Measuring Employment and Unemployment from a Fuzzy Perspective. A Comparative Analysis Across European Countries

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Abstract

According to the standards set by the International Labour Office, people of working age who have performed some remunerated work during a specified short reference period are classified as employed, regardless of how many hours they have worked, while those who have not been able to work at all (despite wanting to) are classified as unemployed. As observed by many experts in the field, this rigid division between employed and unemployed can conceal labour markets with deeply different characteristics. In particular, the average number of hours worked and, more importantly, their distribution across employees may vary significantly across countries. The aim of this paper is to define fuzzy indicators of employment and unemployment by using the available information on the number of hours worked and the satisfaction or dissatisfaction of workers with this. In this approach, each person in the labour force is assigned a degree of employment between 0 and 1, where 0 means full non-membership in the fuzzy set of employed (and full membership in the fuzzy set of unemployed) and 1 means full membership in the fuzzy set of employed. To show the potentiality of the proposed method, we apply fuzzy measures to Labour Force Survey data from 29 European countries and compare the results with the official employment and unemployment statistics published by Eurostat.

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1 Introduction

The 13th International Conference of Labour Statisticians (ILO, 1982) introduced the concepts of employment and unemployment still in force. According to this framework, known as the *labour force framework*, people of working age who have performed some paid work during a specified short reference period are classified as employed, while those who have not been able to work at all (despite wanting to) are classified as unemployed.

This definition makes it possible to include in employment all persons whose labour input has contributed to the production of a country's output, including short-time and irregular workers. However, there are obvious limitations to this approach when labour is considered not only a factor of production, but also a tool for personal fulfilment and a source of income affecting the well-being of workers and their families. Working a number of hours well below one's needs, perhaps poorly paid and in precarious conditions, hardly allows individuals to feel a sense of belonging to the employed group. Additionally, this situation can seriously undermine the standard of living of their families.

Therefore, the employment measurement that accounts for the number of persons working even for one hour is frequently criticised for being too broad, leading to unemployment (its complementary measure within the labour force) being perceived as too narrow. To account for the underutilisation of the productive capacity of the employed, Hauser (1974, 1977) and Clogg (1979) developed the *Labour Utilization Framework*, which includes, along with unemployment, such issues as low-paid employment, time-related underemployment and educational mismatch.

Consistent with this approach, the 18th International Conference of Labour Statisticians (ILO, 2008) urged the statistical community to introduce an employment concept that captures individual workers' insufficiencies (including the low number of hours worked). Furthermore, it emphasised the need to consider the statistical category of unemployed as part of a continuum from employed to unemployed to inactivity.

This paper contributes to the body of research aiming to overcome the limitations of standard employment and unemployment rates. We align with the notion that there exists a continuum between employment and unemployment statuses and propose to use the number of hours worked and the satisfaction status of workers regarding their working hours to determine the degree to which an individual belongs to the employed group (and, symmetrically, the unemployed group). On a logical level, our approach implies a shift from a Boolean to a fuzzy concept. Fuzzy sets theory (Zadeh, 1965) has been widely applied in various research fields. In the socio-economic field, the study of poverty has long been the preferred area of application of the fuzzy method (Cerioli and Zani 1990; Cheli and Lemmi 1995; Betti et al. 2006). Regarding labour market analyses, Gálvez Ruiz and Pino Mejías (2016) proposed the fuzzy approach to capture the imprecision of the employment measure due to the shadow labour. More recently, Cheli et al. (2021) defined fuzzy versions of the employment and unemployment rates and applied them to the Italian labour market.

To show the potentiality of the method proposed in this paper, we apply fuzzy measures to Labour Force Survey data from 29 European countries and compare the results with employment and unemployment statistics published by Eurostat.

The paper is structured as follows. Section 2 reviews existing measures of employment and unemployment. Section 3 describes the fuzzy theory applied to labour force estimates, while Sect. 4 shows the results of the empirical application. Finally, Sect. 5 contains the concluding remarks.

2 Existing Measures of Employment and Unemployment

According to the international guidelines in force, the working-age population can be split into three mutually exclusive groups: the employed, unemployed and the inactive, with the first two groups constituting the labour force.

These groups are identified according to a hierarchical process, starting with identifying the employed, then the unemployed and finally the inactive population. Employed persons are persons aged 15 and over who worked under pay, profit or family gain during the reference week, even if only for one hour, or were temporarily absent from work for specific conditions. The unemployed population includes all persons of working age who were not at work during the reference period (without work) but were available for work in the reference period or shortly after (currently available for work) and had taken concrete steps to seek paid employment or self-employment in a specified precedent period (seeking work). Finally, the inactive population includes working-age persons who were neither employed nor unemployed.

The calculation of employment and unemployment rates is based on the definition of the above mentioned categories. Indeed, the employment rate is defined as the share of employed persons in the working-age population. In contrast, the unemployment rate is defined as the share of unemployed persons in the total labour force.

The International Labour Office (ILO) has highlighted the importance of also estimating the number of people with undefined employment statuses, as they are on the borderline between employment and unemployment or unemployment and inactivity. In particular, it has suggested identifying: the number of employed persons working an insufficient number of hours compared to what is desired (time-related underemployment); the number of persons not classified as unemployed who are available for work but not seeking work during the reference period (discouraged workers); the number of inactive persons who are involuntary inactive or have a certain degree of attachment to the labour force (other inactive persons with labour force attachment). The combination of these three components and the unemployed constitutes the so-called labour slack, which highlights the insufficiency of the volume of labour used in relation to the potentially available labour volume.

Eurostat provides information on the labour market slack and its components in line with those indications. Labour market slack is expressed as a percentage of the extended labour force that includes the potential additional labour force (those who are available for work but not actively seeking it and those who are looking for work but are not immediately available for it).

The introduction of statistics on labour market slack and its components enriches the unemployment rate with information highlighting the different degrees and types of unmet labour demand across countries. In particular, they can highlight differences in the time-related underemployment that underlie similar unemployment rates in other labour markets. This is indeed important in Europe, where the number of underemployed part-time workers and their change over time varies considerably between countries (Eurostat, 2023).

The measure of employment and unemployment may also be influenced by the so-called shadow labour force, which in turn depends on the size of a country's shadow economy (Schneider, 2011). However, one of the objectives of supply-side labour force sample surveys that provide official employment measures is to uncover non-standard

employment situations, such as employment without a contract or social security coverage, or employment in illicit activities.

Two recent proposals consider using the information on the hours actually worked to produce generalised measures of employment and underemployment. Brandolini and Viviano (2016) propose to go beyond the standard employment rate by considering the actual number of hours worked. Indeed, employment is obtained by summing the number of persons employed weighted by their work intensity, namely the ratio between the total number of hours worked and a reference number of hours worked by a full-time worker. In addition, they introduce the α parameter (as an exponent of the weight and varying between 0 and 1) to obtain an estimate of employment that also includes normative evaluations of having a job. The α parameter allows, for example, to differentiate between labour markets with similar labour intensity but with a different incidence of contracts considered less satisfactory for the worker, such as part-time or involuntary temporary contracts.

Bell and Blanchflower (2021) observe how time-related underemployment affects both full- and part-time workers and how its intensity depends on the difference between the number of hours desired (at the current wage) and the number of hours actually worked. Furthermore, the authors argue that overemployment should be considered to fully capture the extent of worker dissatisfaction with current contracted hours. As a result, they propose an underemployment rate computed in hours rather than in people space. Interestingly, this rate corresponds to the ILO unemployment rate in case of complete satisfaction of workers with their worked hours or in case the desired increase in hours expressed by some workers equals the desired reduction in hours expressed by others.

Our proposal shares with those described above the idea of considering hours actually worked as an essential element in measuring employment. It also shares with Bell and Blanchflower (2021) the idea of including the aspect of worker satisfaction for hours worked. However, our proposal goes beyond the definition of an underemployment index or a revised employment index. Indeed, it aims to define a simple fuzzy measure of employment and unemployment that answers the ILO (2008) indication of considering a continuum between employment and unemployment statuses. Our proposal, described in detail in the next section, can be easily applied using the EU Labour Force survey and does not imply any subjective assumption.

3 Methodology

Applying the fuzzy sets theory (Zadeh, 1965) for measuring employment and unemployment intends to overcome the binary classification between employed and unemployed persons resulting from the ILO definitions. Our proposal consists of a fuzzy method to measure the degree of employment of any labour force unit and subsequently arrive at a new estimate of the employment and unemployment rates.

The basic assumption is that workers are employed to a certain degree, represented by a membership function μ_E in the fuzzy subset E of the employed, measured on a scale from 0 to 1, whereby 1 means full membership to the set of employed persons and 0 full non-membership.

Contextually, we define a membership function μ_U in the fuzzy subset U of the unemployed. Among the individuals in the labour force, we assume that the fuzzy set U of the unemployed corresponds to the standard complement of the fuzzy set E of the employed: therefore, the membership function in the fuzzy set U is given by $\mu_U = 1 - \mu_E$.
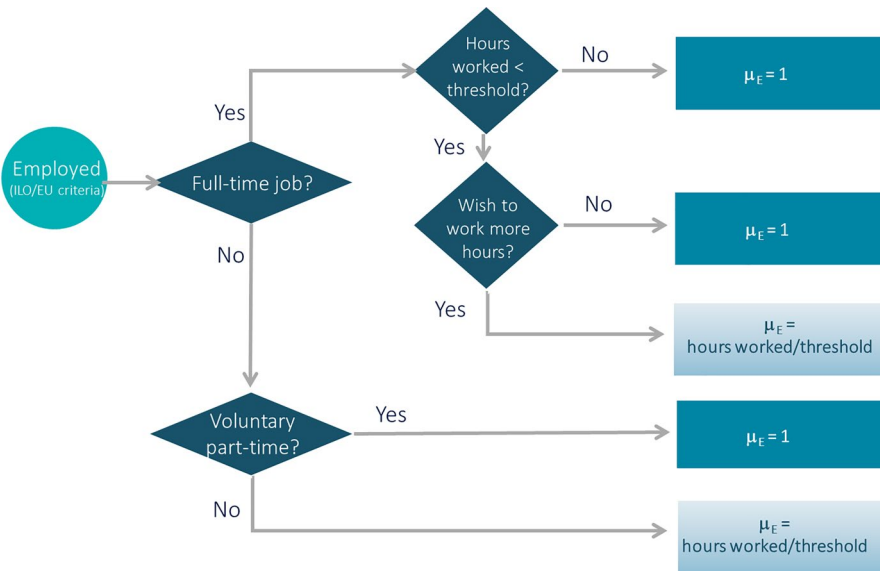
Fig. 1 Specification of the membership function μ_E for individuals who are employed according to the ILO/EU criteria

According to the ILO employment status, inactive people do not belong to the labour force and are assigned membership functions μ_E and μ_U equal to 0.

For the employed (according to ILO employment status), we compute the values of the membership function μ_E by following the assumptions sketched in Fig. 1.

The value assumed by μ_E depends on the number of hours worked (x) by the person concerned and their satisfaction with it. We define an upper bound (threshold) for the weekly worked hours. This threshold (t) acts as a limit set by statutory or collectively agreed-on standards and corresponds approximately to the hours worked on average by full-time workers. The threshold may assume different values in different countries.

We specify the μ_E and μ_U membership functions as follows:

- μ_E is set equal to 1 (and therefore μ_U is set equal to 0) for:

  i) full-time workers whose number of hours worked is not lower than the threshold ($x \geq t$);
  ii) full-time workers who do not wish to work more, even if employed for less than the specified threshold ($x < t$);
  iii) voluntary part-time workers ($x < t$).
      For these categories of workers, $\mu_E(x) = 1$.

- μ_E and μ_U are greater than 0 and lower than 1 ($0 < \mu_E < 1$ and $0 < \mu_U < 1$) for underemployed workers. Part of them is composed of involuntary part-time workers, i.e. people who could not find a full-time job. The remaining part comprises full-time workers who work less than the specified threshold and are willing to work additional hours. For both categories of underemployed workers, the membership function μ_E has been defined as the ratio of the hours actually worked to the threshold value, that is:

Fig. 2 Membership function in the fuzzy set of the employed (μ_E)
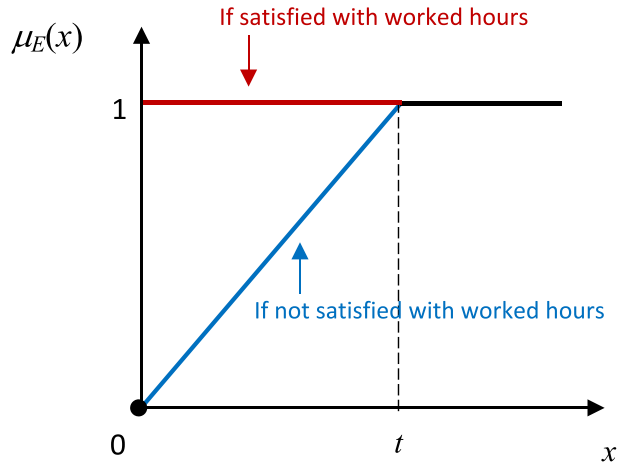
$$\mu_E(x) = x/t \quad \text{for } 0 < x < t$$

- Finally, $\mu_E = 0$ and $\mu_U = 1$ for the individuals classified as unemployed based on ILO criteria, that is: $\mu_E(x) = 0$ for $x = 0$.

In summary, for any individual in working age the membership function μ_E is specified as follows:

$$\mu_E(x) = \begin{cases} 0 & \text{if } x = 0 \\ x/t \text{ or } 1 & \text{if } 0 < x < t \\ 1 & \text{if } x \geq t \end{cases}$$

More precisely, for individuals working fewer hours than the threshold ($0 < x < t$), we check whether or not they are satisfied with the number of worked hours. In the former case, they are considered as belonging to the set of employed with membership function $\mu_E = 1$. In the latter case, they are given a membership function $\mu_E = x/t$.

The graphic representation of the membership function μ_E is shown in Fig. 2.

The fuzzy employment rate (FER) is the weighted arithmetic mean of the individual membership values $\mu_{E,i}$, where the weighting factors w_i are the sample weights of the survey that provides the employment data:

$$FER = \frac{\sum_{i=1}^n \mu_{E,i} \cdot w_i}{\sum_{i=1}^n w_i}$$

This mean is calculated across all the sampled individuals aged between 15 and 64. As such, it can be compared with the official employment rate referring to the same age range. The official rate can be viewed as a particular case of the fuzzy rate, where each employed individual (according to the official definition) is assigned a μ_E value equal to 1.

The weighted mean of the μ_U membership function across the sampled individuals provides the fuzzy unemployment rate (FUR):

$$FUR = \frac{\sum_{i=1}^{n} \mu_{U,i} \cdot w_i}{\sum_{i=1}^{n} \mu_{E,i} \cdot w_i + \sum_{i=1}^{n} \mu_{U,i} \cdot w_i}$$

4 Empirical Application: A Comparative Analysis Across European Countries

We used data from Eurostat, specifically the European Union Labour Force Survey (EU-LFS), for the reference year 2019 (Eurostat 2019).[1] This data allowed us to obtain fuzzy measures of employment and unemployment for individuals between the ages of 15 and 64. The analysis includes 29 European countries: 25 EU countries, the United Kingdom, Switzerland, Norway and Iceland.[2]

Since the analysis essentially aims to illustrate the potential and applicability of the method, we decided to refer to data from 2019, i.e. the last available pre-Covid year. Indeed, the impact of Covid-19 and the consequences of lockdown policies, which have been implemented in various ways by different countries, may potentially affect the comparability of the measures in our analysis.

To determine the threshold for hours worked, we start by dividing the employed into three subgroups: employees in the public sector, employees in the private sector and the self-employed. Subsequently, for each subgroup, the threshold is set at the median value of the hours worked by full-time workers.

The self-employed category also includes family workers, which account for a very small share in many countries. Furthermore, we consider all activities in the categories O (Public Administration and Defence; Compulsory Social Security), P (Education), and Q (Human Health and Social Work Activities) of NACE Rev. 2 classification to be part of the public sector.[3]

Table 1 shows the composition of employed individuals according to the five categories outlined in Sect. 3 and visualised in Fig. 1, for each respective country. A considerable heterogeneity among countries characterises such composition.

Category A in the first column includes full-time workers who work at least the minimum threshold number of hours. Its weight varies considerably between countries, ranging from 40.3% in the Netherlands to 96.8% in Bulgaria, with a median of 68.6%.

Category B includes full-time workers who work less than the threshold but have no desire to work more. The percentages range from approximately 1% in Hungary and Bulgaria to more than 30% in France, Ireland and Finland, with a median of 15.8%.

Category C comprises voluntary part-time workers. The Netherlands rank first for the share of voluntary part-time workers (45.3%), followed by Switzerland (34.1%). The lowest percentages are recorded in Bulgaria (1%) and Romania (2.7%), while the median is 10.6%.

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[1] The responsibility for all conclusions drawn from the data lies entirely with the authors.

[2] Slovenia and Lithuania are excluded, the former due to the unavailability of information on the reasons for working part-time whereas the latter because more than 50% of self-employed and family workers declare zero hours usually worked.

[3] Statistical Classification of Economic Activities in the European Community, NACE Rev. 2 https://ec.europa.eu/eurostat/documents/3859598/5902521/KS-RA-07-015-EN.PDF.

Table 1 Percentage composition of employed (from 15 to 64 years) classified in the categories of Fig. 1, by country

Country	(A) Full-time workers (hours not less than the threshold)	(B) Full-time workers (hours less than the threshold, not willing to work more)	(C) Voluntary part-time workers	(D) Involuntary part-time workers	(E) Full-time workers (hours less than the threshold, willing to work more)	Total
AT	46.6	25.4	24.8	2.4	0.8	100
BE	57.8	15.6	23.8	1.5	1.3	100
BG	96.8	1.1	1	1.1	0.1	100
CH	41.4	20.5	34.1	3.2	0.8	100
CY	68.6	18.7	4.4	5.8	2.5	100
CZ	76	17.9	5.5	0.4	0.1	100
DE	48.4	23.3	24.5	2.5	1.3	100
DK	66.2	8.7	21.5	2.6	0.9	100
EE	80.7	7.7	10.6	0.7	0.3	100
ES	68.2	15.6	6.7	8	1.5	100
FI	44	37.1	10.9	4.9	3	100
FR	44.6	30.3	10.6	6.5	8.1	100
GR	73.8	15.8	3	5.9	1.5	100
HR	73	21.2	3.4	1.4	1	100
HU	94.5	1	3.5	0.9	0.1	100
IE	43.5	31.9	16.6	3.2	4.9	100
IS	67.9	10.1	18	3.4	0.6	100
IT	60.9	19.8	6.4	12.3	0.5	100
LU	77.2	5.4	14.4	2.1	0.9	100
LV	89.7	2.3	5.9	1.7	0.4	100
MT	80.6	6.6	11.5	0.9	0.4	100
NL	40.3	11.4	45.3	2.6	0.4	100
NO	70.5	9.1	16.1	3.9	0.3	100

Table 1 (continued)

Country	(A) Full-time workers (hours not less than the threshold)	(B) Full-time workers (hours less than the threshold, not willing to work more)	(C) Voluntary part-time workers	(D) Involuntary part-time workers	(E) Full-time workers (hours less than the threshold, willing to work more)	Total
PL	90.7	2.8	5.2	0.9	0.4	100
PT	71.8	16.4	4.6	3.5	3.7	100
RO	85.3	7.4	2.7	3.4	1.2	100
SE	61.9	16.1	16.3	4.8	0.8	100
SK	76	19.4	3.3	1.2	0.1	100
UK	47.5	27.1	20.7	3.1	1.5	100

Note: Our elaborations on EU-LFS 2019 data

Table 2 Percentages of workers who fully belong to the fuzzy set of employed ($\mu_E = 1$) by work typologies (Italy, France and Spain)

Work typologies	France	Italy	Spain
Professional status			
Self-employed	91.2	91.1	96.2
Employee with a permanent job	87.3	88.8	93.1
Employee with a temporary job	75.7	73.9	79.3
Skill level			
High-skilled occupation	91.4	93.4	94.3
Low-skilled occupation	83.3	86.4	91.3
Elementary occupation	70.1	70.3	76.9
Industry			
Agriculture	93.8	90.4	95
Manufacturing and construction	88.1	94.8	97.4
Trade, transportation and accommodation	84	84.1	89.6
ICT, finance and insurance, real estate	88.1	85.5	89.5
PA, education, health, arts	84.5	83.8	86.5
Total	85.5	87.1	90.5

Note: our elaborations on EU-LFS 2019 data

Category D consists of involuntary part-time workers. Italy (12.3%) and Spain (8.0%) have by far the highest values and the median is 2.6%.

Finally, category E includes workers who work less than the threshold and want to work more. France stands out with the highest percentage (8.1%), while 17 of the 29 countries record a percentage below 1.

The first three categories described above identify the workers who belong fully to the set of the employed ($\mu_E = 1$). In contrast, the last two categories cover the workers dissatisfied with their condition and therefore employed to a certain degree ($0 < \mu_E < 1$). The percentage of workers with $\mu_E = 1$ ranges from 85.5% in France to 99.5% in Czechia. Table 2 shows how this percentage varies by type of employment in France, Italy and Spain, the three countries with the lowest values of these percentages (85.5%, 87.1% and 90.5%, respectively).

The analysis reveals significant differences between the various types of employment. For all three countries, the lowest percentages are found for temporary workers and workers in elementary occupations. As regards the sector of employment, workers in the service sector (mainly in public administration, education, health and the arts, but also in trade, transport and accommodation) are more likely to be underemployed than workers in agriculture and industry.

4.1 The Employment Rate

The share of workers who only partially belong to the fuzzy set of employed (for which $0 < \mu_E < 1$) is greater than zero for all countries (Table 1, columns D and E). Therefore, the fuzzy employment rate is always lower than the official one. The higher the share of workers who partially belong to the fuzzy set of employed, the lower the fuzzy employment rate compared to the official rate.

Figure 3 shows the official and fuzzy employment rates by country. In the figure, the countries are ordered from top to bottom according to decreasing values of the
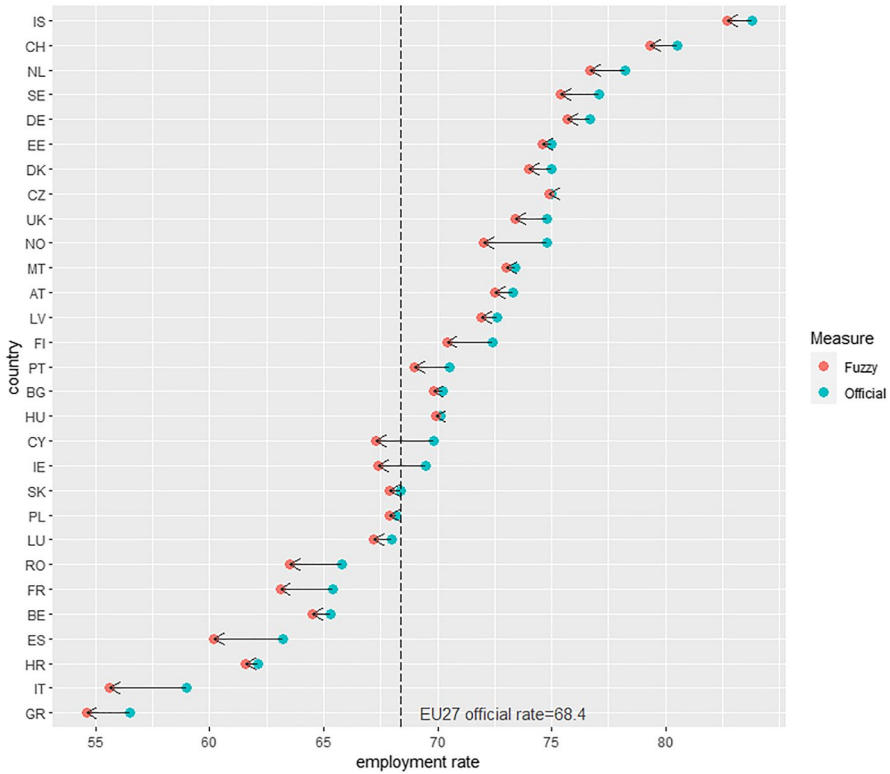
Fig. 3 Comparison between official and fuzzy employment rates (15–64 years) by country - year 2019. Note: our elaborations on EU-LFS 2019 data. The dotted vertical line represents the official employment rate for EU27 countries

official employment rate, from Iceland with a rate of around 84% to Greece with a rate of 56.5%. We can observe that for most of the countries in the lower part of the figure, whose official employment rate is well below the EU27 average, the fuzzy approach leads to the largest reductions in the employment rate compared to the official value. In particular, Greece and Italy record the lowest official employment rates and marked downward corrections when the fuzzy rate is calculated. The fuzzy approach reveals that these countries are doubly disadvantaged. On the one hand, the shares of employed people in the population are at their lowest level, and, on the other hand, jobs tend to be of poor quality in terms of an unsatisfactory number of hours worked.

The size of the difference between the fuzzy (F) and official (O) rate is shown in Table 3. For every country, the figures in the table represent the ratio (F/O)*100, and they are calculated by some characteristics of the worker. Regarding the overall ratio, in the last column of the table, we can notice that for nine countries, most of which are Eastern European countries (namely Bulgaria, Croatia, Czechia, Estonia, Hungary, Latvia, Malta, Poland, and Slovakia), the fuzzy rate is less than 1% lower than the official rate. For another group of eight countries (Cyprus, France, Greece, Ireland, Italy, Norway, Romania, and Spain), the fuzzy method resulted in a contraction of the official rate by more than 3%.

Table 3 Comparison between fuzzy (F) and official (O) employment rates by individual characteristics – European countries, year 2019. The figures represent the ratio (F/O)*100

Country	Gender		Age (years)					Country of birth		Education			Total
	Male	Female	15–24	25–34	35–44	45–54	55–64	Native-born	Foreign-born	Lower secondary	Upper secondary	Tertiary	
AT	99.2	98.4	98.6	98.9	99.1	98.7	98.9	99.2	98.0	98.1	98.9	99.1	98.9
BE	98.5	98.9	98.1	98.5	98.7	99.0	99.0	99.0	97.7	97.6	98.8	99.0	98.8
BG	99.5	99.5	98.6	99.6	99.4	99.5	99.4	99.4	99.0	97.4	99.7	99.9	99.4
CH	99.2	97.8	98.9	98.8	98.6	98.2	98.1	99.0	97.3	96.9	98.4	99.0	98.5
CY	96.8	95.7	96.1	95.7	96.8	96.9	95.9	96.2	96.5	96.5	96.4	96.3	96.4
CZ	99.9	99.7	99.6	99.7	99.8	99.8	99.8	99.7	99.9	98.9	99.9	99.9	99.9
DE	99.1	98.1	98.8	98.9	98.8	98.6	98.4	98.9	97.5	97.2	98.8	99.2	98.7
DK	99.1	98.3	97.1	98.6	99.2	99.3	99.2	98.8	97.4	98.5	98.6	99.0	98.7
EE	99.6	99.3	99.2	99.6	99.5	99.4	99.2	99.5	99.1	99.0	99.5	99.4	99.5
ES	97.2	92.7	89.7	93.8	96.0	95.9	96.2	95.9	92.4	94.0	94.8	96.2	95.3
FI	97.8	96.5	94.5	96.8	98.0	98.0	97.3	97.3	95.5	94.8	96.9	98.0	97.2
FR	97.7	95.4	95.2	96.8	96.8	96.8	95.8	96.9	93.8	93.3	96.2	97.8	96.5
GR	97.6	95.1	91.1	95.0	97.0	97.5	97.7	96.8	93.2	95.9	96.6	97.1	96.6
HR	99.6	98.9	99.3	99.1	99.3	99.3	99.3	99.2	99.2	98.1	99.4	99.3	99.2
HU	99.7	99.4	99.6	99.7	99.7	99.7	99.5	99.7	99.6	99.2	99.7	99.9	99.7
IE	97.1	96.9	94.9	97.1	97.9	97.2	96.4	97.1	96.6	95.0	96.4	98.0	97.0
IS	99.2	98.2	98.0	98.5	99.3	98.8	98.4	98.7	98.7	98.5	98.7	98.8	98.7
IT	96.9	90.4	89.7	92.8	94.4	94.8	95.2	94.7	91.0	92.7	94.5	95.6	94.2
LU	99.4	98.0	96.2	98.8	99.1	99.1	98.6	99.2	98.3	97.2	98.8	99.4	98.8
LV	99.3	98.7	100.0	99.6	99.2	98.7	98.3	99.0	98.5	99.2	98.9	99.3	99.0
MT	99.5	99.4	99.2	99.4	99.8	99.1	99.6	99.6	99.3	99.4	99.6	99.5	99.5
NL	98.5	97.6	96.6	98.3	98.8	98.5	97.7	98.3	96.5	96.9	98.1	98.6	98.1
NO	97.8	94.4	85.0	96.5	97.5	97.8	97.3	96.9	93.1	89.5	96.1	98.3	96.3

Table 3 (continued)

Country	Gender		Age (years)					Country of birth		Education			Total
	Male	Female	15–24	25–34	35–44	45–54	55–64	Native-born	Foreign-born	Lower secondary	Upper secondary	Tertiary	
PL	99.7	99.3	99.1	99.6	99.5	99.5	99.6	99.4	99.6	98.8	99.4	99.7	99.6
PT	98.8	97.0	96.4	97.9	98.3	98.2	97.7	98.0	97.2	97.9	98.2	97.9	97.9
RO	96.0	97.4	91.1	97.1	96.9	96.9	96.4	96.5	98.4	88.5	97.5	99.8	96.5
SE	98.5	97.2	90.0	97.7	98.8	98.9	98.5	98.3	96.0	94.1	97.7	98.9	97.8
SK	99.5	99.0	98.8	99.6	99.2	99.3	99.5	99.4	99.9	92.8	99.5	99.9	99.3
UK	98.5	97.7	96.0	98.5	98.6	98.5	98.2	98.3	97.5	96.9	98.0	98.7	98.1

Note: Our elaborations on EU-LFS 2019 data

Differences between official and fuzzy rates vary according to the demographic and social characteristics of the worker. In general, these differences are more marked for women, young people, immigrants and workers with a low level of education across countries. This finding reflects the fact that such categories are the most affected by underemployment.

In most countries, the fuzzy approach exacerbates the gender gap in the employment rate. Indeed, the fuzzy methodology reveals a larger disadvantage for women compared to men, especially in Italy, Spain, Norway, Greece and France.

The youngest and the least-educated workers show the most marked decline in the employment rate when moving from the official to the fuzzy measure. For the 15–24 age group, the largest changes (greater than 10 percent) are observed in Italy, Norway, Spain and Sweden. Romania and Norway, on the other hand, record the largest downward revision for workers with a low level of education.

In most countries, individuals who were born outside of the country typically have a lower rate of employment compared to those who were born within the country. Additionally, when using a fuzzy measure instead of the official one, the employment rate for foreign-born individuals tends to experience a larger decrease. The exceptions are Italy, Ireland, Portugal, Luxembourg and Malta, where both the fuzzy and official employment rates are higher for the foreign-born than native-born.

Italy has the highest gap between the official and fuzzy employment rates across all worker categories, except for the youngest and those with lower levels of education. In those cases, Norway and Romania show the largest gap.

On the contrary, Czechia shows the smallest reductions in all categories.

4.2 The Unemployment Rate

Both official and fuzzy unemployment rates by country are shown in Fig. 4, with countries ordered by decreasing values of the official rate, from Greece (17.5%) to Czechia (2.1%). In all countries, the fuzzy measure exceeds the official unemployment rate. In general, we observe the largest corrections among the countries whose official rate is above the EU27 average. Exceptions are Romania and Ireland, which display a lower official rate than the EU27 average and a sharp increase after fuzzification.

Table 4 quantifies the relative differences between fuzzy (F) and official (O) measurements through the ratio (F/O)*100, obtained for different individual characteristics. The overall ratio, in the last column of the table, suggests that the increase of the fuzzy rate over the official one ranges from 4.8% in Czechia to 85.0% in Romania. On the one hand, countries with the largest difference (more than 40 per cent) include Cyprus, Germany, Ireland, Italy, Norway, the United Kingdom, and Romania. On the other hand, there are eight countries with an increase of less than 15 percent compared to the official measure, namely Bulgaria, Croatia, Czechia, Estonia, Hungary, Latvia, Malta, and Slovakia.

Considering the classification by age and education level, we observe that the fuzzy approach leads to the largest upward corrections in relative terms for the categories generally less affected by unemployment, namely the oldest and the highly educated individuals. For the age class 55–64, the difference exceeds 80 percent in Ireland, Iceland and Italy. Romania stands out for having fuzzy rate values more than double the official rate for individuals aged 35–64 and those with the lowest educational level.

A separate discussion concerns the analysis of the gender gap in the unemployment rate (computed as the difference between the female and male rates) and its evolution
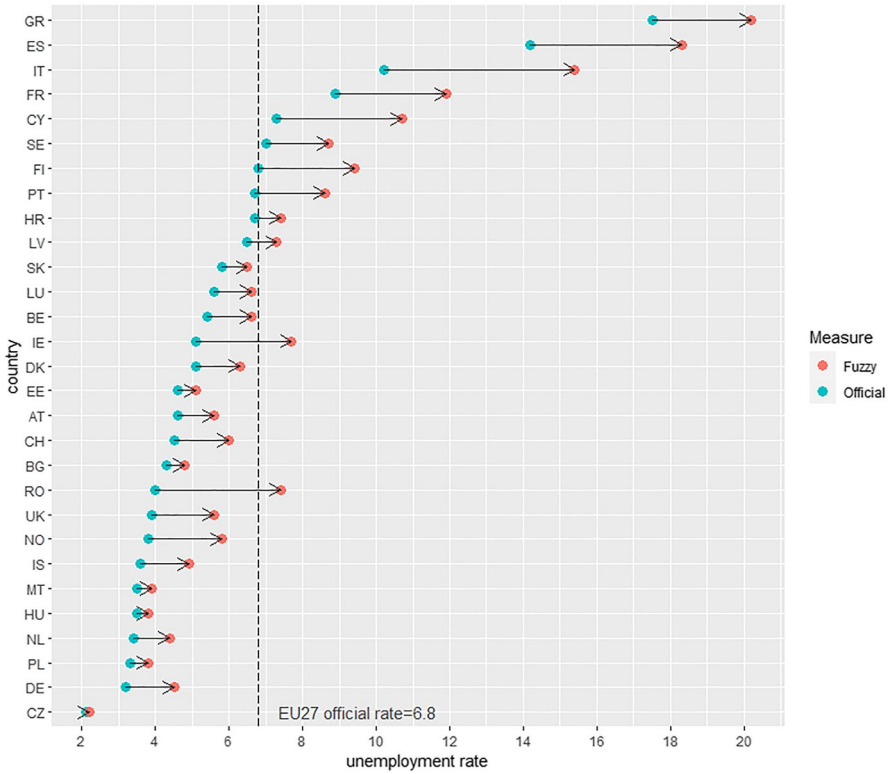
Fig. 4 Comparison between official and fuzzy unemployment rates (15–64 years) by country - year 2019. Note: our elaborations on EU-LFS 2019 data. The dotted vertical line represents the official unemployment rate for EU27 countries

when moving from the official to the fuzzy measure (Fig. 5). In Fig. 5, countries are ordered by decreasing values in the gender gap according to the official unemployment rate. The vertical line drawn for a null gap distinguishes countries where the gap disadvantages women (gap > 0) from those where the gap disadvantages men (gap < 0).

In almost all Mediterranean countries (namely Cyprus, Greece, Italy, Portugal, and Spain), the official unemployment rate is exceptionally higher for females than males, with the highest gap in Greece (7.6).

Conversely, most Continental and Nordic countries show a negative gap, which means that the unemployment rate is lower for females than males.

In most countries, the differences between the fuzzy and official unemployment rates are higher for females than for males (as we have seen in Table 4), which implies a larger gender gap in the unemployment rate when using the fuzzy approach. In particular, Italy has the largest increase in the gap (+ 5.6 percentage points) when switching from the official measure to the fuzzy measure, compared to other countries.

Figure 5 also shows that when the fuzzy approach is used, the sign of the gap reverses for certain countries in Continental and Nordic regions. These countries include Austria, France, Germany, Iceland, Luxembourg, Norway, and the United Kingdom.

Table 4 Comparison between fuzzy (F) and official (O) unemployment rates by individual characteristics – European countries, year 2019. The figures represent the ratio (F/O)*100

Country	Gender		Age (years)					Country of birth		Education			Total
	Male	Female	15–24	25–34	35–44	45–54	55–64	Native-born	Foreign-born	Lower secondary	Upper secondary	Tertiary	
AT	114.9	134.1	115.1	117.6	122.2	140.0	129.4	122.9	122.9	114.7	125.0	133.3	121.7
BE	122.4	122.0	112.0	120.3	128.9	128.6	119.5	124.4	119.1	118.1	119.3	132.1	122.2
BG	110.9	112.8	113.5	107.3	112.8	116.1	118.4	111.6	-	118.1	108.6	105.0	111.6
CH	118.6	143.8	112.5	123.9	137.8	143.6	148.7	128.1	132.9	132.9	133.3	133.3	133.3
CY	144.6	148.1	119.3	154.1	153.6	147.5	161.9	147.3	147.1	140.2	142.0	156.5	146.6
CZ	105.6	112.5	107.1	107.7	112.5	106.7	105.0	110.0	100.0	108.8	111.1	100.0	104.8
DE	125.0	160.7	115.3	128.2	136.7	156.5	155.6	133.3	140.4	129.6	142.9	136.8	140.6
DK	118.4	127.8	125.7	116.4	122.9	126.7	126.5	123.4	128.2	116.1	131.0	122.5	123.5
EE	112.2	111.8	104.5	109.1	113.2	116.2	122.5	111.4	114.5	107.8	110.2	120.7	110.9
ES	119.2	137.9	121.2	132.7	130.4	130.8	126.0	127.3	133.7	122.8	130.3	140.2	128.9
FI	127.4	153.2	126.2	145.5	143.2	141.9	140.6	136.9	131.6	125.3	137.8	147.5	138.2
FR	121.1	146.6	115.3	130.5	138.9	145.9	148.6	133.8	133.8	134.2	131.7	136.5	133.7
GR	114.2	116.6	114.8	116.7	116.3	114.8	113.4	115.3	116.8	114.4	114.2	119.5	115.4
HR	108.1	112.3	103.0	107.3	112.5	118.9	123.5	110.3	111.3	118.8	107.1	111.1	110.4
HU	105.9	114.3	103.5	105.4	113.8	112.0	122.7	108.6	118.5	108.2	113.3	106.3	108.6
IE	146.3	156.3	131.2	148.1	154.1	172.2	188.9	154.2	150.8	142.3	150.8	162.5	151.0
IS	120.0	156.3	120.7	131.8	124.0	175.0	188.2	134.3	127.7	123.3	141.2	145.8	136.1
IT	130.1	174.3	124.0	141.9	154.9	165.8	183.3	150.0	160.3	144.7	152.1	169.5	151.0
LU	105.3	130.9	110.0	118.2	118.8	121.6	119.5	107.5	120.9	123.2	116.1	115.0	117.9
LV	108.2	119.3	100.0	106.1	114.5	121.4	123.1	112.7	114.0	104.5	114.1	116.7	112.3
MT	112.1	113.5	105.4	110.5	108.3	138.1	126.7	108.8	122.2	110.4	112.5	122.7	111.4
NL	123.5	138.2	120.9	141.4	125.9	140.9	132.4	133.3	128.8	121.7	134.4	139.1	129.4
NO	126.8	185.7	153.0	173.0	148.5	154.2	166.7	150.0	158.3	147.1	167.6	150.0	152.6

Table 4 (continued)

Country	Gender		Age (years)					Country of birth		Education			Total
	Male	Female	15–24	25–34	35–44	45–54	55–64	Native-born	Foreign-born	Lower secondary	Upper secondary	Tertiary	
PL	109.7	116.2	108.1	110.8	119.2	116.7	120.8	115.2	105.3	112.8	113.5	120.0	115.2
PT	118.0	137.0	115.8	128.6	132.7	135.8	135.5	127.7	129.8	129.2	121.9	137.0	128.4
RO	188.6	174.3	142.9	160.0	219.2	214.8	241.7	185.0	141.7	251.4	160.0	112.5	185.0
SE	118.8	131.0	127.4	130.2	121.6	121.3	127.7	133.3	118.1	115.3	139.2	126.3	124.3
SK	108.8	115.0	107.5	104.5	116.3	114.0	110.6	110.2	102.8	115.3	108.2	108.0	112.1
UK	137.5	154.1	129.2	146.9	154.2	160.0	155.2	142.1	157.5	142.2	148.7	142.9	143.6

Note: Our elaborations on EU-LFS 2019 data
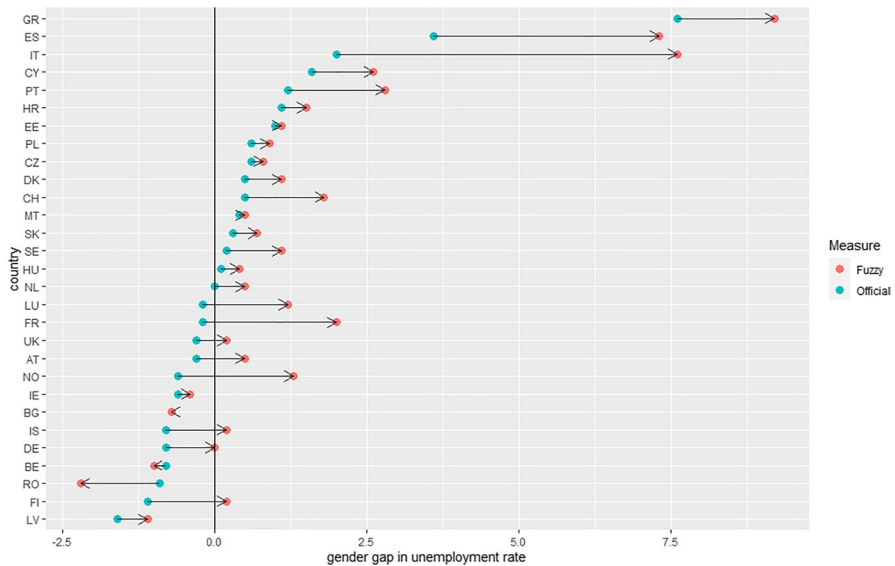
Fig. 5 Gender gap in the unemployment rate (15–64 years) by country - comparison between fuzzy and official measures - year 2019. Note: The gender gap is computed as the difference between the female and male unemployment rate. Our elaborations on EU-LFS 2019 data

5 Conclusions

Classifying the working-age population into the three mutually exclusive and exhaustive categories—employed**,** unemployed and economically inactive—may hide important grey areas, such as underemployment and marginal labour force attachment. These areas require special attention.

This paper explores the margin between employment and unemployment, which refers to individuals who are officially classified as employed but are actually working fewer hours than they would like. Using a fuzzy approach, we assume that these workers are "employed to a certain degree" where the degree depends on the number of hours they actually work in relation to a predetermined threshold. This threshold may vary from country to country and corresponds to the median number of hours worked by full-time workers. It is calculated separately for employees in the public sector, employees in the private sector and self-employed individuals. Measuring the degree of employment (and unemployment) of each labour force unit allows for the calculation of fuzzy employment and unemployment rates. These rates can capture the nuances between the different conditions of workers in terms of their work intensity and satisfaction levels.

Our proposal has several strengths. First, the fuzzy employment and unemployment rates take into consideration underemployment. Second, fuzzy rates are simple to calculate and can be directly compared to official rates. Indeed, they are a generalisation of the official rates and can be computed using the same data source. Third, they follow the ILO (2008) recommendation to consider a continuum between the employed and unemployed categories.

The choice of the threshold involves a certain degree of arbitrariness, and additional sensitivity analysis is required to assess the robustness of the method to changes in the threshold. Furthermore, in our proposal, we assume that the threshold represents the desired number of hours for all employees who express a desire to work more. However, it is important to acknowledge that there may be some employees who prefer to work fewer hours then the threshold. This is a limitation that can be addressed in future research.

According to the analysis of the EU Labour Force Survey 2019 data, the fuzzy approach shows more differences between European countries in employment and unemployment measures compared to the official method. The impact of this effect depends on the proportion of workers who are underemployed and the variability in the distribution of working hours. The larger the percentage of underemployed workers and the more variation there is in the distribution of working hours, the larger the difference between the official and fuzzy rates.

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