

Two groundbreaking works on Natural Deduction (ND) were published in 1934 by Gerhard Gentzen and Stanisław Jaśkowski. They worked out their results independently and the systems they provided differ significantly, but no doubt both realised the same idea — the formalization of the “natural” ways of reasoning in mathematics. Additionally, Gentzen introduced Sequent Calculus (SC) in his paper as a tool for analysing properties of ND proofs. The following 80 years have shown enormous development of the research rooted in the work of both logicians. Their influence may be observed in at least three domains: educational, theoretical and philosophical.

First of all, ND systems became one of the most popular method of teaching logic in introductory courses. As such ND found the way not only to specialists but also to these educated persons who have passed the course in logic. Hundreds of textbooks exhibited vast diversity of forms of ND but despite the differences all are descendants of original systems of Jaśkowski and Gentzen.

But ND is not only a standard working tool but also an interesting domain of theoretical research. In modern proof theory ND and SC play a dominant role and the questions of normalization of ND proofs or elimination of cut rule in SC are sometimes treated as central problems of this field. It is not surprising if one pays attention to a variety of consequences of these results like e.g. (semi)decidability, interpolation, consistency. Many results appeared to be of great utility outside proof theory; for example the investigation on the Curry-Howard isomorphism between ND proofs, types and programs provided a fruitful basis for logical programming.

Finally, the analysis of proper shape of rules of ND or SC led to the development of rich considerations in the philosophy of meaning (of logical constants). In particular, applications of standard solutions to different non-classical logics led to the problem of conditions for being a logical constant.

These sketchy remarks cannot even point out all the areas where Gentzen’s and Jaśkowski’s achievements play an important, if not decisive, role.

Eight papers collected in this special issue devoted to Gentzen's and Jaśkowski's heritage provide a detailed picture of some aspects of their original work, as well as of some consequences and later developments. Let me briefly characterise their content.

Although Jaśkowski's paper from 1934 is often referred to as one of the sources of ND it is rather rarely associated with nonclassical logics. In particular, it is rarely noticed that his rules for first order logic provide a formalization of logic essentially weaker than classical. In modern terms it is Inclusive Logic because the rules admit empty domain. Ermanno Bencivenga in his contribution recalls this pioneering aspect of Jaśkowski's work. Moreover, he shows that straightforward extension of Jaśkowski's solution yields the first system of Universally Free Logic, admitting also terms that do not denote existing objects.

When comparing the approaches of Gentzen and Jaśkowski to ND one can observe that Gentzen is more focused on theoretical matters whereas Jaśkowski's system is devised mainly for practical purposes of constructing proofs. In particular, the way of representing proofs (as trees in Gentzen's approach and sequences in Jaśkowski's approach) has important consequences. Dealing with trees makes simpler the analysis of ready proofs and leads to successful investigations on normalization of proofs. On the other hand it is simpler to search for proofs in Jaśkowski's ND (commonly known as Fitch's approach). Francis Jeff Pelletier and Allen Hazen in their paper focus on the differences between ND in the spirit of Jaśkowski and Gentzen. They pay special attention to the fields where Jaśkowski's approach seems to be better applicable and explain its heuristic value with many examples. In particular, the applications of Jaśkowski's format of ND to formalization of various nonclassical logics are described. The paper contains also extensive discussion of the generalized ND due to Peter Schroeder-Heister and its application to the problem of definability of constants.

Differences between Gentzen's and Jaśkowski's way of representing proofs are also explored by Greg Restall. He is particularly concerned with normal proofs and with the possibility of isolating the effects of structural rules from sequent calculus in the framework of ND. The paper ends with a presentation of Girard's proof nets (in directed form) which are conceived here as a more refined version of ND combining virtues of ND and sequent calculus.

In studies on the origins of Gentzen's work usually the influence of Paul Hertz' papers is underlined. Certainly the idea of using trees as a form of representation of proofs is due to Hertz but not the shape of logical rules. In particular, one may ask what was the source of the idea of characterisation of constants in terms of pairs of int-elim rules. Jan von Plato in his paper

investigates the problem of possible origins of Gentzen's ND. On the basis of handwritten version of Gentzen's thesis and a short manuscript containing five different versions of ND rules, he noticed that ND was in fact deeply rooted in axiomatic formulation of logic. Von Plato compares four different axiom systems and concludes that Heyting's system was possibly the main source of the rules in original system of Gentzen.

It is well known that Gentzen's way of characterising logical constants led to the development of enormous logico-philosophical work on the meaning of logical constants and their proof-theoretical characterisation. One of the key notions of these discussions, but unfortunately often applied in rather vague way, is that of a "harmony" between introduction and elimination rules. The paper of Schroeder-Heister provides precise account of this concept in terms of higher-level rules, extended with propositional quantification. As a result we obtain a system where a canonical elimination rule corresponds to each set of introduction rules and a canonical introduction rule corresponds to each set of elimination rules.

Gentzen's seminal paper introduced not only ND but also Sequent Calculus. Although the latter was proposed only as a technical tool for showing indirectly (via so called *Hauptsatz* or cut elimination theorem) that ND proofs may be reduced to normal form, it became soon the first-class citizen in the field of proof theory. Accordingly Gentzen's *Hauptsatz* became one of the most celebrated result and many logicians (including Curry, Schütte, Tait, Dragalin — to mention just a few) proposed several methods for proving them. Mathias Baaz and Alex Leitsch in their paper do not restrict considerations to comparison of classical methods of Gentzen and Schütte-Tait but also present the new methods of cut elimination by resolution called CERES and its generalization CERESD obtained by addition of so called deletion rules.

Despite unquestioned usefulness of sequent calculi it soon became obvious that their (satisfying) applicability to nonclassical logics is strongly restricted. In particular, very often proposed logical rules did not possess nice properties of rules for classical (or intuitionistic) logic and cut rule is not eliminable. These deficiencies of standard sequent calculi was the starting point for construction of numerous generalised sequent calculi. Two of the most important approaches are surveyed in the paper of Agata Ciabattoni, Revantha Ramanayake and Heinrich Wansing. The first generalization — Hypersequent Calculi — were introduced independently by Garell Pottinger and Arnon Avron, and developed significantly by the latter. The basic idea is quite simple; instead of single sequents we use their (multi)sets called hypersequents. This apparently simple generalization strongly extends the

expressive power of standard SC. The second solution — Display Calculi — was devised by Nuel Belnap on the basis of Curry's analysis of properties of rules of standard SC. In this approach each logic is additionally equipped with structural connectives which form structures from formulae, and sequents are built from structures not from (multi)sets of formulae. Except from extensive presentation of these two rather different generalizations of SC, the paper contains also a detailed comparison of their similarities and differences.

Although the notion of SC has rather definite meaning and even generalised forms of SC tend to reproduce in some way the format of Gentzen's rules and the focus on cut eliminability, one can find also some sequent calculi of rather different character. They use standard sequents (usually in intuitionistic version, i.e. with one formula in succedent) but admit rules of different form than just a pair of (antecedent/succedent) introduction rules. Since they are not a generalizations of SC in the sense of calculi described in the preceding paper we may call them nonstandard SC. The issue ends with a survey of such calculi divided into three types: Gentzen's type (rules based), Hertz' type (primitive sequents based) and mixed type (some constants characterised by means of rules some by means of primitive sequents).

Finally let me add that *Studia Logica* seems to be a particularly nice place for celebrating Gentzen's and Jaśkowski's heritage since the paper of the latter author was published in the very first issue of this journal. I'm also very happy to announce that the XIIIth edition of the conference *Trends in Logic* which will be held in July 2-5 at the Department of Logic of Lodz University in Lodz (Poland) is also devoted to this 80th Anniversary.

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