Preface to “Optimization, Convex and Variational Analysis”

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This collection of works in the honor of professor Terry Rockafellar is a follow-up of the “Workshop on Optimization and Variational Analysis”, dedicated to Terry’s 85th birthday. The meeting, jointly organized by the CMM Center for Mathematical Modeling of the University of Chile (Chile) and the University of Perpignan (France), was held in Santiago on January 20–21, 2020.

That workshop was one of the last meetings we could attend physically, before Coronavirus changed our lives in so many ways. Globetrotting has become virtual since then. Suddenly, the beauty of the world found itself flattened to a screen. Fortunately, some things have not changed: our admiration and appreciation for Terry’s unique career has remained intact, as has the momentum to duly celebrate his birthday, through the edition of this special volume.

We are very grateful to the authors and referees for their valuable contributions and careful work.

The two volumes that make up the special issue in Terry’s honor sample the tremendous breadth of subjects where Terry has made fruitful contributions. This special issue is a modest gift for someone who has gifted us with seminal textbooks, whose content has marked generations of researchers, influencing the way of doing mathematics when it involves “variations”, regarding not only theory or analysis in optimization but also applications.

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Thank you Terry and happy birthday,
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1 The First Volume of this Special Issue Includes the Following Works

In *Continuous Newton-like inertial dynamics for monotone inclusions*, Hedy Attouch and Szilard Lazlo study the convergence properties of a Newton-like inertial dynamical system governed by a general maximally monotone operator in a Hilbert space. When the operator is the subdifferential of a convex lower semicontinuous proper function, the dynamic corresponds to the introduction of the Hessian-driven damping in the continuous version of the accelerated gradient method of Nesterov. The maximally monotone operator is replaced by its Yosida approximation with an appropriate adjustment of the regularization parameter. The introduction into the dynamic of the Newton-like correction term provides a well-posed evolution system for which convergence of the generated trajectories and velocities is shown, even in the presence of perturbations or errors.

In *Bregman Forward-Backward operator splitting*, Minh N. Bui and Patrick Combettes, establish the convergence of the forward-backward splitting algorithm based on Bregman distances for the sum of two monotone operators in reflexive Banach spaces. The proposed approach considers Bregman distances that vary over the iterations together a novel assumption on the single-valued operator, capturing various properties in the literature. For minimization, the obtained rates are sharper than existing ones.

In *Cone-constrained eigenvalue problems: structure of cone spectra*, Alberto Seeger investigates topological properties of cone spectra for different structures, involving both symmetric and non-symmetric matrices, associated to cones with countably and uncountably many faces.

In *Generalized sequential differential calculus for expected-integral functionals*, Boris S. Mordukhovich and Pedro Pérez-Aros, motivated by applications to stochastic programming, introduce and study certain expected-integral functionals, which are mappings given in an integral form depending on two variables, the first being a finite dimensional decision vector and the second one an integrable function. The main goal is to establish sequential versions of Leibniz's rule for regular subgradients by employing and developing appropriate tools of variational analysis.

The work *MPCC: strong stability of M -stationary points*, by Harald Günzel, Daniel Hernandez Escobar and Jan-J. Rückmann, deals with mathematical programs with complementarity constraints. Under the Linear Independence constraint qualification, the authors state a topological/algebraic characterization for the strong stability in the sense of Kojima of M -stationary points. The main tool in their approach is based on considering perturbations of the describing functions up to the second order.

In *Algebraic approach to duality in optimization and applications*, Dinh The Luc and Michel Volle focus on duality of optimization problems in a vector space without topological structure. Topological duality relations are established by means of an algebraic approach without lower semicontinuity or quasicontinuity hypothesis on perturbation functions. Algebraic constraint qualification conditions are also proposed for problems with countably

infinitely many inequality constraints. Applications to the sum of two convex functions, monotropic problems, and infinite convex or linear problems illustrate the approach.

Alternative representations of the normal cone to the domain of supremum functions and subdifferential calculus, by Rafael Correa, Abderrahim Hantoute and Marco López, study new characterizations of the normal cone to the effective domain of the supremum of an arbitrary family of convex functions. These results give new formulas for the subdifferential of the supremum function, which use both the active and nonactive functions at the reference point. Only the data functions are involved in these characterizations, the active ones from one side, together with the nonactive functions multiplied by some appropriate parameters. In contrast with previous works in the literature, the main feature of the subdifferential characterization is that it does not involve the normal cone to the effective domain of the supremum (or to finite-dimensional sections of this domain). A new type of optimality conditions for convex optimization is also established.

In *Enlargements of the Moreau–Rockafellar Subdifferential*, by Malek Abassi, Alexander Kruger and Michel Théra, the focus is on three enlargements for the classical Moreau–Rockafellar subdifferential, originally and mainly used in the framework of convex analysis, called the sup^* -subdifferential, the sup -subdifferential and the symmetric subdifferential. Many properties of the usual subdifferential are shown to hold for these new operators, including convexity, weak^* -closedness, weak^* -compactness, as well as some calculus rules.

In *Resolvents and Yosida approximations of displacement mappings of isometries*, Salihah Alwadani, Heinz Bauschke, Julian Revalski and Xianfu Wang consider maximally monotone operators, which are fundamental objects in modern optimization. The main classes of monotone operators are subdifferential operators and matrices with a positive semidefinite symmetric part. The authors study a nice class of monotone operators: displacement mappings of isometries of finite order. The authors derive explicit formulas for resolvents, Yosida approximations, and (set-valued and Moore–Penrose) inverses, and illustrate some results by considering certain rational rotators and circular shift operators.

In *On existence of solutions of parametrized generalized equations*, Asen Dontchev studies the existence of solutions to a structured generalized equation, defined by the sum of a function subject to perturbations and a set-valued mapping.

In *Constrained Lipschitzian error bounds and noncritical solutions of constrained equations*, Andreas Fischer, Alexey F. Izmailov, and Mario Jellite extend characterizations of error bounds to include, to some extent, additional constraints and to consider mappings with reduced smoothness requirements. In this way, new necessary as well as sufficient conditions for the existence of error bounds are established.