



Trojan Asteroid Satellites, Rings, and Activity

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Abstract

The *Lucy* mission will encounter five Jupiter Trojans during its mission with three of the five already known to be multiple systems. These include a near-equal-mass binary, a small and widely separated satellite, and one intermediate-size satellite system. This chapter reviews the current state of knowledge of Trojan asteroid satellites in the context of similar satellite systems in other small body populations. The prospects for the detection of additional satellites as well as other near-body phenomena are considered. The scientific utility of satellites makes their observation with *Lucy* an important scientific priority for the mission.

Keywords Asteroids · Trojan asteroids · Satellites · Rings

1 Introduction

The existence of asteroid satellites was, for many years, a purely speculative topic that had yielded hints, but no incontrovertible evidence, despite numerous searches (Weidenschilling et al. 1989). But, starting with the serendipitous discovery of S/(243) I Ida by the Galileo spacecraft (Belton et al. 1996), there was a rapid convergence of multiple observational and theoretical developments that led to the paradigm-shifting realization that satellites exist in essentially all small body populations (Merline et al. 2002; Richardson and Walsh 2006; Noll et al. 2008), including the Jupiter Trojans (Table 1).

Satellites and binary systems exist over a wide range of relative sizes and separations, but, when scaled to the region of gravitational influence given by the Hill radius, regularities

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Table 1 Known or Suspected Trojan Satellites and Binaries

Object	d_2/d_1 (%)	a/r_{Hill} (%)	method	discovery
(617) Patroclus	92	1.7	imaging	Merline et al. (2001)
(640) Hektor	4.8	1.1	imaging	Marchis et al. (2006b)
(911) Agamemnon	≥ 3	≥ 3	occultation	Timerson et al. (2013)
(2207) Antenor	-	-	lightcurve	Stephens et al. (2018)
(3548) Eurybates	1.9	11.3	imaging	Noll et al. (2020b)
(15094) Polymele	≥ 20	≥ 3	occultation	Buie et al. (2022)
(16974) Iphthime	76	1	imaging	Noll et al. (2016)
(17365) Thymbræus	~ 100	<i>contact</i>	lightcurve	Mann et al. (2007)
(29314) Eurydamas	~ 100	<i>contact</i>	lightcurve	Mann et al. (2007)

Estimated relative sizes and separations of components. Uncertain values are shown in italics

are evident (see Fig. 1). Widely separated systems with satellites at more than ten percent of the Hill radius are known, but the majority of satellites cluster closer to their primaries, at distances of a few percent of the Hill radius or less. Relative sizes also span the range from near-equal size binaries down to small satellites at observational limits. During the flybys, *Lucy* will be able to identify $d \geq 2$ km objects over the full Szebehely sphere, *i.e.* the inner third of the Hill sphere where satellites can have stable orbits.

Satellites are produced in several ways - systems may be primordial, the product of collisions and subsequent capture, or can arise from rotational fission. The three different formation processes are partially correlated with size and can be grouped by the total angular momentum of the system (Pravec and Harris 2007). Primordial binaries are a natural and expected outcome of the streaming instability which results in a gravitationally-bound collapse (Nesvorný et al. 2010, 2019). Kuiper Belt objects, especially the Cold Classics, are the best examples of likely primordial objects that formed by co-accretion in the nebula and were not subsequently disrupted. Large Trojan and Outer Main Belt asteroids with similar-size binary components, including contact binaries, may also have formed in this way.

A second ubiquitous process that can result in the formation of satellites is collisional evolution. Collisions can result in one or more satellites over a wide range of primary size and relative component size (Durda et al. 2004). Collisions also play an important role in driving intermittent activity (Jewitt et al. 2015) and could be a source mechanism for transient rings or other orbital debris.

Finally, small asteroids can split by fission after being spun up by solar radiation pressure via YORP (see Vokrouhlický et al. 2015). This is the likely origin for most Near-Earth Asteroids (NEA) with satellites as well as the majority of small, rapidly rotating Main Belt asteroid systems (Margot et al. 2015). It remains to be seen whether this mechanism might be important for Trojans, both because few small Trojans are known and because YORP is much less effective at 5 AU (Kalup et al. 2021).

Satellite discoveries provide great scientific opportunities. The relative size, composition, and orbit properties of a satellite are all important constraints on its origin and the dynamical and collisional history of the system. Satellite orbits can be used to determine system mass which can then be combined with volume determinations to yield densities. The *Lucy* encounters provide a unique opportunity to search for satellites in orbits and size scales inaccessible to current Earth-based telescopes. Therefore, the search for satellites during Trojan encounters is a scientific priority for the *Lucy* mission.

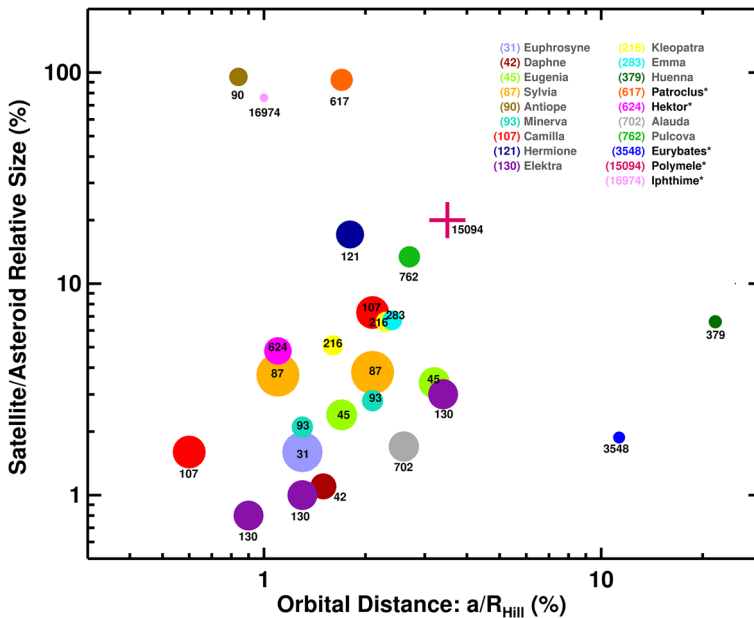


Fig. 1 The relative size and orbital distance of satellites of large, primitive, low albedo asteroids (C, P, and D spectral types) in the Main Belt (MB) and Trojans having primaries with volume-equivalent diameters $d_{veq} \geq 50$ km are compared (plus Polymele with $d_{veq} = 19$ km). Five Trojan (denoted by bold with * in the legend) and fourteen Main Belt systems are shown. The symbol size is proportional to the effective diameter of the system except for Polymele which is shown as a red cross because of the uncertainty in both the satellite size and separation (see text). Five of the nineteen systems shown consist of two satellites and one system has three; the orbital properties of each satellite relative to its primary are shown for these systems

Rings and other circum-body material may be considered as special cases of the more general topic of satellites. Rings are likely collisional in origin, but might also arise from other forms of mass loss. Of the four known small bodies with rings, none are Trojans. Asteroids are also known to exhibit activity driven by a range of processes including impact ejection and disruption, rotational instabilities, thermal fracture, and sublimation of volatile ices (Jewitt et al. 2015). To date, no such activity has been observed among the Trojans, but, observational biases may account for the lack of detection. Taken altogether, the possibility of rings and near-asteroid phenomena must be considered during any small body flyby.

In this chapter we focus primarily on the Jupiter Trojans, and specifically the targets of the *Lucy* mission. However, to understand the Trojans in context it will be important to look at other populations as well, especially carbonaceous (low-albedo, C-, X-, and D-type) asteroids in the Outer Main Belt that potentially share similar origin and evolutionary tracks with the Trojans.

2 Earth-Based Observations

The vast majority of known satellites of small bodies (see Johnston 2019 for compilation) have been discovered from ground-based and Earth-orbiting observatories using one or more distinct observational techniques. Search methodologies that have yielded satellite detections include direct imaging, stellar occultations, and lightcurve analysis. All of these ob-

serving techniques have been used in searches of Trojans, but only a small number of satellites or binary systems have been identified. It remains to be seen whether this represents a fundamental difference in this population compared to other small body populations, or, more likely, is an artifact of observational biases, as described below.

2.1 Direct Imaging

The most straightforward method by which to identify satellites is with direct imaging. The relevant parameters governing detectability are the angular separation of the bodies, their relative brightness, and the mutual orbit period and orientation. The sub-arcsecond angular scale of most known small-body satellite systems and the occurrence of large brightness differences between primary and secondary means that observations from the Hubble Space Telescope (HST) or from large ground-based telescopes with adaptive optics are the most productive search techniques. Such searches in the Main Belt and the Kuiper Belt have yielded the detection of many satellite and binary systems (Noll et al. 2008; Margot et al. 2015; Noll et al. 2020a).

Imaging searches of Jupiter Trojans have led to the discovery of four satellites/binaries (Table 1). Two systems were detected from the ground with adaptive optics (AO) systems - (617) Patroclus has a near-equal mass binary companion, Menoetius (Merline et al. 2001; Buie et al. 2015) and (624) Hektor is a probable contact binary with a small satellite, Skamandrios (Marchis et al. 2006b, 2014). Two additional systems have been found with HST - (16974) Iphthime was found to be a partially-resolved, near-equal mass binary in a search for possible synchronous binaries among Trojans with long rotation periods (Noll et al. 2016), and (3548) Eurybates was found to have a small, widely-separated satellite, Queta, in a deep search of the *Lucy* mission targets (Noll et al. 2020b; Brown et al. 2021). Other searches have had mixed results: a search of 35 moderate-size Trojans with HST's Advanced Camera for Surveys High Resolution Camera did not find any companions, while ground-based observations of a "few dozen" larger Trojans discovered only one, Patroclus (Merline et al. 2001, 2007). Marchis et al. (2006a) reported observations of 20 Trojans brighter than 18th mag in R-band (approximately $d > 30$ km) with only the detection of the satellite of Hektor (Marchis et al. 2006b). Unfortunately, the negative searches of Trojans from ground-based AO systems are not well-documented - neither target lists nor observing details are available - so the significance of the non-detections is difficult to evaluate. In hindsight, however, the non-detections are not especially constraining because, even with Hubble or ground-based AO observations, the expected angular separations of satellites falls below detection limits for all but the largest objects and/or unusually wide satellite separations, even in very deep images. As shown in Fig. 1, the satellite of Eurybates, Queta, is relatively wide compared to the orbits of comparable Main Belt objects, with the exception of the very wide satellite of (379) Huenna. At equivalent diameters of 69.8 km and 57.3 km respectively, the Trojans Eurybates and Iphthime are smaller than all of the Main Belt asteroids where satellites have been directly imaged from Earth-based observatories. As the Hill radius shrinks proportionally with less massive systems, closer and/or smaller satellites are rendered undetectable, even with the superior resolution and contrast of HST.

Main Belt (MB) asteroids with satellites are useful for bounding the kinds of systems the *Lucy* spacecraft might encounter. Among MB asteroids of similar spectral types (Tholen types C-, P-, and D-) approximately 5% of objects with diameters of $d \geq 85$ km have been found to have satellites (Fig. 1). It is worth noting that, if these MB systems were moved to the same distance as the Trojans, many would be undetectable from Earth-based AO or HST. As this figure shows, for this subset of asteroids, satellites are mostly found at a few percent or less of the Hill radius with more weakly-bound, distant satellites being rare. We also note

the high prevalence of multiple satellite systems - five of the MB asteroids host two satellites each and a sixth, (130) Elektra, previously known to have two satellites, has recently been found to have a third (Berdeu et al. 2022). The MB asteroids with multiple satellites are also the largest bodies in collisional families, a correlation that is highly suggestive of a collisional origin for these satellite systems. Likewise, among the Trojans, we note that Eurybates is the largest collisional family known and similarly conclude that its satellite Queta likely originated from the family-forming event. Orbital analysis and photometric observations point to another potential catastrophic family in the Trojans, that of (4709) Ennomos (Nesvorný et al. 2015; Wong and Brown 2023), which, however, has yet to be searched for potential satellites. Thus, based on the prevalence and architecture of known satellite systems, the detection of close-in satellites and multiple satellite systems must be considered in planning the *Lucy* flybys.

2.2 Stellar Occultations

Stellar occultations also have the potential to identify satellites. Timerson et al. (2013) summarize previous reports of possible visual detections and present evidence of a secondary event recorded in an occultation of the Trojan (911) Agamemnon. The projected sky-plane separation was 278 km and the diameter of the satellite was estimated to be 5 km. A satellite this small and close to its primary is not detectable by direct imaging with AO systems or by using HST, leaving stellar occultation as the only means to detect it. With sparse chord sampling, the probability of serendipitous detection is low and suggests that close-in satellites may not be rare among the Trojans.

In preparation for the *Lucy* flybys, there has been an intensive campaign to observe stellar occultations of the *Lucy* mission targets (Buie et al. 2021; Keeney et al. 2021, 2022) with the primary goal of refining size and shape estimates needed to plan the spacecraft encounters. Unexpectedly, during an occultation by Polymele on 27 March, 2022, observers on two adjacent chords detected a secondary event corresponding to a 5–6 km object at a projected separation of 204 km (Buie et al. 2022). The satellite was observed again on 03 February, 2023 in another stellar occultation, this time with a massive deployment of over 90 telescopes tightly spaced across the track, intentionally arranged to redetect the satellite (Buie et al. 2023; Levison et al. 2023). As the discovery of Polymele's satellite demonstrates, intensive observations with close spacing of telescopes across the shadow path can be a successful technique for the discovery of satellites. However, the logistical challenges of this method have, so far, limited its widespread application.

2.3 Lightcurves

Satellites can also be found by detecting the occurrence of mutual events or from the analysis of the lightcurve shape, amplitude, and period (Tedesco 1979; Pravec et al. 2002). The appearance of multiple, asynchronous periods in a lightcurve is one indicator that can be interpreted as evidence of a binary system (Mottola and Lahulla 2000; Pravec et al. 2006). The analysis of lightcurve periods for populations can also reveal patterns that may be interpreted as evidence of binaries. In particular, Trojans show a marked excess of slow rotators (French et al. 2015; Ryan et al. 2017; Kalup et al. 2021) which may be attributable to tidally synchronous binaries (Nesvorný et al. 2020). At the other extreme, there is an apparent spin barrier at a $P \sim 4\text{--}5$ hr (Pravec and Harris 2000; Melita et al. 2010; Ryan et al. 2017; Chang et al. 2021) signaling a maximum rotation rate that is consistent with the observed low bulk densities for the Patroclus and Eurybates (Buie et al. 2015; Noll et al. 2020b). Further acceleration of bodies rotating near the spin barrier leads to the formation of binaries by fission.

The mechanism for acceleration of rotation rates at Trojan distances is not clear as solar radiation that powers the YORP effect in the Main Asteroid Belt is significantly reduced; instead, collisions may play a dominant role.

Large amplitude, slow rotators have been interpreted as being consistent with close- or contact-binary configurations (Leone et al. 1984; Sheppard and Jewitt 2004). Mann et al. (2007) identified two Trojans, (17365) Thymbraeus and (29314) Eurydamas, as likely contact binaries where they also note that the primary component of Hektor meets this criterion as well. Sonnett et al. (2015) used sparse photometric data from WISE to identify 34 Trojans with large amplitude lightcurves out of a sample of 953 Trojans as well as 48 of 554 Hildas. The high fraction of candidates is consistent with other lines of evidence that suggest close binaries may be a relatively common feature of the Trojans, particularly at smaller diameters.

When asteroid satellite systems are suitably aligned, mutual events are observable and can yield the relative sizes of the components in addition to the orbital period. The observation of mutual events accounts for a significant fraction of systems known in the Main Belt and NEA populations because it can be carried out with telescopes with modest apertures. Stephens et al. (2018), Stephens and Warner (2019) report the observation of five or more possible mutual events in the lightcurve of (2207) Antenor observed from Jan. 23-May 22, 2018 and another in March 2019. They were, however, unable to extract a consistent orbital period from the observed events and so are unable to estimate the size of the secondary. The utility of this technique remains mostly unrealized for Trojans, primarily because they are significantly fainter than their Main Belt counterparts owing to their larger heliocentric distances and because the smaller-diameter asteroids that account for the majority of Main Belt and NEA systems remain largely unidentified among the Trojans.

2.4 Orbit Determination and Tidal Evolution

Once satellites are detected by any of the methods described above, the next observational goal is to establish the mutual orbit. This can be accomplished with repeated detections of the satellite with sufficiently accurate relative astrometry to allow for orbit solutions. In practical terms this usually requires at least four additional detections but can require many more (Grundy 2012). In cases where the number of detections is insufficient to uniquely determine an orbit, other orbital constraints can be applied. One of the most useful is the state of tidal evolution of the system which can potentially constrain the orbital eccentricity. In cases where the shape of the primary deviates significantly from spherical, the mutual orbit plane can also sometimes be inferred.

Mutual tides in binary systems lead naturally to a synchronous, circular-orbit end state. The time scale for eccentricity evolution is very strongly dependent on the semimajor axis of the system, a , scaled by the radius of the satellite, R_2 , with $\tau_e \propto (a/R_2)^5$. The timescale is also directly proportional to Q/k , the tidal quality factor scaled by the tidal Love number (Goldreich and Sari 2009). However, because tidal dissipation for porous, rubble-pile-like structures is complex and poorly constrained, the tidal synchronization formulae are most useful in predicting the functional sensitivity, but less so in determining absolute timescales. Fortunately, empirical data from known binary systems can be used to constrain tidal evolution timescales. Kuiper Belt binaries span the transition from eccentric to circularized (Noll et al. 2020a) and show a very regular behavior, transitioning from circularized orbits to random eccentricities at $a/R_{avg} \approx 55$, where R_{avg} is the average of the binary component radii (Nesvorný et al. 2020). The trend is similar, but less clearcut, among the Main Belt systems shown in Fig. 1. This may be due to the prevalence of multi-satellite systems where additional orbital resonances come into play.

Table 2 Orbits of Trojan Satellite Systems

Secondary	a (km)	e	T (days)	a/R_{avg}	reference
(617) I Menoetius	688.4 ± 4.7	0.0043 ± 0.0049	4.28270 ± 0.00007	12.6	Grundy et al. (2018)*
(640) I Skamandrios	623.5 ± 10	0.31 ± 0.03	2.9651 ± 0.0003	9.5	Marchis et al. (2014)
(3548) I Queta	2321.2 ± 35	0.098 ± 0.009	82.47 ± 0.06	131	Brown et al. (2021)**
S/2022 (15094) 1	≥ 204	0	<i>14–17</i>	34	Buie et al. (2022)
S/2016 (16974) 1	≥ 205	0	3.288 ± 0.016	<i>10.1</i>	Noll et al. (2016)

Estimated or assumed values shown in italics. The satellites of (15094) Polymele and (16974) Iphthime do not have established orbits. The eccentricity of both, however, is assumed to be near zero due to the expected short tidal evolution timescales in both cases. The orbital period of (16974) Iphthime is assumed to be the same as the observed lightcurve period Mottola et al. (2011). The range of orbital periods for Polymele's satellite is for density from 700–1000 kg/m³ (Levison et al. 2023) *Unconstrained orbit fit **Updated with additional observations obtained in December 2022

Among the Trojans (see Table 2), the Patroclus-Menoetius binary, with $a/R_{avg} \approx 6$, is fully circularized and synchronous (Grundy et al. 2018). Queta orbits Eurybates at $a/R_{avg} \approx 131$, a distance where it would not be expected to be circularized, consistent with its observed eccentricity of $e = 0.125 \pm 0.009$ (Brown et al. 2021). If Polymele's recently discovered satellite's observed separation of 204 km represents its semimajor axis, it would have $a/R_{avg} \approx 34$, in the range where it would also be expected to be tidally evolved on timescales of order 200 Myr or less. Absent a recent formation or other factors, its yet-to-be-determined orbit is likely to be nearly circular. The close binary Iphthime should be tidally evolved, and multi-epoch HST observations are consistent with this (Noll & Grundy, in preparation). The orbit of Hektor's satellite stands out as an exception. Despite having $a/R_{avg} \approx 9.5$, its orbit is substantially eccentric at $e \sim 0.3$. The satellite orbit is also inclined relative to the primary's equator with $i \sim 50$ (Marchis et al. 2014). It is possible, however, that both unusual features may result from forcing by the elongated, bilobed Hektor primary (Jiang et al. 2018).

Besides mutual tides of the primary/satellite system, it is also important to consider orbital evolution from orbital perturbations caused by the Sun. For example, for the Eurybates-Queta system, a Kozai resonance could force coupled oscillations of the eccentricity and inclination over surprisingly short timescales of ≈ 500 years (Brown et al. 2021). The precise mutual orbit that will be established by the *Lucy* flyby will set the stage for testing whether or not this resonance comes into play in this system.

2.5 Rings and Activity

Rings have been found or have been suggested to be present around four small bodies, the Kuiper Belt objects Haumea and Quaoar, and the Centaurs Chariklo and, possibly, Chiron (Ortiz et al. 2015; Morgado et al. 2023; Braga-Ribas et al. 2014; Ortiz et al. 2017). The evidence for rings comes from stellar occultations that are sensitive to the largest and most optically thick examples of rings. Whether similar structures exist around smaller and less active bodies such as the Trojans is currently a matter for speculation that can be directly addressed by the *Lucy* flybys.

Activity is an umbrella term that covers a wide range of potentially time-variable sources of material found in proximity of any given small body. In addition to satellites and rings, asteroids may intermittently release material from the surface as the result of collisions or the activation of subsurface volatiles. So-called Main Belt comets (MBC) are bodies with

asteroid-like orbits that show comet-like characteristics activated by a collision (Jewitt et al. 2015). No examples of this phenomenon have been detected among the Trojans despite the fact that the Trojans are likely to be compositionally similar to MBCs. The lack of detected activity may be the result of their colder surface temperatures and correspondingly lower sublimation rates, or due to the lower collision rates in the Trojans, or simply due to observational biases. It is worth noting that Chiron and a handful of other Centaurs are known to exhibit episodic activity (e.g., Jewitt 2009) despite the fact that their surfaces are generally even colder than Trojan surfaces and collision rates are also likely lower. Centaur activity might suggest that Trojans are more depleted in near-surface volatiles than either MBCs or Centaurs.

3 Observations of Lucy Targets

Given the widespread evidence of satellites, rings, and activity in asteroid populations, along with the diagnostic scientific value of satellite dynamical and physical properties, the search for and study of these phenomena was identified as one of the top scientific priorities of the *Lucy* mission (Levison et al. 2021). *Lucy*'s close flybys of five Trojan systems puts it in a unique position to make observations that are difficult or impossible to make in any other way. Beforehand, there are ground- and space-based observations that can help optimize *Lucy*'s flyby sequences. At all steps of the process leading up to the flybys, careful planning is of paramount importance.

Satellite-focused observations of the *Lucy* targets can be divided into two main categories - searches for currently unknown satellites, rings, or other near-asteroid phenomena - and observations of currently known satellites. We discuss each below.

3.1 Searches for Unknown Satellites

Lucy will search for currently unknown satellites and rings or other orbital debris with direct imaging. How best to conduct an imaging search involves tradeoffs between sensitivity, coverage, angular resolution and timing. *Lucy* has a requirement to be capable of conducting a search for a 2 km diameter or larger object with an albedo of $p = 0.04$ or greater anywhere in the stable portion of the Hill sphere (Levison et al. 2021). Because such a search covers such a large scale in distance and, at the same time, requires fine resolution and high contrast close to the primary, the search is broken up into two phases as described below.

The volume of space that must be searched is defined by the region around each target where satellites and rings can have stable orbits. This is usually parameterized by the Hill radius where $R_{Hill} = a (m_1/m_{Sun})^{1/3}$ where a is the semimajor axis of the binary and m_1 is the mass of the primary; the Hill radius thus scales with the effective radius of the primary. Hamilton and Burns (1991) showed that some orbits, particularly retrograde orbits, can be exist at separations up to $\sim 1/2 R_{Hill}$, however, there are few known examples of such weakly bound satellites, possibly because the lifetime of any such systems is short. Most orbit at smaller fractions of the Hill radius. On the other hand, the existence of asteroid pairs (Pravec et al. 2019) - formerly bound small asteroids that now orbit the Sun independently - attests to the fact that some very wide systems must exist at times, if only briefly. Fortunately, satellites at such wide separations from the primary can be found with Earth-based observatories such as HST or Keck at the required sensitivity or better. Thus, the Szebehely radius, $1/3 R_{Hill}$, where satellites are most stable, is the region of interest for satellite searches by *Lucy* (Table 3, Fig. 2).

Table 3 Szebehely Radius and Angular Size for *Lucy* Targets

Object	Mass ^a (10 ¹⁵ kg)	r_{sz} (10 ³ km)	range ^b (10 ⁶ km)	$t \pm CA$ ^c days
(3548) Eurybates	151(3)	6.93	2.52	5.12
(15094) Polymele	21.1(1)	2.18	0.79	1.47
(11351) Leucus	40.4(4)	3.73	1.36	3.21
(21900) Orus	50.8(8)	6.91	2.51	4.68
(617) Patroclus	1410(29)	13.9	5.04	11.2

Uncertainty in final digit(s) shown in parentheses ^aEurybates and Patroclus use system masses derived from satellite orbits (Noll et al. 2020b; Grundy et al. 2018), other objects use assumed density of 1000 kg⁻³ and effective diameters of 21, 34, and 63 km for Polymele, Leucus, and Orus respectively ^bThe range at which the angular size of the Szebehely sphere equals 5.5 mrad, the size of the L'LORRI field of view ^cTime from closest approach when Szebehely sphere exceeds the L'LORRI field of view

As detailed in Table 4, deep searches around *Lucy* mission targets have been carried out with HST at two or more epochs and already constrain the presence of distant satellites at separations encompassing the full Szebehely sphere (Noll et al. 2018). The sensitivity of these HST observations exceed that needed to reach the $d = 2$ km requirement. However, at separations of $s < 0.5$ arcsec, where the background is determined by the PSF of the primary, the sensitivity to km-scale satellites rapidly declines. Equal-diameter binaries are detectable at significantly smaller separations of $s > 0.07$ arcsec. Intermediate cases fall between these extremes, although it is worth noting that the satellite of Polymele (plotted in Fig. 1) would not be detectable with HST due to the small diameter of the primary and consequent small angular scale of the Hill sphere. Thus, a complete search satisfying the mission requirement will rely on observations to be made by the *Lucy* spacecraft during each flyby.

Satellite searches with *Lucy* can be attempted once a potential satellite would be bright enough to be detectable with one of the *Lucy* instruments. L'LORRI is the most sensitive instrument for this measurement and will be the primary instrument used for searches. As shown in Fig. 3, L'LORRI has sufficient sensitivity to meet this requirement for approximately one month before or after close approach (CA). This leaves a large window when observations can be scheduled while optimizing coverage and resolution. L'LORRI has two observing modes – in 1×1 mode each pixel is read out for the highest resolution. In 4×4 mode, individual pixels are binned to larger superpixels resulting in higher sensitivity at the expense of resolution. Detection limits for specified exposures times in the two modes are shown in Fig. 3. Observations from *Lucy* will switch from deep exposures in the 4×4 mode to shorter exposures in 1×1 mode with when the expected brightness of a satellite crosses the detection threshold for 1×1 mode. The timing of this switchover varies depending on the size of the target and details of the encounter geometry (see Fig. 3).

In order to differentiate a satellite from a background source, there must be detectable motion of the *Lucy* target and any potential bound satellites relative to distant stars. The apparent rate of Trojan targets as seen from the *Lucy* spacecraft is a function of both the satellite's orbital motion and the spacecraft trajectory. Thus, a successful search strategy requires repeated observations over multiple days while at large distances with increasingly smaller time steps between observations nearer to close approach as the viewing geometry changes more rapidly. *Lucy* will carry out its search using just such a cadence.

Lucy encounters its Trojan targets at phase angles ranging from 63° to 126°, with the phase angle of egress complementing that of the ingress. The lower phase angle leg is the most favorable for satellite searches because the greater fraction of illuminated surface as

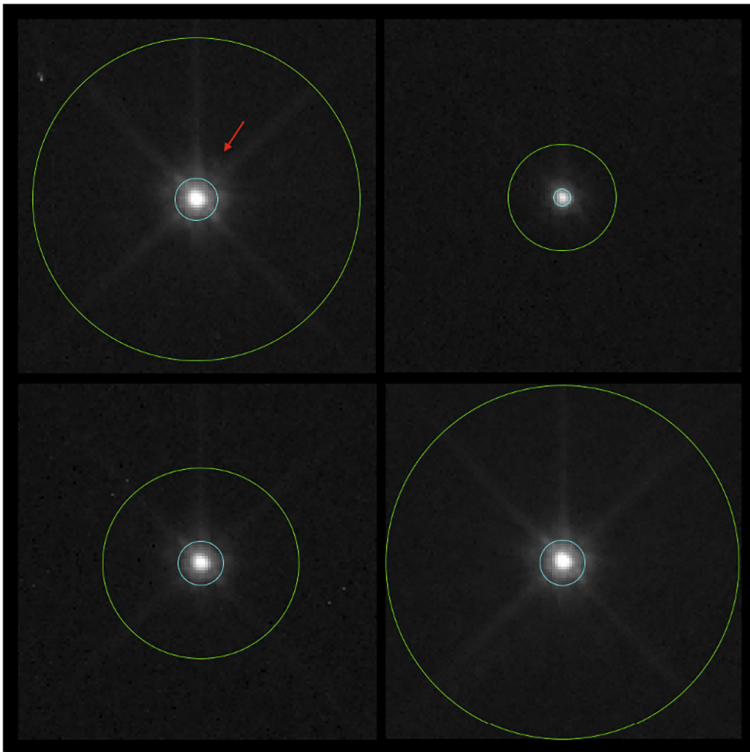


Fig. 2 Four ~ 5 arcsec square regions are shown - clockwise from the upper left, Eurybates, Polymele, Orus, and Leucus. Images are median combined WFC3 images shown in a logarithmic stretch at a common scale with the central region saturated to bring out faint detail in the wings of the PSF. Pixels are ~ 40 mas on a side. The Szebehely radius for a density of $\rho = 1500 \text{ kg/m}^3$ and the geocentric distance at the time of observation is shown in green. The smaller blue circle centered on the target has a radius of 1000 km for Eurybates, Orus, and Leucus, 400 km for Polymele. The red arrow points at Eurybates' 1.2 ± 0.4 km satellite Queta, barely visible at this stretch (Noll et al. 2020a). Other small image artifacts are not repeatable in individual frames

Table 4 HST Satellite Search of *Lucy* Targets

Object	dates		Object	dates	
Donaldjohanson	09 Jan	2021	Orus	07 Aug	2018
Eurybates ^a	12 Sep	2018		08 Aug	"
	14 Sep	"		11 Sep	"
Polymele	13 Sep	2018		12 Sep	"
	14 Sep	"		13 Sep	"
Leucus	17 Jun	2017		20 Sep	"
	30 Aug	2018	Patroclus	13 Feb	2018
	01 Sep	"		15 Feb	"

^aAdditional observations of Eurybates were obtained following the discovery of Queta as described in Noll et al. (2020b) and Brown et al. (2021)

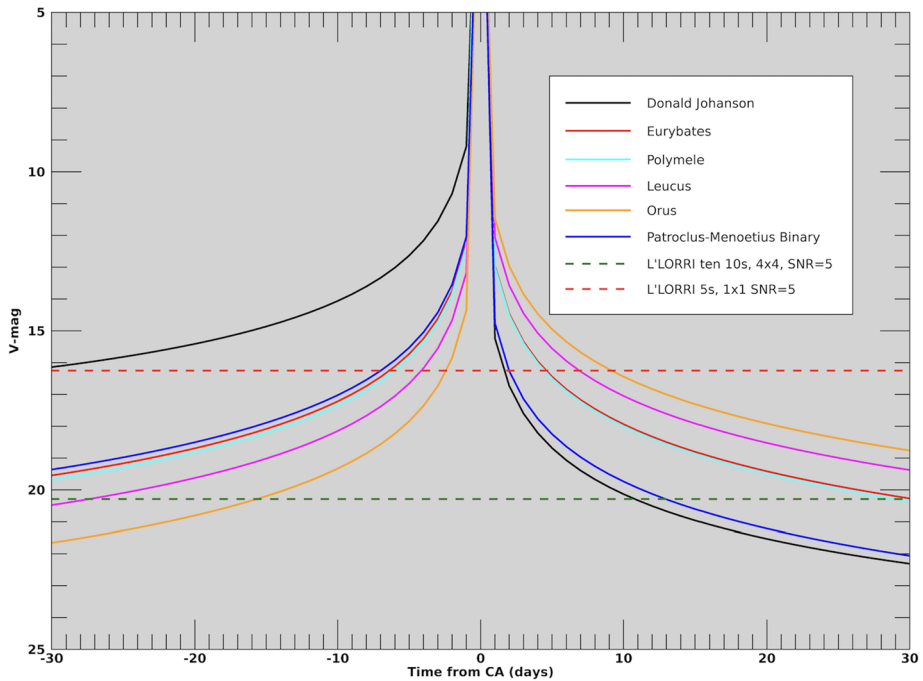


Fig. 3 The brightness of a 2 km diameter satellite as seen by *Lucy* as a function of time from close approach is shown. The satellites are assumed to have an albedo of $p = 0.04$ and a phase function constant of 0.04 mag/deg. Dashed curves show detection limits for ten 10 s exposures using a 4×4 L'LORRI superpixel (green dashed) and a single 5 s exposure in 1×1 pixel mode (red dashed)

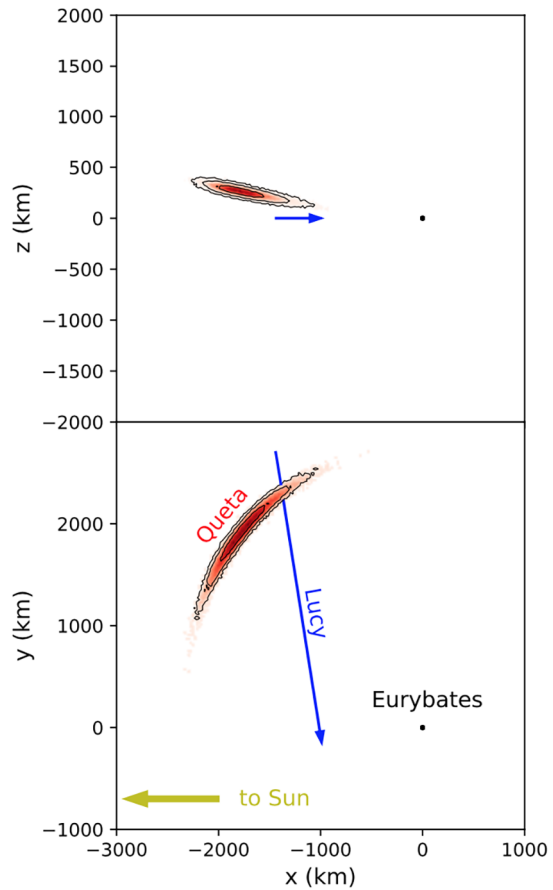
seen from the spacecraft results in a brighter target. The low phase angle leg occurs on ingress for Eurybates, Polymele, and Patroclus. At Orus and Leucus, the phase angle is lowest on egress. The prime satellite search will be scheduled for the optimal leg of the encounter. However, in order to provide backup redundancy, a smaller set of satellite search observations will also be scheduled for the less-favorable encounter leg. An added benefit of this built-in redundancy will be improved orbit determination for any existing or newly discovered satellites.

3.2 Observations of Known Satellites

Three of the *Lucy* targets are known to have a companion. One of these, the near-equal-mass Patroclus-Menoetius binary, was chosen as a mission target, in part, because it was known to be a binary that shares characteristics with many primordial binary systems in the Kuiper Belt (Buie et al. 2015; Levison et al. 2021). The satellites of Eurybates and Polymele were discovered after the selection of the *Lucy* mission as a result of the observations undertaken in preparation for the mission (Noll et al. 2018, 2020b; Brown et al. 2021; Buie et al. 2022). During the flybys, observations of each of these companions will be planned as part of the encounter observing sequence.

The state of knowledge of binary or satellite orbit is the primary driver of what observations can be carried out during encounters. For the Patroclus-Menoetius binary, which *Lucy* will fly by in March 2033, the orbital phase of the binary pair must be precisely known in order for both binary components to be observable during the close approach. The last planned

Fig. 4 Fig. 3 from Brown et al. (2021) is reproduced here showing the Eurybates flyby trajectory of the *Lucy* spacecraft and the predicted location of the satellite Queta. The top panel shows the view in the flyby plane (near-ecliptic) looking away from the Sun. The lower panel shows the view looking down from the plus- z axis (near the north ecliptic pole) with the Sun to the left. In both, the motion of the *Lucy* spacecraft from $t=500$ s to close approach is shown by the blue arrow. The 1-, 2-, and 3- σ error ellipses for Queta's position are shown



deep space maneuver in mid-2029 will set *Lucy* on its trajectory to the binary and lock in the timing of the encounter, thus, the state of knowledge of the binary orbit at that time is critical. In order to recover the lost phasing of the binary Grundy et al. (2018) completed a series of observations using HST that successfully refined the orbit to sufficient precision to allow predictions for mutual events that occurred from 2017 to 2019 (see chapter by Motola et al. in this volume for discussion of the use of mutual events as a probe of object shapes). The timing of mutual events, in principle, offer a significant advantage in constraining the mutual orbit compared to imaging observations. The next mutual event season for the Patroclus-Menoetius binary will take place in 2024. Stellar occultations that detect both binary components are another technique that can address this problem (Keeney et al. 2022). Event prediction and planning that enable additional observations will be needed up to the 2033 encounter to ensure the complex *Lucy* encounter with this binary object can be successfully completed.

During the flyby of the Eurybates system, *Lucy* will have a very close encounter with the satellite Queta as shown in Fig. 4. However, because *Lucy* is passing Queta on the anti-sunward side, it is unlikely that *Lucy*'s instrument platform range of motion will allow for an observation near the time of closest approach. Observations will be planned when *Lucy* is at a larger separation where the geometry is more favorable and where Queta's positional uncertainty can be covered with the L'LORRI field of view or by a small mosaic. Additional

pre-encounter astrometric observations of Queta can further reduce its positional uncertainty enabling observations at closer range and higher spatial resolution.

The orbit of Polymele's satellite is the least well-known of the three, having been detected only twice. However, because of the inherent precision of the two stellar occultation detections, one or two additional detections could be enough to yield an orbit, especially if the orbit has near-zero eccentricity. Either another stellar occultation detection or early detections from *Lucy* during approach may be enough to allow for a late target update that will enable resolved observations of the satellite during the flyby. An important added benefit of tracking the orbit of Polymele's satellite will be to obtain an independent measure of the system mass. Mass determination from the radio science spacecraft system will be carried out for all of the *Lucy* targets. But because Polymele is the smallest Trojan primary that *Lucy* will encounter, the mass determination by this method will likely be less precise than can be obtained from the satellite orbit.

3.3 Searches for Rings and Activity

Lucy will search for evidence of Trojan rings, other dust assemblages, and for Trojan activity on each of its flybys, primarily using high-phase-angle imaging to search for coma particulates. *Lucy* will also use its L'Ralph MVIC instrument Violet filter channel to search for coma gas due to activity; MVIC's Violet filter bandpass includes the 388.3 nm CN resonance line that is often used to trace gas activity in comets. The unusually high fluorescence efficiency (g-factor) of CN makes this species detectable in even minute quantities of gas. Apart from observations *Lucy* will make on its specific flyby targets, it would also be useful to conduct groundbased search for activity using CN filters and for both activity and for rings using stellar occultations on a wide variety of Trojans.

Lucy will have a unique opportunity to search close-in to the bodies it will encounter. The region of stable satellite orbits extends inwards to the Roche limit at $R_{Roche} \approx 2.44 r_1$ where r_1 is the radius of the parent body, much closer than can be accessed by any ground- or space-based observatory (with the exception of stellar occultation observations). Thus, there exists a significant region where satellites could exist that is largely unexplored. Rings are also situated at small separations from their parent body. The known rings around three of the four asteroids have $r_{ring} \approx 3 r_1$ (Ortiz et al. 2015; Braga-Ribas et al. 2014; Ortiz et al. 2017) while the recently identified ring around Quaoar is at $r_{ring} \approx 7.4 r_1$ (Morgado et al. 2023). Satellites are typically found at $a \approx 5 - 15 r_1$ with a few found closer or farther. *Lucy* will fly by well within the Hill sphere of each Trojan target with close approach distances of $s_{min} \geq 20 r_1$ giving it the opportunity to probe portions of the Hill sphere that are otherwise difficult or impossible to access. We note that *Lucy*'s flybys at distances of ~ 450 -1100 km are likely too distant to be able to observe phenomena such as the flurries of cm-scale particles that were seen episodically by OSIRIS-REx at Bennu (Lauretta et al. 2019). However, the appearance of unexpected activity is a possibility that nevertheless must be considered when planning *Lucy* observations.

3.4 Main Belt Asteroid Encounters

The *Lucy* mission will also fly by two Main Belt asteroids on the way to the Trojans (152830) Dinkinesh and (52246) Donaldjohanson. While these objects are not part of the formal science requirements and are primarily intended to test critical spacecraft systems, observations will nonetheless allow for a number of scientific investigations, including a search for satellites. Interestingly, Donaldjohanson has an extremely high amplitude lightcurve of 1.7 mag

and a long period of 251 hr (Levison et al. 2021), consistent with a possible synchronous binary or contact binary configuration. A single epoch of deep HST imaging was obtained near lightcurve maximum on 09 January 2021, but did not resolve a detached binary. This is, however, only a weak constraint because the most likely angular separation for a synchronous secondary would be below HST's resolution limit. The HST observations were unfortunately obtained away from opposition due to a guide star failure during the initial attempt resulting in a reduced spatial resolution. *Lucy* imaging during a successful encounter with Donaldjohanson will directly constrain the configuration of this object. Dinkinesh is a much smaller target with a likely diameter near 1 km and a 0.39 ± 0.02 mag lightcurve amplitude with a period of 52.67 ± 0.04 hours (Mottola et al. 2023). Imaging observations taken as part of optical navigation will also be useful for identifying any possible satellites.

4 Summary

As the number of known satellites of asteroids has grown, their importance and utility in understanding the formation and evolution of small-body populations, and the solar system more generally, has become apparent. Satellites are known in almost every small body population despite observational limitations that leave significant portions of the Hill sphere out of reach of Earth-based searches. Because of their distance and relatively small sizes compared to Main Belt and Kuiper Belt objects, the Jupiter Trojans are one of the least-well explored of the major small body populations. Despite this, satellites have been detected directly and there are indications of many more in the population as a whole. The *Lucy* mission will encounter five Trojans, with at least three of them having a binary companion or a satellite. Two Main Belt asteroids will also be encountered on the way to the Trojans and one of these is a candidate contact binary. The flybys will provide opportunities for close-up studies of these objects, and may reveal the presence of more satellites that have remained undetected. *Lucy* will also be able to search for rings and activity - two less-common, but possible phenomena. Up to now, the only small bodies with satellites that have been studied *in situ* by spacecraft are Ida and its satellite Dactyl in the Main Belt, the Pluto-Charon binary with its retinue of smaller satellites, and Arrokoth, a possible primordial contact binary. *Lucy* will study objects that may include both primordial binaries and collisional satellite systems, thereby significantly expanding our knowledge of this aspect of the planetary system.

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Declarations

Competing Interests The authors declare that they have no competing interests.

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