

Diversity and rights: a social choice-theoretic analysis of the possibility of public reason

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Abstract Public reason liberalism takes as its starting point the deep and irreconcilable diversity we find characterizing liberal societies. This deep and irreconcilable diversity creates problems for social order. One method for adjudicating these conflicts is through the use of rights. This paper is about the ability of such rights to adjudicate disputes when *perspectival disagreements*—or disagreements over how to categorize objects in the world—obtain. We present both formal possibility and impossibility results for rights structures under varying degrees of perspectival diversity. We show that though perspectival diversity appears to be a troubling problem for the prospect of stable social order, if rights are defined properly then disagreements can likely be resolved in a consistent manner, achieving social cooperation rather than conflict.

Keywords Public reason · Liberalism · Diversity · Rights · Social choice theory

1 Introduction

Public reason liberalism takes as its focus the deep and irreconcilable diversity we find characterizing liberal societies. This deep and irreconcilable diversity creates problems for social order: given that we disagree with one another so sharply, we often make conflicting claims on one another, claims that must be adjudicated. Of course, public reason liberalism does not *merely* seek to adjudicate the sorts of disputes that arise

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out of our deep and irreconcilable disagreements—a capricious dictator randomly choosing sides could suffice for that. Rather, public reason liberals hope to solve the problem of social conflict and cooperation in a manner that is justified to all. But still, a focus on the adjudication of conflicting claims is at the core of public reason.

Rights are often proposed as a mechanism for resolving disputes. Consider an example. Althea is a Christian, Bertha a Hindu, and Cassidy an atheist. There is disagreement as to what it is persons should worship (if anything at all) and in what manner persons should worship (if they even should at all). One way of resolving this conflict is to grant dictatorial prerogative to one person's preferences: perhaps Althea gets her way and everyone must be a Christian. And for much of human history such was the case. But a different way of resolving this conflict—and this is the liberal solution—gives each citizen a right to do as she wishes: Althea may spend Sundays in church; Bertha may worship Shiva and refrain from eating cow; and Cassidy can go about her business unopposed by the divine. Here, a right to freedom of conscience resolves the disagreement by giving all persons a private sphere to worship as they please.

Recent work, however, calls into question the ability of rights (and dispute-adjudication mechanisms more generally) to resolve conflicts by focusing on what we shall call *perspectival disagreements*. In short, perspectival disagreements are disagreements over how to categorize objects in the world. Though public reason liberals have never denied the presence of these sorts of disputes, careful focus on perspectival disagreements raises unique problems public reason liberals have hitherto not adequately addressed. One issue concerns whether rights are actually capable of adjudicating our disputes when such disagreements obtain. As we shall see, there is good reason to be skeptical that this is so.

Our paper addresses this new challenge. More specifically, the paper examines to what extent rights are capable of adjudicating disputes when perspectival disagreements obtain. We do this by constructing a formal model of rights that extends the classical model of rights offered in the social choice tradition. This extended model explicitly incorporates perspectival diversity into the domain. The results of our model show that those in the current literature skeptical of the ability of rights to adjudicate disputes when perspectival disagreements obtain are largely incorrect in this assessment. If rights are formulated properly then such disagreements can obtain and our conflicts resolved, achieving social harmony and cooperation rather than conflict and discord.

2 The resolution of conflicting claims

Public reason liberalism is concerned with those disagreements endemic to liberal societies that can potentially lead to conflict and discord. Some method of conflict resolution must be introduced to achieve stable social and political order. Though many do not emphasize it, the resolution of conflicting claims was always a core concern of John Rawls's. Indeed, focus on the resolution of conflicting claims is perhaps the one theme unifying Rawls's entire corpus. In his very first paper, "Outline of a Decision Procedure for Ethics," Rawls asks whether there exists "a reasonable

decision procedure which is sufficiently strong... to determine the manner in which competing interests should be adjudicated; and, in instances of conflict, one interest given preference over another...?” (Rawls 1951/1999: p. 1). That is, from the very beginning Rawls asks whether some kind of “reasonable” procedure can be constructed that is capable of telling us what society should do when citizens make conflicting and incompatible claims on one another.

Rawls’s focus on dispute adjudication is not a feature of his early thought alone, but survives throughout his entire body of work. In *A Theory of Justice* Rawls defines the role or purpose of justice as essentially adjudicative. The very *concept* of justice is defined as a set of principles determining the “proper balance between competing claims,” whereas *conceptions* of justice are defined as “a set of related principles for identifying the relevant considerations which determine this balance” (Rawls 1971: p. 10). Rawls is here saying that the very point of a theory of justice is to adjudicate disputes between parties like Althea and Bertha when they making conflicting claims on one another, and different conceptions will specify in different ways how we are to order Althea’s and Bertha’s claims. Indeed, one of the formal constraints on the concept of right, according to Rawls, is that “a conception of right must impose an ordering on conflicting claims. This requirement springs directly from the role of its principles in adjusting competing demands” (Rawls 1971: pp. 133–134).

From the public reason liberal’s concern with adjudicating those disagreements characterizing liberal societies we may infer a plausible success condition any public reason project must meet:

Conflict Resolution Requirement: Any public reason project, in order to be successful, must propose some kind of mechanism capable of adjudicating disputes.¹

The Conflict Resolution Requirement is necessary but not sufficient for a public reason proposal to be successful. For the public reason liberal is not *only* concerned with adjudicating disagreements that may lead to conflict and discord. Though exterminating all Catholics will indeed resolve disputes between Protestants and Catholics—there are no longer Catholics for Protestants to dispute with!—such a proposal is clearly not an exercise in public reason. The public reason liberal also seeks to show that the method of conflict resolution proposed is *endorsable* by all who are subject to it. Though endorsement of the proposed dispute-adjudication mechanism is a constitutive component of the public reason project, this paper is solely concerned with the Conflict Resolution Requirement which, again, is necessary but not sufficient for a public reason proposal to be successful.

Now there are many different kinds of dispute-adjudication mechanisms public reason liberals introduce in hopes of satisfying the Conflict Resolution Requirement. As a few examples of what such proposals look like, Thomas Hobbes—a public reason liberal according to some interpretations²—proposes that persons “confer all their power and strength upon one man, or assembly of men, that may reduce all their wills, by

¹ For theorists who endorse something like the Conflict Resolution Requirement, see Rawls (1971: pp. 133–134), D’Agostino (2003: p. 61) and Gaus (2011: pp. 297–298).

² For more on interpreting Hobbes as a public reason liberal see Gaus (2013).

plurality of voices, unto one will” (Hobbes 1688/1994: p. 109). For John Locke—like Hobbes, a public reason liberal on some interpretations of his thought³—conflicts are resolved when “all private judgment of every particular individual being excluded, the community comes to be umpire, by settled standing rules, indifferent, and the same to all parties” (Locke 1690/1980: p. 46). On Rawls’s proposal, a political conception of justice is found that “is to be publicly recognized as a final court of appeal for the ordering of conflicting claims” (Rawls 1971: p. 135). And as a final example, for Gerald Gaus the focus is on moral rules that make up our shared social morality: “the rules of social morality structure social interaction in ways that are beneficial to all and make social existence possible” (Gaus 2011: p. 3).

This paper does not focus on one specific proposed mechanism of dispute adjudication, but rather a feature several proposed mechanisms share in common: the use of jurisdictional rights, which can be understood very broadly as rights that grant the rightsholder “authority or sovereignty in relation to some specific manner” (Hart 1955: p. 184). Rawls, we have seen, relies on a political conception of justice to adjudicate between conflicting claims. But one feature *all* political conceptions of justice share in common is that they specify “certain rights, liberties, and opportunities” (Rawls 1993/2005: xlvi). Rights, then, play a key role in conflict resolution according to Rawls’s public reason project.

Though Gaus explicitly rejects the use of political conceptions of justice to adjudicate disputes relying instead on moral rules (Gaus 2011: p. 271), these moral rules include (but by no means solely consist of) rights of various sorts to perform our sought after adjudicative function: “a deeply pluralistic society can effectively cope with many of its disagreements about what evaluative standards to adopt by establishing a system of private property” (Gaus 2011: p. 375). Our concern in this paper is to what extent rights generally speaking are capable of satisfying the Conflict Resolution Requirement. In focusing on rights generally speaking the hope is that we are able to speak to several different public reason proposals, including both Rawls’s model as well as the quite different Gaussian one.

3 Perspectival disagreements

One might initially wonder why we are even asking this question in the first place, for *surely* rights are capable of adjudicating our disputes. Indeed, though economists tend to focus on the incentive effects of rights, philosophers have tended to emphasize the role rights play in conflict resolution. Consider, for instance, Fred D’Agostino’s work in *Incommensurability and Commensuration*. In this book D’Agostino examines how it is that incommensurable disagreements—the kinds of disagreements public reason liberals are concerned with—are in fact resolved. One such method of adjudication is what he calls *separation via devolution*: “Rather than trying to bring decision-relevant factors together in a space of calculation, these sorts of projects aim, in effect, precisely to keep them apart, in order to simplify the basis for decision-making” (D’Agostino 2003: p. 104).

³ For more on interpreting Locke as a public reason liberal see Gaus (2017).

There are many examples of this dispute-adjudication technique forthcoming. But perhaps the core example of separation via devolution is jurisdictional rights. To return to the example appealed to in the introduction of this paper: when faced with the question of what religion persons should worship, one could try to solve what seems like an intractable dispute by appeal to some kind of collective solution where one religious doctrine is chosen for all. Or, one could solve this problem through separation via devolution by implementing a scheme of rights, which indeed is the common solution implemented in liberal societies: “In effect, we say that, in a society with n individual members, there are n separate spheres in which an answer to this question might be sought, each of which is, in theory, inviolable and particular to the individual who, in effect, occupies it” (D’Agostino 2003: p. 105). These remarks should raise the question: why would we ever think that rights *cannot* adjudicate our disputes, thereby calling into question whether a public reason proposal, including as one of its components the use of rights to resolve our conflicts, actually satisfies the Conflict Resolution Requirement?

Recent work by Gerald Gaus and Ryan Muldoon calls into question whether schemes of rights can satisfy the Conflict Resolution Requirement by focusing not on a new kind of diversity per se, but rather a kind of diversity that has hitherto not received due attention in the public reason literature (Muldoon 2015, 2016, 2017; Gaus 2016). To get a better handle on the kind of diversity we are referring to, we introduce the notion of a *perspective*. We can think of perspectives as mental schemata or internal languages we consciously or unconsciously impose on reality—they are ways of carving up and categorizing the world. More formally, Scott Page defines perspectives as mappings between objects in the external world and one’s internal language (Page 2007: p. 31). To borrow Muldoon’s (2016: p. 50) terminology, we can think of these mappings as imposing a sort of ontology: though we all confront the same reality, the way we code this reality in our internal languages can be different for we may employ different mapping functions—when this happens, individuals will categorize objects in the world differently even though, at rock bottom, it is the same external world they confront. Though we all see the same reality we disagree oftentimes about what counts as a person, what counts as a harm, and so on and so forth. This is because our perspectives categorize and carve up the world in different ways.

Now public reason liberals have never denied that persons may disagree in terms of their perspectives. But focusing on the fact that persons may have differing perspectives as much recent work does allows us to better explain some of our most heated social conflicts. In the words of Gaus: “some of our deepest and most intractable disputes are not about values or principles of justice, but about the world to which these principles apply” (Gaus 2016: p. 162). Take abortion, for instance. On one reading, pro-life persons and pro-choice persons just have conflicting preferences over what policy should be. A more charitable reading, though, suggests that many pro-lifers and pro-choicers disagree fundamentally about the nature of reality. According to some, inside a woman’s uterus is a person and bearer of rights. According to others, inside a woman’s uterus is a collection of cells that at first is no different than bacteria and at later stages no different than a parasite. The disagreement between the two parties is over how to categorize objects the world. In other words, the disagreement is over what objects fall

under the category “person”: does a fetus count as a person, or should it be categorized as something else entirely?

Beyond being better able to account for what drives some of our deepest social conflicts, focusing on perspectival disagreements allows us to understand how schemes of rights—and methods of dispute-adjudication more generally—may fail to satisfy the Conflict Resolution Requirement when such disagreements obtain. Returning to the abortion example and to quote a lengthy passage from Gaus:

Advocates of such [abortion] rights see the case as decisively about fundamental rights of personal autonomy: opponents of abortion rights are depicted as having little sensitivity to a woman’s claim to control her own body. But this by no means follows, and often is simply not the case; opponents of abortion can be deeply devoted to such autonomy, but not in cases where it entails overriding another’s right to life. And, of course, in the abstract, most advocates of abortion rights would also draw back in such situations. The dispute is centrally about the social world to which the principles of autonomy and right to life apply: the two social worlds do not have the same set of persons, *and so even perfect agreement about abstract principles of justice would not resolve the dispute* (Gaus 2016: pp. 162–163) (emphasis ours).

The point Gaus is raising here is that *even if* two parties to a dispute (for instance, Althea the pro-lifer and Bertha the pro-choicer) settle on a scheme of rights or method of conflict resolution more generally to resolve their disagreement—for instance, that persons are granted a robust right to personal autonomy except in cases where this conflicts with another person’s right to life—then it can *still* be unclear how to resolve disputes when there are perspectival disagreements. In this case, when there are disagreements concerning how to categorize objects in the external world under the category “person.” This calls into question whether schemes of rights can indeed satisfy the Conflict Resolution Requirement.

Other examples abound.⁴ Suppose Althea and Bertha decide to regulate their disputes according to something like John Stuart Mill’s famous Harm Principle: persons are only obligated to not engage in certain behavior when such behavior causes harm to others. But this principle is guaranteed to adjudicate disputes between Althea and Bertha (thus satisfying the Conflict Resolution Requirement) only if Althea’s and Bertha’s perspectives categorize the same behaviors under the category “harm.” But this might not be so. Robert Paul Wolff, for instance, criticizes the Harm Principle along these lines:

But Mill also seems to think it is obvious that when Smith practices the Roman Catholic faith, or reads philosophy, or eats meat, or engages in homosexual practices, he is not affecting Jones’ interests. Now suppose that Jones is a devout Calvinist or a principled vegetarian. The very presence in his community of a Catholic or a meat-eater may cause him fully as much pain as a blow in the face or the theft of his purse. Indeed, to a truly devout Christian a physical blow counts for much less than the blasphemy of a heretic. After all a physical blow

⁴ See Gaus (2016: pp. 158–165).

affects my interests by causing me pain or stopping me from doing something I want to do. If the existence of ungodly persons in my community tortures my soul and destroys my sleep, who is to say that my interests are not affected? (Wolff 1968: pp. 23–24).

These are not mere philosophical possibilities, but actual issues we face today. Consider, for instance, the recent actions by students at the University of California at Berkeley to violently shut down a speech by provocateur Milo Yiannopoulos. One way of viewing these actions is by understanding the students at Berkeley as illiberal, rejecting the Harm Principle: Yiannopoulos’s speech was not harming anyone, yet students took to violence to shut it down anyways. But on a different and perhaps more charitable interpretation of what happened, Berkeley students could plausibly accept something like Mill’s Harm Principle, but simply categorize actions as “harms” differently than others. Indeed, as one Berkeley alumna characterized Yiannopoulos’s speech: “This is violence. If I know that you are planning to attack me, I’ll do all I can to throw the first punch” (Dang 2017).

One way of thinking about the unique problems perspectival disagreements raise for adjudicating disputes is by returning to D’Agostino’s characterization of separation via devolution as a method of conflict resolution: here, we resolve conflicts by giving each person a separate sphere over which they are sovereign. But if there are perspectival disagreements—if we categorize objects in the world under different categories—then there may be disagreements about what does and does not fall into persons’ separate spheres. Saying that all persons have a sphere of personal autonomy to do as they please so long as they do not violate the autonomy of others will do us no good if one person categorizes ϕ -ing as an act of autonomy and another person categorizes ϕ -ing as going beyond being an act of autonomy, violating the autonomy of another. Saying that all persons have a sphere to practice their religion of choice undisturbed by others will do us no good if Cassidy does not categorize Dupree’s scientology as an actual religion. Here, Cassidy may insist on banning scientology because of the social harms she thinks it causes, whereas Dupree appeals to his private sphere of religious practice that Cassidy, in principle, accepts.

So in cases of perspectival disagreement it cannot be taken for granted that schemes of rights satisfy the Conflict Resolution Requirement. What to do in the face of this challenge? Gaus begins by noting that one often neglected facet of Rawls’s later thought is the role publicly constructed social worlds play: “the parties [in the original position] in effect try to fashion a certain kind of social world; they regard the social world not as given by history, but, at least in part, as up to them” (Rawls 2001: p. 118). On Gaus’s interpretation, part of constructing a public social world entails developing a shared perspective that all citizens adopt for the purpose of adjudicating disputes—though Althea’s and Bertha’s perspectives code objects in the world according to different categorizations thereby leading to potential adjudicative problems, Althea and Bertha can construct a public social world that gives them the *same* categorizations, thereby circumnavigating the sorts of problems we have been considering in this section. In Gaus’s words: “If each perspective can make sense of the categories of the artificial social world and endorse their use, we can have a shared artificial world without normalization” (Gaus 2016: p. 178). If successful, then schemes of

rights—and dispute-adjudication mechanisms more generally—will satisfy the Conflict Resolution Requirement.

Gaus’s proposal is one way of showing how the Conflict Resolution Requirement can still be satisfied even in cases of perspectival disagreement. However, we want to explore whether schemes of rights can satisfy the Conflict Resolution Requirement *without* persons abandoning their unique perspectives, having to adopt a public perspective instead. Though this section raises serious doubts about this possibility, we construct a formal model showing that such pessimism is unjustified.

4 The model

4.1 Individuals, perspectives, and preferences

Rigorously examining the extent to which rights can adjudicate disputes in cases of perspectival disagreement requires introducing some formalism, though we relegate all proofs of our results to the “Appendix”. This section introduces the formal model, Sect. 5 examines results, while Sect. 6 interprets these results. We here employ the tools of social choice theory.⁵ Though social choice theory was first used to examine to what extent rights can adjudicate disputes with Amartya Sen’s famous paradox of the Paretian liberal (1970/1982), we here roughly follow Allan Gibbard’s model of a rights structure (1974), extending the environment to incorporate perspectival disagreements.

We begin with a set of individuals, $N = \{1, \dots, n\}$, and a set of social states, $X = \{x, y, z, \dots\}$, where each social state is understood broadly as “a complete description of society including every individual’s position in it” (Sen 1970/1982: p. 285). Each individual $i \in N$ has a *private sphere*, X_i , defined as the set of all possible descriptions of what happens within individual i ’s private sphere, roughly following D’Agostino’s notion of separation via devolution discussed in the section above. Intuitively, if Althea reads a book, goes to the movies, or buys a new motorcycle, then such behavior affects Althea’s private sphere. Such possible descriptions of states of affairs are thus contained in the set X_A . But it is not only things that Althea may do that can possibly describe what happens in Althea’s private sphere. If Bertha punches Althea in the face, for instance, then surely this affects Althea’s private sphere as well. The description of such a state of affairs would thus also be contained in the set X_A . Though this understanding of private spheres is not precise just yet—Sect. 6 below is entirely dedicated to discussing how to interpret this primitive of our model—we believe it is intuitive enough for the time being.

In trying to model perspectival disagreements we say that each individual $i \in N$ *sees* or *perceives* each social state $x \in X$ from his or her unique perspective. More rigorously, for each $i \in N$, define the mapping $\pi^i : X \rightarrow X_1 \times X_2 \times \dots \times X_n$. We call the mapping π^i individual i ’s *perspective function*. In words, each individual $i \in N$,

⁵ Here we follow the formal model developed in Chung (forthcoming). See also Kogelmann (2017) for a formal model of perspectival disagreement. Kogelmann (2018) also uses the tools of social choice theory in the context of public reason liberalism.

based on his or her specific perspective function π^i , codes each social state $x \in X$ in his or her internal language as a list of descriptions describing what is happening within each individual’s private sphere. Here, $\pi^i(x)$ denotes social state x seen from individual i ’s specific perspective. For notational convenience, we will write $\pi^i(x)$ simply as x^i . When $x^i \neq x^j$ (that is, when $\pi^i(x) \neq \pi^j(x)$), then this means that individuals i and j categorize the social world x differently. Here, there is perspectival disagreement. Inversely, if $x^i = x^j$ (that is, if $\pi^i(x) = \pi^j(x)$), then this means that individuals i and j categorize the social alternative x identically. Here, there is no perspectival disagreement. Let $\pi = (\pi^1, \dots, \pi^n)$ denote a profile of individual perspective functions (one perspective function for each individual $i \in N$), and let Π be the set of all logically possible profiles of individual perspectives.

Though there is a single set of social states X , because individuals have different perspective functions, they may perceive these social states quite differently. To account for this, define $X^i = \{ \pi^i(x) \in X_1 \times X_2 \times \dots \times X_n \mid x \in X \}$ as the set of all social states seen from individual i ’s perspective function π^i . Thus, X^i is how individual i perceives the set of social states X given his or her perspective.

As is typical in social choice theory we assume that persons have preferences over the set of all social states X . But in taking seriously perspectival disagreements we assume that each individual has preferences over social states as they perceive them. That is, each individual $i \in N$ has a *preference relation* R_i defined on X^i (i.e., $R_i \subseteq X^i \times X^i$) that is *complete* and *transitive*.⁶ This means that each individual has a preference relation over social states as they perceive them given their perspective function. When $(x^i, y^i) \in R_i$, this means that individual i judges that the social state x (seen from his or her own perspective) is at least as good as the social state y (seen from his or her own perspective).

In this way, each individual i ’s preference relation R_i on X^i induces each individual i ’s preference relation R_i on X such that for all $x, y \in X$ and for all $x^i, y^i \in X^i, (x, y) \in R_i$ if and only if $(x^i, y^i) \in R_i$. As a notational convention, we will write $(x^i, y^i) \in R_i$ and $x^i R_i y^i$ interchangeably. Each individual’s strict preference relation P_i and indifference relation I_i are defined in the usual way: that is, for all $x^i, y^i \in X^i, x^i P_i y^i$ if and only if $\neg y^i R_i x^i$; and $x^i I_i y^i$ if and only if $x^i R_i y^i$ and $y^i R_i x^i$. It follows that P_i is *asymmetric* and *negatively transitive*, and I_i is an *equivalence relation*.⁷ In other words, each R_i is a weak order on X^i and thus on X . Let $R = (R_1, \dots, R_n)$ denote a profile of individual preference orderings, one for each individual $i \in N$, and let \mathcal{R} be the set of all logically possible profiles of individual preference orderings.

⁶ A preference relation R_i on X^i is *complete* if for all $x^i, y^i \in X^i$ either $(x^i, y^i) \in R_i$ or $(y^i, x^i) \in R_i$; and *transitive* if for all $x^i, y^i, z^i \in X^i, (x^i, y^i) \in R_i$ and $(y^i, z^i) \in R_i$ imply $(x^i, z^i) \in R_i$.

⁷ A preference relation P_i on X^i is *asymmetric* if for all $x^i, y^i \in X^i, (x^i, y^i) \in P_i$ implies $(y^i, x^i) \notin P_i$; and *negatively transitive* if for all $x^i, y^i, z^i \in X^i, (x^i, y^i) \notin P_i$ and $(y^i, z^i) \notin P_i$ imply $(x^i, z^i) \notin P_i$. The indifference relation I_i on X^i is a *equivalence relation* if it is *reflexive* (i.e. $\forall x^i \in X^i (x^i, x^i) \in I_i$), *symmetric* (i.e. $\forall x^i, y^i \in X^i (x^i, y^i) \in I_i$ implies $(y^i, x^i) \in I_i$), and *transitive* (defined in footnote 1).

4.2 Social preference and social choice

Thus far we have defined individuals, different social states to implement, how individuals perceive these social states via their perspectives, and the preference rankings individuals have over these social states. With these primitives defined we can begin looking at how a society makes social choices. Sometimes, when making social choices, we may restrict our attention only to a set $F \subseteq X$ of social states that are feasible. Let \mathfrak{S} be the set of all non-empty subsets of X . Then, \mathfrak{S} is the set of all logically possible feasible sets of social states. A *social choice problem* is a triple $(F, R, \pi) = (F, (R_1, \dots, R_n), (\pi^1, \dots, \pi^n))$. It is characterized by (i) a feasible set of social states to implement; (ii) a profile of individual preference orderings, containing a unique ordering for each individual $i \in N$; and (iii) a profile of individual perspectives, containing a unique perspective function for each individual $i \in N$. From a social choice problem a social choice must be made. But how?

For any social choice problem (F, R, π) , a *social preference function* $P^*(F, R, \pi)$ generates society's preferences (i.e., a social preference relation) over social states in F based on the profile of individual preference orderings and the profile of individual perspectives. For any $x, y \in F$, $xP^*(F, R, \pi)y$ will mean x is socially preferred to y . Derivation of this social preference relation is key in the resolution of disputes as required by the Conflict Resolution Requirement. If our feasible set of social states is x, y , and z , where Althea wants to implement x (i.e., x is ranked at the top of R_A), Bertha wants to implement y (i.e., y is ranked at the top of R_B), and Cassidy wants to implement z (i.e., z is ranked at the top of R_C), then our social preference function orders these conflicting claims by saying that, for instance, x is ordered over y is ordered over z . From this social preference relation a social choice can be made, thus resolving the dispute over whether x, y , or z should be implemented.

So a social preference function derives a social preference relation on some set of feasible social states $F \subseteq X$ by taking as its input the preferences and perspectives of all individuals $i \in N$. From this social preference relation a social choice must be made to resolve disputes over which social state to implement, thus ensuring that the Conflict Resolution Requirement is satisfied. But a social choice cannot be made from just *any* social preference relation. If the social preference relation says that x is ordered over y is ordered over z is ordered over x then it is unclear how a social choice can be made, for here every option is worse than some other option. Thus, social preference relations generated by our social preference function must have a certain structure for a social choice to be made and the Conflict Resolution Requirement satisfied. But what is this structure?

At a bare minimum, we say that an *admissible social choice* is a social state that is not dominated by another social state with respect to the social preference relation $P^*(F, R, \pi)$.⁸ More formally:

Definition (Admissible Social Choice) Let $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$ be any social choice problem. Define the set of *undominated* social states $UD(F, P^*(F, R, \pi))$ as

⁸ We thus follow Sen's (1997) reliance on maximal sets rather than choice sets.

$$UD(F, P^*(F, R, \pi)) \equiv \{x \in F \mid \nexists y \in F \text{ such that } P^*(F, R, \pi) x\}.$$

Then, we say that a social state $a \in F$ is an *admissible social choice* if and only if

$$a \in UD(F, P^*(F, R, \pi)).$$

In short, an admissible social choice is a social state not ranked below some other social state in our social preference relation. Clearly, a society will *always* be able to generate an admissible social choice, and, thereby, *always* satisfy the Conflict Resolution Requirement, whenever the set $UD(F, P^*(F, R, \pi))$ is *non-empty*. And, all that is needed for $UD(F, P^*(F, R, \pi))$ to be non-empty is for our social preference relation $P^*(F, R, \pi)$ to be *acyclic*.⁹ That is, the social preference relation induced by the social preference function must not contain any cycles.¹⁰ The following proposition states this formally.

Proposition 1 Let $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$ be any social choice problem. Then, $UD(F, P^*(F, R, \pi))$ is non-empty if $P^*(F, R, \pi)$ is *acyclic*.

Proof See “Appendix”.

Since (i) the Conflict Resolution Requirement requires that an admissible social choice can always be made, and since (ii) an admissible social choice can always be made when the social preference relation generated by our social preference function is acyclic (as Proposition 1 shows), it follows that the Conflict Resolution Requirement will always be satisfied by a social preference function satisfying the following condition:

Condition NC (No Cycles): For all admissible social choice problems $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$, $P^*(F, R, \pi)$ is *acyclic*.

In words, Condition NC or No Cycles says that our social preference function cannot induce a cycle over options in the feasible set of social states. As we have seen, the satisfaction of Condition NC guarantees the satisfaction of the Conflict Resolution Requirement, for Condition NC is sufficient for an admissible social choice to always be made. Thus, we will be interested in whether schemes of rights are capable of generating a social preference relation satisfying Condition NC. If they can, then schemes of rights satisfy the Conflict Resolution Requirement.

4.3 Jurisdictional rights

Our task is to see whether schemes of rights are capable of satisfying the Conflict Resolution Requirement given perspectival disagreements. Rights can do this if they

⁹ Formally, the social preference relation $P^*(F, R, \pi)$ is *acyclic* if and only if $\forall n \in \mathbb{N}, \forall x_1, x_2, \dots, x_n \in F : x_1 P^*(F, R, \pi) x_2 P^*(F, R, \pi) \dots P^*(F, R, \pi) x_n \Rightarrow \neg x_n P^*(F, R, \pi) x_1$.

¹⁰ Note that if the social preference relation $P^*(F, R, \pi)$ is complete and transitive, then it will be acyclic as well. Hence, acyclicity is a weaker requirement than the conjunction of completeness and transitivity.

generate a social preference relation satisfying Condition NC or No Cycles. Yet how exactly do rights generate a social preference relation in the first place? This is where we model jurisdictional rights. Before offering our definition of rights, we need to introduce two concepts: the idea of an *i*-variant and the idea of *unconditional preferences*. After introducing these two concepts we then offer a formal definition of jurisdictional rights made to function in cases of perspectival disagreements specifically.

For each $i \in N$ and each $x^i \in X^i$, let $x^i_{-i} \equiv (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$. Then, for all $x^i, y^i \in X^i$, we say that x^i and y^i are *i*-variants whenever $x^i_{-i} = y^i_{-i}$. In plain words, when two social states x and y seen from individual i 's perspective are *i*-variants, this means that the two social states seen from individual i 's perspective are identical in all aspects *except* for those aspects which individual i regards to be within his or her private sphere. In other words, the only difference between social states x and y are the respective descriptions of i 's private sphere. Importantly, individuals can disagree over whether two social states x and y are *i*-variants when they have different perspective functions—that is, they can disagree over whether social states x and y differ according to whose private spheres are affected.

As an example of this, suppose the only difference between social states x and y is that Althea reads pornography in social state x but does not read pornography in social state y . Althea understands reading pornography as only affecting *her* private sphere. This means that, according to Althea's perspective, social states x and y are A-variants: they only differ with regards to her private sphere. But Bertha, given her perspective, has a different take on pornography: she sees it as a patriarchal form of oppression and, when one reads it, one harms *all* women. As such, Bertha will *not* view social states x and y as A-variants. According to Bertha's perspective, reading pornography affects the private spheres of *all* women, not just Althea's. The two social states are thus not A-variants according to Bertha's perspective.¹¹

We now introduce the notion of unconditional preferences, to be used in our definition of jurisdictional rights below.¹² Suppose $a, b \in X_i$ —that is, a and b are two different descriptions of what is happening within individual i 's private sphere. We say that individual i prefers a to b *unconditionally* if and only if for every pair of *i*-variants x^i and y^i in X^i (that is, $x^i_{-i} = y^i_{-i}$), if $x^i_i = a$ and $y^i_i = b$, then $x^i P_i y^i$. In plain words: individual i unconditionally prefers description of his or her private sphere a over description of his or her private sphere b just in case they prefer *every* *i*-variant containing description a of his or her private sphere over *every* *i*-variant containing description b of his or her private sphere. Again, this notion of unconditional preferences is used in our definition of jurisdictional rights below.

With these definitions out of the way we can now offer our definition of rights. In the standard social choice literature (e.g., Sen 1970/1982; Gibbard 1974), rights are modeled in terms of decisiveness over *i*-variants: for individual i to have a right

¹¹ For deeper philosophical exploration of issues related to private spheres and pornography specifically see Gaus (1997) and Muldoon (2017).

¹² The use of unconditional preferences is a standard move in the literature. On this point see Gibbard (1974).

over social states x and y , where both x and y are i -variants, means that individual i 's preferences over x and y *solely determine* the social ordering of x and y . That is, if individual i has a right to x over y or y over x , then i 's preferences over x and y determines the social ordering of x and y , *regardless anyone else's preferences*. Everyone else in society could prefer y over x , but if individual i has a right to social states x and y , then if i prefers x to y the social preference relation will say x is preferred to y , *contra* everyone else. This is in line with the intuition that rights are “trumps.”

This understanding of how rights work in relation to social preference functions is fine when there are no perspectival disagreements. But it breaks down when we *do* introduce perspectival disagreements. To see why consider an example. Suppose everyone has a right to personal autonomy so long as they do not infringe upon the autonomy of others. In social state x Althea reads pornography, and in social state y Althea does not read pornography. If all individuals see social states x and y as A-variants—if all individuals see x and y as being solely about Althea's right to personal autonomy so long as she does not infringe upon the autonomy others, thus differing only according to Althea's private sphere—then Althea's preference ordering over social states x and y will resolve conflicts over whether x is to be ordered over y or y is to be ordered over x . Everyone agrees that whether x is to be ordered over y or y is to be ordered over x reduces to Althea's right to personal autonomy, so Althea's preferences over x and y are sovereign.

But some might not see x and y as A-variants when perspectival disagreements obtain, something we noted above. As a result, when these individuals consider the social choice of x and y they will *not* see this as being resolved by appeal to Althea's right to personal autonomy. If Bertha sees reading pornography as harming all women, then x and y will not be A-variants: other persons besides Althea will have their private sphere affected when Althea decides to read pornography. As a result, Althea's claim to decisiveness over x and y by appeal to her right to personal autonomy will not persuade Bertha. Though Bertha grants that Althea has such a right, she does not think that Althea's choice in this case is an exercise of such a right, because Althea's choice harms all women, and Althea only has a right to personal autonomy so long as she does not infringe upon the autonomy of others. The conflict over whether x should be ordered over y or y should be ordered over x goes unresolved when Althea appeals to her right of personal autonomy in this case—for, again, Bertha's perspective deems that the current example is not a *bona fide* instance of an exercise of such a right.

How, then, should jurisdictional rights be defined to operate in cases of perspectival disagreement? We now propose a definition, and then explain and justify it below.

Definition L (*Jurisdictional rights under perspectival diversity*) Individual i has a right if and only if for all social choice problems $(F, R, \pi) \in \mathfrak{S} \times \mathfrak{R} \times \Pi$ and for all $x, y \in F$ such that x^i and y^i are i -variants (that is, $x^i_{-i} = y^i_{-i}$), $xP^*(F, R, \pi)$ y holds, if and only if i prefers x^i_i to y^i_i unconditionally and either

- (α) $\forall j \in N, x^i = x^j$ and $y^i = y^j$ or
- (β) $\forall j \in N$ such that $x^i \neq x^j$ or $y^i \neq y^j, x^j R_j y^j$.

Condition (α) explains how jurisdictional rights operate in cases of no perspectival disagreement: when Althea unconditionally prefers x to y , then Althea is allowed to determine that social state x is socially preferred to social state y (or vice versa) so long as x and y are A-variants. Note that when there is no perspectival diversity, our definition of jurisdictional rights operates in the way we normally think rights should operate: the rights-holder's preferences are socially decisive regardless the preferences of others. This resolves the dispute—everyone agrees that Althea has a right to x over y or y over x , and then Althea chooses one social state over the other. What anyone else prefers is simply irrelevant.

Condition (β) explains how jurisdictional rights operate in cases when there is perspectival disagreement. In this case, even though Althea and Bertha see social states x and y differently—Althea sees x and y as social states that only affect her private sphere whereas Bertha sees social states x and y as affecting private spheres beyond Althea's—if Althea unconditionally prefers x to y , and if Bertha agrees that x is at least as good as y , then Althea is allowed to determine that x is socially preferred to y . Here, though Althea and Bertha disagree about the features of social states x and y respectively, rights can still be operative *so long as* Bertha minimally goes along with Althea's preferences: if Althea unconditionally prefers not reading pornography to reading pornography, and Bertha thinks a social state in which Althea does not harm other women is at least as good as a social state in which Althea harms other women, then Althea's right to x over y goes through.

Even though Althea's and Bertha's perspectives disagree about the relevant categorizations of social states in case (β), Althea's right to x over y is allowed to go through because there are no conflicting claims remaining after exercise of the right, thus meaning that the Conflict Resolution Requirement is satisfied. Recall the very reason why we are focusing on jurisdictional rights is because they are plausible mechanisms for adjudicating disputes. A natural test in deciding whether jurisdictional rights actually perform this task is to ask whether there remain conflicting claims in the space that the right was intended to regulate after the right in question has been exercised. In case (β) there is no such conflict—after Althea's right to x over y is implemented both Althea and Bertha are satisfied with the resulting state of affairs and thus do not make conflicting claims on one another.

But this may not be so if (i) Althea and Bertha see different social states x and y , yet (ii) Bertha's preferences over x and y do *not* minimally accord with Althea's. For in this case, after we implement Althea's right to x over y there may still be discord. Even though Althea thinks she has merely exercised her rights in the relevant case, because Bertha sees these social states differently *and* disagrees with Althea's decision, Bertha will press a claim on Althea: she does not see Althea's actions as a *bona fide* instance of an exercise of rights. When this occurs, we still have social conflict even after a right has been exercised, meaning that the Conflict Resolution Requirement goes unsatisfied. In such a case jurisdictional rights fail to perform their intended function. This is why we do not model jurisdictional rights as being operative in these sets of cases. Rights are only operative in cases of perspectival disagreement when (β) obtains, for in such cases exercising the right clearly settles the relevant dispute, allowing rights to perform their intended function.

Having explained our definition of rights for cases of perspectival diversity, we now introduce our second and final condition on social preference functions. It is as follows:

Condition L (Liberalism): Each individual $i \in N = \{1, \dots, n\}$ has jurisdictional rights in the sense defined in Definition L.

Condition L simply says that all individuals have jurisdictional rights as we have defined and explained them above. Clearly this is intuitive: it would be deeply illiberal if we held that only *some* persons have rights and others do not. With the model sufficiently explicated, we now turn to our guiding question, which has been refined a bit: can a social preference function satisfy both Condition NC (No Cycles) and Condition L (Liberalism)? If a social preference function *can* simultaneously satisfy both conditions then rights satisfy the Conflict Resolution Requirement. If a social preference function *cannot* simultaneously satisfy both conditions then we cannot conclude that rights satisfy the Conflict Resolution Requirement.

5 Results

Let us begin with the easy case to give ourselves a baseline: cases where there is *no* perspectival diversity. We define this restriction below, and then offer a condition social preference functions meeting this definition satisfy.

Definition ND (No Diversity): A profile $\pi = (\pi^1, \dots, \pi^n)$ of individual perspectives satisfies ND (No Diversity) if and only if $\forall x \in X, \forall i, j \in N, x^i = x^j$. That is, all individuals categorize any given social alternative in the same way, and, hence, there exists no perspectival diversity.

Condition ND (The Domain of No Diversity): All social choice problems $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$, where $\pi = (\pi^1, \dots, \pi^n)$ satisfies Definition ND, are admissible.

Condition ND simply says that all persons adopt the *same* perspective. That is, all persons categorize objects in the external world the same way. Because this is so, all persons will agree where the boundaries of each person's private sphere ends: if Althea engages in some action φ , then whether φ -ing affects only Althea's private sphere or the private spheres of others is something all individuals agree on when Condition ND obtains.

What happens when Condition ND characterizes our shared social world? It can be shown that when Condition ND holds there exists a social preference function satisfying Condition L that does not induce a cycle, thereby guaranteeing Condition NC and thus the Conflict Resolution Requirement is satisfied.

Theorem 1 *Under Condition ND (Domain of No Diversity), Condition L (Liberalism) implies Condition NC (No Cycles).*

Proof See “Appendix”.

Note that Theorem 1 should really be of no surprise at all. As we noted in at length in Sect. 3, many in the philosophical literature on rights stress the adjudicative function rights perform in resolving our conflicts. It is only the recent focus on perspectival disagreements in the work of Gaus and Muldoon that calls into question whether rights can perform this crucial function. As a result, it should be no surprise that, absent the perspectival disagreements Gaus and Muldoon emphasize, rights are capable of adjudicating our disputes by *always* generating a social preference relation that does not admit cycles, thus satisfying the Conflict Resolution Requirement.

Our Condition ND is what social choice theorists call a *domain restriction*, but with an important twist. Typically, a domain restriction says that certain kinds of *preferences* are not admissible into the social choice problem. Indeed, one of the most common responses to Kenneth Arrow’s famous impossibility theorem is that if we restrict the domain of preferences—if we only allow certain kinds of preferences to enter the aggregation exercise—then the impossibility theorem goes away, and we generate a possibility theorem instead.¹³ Our Condition ND is a bit different than this: instead of putting a restriction on *admissible preferences* (Condition ND allows for all logically possible preference profiles), we are instead putting a restriction on *admissible perspectives*. If we restrict the kinds of perspectives admitted into our social choice problem such that all persons categorize objects in the external world in the same manner, then schemes of jurisdictional rights are guaranteed to satisfy Condition NC.

With Condition ND only *some* kinds of perspectives are admitted into the social choice problem—those that all agree with one another. What happens when we allow *all* logically possible perspectives to be admitted? We first define the relevant condition before examining results.

Condition UD (Unrestricted Domain): All social choice problems $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$ are admissible.

Recall that in our model, a social choice problem includes a profile of individual perspectives along with a set of feasible social states and a profile of individual preferences. Hence, under Condition UD, not only is any profile of individual preferences or any set of feasible social states allowed, but any profile of individual perspectives is allowed as well. When Condition UD holds, we allow individuals to categorize social reality in radically different and incompatible ways. More specifically, since perspectives are defined in our formal model as ways of describing what is happening in the private sphere of all individuals for any social state x , Condition UD says that all logically possible ways of describing the private spheres of individuals are admissible into the social problem.

¹³ For an extensive overview see Gaertner (2001).

What happens when Condition UD obtains? In contrast to Condition ND, when Condition UD holds there exists no social preference function satisfying both Condition NC and Condition L. That is, we cannot guarantee that schemes of rights will satisfy the Conflict Resolution Requirement. This is a deeply troubling result for *all* those public reason proposals relying on schemes of rights to adjudicate disputes—which includes both the Rawlsian and Gaussian models.

Theorem 2 *Suppose $|X| \geq 4$ and $n \geq 4$. Then, there exists no social preference function $P^*(F, R, \pi)$ that satisfies Condition UD (Unrestricted Domain), Condition L (Liberalism), and Condition NC (No Cycles).*

Proof See “Appendix”.

We can think of Theorem 2 as simply confirming our previous philosophical intuitions. For as we noted in our explication of the problems perspectival disagreements raise in Sect. 3 above, much recent work points out that, when such disagreements obtain, it is possible that schemes of rights—and dispute-adjudication mechanisms more generally—may fail to satisfy the Conflict Resolution Requirement. We can think of Theorem 2 as formally proving this: that when Condition UD obtains, there exists no social preference function satisfying both Condition L and Condition NC, meaning that a social preference relation generated by rights cannot be guaranteed to be free of cycles. Hence, we cannot guarantee that the Conflict Resolution Requirement is satisfied.

What to do in the face of Theorem 2? Recall that Gaus’s solution to the adjudicative problems caused by perspectival disagreements is to construct a public social world that all citizens adopt for the purpose of resolving conflicts. Using our formal apparatus, we can interpret this strategy as a domain restriction. That is, we can interpret Gaus’s strategy as changing our domain from Condition UD to Condition ND by constructing a public social world. And, indeed, as Theorem 1 shows, this solves the problems perspectival disagreements raise. But this has the flavor of solving the problem by fiat. On one way of looking at what is going on here, Gaus introduces interesting problems that perspectival diversity may generate for public reason liberalism, but then asks us to temporarily bracket away such diversity by constructing an artificial public social world that all endorse regardless his or her unique perspective. To some readers, this might feel as though Gaus simply avoids or circumnavigates the problem of perspectival diversity without fully resolving it. Yet in the face of those troubling results of Theorem 2, this solution is clearly preferred to no solution.

Our final result examines whether it is possible for a scheme of jurisdictional rights to satisfy Condition NC without *completely* bracketing our perspectival disagreements, as Gaus’s solution does. To this end, we introduce the notion of *restricted perspectival diversity*, defined below, along with the corresponding condition on social preference functions.

Definition RD (Restricted Perspectival Diversity): A profile $\pi = (\pi^1, \dots, \pi^n)$ of individual perspectives satisfies RD (Restricted Perspectival Diversity) if and only if for all individuals $i \in N$ and for all social states

$x, y \in X$, if x^i, y^i are i -variants, then, for all $j \in N$ such that $x^i \neq x^j$ or $y^i \neq y^j$ (or both), x^j and y^j are i -variants.

Condition RD (Domain of Restricted Perspectival Diversity): All social choice problems $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$, where $\pi = (\pi^1, \dots, \pi^n)$ satisfies Definition RD, are admissible.

When Condition RD obtains it means that if one individual deems that two social states differ only according to what happens with respect to his or her private sphere, then *everyone* agrees with this individual, in the sense that they agree that the two social states differ only with respect to what happens in that specific individual's private sphere. In other words, under Condition RD, everyone in society agrees with the public demarcation of each individual's private spheres, even though these individuals might *still* disagree about how to categorize what it is that is happening in said spheres. Now Condition RD is a *bona fide* domain restriction, of the kind Gaus introduces. But, unlike Gaus's artificial social world that is characterized by the domain restriction Condition ND, Condition RD does not require society to completely do away with all perspectival diversity. Citizens can still categorize social states in very different ways when Condition RD obtains.

Indeed, the only thing that Condition RD requires is that whenever a jurisdictional right is exercised, everyone sees the two social alternatives as being different only with regards to the private sphere of the person exercising the right. However, Condition RD does not restrict what specific features each person assigns to the two different social alternatives—that is, Condition RD does not restrict how persons understand what is happening within each individual's private sphere. So long as everyone sees the two social alternatives as being different only with regards to the private sphere of the person exercising the right, any two people can see the two social alternatives in very different ways. The following example illustrates how there can still be radical perspectival diversity even under Condition RD:

Example: RD still allows radical perspectival diversity

- There are two individuals, Althea and Bertha, i.e. $N = \{A, B\}$.
- Let $F = X = \{x, y\}$.
- Suppose that $X_A = X_B = \{a, b, c\}$ (where a, b, c denote unique descriptions/events.)
- Suppose Althea has the following perspective:
 $\pi^A(x) = (a, a)$ and $\pi^A(y) = (b, a)$.
- Suppose Bertha has the following perspective:
 $\pi^B(x) = (b, c)$ and $\pi^B(y) = (c, c)$.

We can see here that $\pi^A(x) = (a, a) \neq (b, c) = \pi^B(x)$ and $\pi^A(y) = (b, a) \neq (c, c) = \pi^B(y)$. That is, Althea and Bertha perceive the two social states x and y in completely different ways. Yet, their perspectives satisfy condition RD. Specifically, both Althea and Bertha at least agree that the two social states x and y differ only in regards to what happens within Althea's private sphere (even though their interpretations of what is actually happening in the two social states are radically different).

Hence, we can see that Condition RD is by no means trivial. It still allows for much disagreement.

Recall that allowing unrestricted perspectival diversity (Condition UD) led us to an impossibility result in which jurisdictional rights fail to satisfy the Conflict Resolution Requirement. On the face of it, since, under condition (β), those who perceive the social alternatives differently from the rights-holder *in any way* must minimally go along with the rights-holder's preferences in cases of perspectival disagreement, one might be skeptical that restricting our domain to Condition RD might suddenly generate a positive outcome, for Condition RD *does* allow for persons to perceive social alternatives differently (as we have just seen). Yet the following result says otherwise.

Theorem 3 *Under Condition RD (Domain of Restricted Perspectival Diversity), Condition L (Liberalism) implies Condition NC (No Cycles).*

Proof See “Appendix”.

In other words: even when there is *some* perspectival diversity present in society, the use of jurisdictional rights to order conflicting claims can *still* resolve these claims by *guaranteeing* a non-cyclical social preference relation, so long as this perspectival diversity manifests itself in a particular way. That is, we can still *guarantee* that the Conflict Resolution Requirement will be satisfied so long as perspectival diversity looks like Condition RD rather than Condition UD. This, we think, is a surprising result.

6 The possibility of public reason

Theorem 3 says that under a certain domain restriction on admissible perspectives—what we have called Restricted Perspectival Diversity (Condition RD)—schemes of jurisdictional rights will always satisfy Condition NC (No Cycles), which, we have seen, is sufficient for the satisfaction of the Conflict Resolution Requirement. But the introduction of Condition RD might seem unmotivated. Why care about this arbitrary domain restriction specifically? What does it teach us about public reason liberalism and the adjudication of conflicting claims?

Any time some kind of restriction changes an impossibility result to a possibility result (as Condition RD does) it calls for some kind of interpretation. Whether the possibility result should be a cause for optimism depends on this interpretation. As an example, with Arrow's theorem (we have already noted) it has been shown that there are many different ways of restricting the domain of preferences to induce a possibility rather than an impossibility result. This in itself is no reason for optimism. But it has been suggested that preferences in the real world more often than not resemble these restrictions rather than those preference profiles that cause the trouble (e.g., Mackie 2003: ch. 5). This specific interpretation of the relevant domain restrictions allows for optimism. As another example, Sen's paradox of the Paretian liberal dissipates if preferences satisfy a certain structure. Again, this alone gives one no reason for optimism. But, it has been suggested that such restricted preference patterns simply mean that persons are “non-meddlesome,” suggesting that liberalism and Paretian

welfarism can co-exist with one another so long as persons mind their own business (Blau 1975). Like before, this specific interpretation of the restriction is a cause for optimism.

To end, then, we would like to offer an interpretation of Condition RD and discussion of Theorem 3 more generally that, we think, should be a cause for optimism. There are two possible takeaways of this result: one obvious and perhaps a bit deflationary, the other less obvious and more optimistic. The obvious interpretation of Theorem 3 and Condition RD is that we need not jump to Gaus's reliance on public social worlds quite as quickly as one might initially think. After a cursory glance at the emerging literature on perspectival disagreements one might think that once perspectival disagreements characterize society then we must immediately adopt a publicly constructed social world in order to ensure that the Conflict Resolution Requirement is satisfied. Theorem 3 says otherwise. So long as perspectival disagreements obtain in the form of Condition RD then we can still rely on the old methods of adjudication to generate a social preference relation absent cycles. But if society's diversity begins to look more like Condition UD then Gaus's reliance on publicly constructed social worlds is necessary to ensure that our social preference relation does not contain cycles. This much is clear but also does little to inspire hope, for there is no reason to think that perspectives will just happen to satisfy Condition RD rather than Condition UD. Here, Condition RD is unmotivated. But what about the less obvious, optimistic interpretation of Condition RD and Theorem 3?

Now thus far we have not said much about each individual's private sphere—one of the core primitives of our model. There are, we think, two ways of interpreting this primitive. First, one can understand private spheres in a moralized notion—it is simply a moral fact that persons possess certain basic rights to do as they wish, which sets the boundaries of each person's private sphere. This seems to be how Jonathan Riley interprets private spheres as employed by Mill (Riley 1989: pp. 128–129). Or, one can understand private spheres as being determined by law: a society's legal system determines what each person's private sphere is. On this understanding of private spheres, persons residing in the United States have different private spheres when compared to persons residing in South Korea, who have different private spheres when compared to persons residing in North Korea, who have very limited private spheres indeed. On the moralized notion of private spheres, however, persons have the same private sphere regardless the legal system they reside in, for it is simply a fact about morality that persons have certain rights, regardless what the law says.

Perspectival disagreements are compatible with both interpretations of this primitive. With the moralized notion of private spheres, perspectival disagreements are cashed out in terms of persons categorizing differently whether certain actions are or are not in line with the requirements of *morality*. For example, if there is a moral right to free speech, then Althea and Bertha can disagree about whether φ -ing is protected by this moral right or not, for they categorize the action φ -ing differently. On the legal interpretation of private spheres, perspectival disagreements are cashed out in terms of persons categorizing differently whether certain actions are or are not in line with the requirements of *law*. As an example of this, Althea and Bertha can look at the First Amendment and disagree about whether ψ -ing is an action protected by this amendment or not, for they categorize the action ψ -ing differently. Indeed, as the

great legal theorist Alexander Bickel notes, when it comes to legal rights like the First Amendment, “men may in full and equal reason and good faith hold differing views about... [their] proper meaning and specification” (Bickel 1962/1986: pp. 36–37).

Note that if one adopts a moralized notion of private spheres then one must also adopt our obvious yet deflationary conclusion concerning the significance of Theorem 3. Whether perspectival diversity manifests in manner Condition RD or Condition UD is an *exogenous* parameter, for it is just a fact about morality what is and is not included in each individual’s private sphere. Sometimes perspectival diversity will manifest itself in a manner such that each person agrees on the boundaries of private spheres (though they can disagree about what it is happening in each sphere), in which case Condition RD obtains. But when perspectival disagreements go beyond this—when persons disagree about the boundaries of private spheres—then Condition UD obtains. Since we work within the public reason liberalism tradition, we reject this controversial notion that there is simply a moral fact concerning private spheres. We thus endorse the view that private spheres are determined by legal systems.

On the legal interpretation of private spheres, though, the exact nature and extent of our perspectival disagreements is an *endogenous* parameter: for when persons categorize ϕ -ing as either falling within or without an individual’s private sphere, they look to the legal system to see whether this is so. Crucially, though, this judgment depends on the nature of the current legal system, which is a contingent fact about the particular legal system in question. If, for instance, our legal system says that persons have a right to personal autonomy so long as they do not violate the autonomy of others then Althea may categorize reading pornography as being completely within her private sphere (for, according to her perspective, reading pornography is an act of autonomy that does not violate the autonomy of others), whereas Bertha may categorize reading pornography as affecting the private spheres of all women (for, according to her perspective, reading pornography does indeed violate the autonomy of others). Here, Condition UD obtains.

But suppose our legal system instead says quite specifically that persons have a right to read pornographic material regardless the effect this has on anyone else—it is this legal right that now what sets the boundaries of private spheres. Here, both Althea and Bertha likely now characterize Althea’s reading pornography as only affecting Althea’s private sphere, given how our society’s legal system now sets the boundaries of private spheres. Though Althea and Bertha likely categorize what it is Althea is doing in different ways, the two will now see the social state in which Althea reads pornography and the social state in which she does not as only differing in terms of Althea’s private sphere. Now, Condition RD obtains. This, then, is our second and more optimistic interpretation of Condition RD and Theorem 3: if rights are defined in an appropriate manner, then we are more likely to face cases of Condition RD rather than Condition UD. When this occurs, persons need not set aside their diverse perspectives in order for rights to cleanly adjudicate disputes.

Thus, Condition RD is an important domain restriction because whether it obtains or not is something we can control. Rights can be defined in a manner such that perspectival diversity manifests itself in a manner resembling Condition RD rather than Condition UD. When this happens we achieve social harmony and cooperation rather than conflict and discord without bracketing away our diversity.

Appendix: Proofs

Proposition 1 *Let $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$ be any social choice problem. Then, $UD(F, P^*(F, R, \pi))$ is non-empty if $P^*(F, R, \pi)$ is acyclic.*

Proof of Proposition 1 Let (F, R, π) be any social choice problem. Suppose $|F| = n \in \mathbb{N}$ and suppose $P^*(F, R, \pi)$ is acyclic. We want to show $UD(F, P^*(F, R, \pi)) \neq \emptyset$. Suppose, for a proof by contradiction, that $UD(F, P^*(F, R, \pi)) = \emptyset$. Pick any $x_1 \in F$. Since $UD(F, P^*(F, R, \pi)) = \emptyset$, we have $x_1 \notin UD(F, P^*(F, R, \pi))$. Hence, there exists another social state, say $x_2 \in F$, such that $x_2 P^*(F, R, \pi) x_1$. However, since $UD(F, P^*(F, R, \pi)) = \emptyset$, we must have $x_2 \notin UD(F, P^*(F, R, \pi))$ as well. This implies that there exists another social state, say $x_3 \in F$, such that $x_3 P^*(F, R, \pi) x_2 P^*(F, R, \pi) x_1$. Continue this process n times. Then, we have $x_{n+1} P^*(F, R, \pi) x_n P^*(F, R, \pi) \dots x_2 P^*(F, R, \pi) x_1$. Since $|F| = n$, there must exist $j, k \in \{1, \dots, n+1\}$ such that $x_k P^*(F, R, \pi) x_{k-1} P^*(F, R, \pi) \dots x_{j+1} P^*(F, R, \pi) x_j$ where $x_k = x_j$. But, then, we have a cycle, contradicting our assumption that $P^*(F, R, \pi)$ is acyclic. Hence, $UD(F, P^*(F, R, \pi)) \neq \emptyset$. \square

Theorem 1 *Under Condition ND (Domain of No Diversity), Condition L (Liberalism) implies Condition NC (No Cycles).*

Proof of Theorem 1 Assume ND (Domain of No Diversity) and L (Liberalism). For a proof by contradiction, suppose condition NC (No Cycles) is violated. Then, there exists a social choice problem $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$ for which the social preference function $P^*(F, R, \pi)$ satisfying L produces a cycle of some length $n \in \mathbb{N}$. We will argue that this cannot be the case for all $n \in \mathbb{N}$.

Suppose $n = 1$. Then, there exists some $x \in F$ such that $x P^*(F, R, \pi) x$. Since $x P^*(F, R, \pi) x$, there exists an $i \in N$ such that i unconditionally prefers x^i to x^i (note that x^i and x^i are trivially i -variants as $x^i_{-i} = x^i_{-i}$), which implies $x^i P_i x^i$. This contradicts that P_i is asymmetric, and, thereby, irreflexive. Hence, the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = 1$.

Now, suppose that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k$. We wish to show that this implies that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k + 1$. So, for a proof by contradiction, suppose that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k$, but produces a cycle of length $n = k + 1$. Then, there exists $x_{(1)}, \dots, x_{(k+1)} \in F$ such that $x_{(1)} P^*(F, R, \pi) x_{(2)} \dots x_{(k)} P^*(F, R, \pi) x_{(k+1)} P^*(F, R, \pi) x_{(1)}$.

Since $x_{(1)} P^*(F, R, \pi) x_{(2)}$, there exists an $i \in N$ such that $x^i_{(1)}$ and $x^i_{(2)}$ are i -variants, i unconditionally prefers $x^i_{(1)}$ and $x^i_{(2)}$, and either

- (i) $\forall j \in N, x^j_{(1)} = x^j_{(1)}$ and $x^j_{(2)} = x^j_{(2)}$; or

(ii) $\forall j \in N$ such that $x_{(1)}^i \neq x_{(1)}^j$ or $x_{(2)}^i \neq x_{(2)}^j: x_{(1)}^j R_j x_{(2)}^j$.

By ND, (ii) cannot happen. So, we have $\forall j \in N, x_{(1)}^i = x_{(1)}^j$ and $x_{(2)}^i = x_{(2)}^j$.

For any $y \in F$, let y^* be the i -variant of $x_{(1)}^i$ such that the i -component of y^* is the i -component of y . Since $x_{(1)}^{*i} = x_{(1)}^i$ and $x_{(2)}^{*i} = x_{(2)}^i, x_{(1)}^i$ and $x_{(2)}^i$ are i -variants, and individual i prefers $x_{(1)i}^i$ to $x_{(2)i}^i$; unconditionally, we have $x_{(1)}^i P_i x_{(2)}^i$.

Now, since $x_{(2)} P^*(F, R, \pi) x_{(3)}$, there exists an individual, say $k \in N$, that $x_{(2)}^k$ and $x_{(3)}^k$ are k -variants, unconditionally prefers $x_{(2)}^k$ to $x_{(3)}^k$, and either

(iii) $\forall j \in N, x_{(2)}^k = x_{(2)}^j$ and $x_{(3)}^k = x_{(3)}^j$; or

(iv) $\forall j \in N$ such that $x_{(2)}^k \neq x_{(2)}^j$ or $x_{(3)}^k \neq x_{(3)}^j: x_{(2)}^j R_j x_{(3)}^j$.

Again, by ND, (iv) cannot happen. So, we have $\forall j \in N, x_{(2)}^k = x_{(2)}^j$ and $x_{(3)}^k = x_{(3)}^j$, and in particular we have $x_{(2)}^k = x_{(2)}^i$ and $x_{(3)}^k = x_{(3)}^i$. There are two cases to consider: when $k = i$ and when $k \neq i$. If $k = i$, then, just as before, since $x_{(2)}^i$ and $x_{(3)}^i$ are i -variants and individual i ($= k$) prefers $x_{(2)i}^i$ to $x_{(3)i}^i$ unconditionally, we have $x_{(2)}^i P_i x_{(3)}^i$. Since $x_{(2)}^i = x_{(2)}^{*i}$ and $x_{(3)}^i = x_{(3)}^{*i}$, we have $x_{(2)}^{*i} P_i x_{(3)}^{*i}$, which implies $x_{(2)}^{*i} R_i x_{(3)}^{*i}$. If $k \neq i$. Then, since $x_{(2)}^k = x_{(2)}^i$ and $x_{(3)}^k = x_{(3)}^i$ are k -variants, the i -components of $x_{(2)}^k = x_{(2)}^i$ and $x_{(3)}^k = x_{(3)}^i$ will be the same. Hence, $x_{(2)}^{*k} = x_{(2)}^{*i} = x_{(3)}^{*i} = x_{(3)}^{*k}$, which implies $x_{(2)}^{*i} R_i x_{(3)}^{*i}$. So, in either case, we have $x_{(2)}^{*i} R_i x_{(3)}^{*i}$.

By repeating the same argument, we will arrive at $x_{(1)}^{*i} P_i x_{(2)}^{*i} R_i x_{(3)}^{*i} \dots R_i x_{(k+1)}^{*i} R_i x_{(1)}^{*i}$. Since R_i is an order, this implies $x_{(1)}^{*i} P_i x_{(1)}^{*i}$, which contradicts that P_i is asymmetric, and, thereby, irreflexive. Hence, the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k + 1$.

By mathematical induction, we conclude that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of any length $n \in \mathbb{N}$. □

Theorem 2 *Suppose $|X| \geq 4$ and $n \geq 4$. Then, there exists no social preference function $P^*(F, R, \pi)$ that satisfies Condition UD (Unrestricted Domain), Condition L (Liberalism), and Condition NC (No Cycles).*

Proof of Theorem 2 Let $F = \{w, x, y, z\}$ and suppose individuals 1 to 4 have the following preferences:

$$\begin{aligned} z^1 R_1 w^1 P_1 x^1 R_1 y^1; \\ w^2 R_2 x^2 P_2 y^2 R_2 z^2; \\ x^3 R_3 y^3 P_3 z^3 R_3 w^3 \\ y^4 R_4 z^4 P_4 w^4 R_4 x^4 \end{aligned}$$

where individual 1 prefers w^1 to x^1 unconditionally; individual 2 prefers x^2 to y^2 unconditionally; individual 3 prefers y^3 to z^3 unconditionally; and individual 4 prefers z^4 to w^4 unconditionally.

For all $j \in N \setminus \{1, 2, 3, 4\}$, suppose $w^j I_j x^j I_j y^j I_j z^j$. Call this profile of individual preferences R and the profile of perspectives π . By condition UD, the social choice problem (F, R, π) is in our domain. Now, assume

- w^1, x^1 are 1-variants;
- x^2, y^2 are 2-variants;
- y^3, z^3 are 3-variants;
- z^4, w^4 are 4-variants

Also, suppose

$$\begin{aligned}
 w^1 &= w^3 \neq w^2 = w^4; \\
 x^1 &= x^3 \neq x^2 = x^4; \\
 y^1 &= y^3 \neq y^2 = y^4; \\
 z^1 &= z^3 \neq z^2 = z^4.
 \end{aligned}$$

So, individuals 1 and 3 share the same perspective, while individuals 2 and 4 share the same perspectives, and all other individuals share the same perspective that is different from any perspective that individuals 1, 2, 3 and 4 have.

- Since w^1, x^1 are 1-variants; individual 1 prefers w^1_1 to x^1_1 unconditionally; $w^2 R_2 x^2$ (and $x^1 \neq x^2$); $x^3 P_3 w^3$ (and $w^1 = w^3$ and $x^1 = x^3$); $w^4 R_4 x^4$ (and $w^4 \neq w^1$); and $w^j R_j x^j$ for all $j \in N \setminus \{1, 2, 3, 4\}$, by L, individual 1 has a liberal right to be socially decisive over w and x . Hence, we must have $w P^*(F, R, \pi) x$.
- Since x^2, y^2 are 2-variants; individual 2 prefers x^2_2 to y^2_2 unconditionally; $x^3 R_3 y^3$ (and $y^2 \neq y^3$); $y^4 P_4 x^4$ (and $x^2 = x^4$ and $y^2 = y^4$); $x^1 R_1 y^1$ (and $x^2 \neq x^1$); and $x^j R_j y^j$ for all $j \in N \setminus \{1, 2, 3, 4\}$, by L, individual 2 has a liberal right to be socially decisive over x and y . Hence, we must have $x P^*(F, R, \pi) y$.
- Since y^3, z^3 are 3-variants; individual 3 prefers y^3_3 to z^3_3 unconditionally; $y^4 R_4 z^4$ (and $z^3 \neq z^4$); $z^1 P_1 y^1$ (and $y^3 = y^1$ and $z^3 = z^1$); $y^2 R_2 z^2$ (and $y^3 \neq y^2$); and $y^j R_j z^j$ for all $j \in N \setminus \{1, 2, 3, 4\}$, by L, individual 3 has a liberal right to be socially decisive over y and z . Hence, we must have $y P^*(F, R, \pi) z$.
- Since z^4, w^4 are 4-variants; individual 4 prefers z^4_4 to w^4_4 unconditionally; $z^1 R_1 w^1$ (and $w^4 \neq w^1$); $w^2 P_2 z^2$ (and $z^4 = z^2$ and $w^4 = w^2$); $z^3 R_3 w^3$ (and $z^4 \neq z^3$); and $z^j R_j w^j$ for all $j \in N \setminus \{1, 2, 3, 4\}$, by L, individual 4 has a liberal right to be socially decisive over z and w . Hence, we must have $z P^*(F, R, \pi) w$.

As a result, we have $w P^*(F, R, \pi) x P^*(F, R, \pi) y P^*(F, R, \pi) z P^*(F, R, \pi) w$, a cycle in the social preference relation $P^*(F, R, \pi)$, violating NC. □

Theorem 3 *Under Condition RD (Domain of Restricted Perspectival Diversity), Condition L (Liberalism) implies Condition NC (No Cycles).*

Proof of Theorem 3 The proof follows the general strategy of the Proof of Theorem 1, but, now, with perspectival diversity, we need to consider additional cases.

Assume RD and L. For a proof by contradiction, suppose NC is violated. Then, there exists a social choice problem $(F, R, \pi) \in \mathfrak{S} \times \mathcal{R} \times \Pi$ for which the social preference function $P^*(F, R, \pi)$ satisfying L produces a cycle of some length $n \in \mathbb{N}$. Again, we will argue that this cannot be the case for all $n \in \mathbb{N}$.

Suppose $n = 1$. Then, there exists some $x \in F$ such that $x P^*(F, R, \pi) x$. Since $x P^*(F, R, \pi) x$, there exists an $i \in N$ such that i unconditionally prefers x^i to x^i (note that x^i and x^i are trivially i -variants as $x^i_{-i} = x^i_{-i}$), which implies $x^i P_i x^i$. This contradicts that P_i is asymmetric, and, thereby, irreflexive. Hence, the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = 1$.

Now, suppose that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k$. We wish to show that this implies that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k + 1$. So, for a proof by contradiction, suppose that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k$, but produces a cycle of length $n = k + 1$. Then, there exists $x_{(1)}, \dots, x_{(k+1)} \in F$ such that $x_{(1)} P^*(F, R, \pi) x_{(2)} \dots x_{(k)} P^*(F, R, \pi) x_{(k+1)} P^*(F, R, \pi) x_{(1)}$.

Since $x_{(1)} P^*(F, R, \pi) x_{(2)}$, there exists an $i \in N$ such that $x^i_{(1)}$ and $x^i_{(2)}$ are i -variants, i unconditionally prefers $x^i_{(1)}$ and $x^i_{(2)}$, and either

- (i) $\forall j \in N, x^j_{(1)} = x^j_{(1)}$ and $x^j_{(2)} = x^j_{(2)}$; or
- (ii) $\forall j \in N$ such that $x^j_{(1)} \neq x^j_{(1)}$ or $x^j_{(2)} \neq x^j_{(2)}$: $x^j_{(1)} R_j x^j_{(2)}$.

Again, for any $y \in F$, let y^* be the i -variant of $x^i_{(1)}$ such that the i -component of y^* is the i -component of y . Since $x^*_{(1)} = x^i_{(1)}$ and $x^*_{(2)} = x^i_{(2)}$, $x^i_{(1)}$ and $x^i_{(2)}$ are i -variants, and individual i prefers $x^i_{(1)}$ to $x^i_{(2)}$ unconditionally, we have $x^i_{(1)} P_i x^i_{(2)}$.

Now, since $x_{(2)} P^*(F, R, \pi) x_{(3)}$, there exists an individual, say $k \in N$, that $x^k_{(2)}$ and $x^k_{(3)}$ are k -variants, unconditionally prefers $x^k_{(2)}$ to $x^k_{(3)}$, and either

- (iii) $\forall j \in N, x^j_{(2)} = x^j_{(2)}$ and $x^j_{(3)} = x^j_{(3)}$; or
- (iv) $\forall j \in N$ such that $x^j_{(2)} \neq x^j_{(2)}$ or $x^j_{(3)} \neq x^j_{(3)}$: $x^j_{(2)} R_j x^j_{(3)}$.

Case 1: $x^k_{(2)} = x^i_{(2)}$ and $x^k_{(3)} = x^i_{(3)}$.

Again, there are two cases to consider: when $k = i$ and when $k \neq i$. If $k = i$, then, since $x^i_{(2)}$ and $x^i_{(3)}$ are i -variants and individual $i (= k)$ prefers $x^i_{(2)}$ to $x^i_{(3)}$ unconditionally, we have $x^i_{(2)} P_i x^i_{(3)}$. Since $x^i_{(2)} = x^*_{(2)}$ and $x^i_{(3)} = x^*_{(3)}$, we have $x^*_{(2)} P_i x^*_{(3)}$, which implies $x^*_{(2)} R_i x^*_{(3)}$. If $k \neq i$. Then, since $x^k_{(2)} = x^i_{(2)}$ and $x^k_{(3)} = x^i_{(3)}$ are k -variants, the i -components of $x^k_{(2)} = x^i_{(2)}$ and $x^k_{(3)} = x^i_{(3)}$ will be the same. Hence, $x^*_{(2)} = x^*_{(2)} = x^*_{(2)} = x^*_{(3)}$, which implies $x^*_{(2)} R_i x^*_{(3)}$. So, in either case, we have $x^*_{(2)} R_i x^*_{(3)}$.

Case 2: $x_{(2)}^k \neq x_{(2)}^i$ or $x_{(3)}^k \neq x_{(3)}^i$.

Then, $k \neq i$. By RD, $x_{(2)}^i$ and $x_{(3)}^i$ are k -variants (where $k \neq i$). This means that the i -component of $x_{(2)}^i$ and $x_{(3)}^i$ are the same. Hence, we have $x_{(2)}^{*i} = x_{(3)}^{*i}$, implying $x_{(2)}^{*i} R_i x_{(3)}^{*i}$.

So, in all cases, we have $x_{(2)}^{*i} R_i x_{(3)}^{*i}$. So, we have $x_{(2)}^{*i} R_i x_{(3)}^{*i}$ and $x_{(1)}^{*i} P_i x_{(2)}^{*i}$. (Note that $x_{(1)}^{*i}, x_{(2)}^{*i}, x_{(3)}^{*i} \in X^k$)

By repeating the same argument, we will arrive at $x_{(1)}^{*i} P_i x_{(2)}^{*i} R_i x_{(3)}^{*i} \dots R_i x_{(k+1)}^{*i} R_i x_{(1)}^{*i}$. Since R_i is an order, this implies $x_{(1)}^{*i} P_i x_{(1)}^{*i}$, which contradicts that P_i is asymmetric, and, thereby, irreflexive. Hence, the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of length $n = k + 1$.

By mathematical induction, we conclude that the social preference function $P^*(F, R, \pi)$ satisfying L cannot produce a cycle of any length $\in \mathbb{N}$. \square

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