seed can be put in one-to-one correspondence with the distinct objects *d*, *e*, and *f* of another collection, and then the phrase “rubber tree seed quantity” can be used to refer to the cardinality of the latter. It must be noted that this one-to-one correspondence operation does not need to be actively conducted by the agent. By relying on the parallel individuation system, the agent can do this automatically. The only thing agents needed to actively do was to select the rubber tree seed as a canonical collection. In doing so, over time the association between the phrase “rubber tree seed quantity” and collections of three would cause enriched parallel individuation to store a long term representation of a collection of the three lobes of rubber tree seeds, or of generic objects *a*, *b*, *c*, which then becomes the meaning of the phrase/word “rubber-tree-seed-quantity.” In Dehaene's account of number acquisition, the ANS representation of numerosities consisting of approximately three items would be sharpened by the association of the phrase “rubber tree seed quantity” with collections of three, giving rise to the concept of three.

The use of canonical collections to designate the numbers two and three is also present in the other Nadahup languages (except for Nadëb) and in other Amazonian languages. For example, in Xerênte, a language of the Je family spoken thousands of kilometers away from the region where Nadahup languages are spoken, the etymology of the word for two—*ponkwanē*—traces back to a phrase that translates into English as “deer footprint,” and the etymology of the word for three—*mrēpranē*—traces back to a phrase for “rhea bird footprint” (Melo 2007, p. 102). Deer footprints have two distinctive toes, and rhea birds' footprints have three toes.

The strategy of using canonical collections is less evocative for one, since each clearly distinguishable individual object may be as salient as every other when it comes to quantity. In Nadahup languages, the etymology of the words for one refers to “that” or “other individual,” as mentioned above. Epps et al. (2012, p. 66) report that in many Australian languages the word for one also means “alone” or “together” (no parts); in the Amazonian language Xerênte the word for one—*smĩsi*—also means “alone” (Melo 2007, p. 102). Although the meanings of all of these words involve the idea of unit, notice that the application of these words do not require numerical competence, since subitizing suffices. Only when one of these words is selected to refer to the perceived numerosity of singletons does the process of number concept formation take place and the concept of one arise (according to Carey's account, at least).

Grammatical number and canonical collections are two ways through which the first pairs of number words/concepts may have appeared. The same strategies, however,

cannot give rise to words/concepts for higher numbers. Grammatical number is not apt to make precise distinctions above the subitizing range, since it is bounded by the limits of subitizing. For the same reason, the selection of canonical collections of four or more items is not viable without the development of an active one-to-one correspondence procedure. The creation of tallying techniques is the answer.

5 Tallies: active one-to-one correspondence

Through subitizing, anumeric people can establish one-to-one correspondences between collections of up to three or four elements. It is controversial, though, if they can do the same when it comes to larger collections (Gordon 2004; Frank et al. 2008; Everett and Madora 2012). In what follows, I assume the hardest scenario for an explanation of the emergence of tallying techniques, wherein anumeric people are not naturally able to establish one-to-one correspondences between collections with more than three or four items and therefore have to create a specific technique for this.

A first thing to note is that one-to-one correspondence provides a non-numerical method of determining the cardinal size of collections. This is easily illustrated as follows. Imagine we want to know whether the cardinality of the set of people in a room is equal to the cardinality of the set of chairs in the same room. We do not need to count people and chairs. We can just ask people to sit down, each person in a single chair. If no person remains standing and no chair remains empty, we conclude immediately that both sets have the same size. If someone remains standing, then the set of people is larger than the set of chairs; inversely, if any chair remains empty, then the set of chairs is larger than the set of people. No number is involved in this procedure. It can be carried out recruiting only the notion of one-to-one correspondence or equinumerosity, which, despite its name, is defined in mathematics without invoking any numerical concept (Enderton 1977, p. 129).

When we evaluate cardinal sizes by one-to-one correspondence, we use a collection—call it the *model collection*—to “measure” the size of the *target collection*. For example, I can use the set P of pencils on my desk as a model collection, and then say that the cardinality of the set of sections in this paper is equal to P ’s. P is not a number, but I just used it to express a cardinal size in the same way I could have used the number eight.⁷

Tallying techniques, in which materials such as bones, sticks, knotted cords, pebbles, fingers, and toes constitute model collections, have been used by many cultures across the world. Virtually anything that lends itself to be seen as a collection of discrete, easily individualizable objects or marks can be used as a model collection for

⁷ It could be objected that, by using P and one-to-one correspondence, I’m not actually *determining* the cardinality of the set of sections in this paper, but just *comparing* it with P ’s cardinality. But notice that, by counting, what we do is just a comparison as well: we compare the cardinality of the target collection with the cardinality of the set of numbers, as if the sequence of numbers (the natural number line) were a “ruler.” When we say that the cardinality of the target collection is eight, this means that there is a one-to-one correspondence between the elements of the target collection and the segment of the sequence of numbers from one to eight. Therefore, assessments of cardinality always involve a comparison by one-to-one correspondence. When we use numbers, the difference is that the comparison is made against a standard ordered collection.

tallying. But how could anumeric people, or people who know only numbers from one to three or four, have invented tallies?

Anthropologists who study the processes of innovation in human cultures and biologists who study non-human animals' capacity to innovate concur that inventions usually result from the recombination of preexisting elements motivated by necessity (O'Brien and Shennan 2010; Reader et al. 2016; Laland 2017). "Necessity is the mother of innovation" is an often cited motto in the area, although there are other factors, such as opportunity and luck, which can prompt innovation as well. The invention of tallying techniques, however, does not seem to be an exception to the rule; it was probably motivated by necessity.

Although it is difficult to determine the specific necessity that led to the invention of the first tallies, we may speculate that a problem posed by the quantal abilities themselves may have contributed to this. Some have suggested that the exact perception of numerosities provided by the PIS can be felt as conflicting with the fuzzy perception of numerosities provided by the ANS. Thus, "[a]s we apply these different systems to the same objects, events and scenes, we appear to be driven to reconcile the representations that they yield" (Feigenson et al. 2004, p. 313). Barner (2017, p. 554) elaborates on this point:

perception provides humans with an explanatory problem that the creation of symbolic number systems is meant to solve. This problem, confronted by humans from the beginning of our shared cultural history, can be expressed as follows: whereas our perception of quantity is noisy and subject to error, our perception of individual things is not. Consequently, despite our noisy representation of number, we have a strong intuition that collections in the world are made up of distinct individuals, such that they must contain determinate numbers of things that are subject to precise measurement. ... Counting systems, I propose, were constructed by our ancestors to resolve this explanatory gap—to measure and keep track of the precise quantities that we knew to exist in the world, but otherwise are unable to precisely quantify (Barner 2017, p. 554).

Although the disparity between the exactness of subitizing and the fuzziness of estimation may, in itself, be seen as a problem to be solved, it is unlikely that, without other practical needs, our ancestors would have bothered to solve this problem. As the existence of the anumeric Pirahã (Gordon 2004; Frank et al. 2008) shows, this internal "conflict" is not sufficient to prompt the invention of counting. The natural attitude towards large collections is estimation, and estimation usually suffices, unless there is a real need for exactness. Once such a necessity appears, then people probably will notice the internal "conflict" and try to extend the exactness of subitizing to the range of estimation.

There are a number of activities in small-scale societies which may ask for exactness. One of the most obvious is the tracking of time, and many small-scale societies have developed tallying systems specially for this purpose. For example, the Korowai from New Guinea use a simple tallying device, called a *saiindal*, to keep track of the passing days. De Vries (1994, p. 562) describes it.

It consists of the rib of a leaf of the sago palm tree, into which a number of pegs or bits of wood have been inserted. When somebody invites someone else to a feast, for example, he will hand over [a] *saiindal* together with the invitation; the person invited will take one peg out of the *saiindal* every day, and when he has reached the last peg, which is twice as long as the others, the day of the feast will have arrived.

The Korowai are far from being anumeric (they use a body-part counting system whose upper limit is 38), but it is easy to see that anumeric people could operate devices such as the *saiindal*. To operate it, it is sufficient to understand that to each peg corresponds a day, and to be able to take one peg out every day. Both abilities are provided by the parallel individuation system. The operator of a *saiindal* does not need to count or to use any other technique; the only thing she has to do is to take one peg out every day. When there are no remaining pegs, she knows that the feast day has come.

In fact, subitizing seems to be the ability that makes the establishment of one-to-one correspondences possible. In an experiment with the anumeric Pirahã from the Brazilian Amazon, Gordon (2004) tested them in a one-to-one matching task where a number of AA batteries were disposed in a line, and participants were asked to produce an identical row of batteries exactly below the presented one. The Pirahã succeeded when there were up to three batteries, but failed with larger quantities. For larger quantities, their performance was compatible with the use of estimation (Gordon 2004, p. 496). Their failure with numbers above the subitizing limit indicates that they did not decompose the task into smaller tasks manageable with subitizing. For example, if they had thought of a row of six batteries as a shorter row of three juxtaposed with another row of three, they could have succeeded. More generally, any collection of objects can be decomposed into singletons, which can be easily dealt with using subitizing. Instead of proceeding in this way, though, the Pirahã estimated the numerosity of the whole line of items and then tried to deploy a set with the same quantity by estimating it again. What the Pirahã did not realize is that they could have shifted their attentional focus from the whole collection to its parts.⁸ The ability to shift the attentional focus in this way does not seem to require any kind of innovation. However, that this can be done in order to successfully complete the task seemingly requires an insight.

Tallying techniques may have been invented when people realized that any two collections, no matter how big they are, can be compared if decomposed into singletons, which can be successively paired with each other, relying on successive subitizations for this. If inventions come from the recombination of previously available elements motivated by some form of need, then anumeric people, or people who know only numbers from one to three or four, are in the position to invent tallies once the need arises, given that they already have the elements to recombine: (i) the ability to shift the

⁸ When tested in the same task by Frank et al. (2008), the Pirahã succeeded for quantities larger than three. Based on this result, Frank et al. (2008) concluded that the Pirahã can naturally establish one-to-one correspondences in linear arrangements, and attributed their failure in Gordon's experiment to the AA batteries moving around inadvertently, what may have distracted them. However, in a follow-up study, Everett and Madora (2012) raised the suspicion that participants in Frank et al. (2008) have been previously trained in one-to-one matching tasks by Madora without Frank et al. knowing this.

attentional focus from a collection to its elements, (ii) subitizing, and (iii) collections of discrete objects that can be used as model collections to measure the cardinal size of other collections. There is no reason to doubt that human creativity is capable of assembling these elements to develop tallying techniques.

Once tallies are in use, people who master a tallying procedure are on track to acquiring number concepts. Tallying methods can be seen as precursors to counting because they are cognitive tools governed by three out of the five rules that govern counting: the principle of one-to-one correspondence (obviously), the principle of order irrelevance of objects, and the principle of abstraction. In its tallying version, the principle of order irrelevance can be rephrased as stating that the order in which items of the target collection are paired with items of the model collection is irrelevant, i.e., it may change across different tallying events. The principle of abstraction, in turn, regulates tallies as long as people realize that collections of different sorts of objects can be tallied.

But two of the counting principles may still be missing in tallies. One of them is the cardinality principle, which in its tallying version would say that the model collection obtained at the end of a tallying event should be seen as representing the cardinal size of the target collection. This does not seem to be the case with certain tallies. When using a *saündal*, for example, the agent may not be especially interested in how many days have to go until the feast takes place, but only in getting there on the right day. The *saündal* is used to compute the right day, rather than represent a cardinal size. Although the operator of a *saündal* could think of the collection of remaining pegs as representing a number, she does not need to do so in order to succeed in attending the feast on the right day. The other principle missing is the stable order of counting words. In its tallying version, this principle would say that the items of the model collection should be used in a stable order across different tallying events. In certain tallies, this would be not only counterproductive, but also very hard to do, given that the items (e.g., pegs) of the model collections are hardly distinguishable from each other.

Lacking these two principles, tallying systems are not likely to give rise to proper counting and number words/concepts.⁹ As Hurford (1987, p. 79) points out, this is because “sticks, or buttons, or whatever, have no names, and therefore th[ese] system[s] provid[e] no names for the numbers themselves.” Without symbols associated with quantities on a regular basis, number concepts are unlikely to be created in the mind. What is needed for the emergence of number words/concepts is a tallying technique based on named objects, which could be paired with the items of target collections

⁹ Overmann et al. (2011) and Malafouris (2013), Malafouris (2010) claim that the sole use of material artifacts—such as tally sticks, clay tokens, etc.—could give rise to number concepts. However, the lack of two counting principles in tallying systems that rely exclusively on hardly distinguishable material artifacts makes this possibility unlikely, as I argued above. Model collections with distinguishable items, such as those employed in the system of clay tokens in Sumer (Schmandt-Besserat 1992) could overcome this difficulty, but Malafouris’s explanation of the emergence of number concepts by means of the Sumerian system of clay tokens is probably anachronic. Given the complexity of Sumerian society by the time the system of clay tokens was being developed, it is highly unlikely that they hadn’t yet developed number words for larger numbers (Overmann (2016) calls attention to this point). The development of the system of clay tokens in Sumer is better seen as a response to the need to record numerical quantities in permanent media, rather than to the need to count, which could have been solved earlier with words or simpler techniques.

always in the same order, and whose names could be recited along with pairing acts. Body-part tallying systems display these features.

Typically, the fingers and other bodyparts involved [in tallying] have verbal names. While pointing at a bodypart sign for a number, the verbal name of that bodypart can be uttered (Hurford 1987, p. 80).

With this method [body-part tallies] ... there are still no verbal names for numbers, but the various bodyparts, fingers, and so on could themselves be taken as signs for the numbers. And there is, at this stage, a conventional ordering of these bodypart signs (Hurford 1987, p. 80).

In the next section we will see the relationship between body-part tallying systems and the emergence of words/concepts for numbers larger than three or four.

6 Four and beyond: tallies with body parts

Fingers, toes, and other body parts seem to have played a central role in the emergence of number words (Owens et al. 2018). The origins of these systems can be seen as quite similar to the origins of tallying techniques based on pegs, sticks or notches, discussed in the previous section. Body-part tallies differ only with regard to the model collection their creators have chosen: fingers and, if necessary, toes and other body parts. This choice, however, had the fundamental consequence of paving the way for the creation of number words. Hurford's proposal is that, through a gradual process of lexicalization, the names of the body parts uttered along with pairing acts in body-part tallies gave rise to numerals.

The normal processes of language-change over a long period would lead to words which originally had bodypart associations losing these associations and becoming pure numeral, or at least counting, words. There would tend to be a phonological split, reflecting the clear semantic difference between number and bodypart concepts (Hurford 1987, p. 82).

At the same time that a process of lexicalization may have given rise to number words, a process of re-semanticization may have given rise to number concepts. Body-part tallies provide all the sufficient conditions for the emergence of number concepts according to the accounts of number acquisition we saw in section 2. The first thing to notice is that, just as tallying with notches in bones or bundles of sticks, tallying with fingers and other body parts does not require previous familiarity with number concepts. Thus, anumeric or few-number cultures can develop such techniques. However, even if conceptual understanding of numbers is absent in the beginning, the point is that number concepts can arise after the mastery of a body-part tallying system is obtained. This happens because body-part tallying naturally suggests the two counting principles that are absent in other methods of tallying: stable order and cardinality. Wiese (2007) explains how, starting with the principle of stable order.

The use of fingers (and other body parts) as tallies can lead to the emergence of a stable conventional order and hence give rise to a second stage in number development: when fingers are used to represent elements of another set, they tend to be singled out successively, following the natural order of fingers on each hand. ... In this order one could, for instance, start with the thumb on one hand, go all the way to the little finger, and then use the fingers of the second hand in the same way. As the differences in finger counting in modern cultures show, other orders are possible as well, of course; what is important here is that the salient order of fingers on each hand will support a convention for singling out individual fingers successively in a fixed order. ... Given that body tallies are frequently accompanied by verbal tallies (namely the names for the body parts in question), a stable conventional order of fingers used in cardinality icons will lead to a stable conventional order of words (Wiese 2007, p. 766).

Once there is a stable order of words being used regularly in body-part tallying events, the cardinality principle may be seen as a natural consequence.

The final word in a sequence is always more salient and more accessible than the others. This leads, for instance, to ‘recency effects’ shown in memory experiments: the last word in a list can be better recalled and memorised than the others (probably based on a buffer in short-term memory).

This leads to a prominent status of the final word that is used in an iconic cardinality representation. Once the words are used in stable order, for a set of a given cardinality, the same word will always come last and hence be particularly salient for the representation of this cardinality. This, then, will support the emergence of indexical links between individual words and sets of a certain cardinality (Wiese 2007, p. 767).

Cognitively, these indexical links take the form of number concepts, i.e., the idea of exact cardinal sizes that are associated with the last word used in a body-part tally. Once the five counting principles are being used in a tallying procedure, number concepts will emerge by the same means as they emerge in today’s children. Then, once number concepts for numbers larger than three or four are in place, the body-part names uttered along with pairing acts will be re-semanticized—they will start referring to numbers in the context of tallying—and thus the first pairs of words/concepts for numbers larger than three or four come to light.

A vivid illustration of the different stages through which number words/concepts may have stemmed from body-part tallying systems can be seen in Nadahup languages. Dâw, one of the Nadahup languages, has consolidated number words only for numbers from one to three (Epps 2006). The Dâw use a body-part tallying system, but one which lacks a crucial feature and thus has prevented them from developing other number words/concepts. For quantities above three, Dâw speakers use a tallying technique that Epps (2006) calls the “fraternal strategy” and Martins (1994) calls the “even/odd system.” As Martins describes it, in this technique, a representation of a collection of four items is made by separating four fingers of one hand (the thumb is kept bent) into two groups of two fingers. This gesture is accompanied by the words *mẽ’n mab*, which translate into English as “has a sibling.” The reason is that, in this configuration, each

raised finger is accompanied by another finger, i.e., each finger has a “sibling.” To obtain a representation of a collection of five items, the thumb is raised, accompanied by the words *mē’n mab mēr*, i.e., “has no sibling,” indicating that the thumb is alone. For six, the thumb of the other hand is placed against the first thumb to make a new pair, and then it is said again that it “has a sibling,” and so on up to ten, when all fingers of both hands are grouped in five pairs and the process finishes with the words “has a sibling.” In this system, one and the same expression is associated with multiple cardinalities. The phrase *mē’n mab* is associated with four, six, eight, and ten, whereas *mē’n mab mēr* is associated with five, seven, and nine. Without a single expression for each cardinality, the corresponding number words/concepts cannot arise.¹⁰

Despite the fact that in Dâw the fraternal strategy did not give rise to words for four and up, in Hup and Yuhup the etymology of the word for four is clearly traceable back to this tallying technique. Importantly, Hup and Yuhup speakers no longer use the fraternal tallying system, and thus the expression “has a sibling,” in these languages, is uniquely associated with four. Currently they use a base-five body-part tallying system which starts with their words for one, two, and three, includes a fraternal word for four, and goes on with phrases that translate into English as “one hand” (five, the base), “one other finger stands up” (six), “two other fingers stand up” (seven), “three other fingers stand up” (eight), “four other fingers stand up” (nine) and “five other fingers stand up” or “both hands” (ten). Above ten, counting goes to toes and the system becomes ambiguous. The phrases that accompany pairing acts in the interval between 11 and 14 are repeated for pairing acts between 16 and 19.

Not all of these phrases are already lexicalized and constitute real couples of number words/concepts detached from the tallying system. Epps (2006, p. 270–271) reports that

Numerals greater than ‘five’ are more likely to receive an accompanying gesture (usually a tally on the fingers), and although most Hup numerals do not depend on a gesture for their exact value to be understood, it may be difficult to label gesture as secondary (at least for those speakers who always combine the two). In the case of the ambiguous numerals like those between 11–14 and 16–19 (as with 4–10 in Dâw), however, the spoken forms are themselves not enough to indicate an exact value, and can therefore be considered dependent on gesture. Much as S. Martins (2004: 392) notes for Dâw, Hup speakers rarely make use of their numeral terms over ‘five’. The expression *dab* “many” is commonly used, and borrowed Portuguese numerals are typically preferred (particularly by younger speakers) when a specific numeral larger than ‘five’ is required. ... Ospina (2002: 455) notes that the Yuhup numerals are frequently accompanied by gestures involving the fingers (and sometimes even the feet for higher values).

These data suggest that, in Hup and Yuhup, only the words for numbers from one to five are fully associated with consolidated cardinal values. The other “numerals” are still seen just as phrases that accompany tallies. This suggestion is corroborated by a closer look at the level of lexicalization of candidate numerals. The

¹⁰ That *mē’n mab* and *mē’n mab mēr* are not number words nor are associated with number concepts is further indicated by the observation that young Dâw speakers use a hybrid system where words for numbers above three are borrowed from Portuguese (Martins 1994, p. 93).

phrases that originated the words for four and five are already lexicalized as true numerals. In Hup, four is *hi-bab'ní*, whose etymology is analysed by Epps as “(fact)-have.sibling/accompany.nmlz.” It is interesting to note that the word ends with a nominalizer (nmlz), which converts the original phrase into a noun or, more precisely, a numeral. This clearly shows that Hup speakers see the cardinal value corresponding to four as an independent concept. In Yuhup, four is *bab-ní-w'ǎp*, whose etymology is analysed by Epps as “has-sibling-quantity.” Here the use of the suffix *-ǎp* (quantity) is what shows that Yuhup speakers are referring to the cardinal value of four, and not to the gesture or to the idea of having a sibling. The word for five in Hup has a few variants. One variant is not a word, but the phrase “one hand” (*ʔayǎp d'apǎh*). But there is also another variant that displays a process of lexicalization through phonological reduction: *ʔadapǎh*. The Yuhup word for five has only one variant—*cāh-pōh-w'ǎp*—where the suffix *ǎp* shows that speakers are referring to the cardinal value of five. In contrast to the words for one to five, the phrases that accompany tallies for values above five present several variants and do not show signs of lexicalization. To give just one example, Epps (2006, p. 271) identified the following variants in Hup for the phrase accompanying the gesture for six:

<i>cāp cob cakg'et ʔayǎp</i>	“other finger stands up one”
<i>ʔayǎp cob cakg'et</i>	“one finger stands up”
<i>cāp cob popǒg</i>	“other finger RED-big (=thumb)”

These are full phrases, with no signs of lexicalization. This observation, along with the fact that Hup and Yuhup speakers usually show their fingers when referring to six and up, and that youngsters prefer to use Portuguese numerals above five, suggests that five is the limit of consolidated number words in Hup and Yuhup. To reach that many, though, they must already have broken the barrier of subitizing, and they managed to do so by using a body-part tallying system.

In Hup and Yuhup, the expressions for numbers from six to ten are not ambiguous; all the sufficient conditions for the emergence of number concepts corresponding to these expressions (and their subsequent lexicalization) seem to be in place. However, judging by the absence of lexicalization of the corresponding expressions, it seems that cardinal values for numbers above five have not yet emerged. Why? Ontogenetic studies have shown that the speed of development of number concepts depends on the amount of exposure to situations where numbers are used. Piantadosi et al. (2012) suggest that the transition across stages of number knowledge (from one-knower to CP-knower) is driven to a large extent by the input a child receives: the greater the exposure to numbers, the faster the acquisition. This has been confirmed in a study conducted with Tsimané children (Piantadosi et al. 2014). The Tsimané are a farming-foraging group who lives in the Bolivian Amazon. Tsimané children undergo the same knower levels as English speaking children do, but between two and six years later, because number words are used less frequently in their culture. Thus, we can suppose that among the Hupd'əh and the Yuhup, hunter-gatherers who live in even smaller societies, the use of numbers above five (and the production of tallies for quantities greater than five) is so rare that people never develop concepts for them, even if they have mastered a process which, if used more frequently, could lead them to acquire such

concepts. The Hupd'əh and the Yuhup may be in the stage identified by Davidson et al. (2012) where the subject has generalized the cardinality principle only as a mechanical procedure. That is, the Hupd'əh and the Yuhup may be able to realize that the final gesture made and the final phrase uttered at the end of a tallying event represent the cardinality of the whole collection, and thus can be used to denote it, even if they have not yet formed the corresponding idea of an abstract number designated by these expressions. This would explain why the phrases for six and up were not lexicalized. There is no point in using the suffix *-ǎp* (quantity) or nominalizing the phrases to refer to a concept that they do not have. In the absence of a new concept, the phrases are not re-semanticized and speakers simply give the literal description of the tallying gesture.

If my analysis is correct, the Nadahup languages give us the opportunity to see a numeral system under construction. The building blocks are words brought in from a non-numerical context—descriptions of gestures—and a tallying procedure. The operational use of these words for tallying leads to their re-semanticization, and eventually the new concepts so produced prompt the transformation of the originally non-numerical words into proper numerals. Let us take a closer look into the process of re-semanticization that makes number concepts appear where they had not been before.

7 A closer look at re-semanticization

One condition for re-semanticization is the use of symbols for the performance of cognitive operations, instead of denotation or communication, since this opens up the possibility of them being de-semanticized. This condition is met by the words recited during the operation of a body-part tally. In the Hup and Yuhup systems, the recited words are literally describing the gestures the operator makes with her hands (e.g., “one other finger stands up”). These descriptions can be de-semanticized because they do not aim at communicating or denoting anything. The words recited during a tallying event are best seen as being addressed to the operators themselves, probably with the same purpose as when we subvocalize counting words: to help single out objects and/or raise fingers sequentially, so that the counter/tallier does not lose track of the one-to-one matches she is making.

There is experimental evidence that raising the exact number of fingers to represent the size of a collection can be difficult for people who have not undergone adequate training. In an experiment with deaf individuals who did not learn to count nor use a conventional sign language (but who are able to communicate with their families through a home-developed system of signs), Spaepen et al. (2011) showed that, although participants were able to use fingers to express quantities in certain circumstances, “they do not consistently extend the correct number of fingers when communicating about sets greater than three” (Spaepen et al. 2011, p. 3163). These home-signers learned to use fingers to represent the size of collections, but they did not learn a method to do so consistently. Without following a stable, coordinated procedure, they cannot raise the right number of fingers for quantities above the subitizing limit. The act of raising fingers to tally, which is easy for trained people, may be demanding for untrained ones.

There is also experimental evidence for the positive effect of private speech (speech spoken to oneself) on self-regulation. Research on this topic started with Vygotsky (1978), who highlighted the operational role of language in serving as a self-regulatory tool for developing children. In a more recent study, Winsler et al. (2007) tested five-year-old children in a counting task. They asked children to tap a peg with a toy hammer a certain number of times. They instructed children either to count aloud or to keep silent while performing the task, and observed that children's performance improved significantly when they counted aloud, probably because speaking the counting words aloud helped them more efficiently self-regulate their actions. It has also been observed that both children and adults use private speech more in tasks that they find more difficult, and that the use of private speech decreases as subjects become more skillful at the task (Winsler 2009).

Taken together, the fact that singling out fingers sequentially in a tallying procedure may be difficult for people with little practice at this task (as is the case for people who are just inventing the procedure or perform it only occasionally), and the fact that private speech helps self-regulation in demanding tasks, are in line with the hypothesis that the words recited in body-part tallying serve to self-regulate tallying gestures and thus improve performance. If this is so, these words are used for operation (rather than for communication or denotation), and one condition for de- and re-semanticization is met.

A second condition for de- and re-semanticization is the use of symbols in a mechanical procedure. The concepts of de-semanticization and re-semanticization were originally introduced in the context of operative writing and formal systems. Body-part tallying techniques do not involve written language and are not like formal systems in many aspects. Nevertheless, body-part tallying and formal systems have at least two relevant similarities that make their operation automatable. First, both are regulated by clear-cut operational rules. As we saw above, tallying systems in general are regulated by modified versions of three counting principles (one-to-one correspondence, order irrelevance of pairing acts, and abstraction), and body-part tallying adds two more principles, stable order and cardinality. Second, the phrases recited and the gestures made during the production of body-part tallies, just like formulas of a formal system, are produced by clear-cut syntactic rules. For example, the rule in the Hup and Yuhup base-five strategy says that fingers of the second hand are raised one by one followed by a phrase formed through the pattern “(1, 2, 3, 4, 5) other finger(s) stand(s) up.” These simple rules, once mastered, allow for the mechanical execution of the tallying procedure, so that the operator does not have to pay attention to the meaning of the words she is uttering. The more automatized the procedure, the more “ritualized” the role of the words uttered becomes, and the weaker their association with their original meanings. In this way, the words may end up de-semanticized. In fact, linguists have proposed that number words may have originated from the ritualistic execution of a procedure involving meaningless words. The following passages by Hurford summarize what he calls “the ritual hypothesis:”

The Ritual (or ‘Eeny, meeny, miny, mo’) Hypothesis is that at a stage before the development of proper numeral words, rituals exist in which sequences of words which have no referential, propositional, or conceptual meaning are recited while

the human actor simultaneously points (in some way) to objects in a collection (Hurford 1987, pp. 102–103).

The Ritual Hypothesis being put forward for examination here is that numeral systems arose out of counting, developed as a method of achieving a practical purpose simply and reliably, using a conventional sequence of recited words ... The sequences of words used in such rituals would become interpreted numerically (Hurford 1987, p. 104).

Hurford's ritual hypothesis is quite similar to what I am proposing here, with the difference that, in my account, the words employed in the ritual were not originally meaningless. They were descriptions of the actions performed in tallying events that, because of their operational role in a procedure executed ritualistically (mechanically), were de-semanticized. Importantly, the ritualization/automatization of the procedure is crucial for its successful execution, since it prevents distraction. If at each step the operator stopped to grasp how many fingers she had already used, she could lose track of the tally. The procedure is more accurate if executed uninterruptedly.

Words de-semanticized by the ritualistic production of tallies provide all the psychological conditions for the emergence of number concepts. Like today's children in the earliest stages of number learning, people using a body-part tallying system are reciting a sequence of meaningless words during the execution of a procedure governed by the five counting principles. At a certain point, the de-semanticization of the final word of a tallying event makes room for its association with a new meaning. In Dehaene's account of number acquisition, this new meaning is the fuzzy ANS representation of the numerosity displayed in the fingers. Over time, the recurring association between symbol and numerosity will sharpen a representation of the latter and give rise to a proper number concept. In Carey's account, the new meaning associated to the re-semanticized word is a number concept inferred from the previously formed concepts for smaller numbers. Either way, as new concepts emerge and the original words are re-semanticized, the creation of new words through processes of lexicalization is encouraged.

Tallying techniques enabled our ancestors and enable contemporary people who live in few-number cultures to evaluate the cardinality of collections with more than three or four items precisely, which they could not do by relying solely on their quantal abilities. Tallying techniques give access to information otherwise unavailable; once this information surfaces, new concepts and words to designate them can be formed.

8 Conclusion

As quoted in the introduction to this paper, Pelland asks: "how can we rely on external symbols for numbers in our explanation of the development of numerical content when the existence of such symbols in turn depends on the existence of number concepts?" The story presented above partially confirms Pelland's concern. In fact, we cannot rely on external symbols *for numbers* in our explanation of the emergence of numerical content for the first time. The point, however, is that numerical content

may have emerged from originally *non-numerical* external symbols, as shown above. When used in tallying techniques of the proper kind—body-part tallying systems in conformity with the five counting principles—non-numerical symbols may have triggered the cognitive processes that generate number concepts (the same that act in today’s children’s minds), leading to their re-semanticization as true numerals. In this way, the first pairs of number words and number concepts may have appeared *together*. Crucial for this is the fact that tallying techniques are non-numerical, since they are based on the establishment of a one-to-one correspondence between a model collection and a target collection, neither of which consisting of numbers or numerals. Thus, one can do a tally and use it to keep track of the cardinality of large collections of discrete objects or events even if she does not know the corresponding numbers. The ability to subitize, in conjunction with the treatment of large collections as a sequence of singletons, suffices.

The historical account presented here is in line with Menary’s enculturation framework because, here, the emergence of numerical content for the first time, among anumeric people, still comes from an integration between internal factors (such as the ability to subitize) and external, culturally created factors, such as gestures, words describing gestures, and a mechanical practice. The enculturation thesis is usually presented as also involving changes in the brain carried out by processes described as neuronal recycling or neural reuse (Jones 2020). As we have seen, these processes are key in Dehaene’s account of the acquisition of number concepts, but play no role in Carey’s account. The historical account presented here is compatible with both Carey’s and Dehaene’s accounts (and even with Laurence & Margolis’s nativistic account) and, as such, makes no particular claims about the role of neuronal reuse/recycling in the emergence of numeric content for the first time.

Notwithstanding the accommodation proposed here of the historical origins of number words and concepts within the enculturation framework, Pelland is right in pointing out that the existence of people in the remote past who were able to develop numerical content without the aid of previously available symbols *for numbers*,

goes against Menary’s claim that mathematical practices “are part of the niche that we inherit—they are part of our cultural inheritance” (Menary 2015, p. 16), since such practices have not been a part of everyone’s cultural niche. People nowadays only inherit mathematical practices because they were invented (and thus not inherited) by other people in the past (Pelland 2020, p. 11).

This criticism applies not only to the invention of symbols for numbers, but also to every other aspect of mathematical cognition that is believed to result from a process of enculturation. For example, if the idea of negative numbers results from the enculturation of certain symbolic practices involving symbols we currently use to refer to negative numbers, an explanation of how these symbols may have been created before the idea of negative numbers had emerged has to be provided. Processes of cumulative cultural evolution may help explain how arithmetic developed from tallying techniques such as the ones presented above into its current state (Pantsar 2019, p. 10). As Pantsar puts it, “the real challenge [for the enculturation framework] is in moving from the general idea that mathematical cognition is enculturated to satisfactory explanations of how this actually happens,” at both ontogenetic and historical levels. Here I pre-

sented an account of how the process of enculturation that resulted in the emergence of numerical cognition may have started in the remote past.

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