Science & Education (2023) 32:1287–1320
https://doi.org/10.1007/s11191-023-00439-2

SI: FAMILY RESEMBLANCE APPROACH

Exploring the boundaries in an interdisciplinary context through the Family Resemblance Approach: The Dialogue Between Physics and Mathematics

Sara Satanassi[1] · Laura Branchetti[2] · Paola Fantini[1] · Rachele Casarotto[1] · Martina Caramaschi[1] · Eleonora Barelli[1] · Olivia Levrini[1]

Accepted: 12 March 2023 / Published online: 27 April 2023

Abstract

Among the relevant aspects of the family resemblance approach (FRA), our study focuses on the potential of the approach to elaborate on disciplinary identities in an interdisciplinary context, specifically regarding the interplay between physics and mathematics. We present and discuss how the FRA wheel can be used and intertwined with the framework of boundary objects and boundary crossing mechanisms (Akkerman & Bakker, *Review of Educational Research*, *81*, 132–169, 2011), which is well-known in STEM education for dealing with interdisciplinarity. The role of the FRA discussed in the article is dual: both practical and theoretical. It is practical in that we show how its use, in combination with the Akkerman and Bakker framework, appears effective in fostering productive discussions among prospective teachers on disciplinary identities and interdisciplinarity in historical cases. It is theoretical in that the combination of the two frameworks provides the vocabulary to characterise the 'ambiguous nature' of interdisciplinarity: like boundaries, interdisciplinarity both separates disciplines, making their identities emerge, and connects them, fostering mechanisms of crossing and transgressing the boundaries. This empirical study reveals how the theoretical elaboration took advantage of the prospective teachers' contributions. We initially presented the FRA to characterise disciplinary identities, but the prospective teachers highlighted its potential to characterise also the boundary zone and the dialogue between physics and mathematics. The data analysis showed that the combination of the two frameworks shaped a complex learning space where there was room for very different epistemic demands of the prospective teachers: from those who feel better within the identity cores of the disciplines, to those who like to inhabit the boundary zone and others who like to re-shape boundary spaces and move dynamically across them.

✉ Sara Satanassi
sara.satanassi3@unibo.it

[1] Department of Physics and Astronomy 'A. Righi', Alma Mater Studiorum—University of Bologna, Via Irnerio 46, 40126 Bologna, Italy

[2] Department of Mathematics 'F. Enriques', University of Milan, Milan, Italy

1 Introduction

Interdisciplinarity is a keyword today in educational, research, political and institutional contexts. The complexity of contemporary society and its challenges, such as climate change and pandemics, as well as frontier research, cannot be tackled simply by adopting a monodisciplinary perspective (Nyboer et al., 2022; Barelli et al., 2022, Pharo et al., 2012; Brown et al., 2010; Schmitz et al., 2010; Hjorth & Bagheri, 2006). New fields are emerging that require new professional profiles: people who must be able both to guarantee rigour and expertise in a specific disciplinary domain and to display openness to other languages, needs and issues posed by colleagues with other backgrounds or stakeholders who are completely unfamiliar with the disciplines in question (e.g. Palonen et al., 2014). Schools and universities should contribute to rethinking education accordingly.

The actual organisation of teaching at schools and universities is still based on the division of knowledge into disciplinary and intradisciplinary domains (subjects at school, disciplines and research sectors at the university), leading to the creation of discipline-based communities of professionals (teachers, teacher educators and researchers in disciplinary education). Although this organisation can be motivated by the development of expertise in disciplinary fields, it often leads to a failure in promoting collaboration (e.g. Pharo et al., 2012) or creating awareness in people about their similarities and differences and developing a common language to understand each other. Indeed, the demand for institutional contexts where specialists are trained in individual disciplines can lead to the construction of boundaries that easily and frequently become barriers between disciplinary fields (e.g. Lélé & Norgaard, 2005; Sillitoe, 2004).

This issue is particularly relevant when considering the case of prospective teacher education in universities (Ryu et al., 2019; Samson, 2014). In fact, secondary school prospective teachers are students at the university who have been educated within institutions where knowledge is traditionally organised in disciplines and who are expected to have, in turn, a disciplinary-based conception of knowledge (e.g. Gibbons, 1998). While the organisation of knowledge in disciplines is contextual (it changes at school and university, it varies to some extent between different countries), students are usually not aware of the institutional choices; since their only experience of learning happened in such contexts, they often have a stereotyped and uninformed conception of the forms of organisation of knowledge (e.g. Morgan, 2004).

The challenge of training prospective teachers as professionals with disciplinary expertise, who are aware of their personal conceptions of the disciplines and able to question the traditional disciplinary organisation of knowledge at school, is thus rather innovative and complex. To tackle these challenges, the IDENTITIES project coordinated by the university of Bologna is exploring a ‘third route’ midway between saving the advantages of the current organisation of teaching in disciplines and avoiding the risk of moving toward a disciplinary organisation of knowledge, in order to exploit prospective teachers’ education as an opportunity to establish a fruitful dialogue between disciplines. To investigate the potentialities of our approach for dealing with these concrete issues emerging in prospective teachers’ education, we carry out data-driven research focusing on a particular case, the interdisciplinarity between physics and mathematics. We discuss the contribution of our approach to this topic, which has raised a lot of interest also in recent times among researchers in physics and mathematics education, but has been investigated solely from a monodisciplinary perspective (the role of mathematics in physics; the application of mathematics to other disciplines).

We describe a teaching module on the theme of the parabola and parabolic motion and analyse data collected during an implementation that involved 57 Italian prospective secondary school teachers. Both in designing the module and analysing data, we adopted two theoretical frameworks developed within the research communities in science and mathematics education: the family resemblance approach (FRA; Erduran & Dagher, 2014a, b; Irzik & Nola, 2011) and boundary objects and boundary crossing (Akkerman & Bakker, 2011). Furthermore, some basic elements of these frameworks were explicitly introduced to prospective teachers to reflect on the disciplinary and interdisciplinary aspects emerging from two historical cases: the discovery of the parabolic shape of the trajectory of the projectile motion, and the unification of conics as loci of points.

The main objective of the paper is to investigate whether, to what extent and how the FRA framework, together with the boundary objects and boundary crossing one, can promote reflections about disciplinary and interdisciplinary aspects in the historical cases, specifically regarding the interplay between physics and mathematics.

The paper is articulated as follows. Firstly, we present a selection of research works about interdisciplinarity in physics and mathematics education (Sect. 2.1) and the approach to interdisciplinarity developed within the IDENTITIES project (Sect. 2.2). Then, the different approaches to interdisciplinarity are discussed in light of the main assumptions of our approach (Sect. 2.3). After introducing the FRA and the boundary object and boundary crossing frameworks (Sects. 3.1, 3.2), we present the context of the study and the implementation we carried out (Sect. 4). Finally, we present the data analysis. After showing students' initial reactions to FRA, we focus on three case studies in which students use the FRA wheel to characterise the boundaries between physics and mathematics in very different ways.

2 Literature Review

2.1 Interdisciplinarity Between Mathematics and Physics

The theme of interdisciplinarity between mathematics and physics has been investigated relatively often in science education research. Many research studies try to overcome a dichotomy that often emerge from the standard ways of teaching and the disciplinary-based organisation of knowledge: in physics education, mathematics is usually conceived as a mere tool to manipulate, calculate and describe the physical world, while in mathematics education, physics is conceived as a possible context for the application of abstract mathematical concepts (Karam, 2015; Tzanakis, 2016; Tzanakis & Thomaidis, 2000).

Different approaches and models have been developed to address this dichotomy and to value the role of mathematics in physics and vice versa. One of these approaches consists of highlighting the interplay between mathematics and physics through the analysis of historical cases, since it 'frequently broadens our understanding of this interplay' (Karam, 2015, p. 487).

Other approaches are more related to how to rethink teaching in order to include maths in physics effectively. For example, Redish and colleagues (Redish, 2006; Redish & Bing, 2009; Redish & Kuo 2015), presented a process of modelling ‘for the purpose of thinking about teaching mathematical physics’ (Redish & Kuo, 2015, p. 567) and of giving sense to physical phenomena, consisting of four steps in a cycle relationship: modelling, processing, interpreting and evaluating. The process begins with the choice of variables and parameters to describe a particular physical system that is quantifiable through, at least in principle, a measurable process, and the appropriate mathematical structures. The physical system is then modelled by mapping these measurements into mathematical symbols and describing the physical–causal relationships between the measured quantities in terms of mathematical operations between the symbols. Based on the chosen mathematical structures, transformational rules and methodologies are assumed to transform relationships and solve equations. Then, mathematics allows the solving of problems, leading to answers that can be directly seen from physical understanding of the system. At this stage, it is only a matter of mathematical symbols and operators. The results are then to be interpreted in the physical system. Finally, there is the evaluation of whether the results are compatible and support the initial model’s choice when compared with the evidence/observations, or if the model needs to be modified.

Another model was developed by Uhden et al. (2012) starting from the model developed by Redish and colleagues (Redish, 2006; Redish and Bing 2009). Its aim is ‘to serve as a guiding framework when facing aspects related to mathematical reasoning in physics education […] [there is] the need to have an underlying conceptual position towards the role of mathematics in physics education’. Thus, this model sheds light on the different roles of mathematics in physics (*technical* and *structural*) and the different types of skills that are needed to acknowledge and manage the two different roles (technical and structural skills). They associated technical skills with pure mathematical manipulations and structural ones with ‘the capacity of employing mathematical knowledge for structuring physical situations’ (ibidem, p. 493). In their model, the technical skills, as in Redish’s one, do not have any physical reference but are purely mathematical abilities. The key processes that the model describes are mathematisation, which consists in turning a physical problem into mathematics at different degrees; interpretation, namely ‘the ability of “reading” equations, stating their meaning with the use of words and schemes, identifying special or limiting cases and making physical predictions from the formalism’ (ibidem, p.489); and technical mathematical operations related to the technical skills. Therefore, the authors provide this model ‘1) to analyse physical–mathematical reasoning of students or experts more closely, 2) to discern between meaningful and instrumental use of mathematics, and 3) to help guide teaching and learning of conceptual mathematical physics, emphasizing a conceptual physical understanding also for the mathematical aspects’ (Uhden et al., 2012, p.488).

Moving from the theoretical perspective about the role of mathematics in physics to a cognitive account, Tuminaro and Redish (2007), generalising the work of Collins and Ferguson (1993), elaborated a framework to describe cognitive structures and grammar in the context of solving physics problems. By epistemic game, the authors mean ‘a coherent activity that uses particular kinds of knowledge and processes associated with that knowledge to create knowledge or solve a problem’ (Tuminaro & Redish, 2007, p. 24). The authors provide a model to investigate what knowledge students put into play to solve physics problems at a given time. Furthermore, it allows analysis of students’ tacit expectations about how to approach physics problems and their understanding and use of mathematics. In particular, they identified six epistemic games (ranked from most to least intellectually complex): (i) mapping meaning to mathematics; (ii) mapping mathematics to meaning; (iii) physical mechanism game; (iv) pictorial analysis; (v) recursive plug-and-chug; (vi) transliteration to mathematics.

Another example in the literature is the paper by Tzanakis (2016), where the author elaborated a comprehensive approach, the history-pedagogy-mathematics/physics (HPM/Ph), to stress the relevance of the interplay between the two disciplines as the essence of their authentic historical evolution. Historical examples are used to prove the assumptions of the approach, that is, the 'historicity' of both mathematics and physics and their co-evolution, and their close interweaving in a bidirectional manner. On one hand, 'Mathematics is the language of physics, not only as a tool for expressing, handling and developing logically physical concepts, methods, and theories, but also as an indispensable, formative characteristic that shapes them, by deepening, sharpening, and extending their meaning, or even endowing them with meaning'. On the other hand, 'Physics constitutes a (or maybe, the) natural framework for testing, applying and elaborating mathematical theories, methods, and concepts, or even motivating, stimulating, instigating and creating all kinds of mathematical innovations' (ibidem, p.8).

2.2 Main Assumptions Behind Our Approach to Interdisciplinarity

In this paragraph, we briefly resume the main choices that characterise our approach to interdisciplinarity. It takes a specific stance toward interdisciplinarity, that is, it respects the needs that lead to disciplinary-based knowledge organisation: the importance of disciplines to talk about interdisciplinarity.

The first issue is to understand the nature of interdisciplinarity. According to Klein (2004, 2010), interdisciplinarity requires 'integrating, interacting, linking, and focusing' different disciplinary domains that lead to the building of new knowledge. Looking at the word's etymology, interdisciplinarity contains the word 'discipline', whose Latin root, 'discere', refers to learning. Hence, disciplines can be thought as a body of knowledge and skills that ground their roots into the educational necessity to re-organise knowledge to teach and learn it (Alvargonzález, 2011). The re-organisation has to be such that students, in building their knowledge, can also develop epistemic skills like problem-solving, modelling, representing, arguing, explaining, testing and sharing, highlighting the specific meaning that these terms have in every discipline (Branchetti & Levrini, 2019). This need led us to choose the FRA framework since it allows us to reflect on what characterises scientific disciplines. This approach is powerful for characterising 'disciplinary identities' since it allows us to do so both in terms of epistemological status (namely, forms of organisation of knowledge, practices, methods and a range of unique individual cognitive skills) and of aims and values, which oblige us to consider a wider interaction between people, individually or within institutions (Barelli et al., 2022).

An important feature of our approach consists in encouraging an 'egalitarian' perspective to interdisciplinarity, which means that we aim to develop an authentic comparison and integration between disciplines in terms of their identities, exploring the disciplines' epistemologies.

This approach promotes reflection and meta-reflection on interdisciplinarity, to highlight crucial disciplinary aspects that may remain implicit within disciplinary contexts.

For example, the awareness of the nature of the disciplines involved is recognisable when 'one becomes aware of the root disciplines in their relation and difference within the inquiry, e.g., when the nature of "using evidence" in history and in science becomes contrasted. Thereby the epistemic qualities of the disciplines become clearer, but this is the stage at which conscious, theoretical control of the disciplines becomes possible' (Williams & Roth, 2019, p. 15). According to Williams and Roth (2019), 'this kind of meta-knowledge can emerge from reflection on the relationship of mathematics or other disciplines with other knowledge' (ibidem, p. 15). Moreover, the foundational questions about the different epistemologies of disciplines shed light on what is 'knowingly un-disciplined', that is to some extent freed from the disciplines that bind problem-solving and inquiry to disciplinary norms and their limits (Williams, 2019). This potential is not visible in a monodisciplinary paradigm, where the other disciplines appear in a subordinated role and their epistemologies are not questioned by the interdisciplinary reflections at stake. We consider this aspect particularly relevant in prospective teacher education since epistemological awareness of disciplines is crucial for teaching.

To promote dialogue between disciplines and interdisciplinary reflections, that is, on how different disciplinary domains are integrated and linked, the boundary objects and boundary crossing framework (Akkerman & Bakker, 2011) has been chosen since it both provides a rich vocabulary and focuses on interaction mechanisms.

2.3 Problematisation of the Approach to Interdisciplinarity Between Mathematics and Physics in Educational Research

A comparison between some existing research studies and the main features of our own approach paves the way to reflect on some missed opportunities in the traditional approach to interdisciplinarity between mathematics and physics, which can be extended, *mutatis mutandis*, to other domains. In particular, we identified some limitations about how some research works take into account the notion of discipline.

The approaches presented by Uhden et al. (2012) and Redish and colleagues (2006), (Redish & Bing, 2009), (Redish & Kuo, 2015) explore in depth the role of mathematics in physics. On one hand, Udhen and colleagues provide a model to emphasise the translation process between physics and mathematics and the structural role of the latter. On the other hand, Redish and colleagues provide a model for investigating what kind of knowledge students put into play in solving physics problems. In both cases, there is an asymmetry between the two disciplines: the focus is on physics and the role of mathematics in physics. Furthermore, neither case promotes explicit disciplinary meta-reflections on both disciplines' epistemologies.

Tzanakis goes beyond the asymmetrical approach, promoting an account that recovers the roots of disciplines, exploiting historical development as an argument to question the actual boundaries between disciplines. In this perspective, the epistemologies of the disciplines are considered according to the historical-epistemological paradigm (Tzanakis, 2016). Nevertheless, it leaves some issues open concerning the actual disciplinary background of prospective teachers and the deep relationships between teaching and disciplines. Indeed, any comparison between historical sources and textbooks needs to take into account issues related to the goals behind the organisation of knowledge and the differences are often very large (Bagaglini et al., 2021). This huge difference can mean the historical cases are not always effective *per se* for investigating prospective teachers' views about the roots of the disciplines. In our approach, we use historical cases as sources to promote reflection on the actual organisation of knowledge in textbooks and the implicit epistemology of disciplines and interdisciplinarity that the different texts embed (Bagaglini et al., 2021). A first step in this direction is presented by Branchetti et al. (2019), starting with an analysis of historical primary sources (articles by Planck about blackbody radiation) carried out through the model developed by Uhden and colleagues (Uhden et al., 2012). Branchetti et al. (2019) developed a tutorial for mathematics and physics university students to expand their view of the relationship between mathematics and physics, adapting the language and encouraging interdisciplinary meta-reflection.

In this paper, we show an example of the design and implementation of a module about a particular case (parabola and parabolic motion), which suitably represents our approach, and we discuss further issues about the interplay between disciplines in interdisciplinary teaching as raised by our study.

3 Theoretical Frameworks

3.1 The Meta-theory of Akkerman and Bakker

The framework elaborated by Akkerman and Bakker (2011) is developed to systematise literature in the educational sciences and, hence, its main objective is broader than modelling learning in interdisciplinary contexts. However, interdisciplinarity is a very natural context to be modelled as a boundary zone and analysed through the rich vocabulary introduced by Akkerman and Bakker. This vocabulary is very fruitful in unpacking the mechanisms of interactions between different contexts (Kapon & Erduran, 2021). We now introduce the main ideas of the framework that we take and reconceptualise in our approach.

The authors start from the assumption that both learning and work involve *boundaries*.

> Whether we speak of learning as the change from novice to expert in a particular domain or as the development from legitimate peripheral participation to being a full member of a particular community (Lave & Wenger, 1991), the boundary of the domain or community is constitutive of what counts as expertise or as central participation. When we consider learning in terms of identity development, a key question is the distinction between what is part of me versus what is not (yet) part of me.
> Boundaries are becoming more explicit because of increasing specialization; people, therefore, search for ways to connect and mobilize themselves across social and cultural practices to avoid fragmentation (Hermans & Hermans-Konopka, 2010). (Akkerman & Bakker, 2011, p.132).

Both in education and work, the challenge consists in creating possibilities to collaborate across a diversity of sites. Different researchers have studied the nature of boundaries (e.g. Engeström et al., 1995; Suchman, 1994; Bernstein, 2018). From Akkermann and Bakker's perspective, it can be seen as a 'sociocultural difference leading to discontinuity in action or interaction. Boundaries simultaneously suggest a sameness and continuity in the sense that within discontinuity two or more sites are relevant to one another in a particular way' (Akkerman & Bakker, 2011, p. 133). Therefore, according to the authors, boundaries, seen as spatial intersectional space, have an intrinsic ambiguity: they lead to continuity that unites two or more realities in their discontinuity and each with its own identity. The boundaries make them interconnected in a particular way.

The authors, to clarify the boundary crossing, introduce two other important notions: the *boundary object* and the *boundary crossing*.

The *boundary object* refers to 'objects that enact the boundary by addressing and articulating meanings and perspectives of various intersecting worlds or that move beyond the boundary in that they have an unspecified quality of their own' (p.150). The boundary objects have an intrinsic ambiguity: they 'belong to *both* one world *and* another' and, at the same time, 'they belong to *neither* one *nor* the other world'. In such a way, boundary objects have both the power to *divide* two worlds and to *connect* sides. As stressed by the authors, 'Both the enactment of multivoicedness (both–and) and the unspecified quality (neither–nor) of boundaries create a need for dialogue, in which meanings have to be negotiated and from which something new may emerge' (Akkerman & Bakker, 2011, p. 142).

While we maintain the meaning of boundary objects as Akkerman and Bakker proposed, we give a less human-centred interpretation in the case of boundary crossing. In referring to boundary crossing, in fact, the authors refer to 'a person's transitions and interactions across different sites' (p.133). In this perspective, another key term for them is *boundary people*, referring to 'marginal strangers who sort of belong and sort of don't' (ibidem, p. 460), and helps to describe the difficult but necessary experiences with alterity that some people have in crossing the boundaries and bridging communities, whose identities are perceived as strongly defined and 'defended' by their members.

Even if the social dimension is pivotal, by reconceptualising the notion of boundary crossing within interdisciplinarity, we have decided to consider mechanisms of boundary crossing as what the authors called *learning mechanisms*. They identified four *learning mechanisms*. The first is the *identification mechanism* that does not question the identity of disciplines but helps to understand their relationship and, at the same, sheds light on their identities. In this process, the boundaries between practices are encountered and reconstructed, without necessarily overcoming discontinuities. The learning potential resides in a renewed sense-making of different practices and related identities (Akkerman & Bakker, 2011). The other three *learning mechanisms* imply an effort in rethinking the relationships and exchanges between disciplines, or even their foundations, leading to some innovation. *Coordination processes* aim to promote communication and exchanges between communities (e.g. *efforts at translation* or *increasing boundary permeability*); *reflection processes* consist in making explicit one's perspective in understanding and knowledge or taking a different perspective than one's own on the same issue. This mechanism leads not only to perspective making (Boland & Tenkasi, 1995), that is, making explicit one's understanding and knowledge, but also to enriching people's ways of looking at the world so that one improves one's identity beyond its current status. Finally, *transformation processes* constitute a further step, involving the collaboration and co-development of new practices that are meaningful in both worlds and have evolved with regard to the original practices from which they emerged.

This framework was introduced in class, and, in the analysis, we will show examples of how the students elaborated on it to describe their learning experience in an interdisciplinary context.

3.2 The FRA Wheel and Its Theoretical and Empirical Role for Interdisciplinarity in STEM Education

The FRA framework was first elaborated by Irzik and Nola (2011, 2014) and then reconceptualised within science education by Erduran and Dagher (2014a). Starting from Wittgenstein's idea of family resemblance, Irzik and Nola (2011) took a specific stance concerning the delicate methodological problem of defining science, accounting both for the diversity of scientific disciplines and their reciprocal resemblances that create the 'science family'. The approach assumes that 'there is no fixed set of necessary and sufficient conditions which determine the meaning of [science]' (Irzik & Nola, 2011, p. 594). Yet, just as in a family, each member (outside the metaphor, each discipline) resembles some family members in some aspects and other members in other aspects. So, the potential of the framework lies in avoiding an attempt to define what science is and providing instead

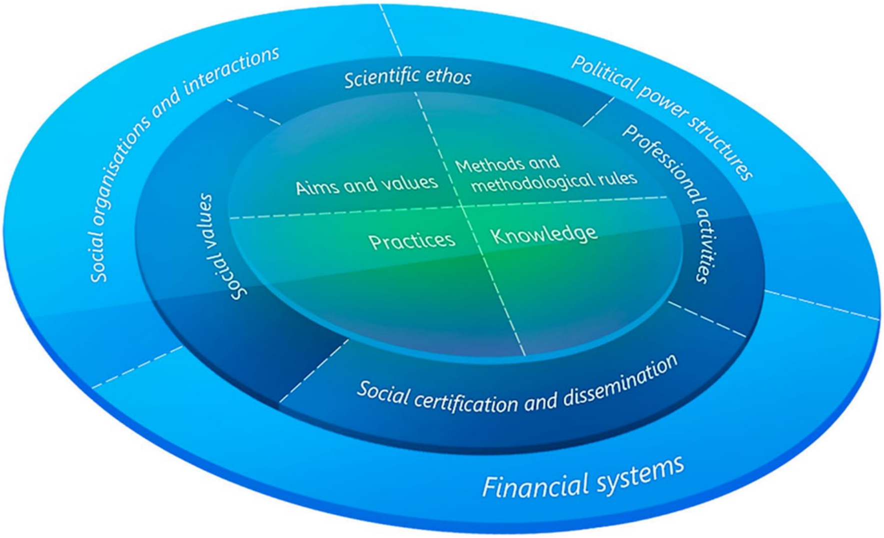


Fig. 1 FRA wheel designed by Erduran and Dagher (2014a, p. 28)

an overall picture of the many aspects that characterise sciences (Erduran & Dagher, 2014a; Irzik & Nola, 2011). On this basis, FRA allows us to characterise scientific disciplines by setting broad categories that can be both domain-general (i.e. with common and shared characteristics to all the sciences and the activities carried out within them) and domain-specific (i.e. characteristics that make the different disciplines unique). For example, experimenting is a common feature in chemistry and biology, but usually not in astronomy unless in modelling contexts. However, experimenting can be considered one of the categories of NOS since it is not necessary that a science feature be generalised in all disciplines of science (Cheung & Erduran, 2022).

There is therefore an intrinsic ambiguity also within the FRA framework: the search for broad categories that can also be differently declined to grasp the disciplines' peculiarities.

In particular, Irzik and Nola (2011) organised the characteristics into a structure composed of a cognitive-epistemic and a social-institutional system. Taking Irzik and Nola's organisation (2011) as a starting point, Erduran and Dagher (2014a) widened the categories and provided a representation of the wheel (Fig. 1) as an inclusive, systemic, diverse, comprehensive and meta-level perspective of disciplines (p.25).

As the wheel shows, the authors identified 11 categories that characterise the nature of science. The cognitive-epistemic system, the core of the wheel, is articulated in 4 categories: aims and values; methods and methodological rules; practices; and scientific knowledge. The first category refers to the set of values, underpinned by the scientific enterprise, that guide scientific practices such as objectivity, consistency, scepticism, rationality, simplicity, empirical adequacy, prediction, testability and novelty. The second category refers to the variety of cognitive, epistemic and discursive practices that characterise scientific enterprises, such as observation, classification, experimentation, argumentation, modelling and reasoning. The third category refers to the wide range of observational, investigative and analytical methods used by scientists in disciplinary inquiry and guided by particular methodological rules to generate reliable evidence and construct theories, laws and models in a given science discipline. Finally, the fourth category refers to knowledge as ‘an interrelated network of theories, laws, and models’, TLM (Erduran & Dagher, 2014a, p.5), as a ‘product of a collective human enterprise to which scientists make individual contributions which are purified and extended by mutual criticism and intellectual co-operation’ (Ziman, 1991, p. 3). Scientific knowledge is holistic and relational, and TLM is conceptualised as a coherent network rather than discrete and disconnected fragments of knowledge (Erduran & Dagher, 2014a adapted from Yeh et al, 2019, p. 295).

The socio-institutional system, represented by the two external circular levels around the core (Fig. 1), highlights the socio-institutional nature of science, which ‘involves individual scientists working in social groups in social institutions, exercising social values and activities’ (Erduran & Dagher, 2014a, p.137).

The 11 components of the framework, represented by the FRA wheel (Fig. 1), provide an image of science as a holistic and dynamic system that visually represents the relationship between its components as parts of a larger whole. ‘The boundaries (represented by dotted lines) between the two circles (or spaces) and their compartments are porous, allowing fluid movement across’ (Erduran & Dagher, 2014a, p.29), indicating that the categories affect each other, regardless of the position they occupy on the FRA wheel.

In recent years, the introduction of the family resemblance approach (FRA) to the nature of science (NOS) in science education (Erduran & Dagher, 2014a, b; Irzik & Nola, 2011, 2014) has aroused increasing interest, inducing experts in science education research to explore how FRA can be used in science education from empirical, practical and theoretical points of view.

For example, the FRA was used as a framework to analyse science and physics curricula in different countries including Ireland (Erduran & Dagher, 2014a), Turkey (Kaya & Erduran, 2016), Italy (Caramaschi et al., 2022), Taiwan (Yeh et al., 2019) and South Korea (Park et al., 2020a, b). Dagher (2020) used the FRA components as functional tools for bridging current gaps between science education studies focused on NOS and studies focused on social justice (Erduran & Dagher, 2014a). In the same field, Erduran et al. (2020) investigated how the FRA framework can be linked to broader curricular goals related to social justice, highlighting potential overlaps of social justice and NOS concepts (e.g. diversity, equity) and providing recommendations to contextualise social justice in science education. FRA and RFN approaches are also used as analytical tools for analysing textbooks (e.g. McDonald & Abd-El-Khalick, 2017), or as a framework to investigate preservice teachers’ understanding of NOS, and develop teaching strategies in teacher education (e.g. Cullinane, 2018; Kaya et al., 2017, 2018; Kelly & Erduran, 2018). Erduran and Kaya (2018) investigated how aspects of NOS such as the nature of ‘scientific knowledge’ and ‘scientific practices’ can be represented visually and how they could be exploited to promote teachers’ learning of NOS.

This present paper paves the way for two novel uses of FRA. The first concerns its use as a teaching tool in university classes to reflect on characteristics of physics and mathematics. The second concerns the theoretical contribution that FRA can provide to the discussion on interdisciplinarity in contexts of prospective teacher education. Both the empirical application of FRA and the theoretical contribution are illustrated by referring to the design and implementation of a module on the parabola and parabolic motion.

The implementation has been analysed to address the following research questions:

- To what extent can FRA be used as a teaching support and provide an effective and rich vocabulary to broaden prospective teachers’ ideas on disciplinary identities?

- Does the FRA wheel hold the potential to help prospective teachers to unpack, make visible and keep under control the process of crossing disciplinary boundaries? If so, what specific potential can be recognised?

An important aspect of the study that we will present in the following is that the theoretical contribution related to the second research question arose through the interaction of the students with the analytical tools—in ways and with insights that the authors themselves did not have in advance. In this sense, the students become both an engine and a source of new analyses and understanding.

4 The Context of the Study

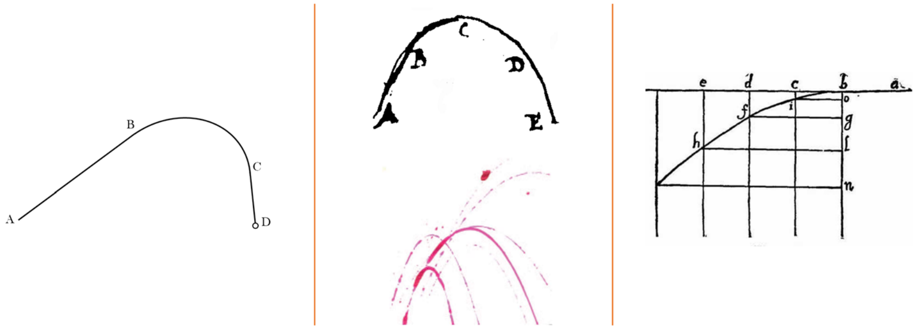
The module was implemented between October and November 2021 in the 60h course on Physics Teaching at the University of Bologna. Within this course, the implementation of the parabola and parabolic motion module lasted 24h and was organised into 8 meetings of 3h each in a blended modality. Fifty-seven Italian university students with different backgrounds attended the course. There were 20 undergraduate students of physics, 17 Master's students from the programme in History of Physics and Physics Education, 15 from the Master's programme in mathematics education (15) and 5 Master's students from other physics curricula (from astrophysics and applied physics, to nuclear physics). Other participants were enrolled in natural science programmes and the Master's degree in Education and Communication of Natural Sciences.

The parabola and parabolic motion module focuses on two curricular themes that represent two important historical episodes: the discovery of the parabolic form of the motion of a projectile, a key moment that led to the birth of the scientific method and the development of physics as a discipline, and the re-classification of conic sections by Kepler (1603) that contributed to the development of projective geometry and the reconceptualisation of parallelism as a particular case of incidence. Despite their historical and epistemological richness, both episodes were impoverished when turned into school narratives where the teaching goals and values mainly concern the development of technical skills (Bagaglini et al., 2021). The topics were chosen as prototypical 'symmetric' cases of interdisciplinarity: on one hand, mathematics started to structure the physical argumentation, and on the other, physics triggered the rethinking of an epistemological pillar in mathematics.

The FRA wheel and the boundary objects and boundary crossing framework were used as a 'design tool' to revitalise and enhance the value of these episodes, offering the opportunity to reflect on interdisciplinarity mathematics and physics, and the disciplinary epistemic cores. Moreover, in the project, we decided to wager on the potential of FRA as an 'epistemological tool' for students to analyse historical cases and carry out personal reasoning inspired by such analysis.

In the module, both the historical cases have been addressed: the discovery of the parabolic shape of projectile motion, and the re-classification of conics carried out by Kepler.

The discovery of the parabolic shape of projectile motion is framed within the scientific revolution of the seventeenth century (Renn et al., 2001), whose revolutionary aspect concerned the discovery of the decomposition of the bullet's motion into two 'simpler' motions—the uniform rectilinear motion and the uniformly accelerated motion. This represented a deep ontological change in the global description: their decomposition overcame the medieval distinction between natural and violent motions and paved the way for a new



Tartaglia Guidobaldo Galileo

Fig. 2 Curve and proof as epistemological activators

conceptualisation of the relationship between matter, space and time (for a synthesis of this historical episode for educational purposes, see Gilbert & Zylbersztajn, 1985). The main protagonists of this discovery were Guidobaldo del Monte and Galileo Galilei, whose texts were analysed with the prospective teachers to identify the epistemological core elements of their discourses. Great attention was paid to the concepts of the 'curve' (firstly reproduced through an experiment invented by Guidobaldo) and 'proof' (carried out by Galileo). Curve and proof were discussed as special boundary objects able to 'activate' the epistemological dimension (epistemological activators): they have the potential to activate a rich comparison between the disciplinary epistemic cores of physics and mathematics, and to recognise the respective structural roles played in these historical cases. The curve as an epistemological activator was mainly discussed through the comparison of three pictures taken from historical texts dealing with the investigation of the trajectory of a projectile. Figure 2, from the left to the right, reports Tartaglia's representation of the projectile motion in his Nova Scientia (1537); the sketch from Guidobaldo's notebook (1592) on the experiment with a ball coloured with ink (in Damerow et al., 1992, p.151–152; Cerreta, 2019); the geometrical drawing of Galileo aimed to demonstrate the parabolic share of the trajectory in the *Two New Sciences* (Galilei, 1638).

In Tartaglia's representation, the trajectory of a projectile consists of three parts (a straight part, a section of a circle and a straight vertical line) that, as emphasised in Renn et al. (2001), represent a projection of the Aristotelian paradigm to projectile motion in the case of artillery.

> The first part of the trajectory was conceived by Tartaglia as reflecting the initially dominant role of the violent motion, whereas the last straight part is in accord with the eventual dominance of the projectile's weight over the violent motion and the tendency to reach the center of the earth. The curved middle part might have been conceived of as a mixed motion compounded of both violent motion in the original direction and natural motion vertical downward. (Renn et al., 2001)

Guidobaldo's sketch comes from what is today called an experiment. His representation shows how the 'symmetry', which was found experimentally, in fact challenged the medieval perception of motion and corroborated the idea that motions compose each other. Galileo's representation completes the process by grounding reasoning on a rigorous mathematical proof that assumed, at the rank of presuppositions, that the two motions which have to be composed are the ‘equable (rectilinear uniform)’ on the horizontal and the uniformly accelerated on the vertical (and not the ‘natural and violent’ ones). As for the term ‘proof’, an analysis of the different meanings of proof in mathematics was carried out, starting with analysis of the Pythagorean theorem, and then adapted to physics to identify criteria suitable to analyse Galileo’s proof of the parabolic motion. This choice was made in order to switch from the more traditional paradigm where mathematics is used in physics only as a technical tool to solve problems to a paradigm where mathematics appears as an actual discipline, with its peculiar habits and values, and is shown to interact with physics in a meaningful interdisciplinary way.

The proof was also a major threat to the second part of the module, represented by reconstruction of the history of the conics, from Euclid to Kepler by way of Apollonio.

Here, the discussion on proof was situated within a wider story on the evolution of the definition of conic sections and related theorems, where the contribution of studies that nowadays belongs to the field of physics was highlighted. Conic sections figured as crucial themes in the history of mathematics and physics because they were the most important curves, different from straight lines and circles, and used to solve geometrical problems, as testified by Pappo, or to represent natural shapes. Their relevance was established definitively by the work of Apollonius, whose work on conics is a masterpiece of the history of geometry and was used as a source of reference until the nineteenth century. At that time, two key elements involved in this definition were challenging in mathematics from an epistemological point of view: the use of the infinity and parallel lines. Thus emerged the main contribution of physics: Kepler, following Witelo’s approach and looking for an analogy between reflection and refraction, did not find a way to overcome the epistemological issues and create a unified classification of all the conics as loci, but did contribute to the development of projective geometry (De Centina, 2016). The conics were thus crucial, from Kepler to Newton, for the establishment of the new cosmological view that arrived to unify the ‘Earth and the Skies’ and that represents the main achievement of the scientific revolution of the sixteenth century (Koyré, 1939, 1965).

Exploring the details of the teaching and learning sequence, the prospective teachers were engaged in different kinds of activities: from lectures and collective discussions, to teamwork and individual activities. In Table 1, we reported schematically the content and the activities we carried out during the meetings.

The module started with the introduction of the FRA framework to NOS, its main aims and the characteristics of the FRA wheel (Fig. 1). Then, the boundary objects and boundary crossing framework was introduced, putting particular emphasis on the three keywords and their meanings: boundary object, boundary people and the four learning mechanisms as processes of boundary crossing. To start dealing with the new vocabulary, we involved the prospective teachers in an ice-breaker activity that consisted in sharing personal interdisciplinary experiences as boundary people, the main barriers they encountered and the most enriching aspects. On the same day, students, who were divided into groups, became acquainted and started to reflect on the main characteristics of mathematics and physics by reflecting on what shape the three images in Fig. 2 (separately) had and how they can demonstrate this.

On the second day, students were introduced to the first historical case, the discovery of the parabolic shape of the projectile trajectory and the birth of modern physics, passing from the fall of the Aristotelian paradigm to the establishment of the experimental method with the works of Guidobaldo and proof of the parabolic shape by Galileo.

Table 1 Table of the contents and activities of the parabola and parabolic motion module

Day	Contents and activities of the module
1	Introduction to the keywords and aspects of FRA and boundary object and boundary crossing frameworks and collective sharing of interdisciplinary experiences. Teamwork 'Which curve is represented in the images in Fig. 2? Why?' in which students in the group discuss the three different images separately and explain why they associate a certain shape to the image
2	First interdisciplinary case. Introduction to the main steps that led to the birth of modern science: the shift from Tartaglia's motion representation as embodying the Aristotelian paradigm to Guidobaldo and Galileo's representations (Fig. 2). Collective discussion about which characteristics of physics and mathematics can be recognised in the historical case
3	Introduction of proof as a mathematical and physical method. Collective exercises and discussion to explore the characteristics, differences and similarities between mathematical and physical proofs. Presentation of Galileo's proof of the parabolic shape of projectile motion
4	Second interdisciplinary case. Introduction to the main steps that led to the birth of projective geometry: history of the conics, from Euclid to Kepler, by way of Apollonio
5	Introduction to linguistics tool: presentation of linguistic elements (lexicon, syntax and textuality) to textbook analysis (of the specific chapter about parabolic motion) and collective analysis of a section of two secondary school textbooks (Cutnell et al., 2015; Walker, 2017)
6	Introduction to the linguistic tool to analyse historical papers and books: analysis of a part of 'The Discourses and Mathematical Demonstrations Relating to Two New Sciences' (Galileo)
7	Introduction to FRA as an epistemological tool to analyse textbooks and historical papers and books: collective analysis of a section of the Physics of Walker (Two-Dimensional Kinematics) and a part of 'The Discourses and Mathematical Demonstrations Relating to Two New Sciences' (Galileo)
8	Collective discussions about the disciplinary and interdisciplinary characteristic belonging to both the cognitive-epistemic nucleus and the socio-institutional nucleus (referring to the FRA wheel in Fig. 1) in preparation for the final activity

On the third day, we introduced proof as a mathematical and physical method. After a collective discussion about the different proofs of Pythagoras' theorem, students were guided to reflect on the rational structure of proof and its different roles.

On the fourth day, we presented the second historical case: the history of the conics, from Euclid to Kepler, by way of Apollonio, paving the way to the birth of projective geometry.

The fifth, sixth and seventh days were dedicated to a comparative analysis of the historical text of Galileo and textbook excerpts on parabolic motion, carried out to identify the epistemological core of physics and mathematics as disciplines in different kinds of text. The analysis was carried out both with tools from linguistics (lexicon, syntax and textuality) and with the FRA categories (Satanassi et al., in preparation). The course ended with a wrapping-up lesson to rethink the disciplinary identities, the roles of disciplines and their dialogue in the historical case.

5 The FRA in the Module's Activities

During the entire course of Physics Teaching and, in particular, during the implementation of the parabola and parabolic motion module, the reflection on disciplinary identities through the FRA framework was explicitly activated with the students many times through different kinds of activities (e.g. individual and 'extracurricular' tasks, collective task,

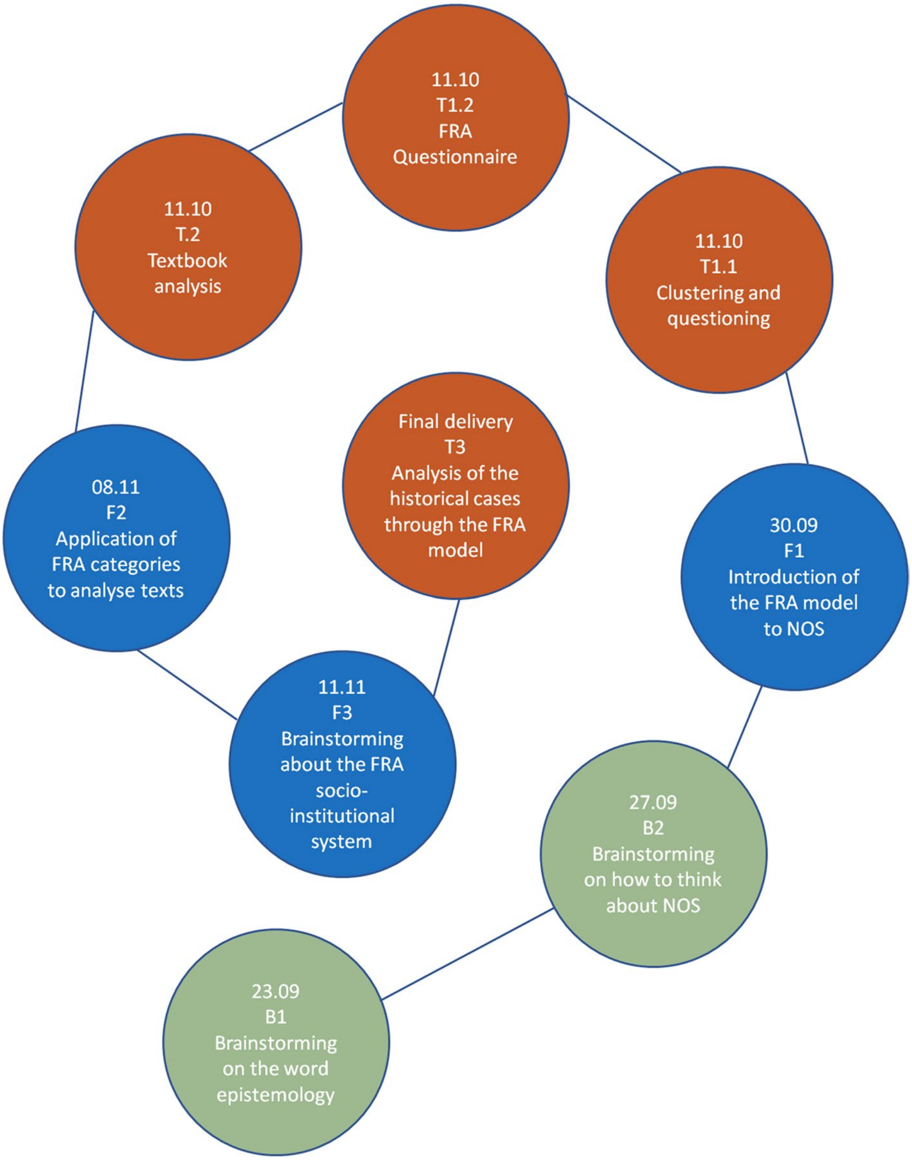


Fig. 3 The NOS and FRA activities within the parabola and parabolic motion module

collective discussions, teamwork). The first challenge to address was to trace and organise all activities related to the NOS so as to identify the kind of data necessary to answer our research questions. We articulated the reflection on disciplinary identities in the spiral shape in Fig. 3. This shape was chosen to convey the idea that the introduction of the FRA framework in the module was meant to progressively support students as they explored the disciplines more closely in order to reflect on their identities at a deeper level. To each activity, identified through an ID abbreviation (e.g. F1), a data collection tool and a dataset have been associated; this correspondence is explored in the following section while here we aim to provide the overall articulation of the 'NOS and FRA thread' in the course.

In green, in the outer arm of the spiral, there are the two initial activities designed to make students focus on some aspects of NOS. In particular, in B1, we asked them to share their ideas about the word 'epistemology' on an interactive digital whiteboard. By commenting on the answers on the board, a collective discussion was spontaneously triggered (B1_cd) on the criterion of objectivity to demarcate science with respect to other disciplines. In B2, the students engaged in an activity of questioning (Winsløw et al., 2013, 2014) to articulate a question such as 'What is science?' in more specific and addressable issues.

The blue circles represent the activities in which the students were explicitly asked to reflect on the FRA wheel. In F1, they were introduced to the FRA framework and, through a real-time survey, were asked to react to what had been presented, in particular on the degree of proximity, aims and values, scientific practices, methods and methodological rules used to characterise science. A collective discussion followed this activity, in which the students commented on their responses and overall results (F1_cd). In F2, students were asked to identify markers to analyse an excerpt of the Walker textbook using the FRA model to recognise the disciplinary characteristics of physics that emerge. In F3, students were involved in a collective discussion concerning the socio-institutional system of the FRA wheel.

Finally, in orange, there are the four individual activities designed to help students to grapple with the FRA framework. In task T1.1, students were asked to cluster the items of the virtual board produced in B2 and to write a text explaining the clustering criteria. In T1.2, they were asked to look back at their initial feelings on the FRA model. In T2, they had to analyse a textbook through the FRA categories, producing a written text. The final activity, T3, required the participants to reflect on the potential to deal with the parabola and the parabolic motion from a historical and interdisciplinary point of view, on the aspects that characterise physics and mathematics as disciplines and on the idea of interdisciplinarity that emerges from this case.

6 The Dataset

The participants' engagement in the activities described in the previous section produced an articulated dataset that included video-recording of the lessons, responses to open questionnaires or surveys and written essays. Data were collected through a large variety of tools for checking one another, corroborating evidence and evaluating the extent to which all evidence converges (Anfara et al., 2002). After being collected, all data were anonymised. In Table 2, we summarise the data collected in the module activities, labelling them with a code that links them with the spiral in Fig. 3, as well as indicating the types of data, the main requests posed to the students and the number of responses collected.

After a preliminary analysis of all the collected data, we decided to focus on those in which the role played by the FRA framework emerged most clearly. In particular, we first investigated if, and to what extent, the FRA could work as a teaching support in class and promote a change in the attitude toward science and broader, more articulated, images of disciplines (RQ1). Then, we investigated the potential of the model to help prospective teachers to unpack, make visible and keep under control the process of crossing disciplinary boundaries (RQ2).

Table 2 Data collected during implementation of the module, listed in chronological order

ID	Data type	Main requests or prompts	Number of responses	Contribution to part studies
B1	Closed and open-ended responses to the questions	What emotions does the word 'epistemology' trigger? (perplexity/curiosity/nothing specifically) If you are perplexed, which words do you associate with the word epistemology? If you are curious, which words do you associate with the word epistemology?	46	/
B1_cd	Transcript of collective discussion	Discussion about the word clouds of the previous activity	1	(2)
B2	Items on a virtual whiteboard	How can we inquire about the issue 'what is science?'? What kind of questions (simpler and more addressable) can we ask to investigate the issue?	34	
F1	Open-ended responses to a questionnaire organised in a word cloud	Which values and aims of science do you feel close to and believe motivate your choice for a science curriculum? Which activities/practices do you think belong to science? Which methodological aspects characterise science?	38	(1)
F1_cd	Transcript of collective discussion	Collective discussion about the word clouds obtained in the previous activity	1	(1)
T1.1	Essays	Choose a criterion that is suitable to you and re-organise in clusters the responses of the virtual board (B2)	33	(2)
T1.2	Closed and open-ended responses to a questionnaire	Before this experience, had you ever reflected on the NOS? To what extent have you understood the following aspects of the FRA that characterise its way of dealing with the NOS (the choice of characterising rather than defining what science is; articulation of the epistemic core; the meaning of aims and values, methods, practices and knowledge; the external wheels)? Which aspects did you find most interesting or struck you most?	43	(1)
T2	Essays	- What structure of the argument emerges? Reformulates the underlying reasoning of the chapter by highlighting the structure of the argument - What role is attributed to mathematics in the text? Are there typical features of mathematics as a discipline? If so, which ones? Referring to the FRA model, do aims and values, methods, practices and knowledge emerge? If so, which ones? - Which characteristics of physics (referring to the FRA model: aims & values, methods, practices, knowledge emerge) as a discipline emerge?	37	/
F2	Transcript of collective discussion	How was the FRA wheel useful for analysing the textbook? Which aspects of physics as a discipline did it contribute to identifying?	1	/

Table 2 (continued)

ID	Data type	Main requests or prompts	Number of responses	Contribution to part studies
F3	Transcript of collective discussion	Do you have any doubts about the FRA wheel? What is the contribution of its external parts (regarding the socio-institutional systems) to the characterisation of science?	1	/
T3	Essays	- What added value to knowledge and reasoning can the collocation of the study of motions and conics in the ‘Great history of physics and mathematics’ give? - What added value to knowledge and reasoning can address the study of motions and conics from an interdisciplinary perspective? - What aspects and characteristics of physics and mathematics as disciplines can be brought to light if this ‘historical case’ is analysed by means of FRA? Consider the internal FRA wheel—both the core represented by the ‘cognitive-epistemic system’ (aims and values, practices, methods and methodological rules and knowledge) and the ‘socio-institutional system’. If you wish, you can use a table to report the FRA aspects that you have recognised as characterising physics and mathematics - In what sense can the curve and the proof be cases of ‘boundary objects’ and/or cases of ‘epistemological activators’? What ‘boundaries crossing mechanisms’ can be put in place? How?	37	(3)

To address the research questions, we developed two studies that we report in the following section. The presentation of each study is articulated in (a) the description of the considered dataset; (b) the goal and methods of the analysis; (c) the data analysis and results; and (d) the partial discussion.

7 The Effectiveness of the FRA Framework and Vocabulary in Widening Prospective Teachers' Ideas About Science

7.1 Description of the Dataset

The data we discuss hereafter were collected in B1, F1 and T1.1 activities. B1 refers to a lecture in which students were asked to share words they associated with the word epistemology. During that activity, the students started a spontaneous debate (B1_cd), coordinated by the teacher, about the meaning of objectivity in science. The students oriented the discussion in the search for which criteria could be used to define 'science'. F1 is an activity carried out, after the FRA introduction, through Wooclap, an online platform that aims to engage and involve students during classroom-taught activities. Students were first asked to individually reflect and write the aims and values, the scientific practices and methods that characterise physics and mathematics as disciplines and then to discuss collectively (F1_cd) the word clouds that emerge from the Wooclap platform. T1.1 is an individual task in which students were asked to group some sticky notes that they had written on a digital whiteboard in a previous activity (B2, the resulting whiteboard is shown in Annex). The T1.1 activity consisted of the students' written explanation of their clustering and re-organisation of the sticky notes, carried out according to criteria each student considered appropriate.

7.2 Goals and Methods of the Analysis

We decided to look at these activities to analyse the first students' reactions to the FRA vocabulary and their attitudes toward NOS before and after the introduction to the FRA wheel. The activities are not strictly comparable, both because students were asked to reflect on different aspects (on one hand, the discussion concerns the objectivity of science and, on the other, focuses on a broader idea of science) and because of the kind of activities. The excerpt of B1_cd, F1 (and F1_cd) and the written texts of T1.1 particularly struck us for the difference in *attitude* that the students display.

The different attitudes were highlighted by observing the focus and the articulations of students' discussions (B1_cd) and descriptions (T1.1) and the kind of language used in the two cases (verbs, words). Finally, in T1.1, we investigate if and to what extent students used (explicitly or implicitly) the FRA framework as a criterion for re-organisation of the sticky notes.

The students in B1_cd are anonymous and numbered in the order they entered the discussion, while in T1.1, the enumeration follows a pseudo-anonymisation strategy implemented to create the dataset.

7.3 Data Analysis and Results

In the B1_cd, the discussion revolves around the objectivity of science. In the very beginning, students focused on the meaning of the word ‘objective’, contributing to the debate by presenting their personal view of science. Three positions emerged:

- As concerning the object of the study *per se*, and independent of who thinks it (S13: “*I would associate objective to something that is independent of those who think it*”)
- As concerning a ‘common idea’ that is sharable among a group of people (S10: “*science is the tool that connects people on a common idea and then [science] becomes objective when everyone thinks it*”)
- As concerning the replicability of an experiment, the invariance of results (S12: “*when I perform an experiment, I see that some results do not depend on other conditions. I ask another person to do it again and I see that he* [finds it] *too…*”)

The students continued the discussion by questioning whether or not the word ‘objective’ was the right word to describe science. For some, the “best word” was “universal” (S12: “*rather than the term objective for science, or in our case for physics, the term universal’ is better*”), for others “inter-subjective” (S13: “*I thought of maybe talking about inter-subjectivity rather than objectivity*”).

Instead of juxtaposing several possible characteristics, as the FRA instead suggests, students tried to discard those features already proposed with the aim of finding the better one (S10: “*I don’t really agree with the word objectivity. Because in my opinion...*”*; S10:* “*in my opinion science is more the instrument* […] *than*”; S11: “*Maybe the right word is invariance*”). This can suggest an attempt to define the exact nature of science, demarcating it with respect to non-science. This emerged also from the use of expressions like “*science is*”, “*science may be*” and “*science is not…*”.

After the FRA introduction (collective discussion F1_cd), we noted immediately a change in manner of speaking about science: students’ reactions show they have quite naturally grasped the inner value of the model. In particular, one student (S3), describing the aspects that struck him/her most, said:

> “*It could be said in a certain sense that this approach to attempting to define science is phenomenological, to the extent that a definition cannot be given ... that is, a compartmentalized definition, but we can say what science is not or how it should behave. So… [...] This is the thing that strikes me the most because actually not only is it already difficult, in our case, to frame physics [...], it is also difficult to do so by understanding physics at a certain historical moment. The last century could be said to have been a moment in which even physics itself changed radically in many aspects. So, it is already an impossible task to do if you have a certain [idea of] physics in mind. I dare not think about doing it at all [...] Attempting to define precisely, [...], strictly in quotation marks what physics is, what science is and so on.*” (S3)

The student tried to describe in his own words the main aims of the FRA framework, focusing on the idea of a phenomenological approach and stressing the impossibility of defining physics, of finding a ‘compartmentalised definition’. This impossibility derives not only from the difficulty of ‘framing physics’ itself but also from considering it in its historical evolution. In the collective Wooclap activity and follow-up discussion, students were asked to think about (and write) possible aims and values, scientific practices, methods and methodological rules that characterise science. Students highlighted a very rich variety of aspects belonging to both the epistemic-cognitive system (aims and values, scientific practices and methods) and the socio-institutional system. In discussing the multidimensionality of characteristics, the students recognised a plurality of dimensions typical of the FRA model itself. For example, S5 commented: 'What strikes me most is that the biggest word is curiosity […] and curiosity refers to a more emotional sphere, while the others […] refer to a rational sphere'. S6 pointed out "*the rational approach to science*", S8 the "*sphere of learning and understanding*" and S7 the "*social dimension*". Continuing on the social dimension, S10 added"*I particularly like [the idea of] peer review. That is, the idea that in science our work must be verified by other people who have the same skills as us* […]".

In the data analysis of T1.1, we explicitly asked students to clarify the criteria of question clustering. Some of them clustered the sticky notes on the digital whiteboard by selecting some principal questions or key points that they considered important (e.g. S6, S23, S24, S25). Others preferred to describe in a single text their organisation of the questions (e.g. S17, S20). About 16 out of 33 students explicitly used the FRA framework as clustering criteria; 10 students, despite not mentioning it specifically, implicitly referred to the FRA framework by citing its categories. Many students explicitly wrote that in grouping the questions, they attempted to characterise and not define science. They often used the words "*features*", "*characteristics*" or verbs like "*to characterise*" and "*to delimit*" to describe science (e.g. S24: "*The characteristics: What is peculiar to science?*" (S24); S20: "*A greater awareness of what science is, what it is characterised by, how it proceeds and how it is built, can be an important key to…'; S17: 'Let's try to give an order to all this, always bearing in mind how unattainable the concept in question is and thus trying to find a way to delimit a concept without drawing its boundaries*"). One student also expressed the impossibility of finding a definition of science (S23: "*All the questions try in some way to characterize the science circumscribing the attempt, established impossible, to define it*"). The students who more or less explicitly used FRA and the students that used other cluster criteria proved that they can discuss science and its nature in its complexity, in a multidimensional structure. They took into consideration aims and values, methods, practices, methodologies, the distinction between different disciplines and between science and non-science, scientific knowledge and scientific culture.

7.4 Partial Discussion: the Leading Argument

From this analysis, we pointed out some differences, before and after the FRA introduction, mainly in terms of *attitude*. In the collective discussion (B1), talking about the term 'objectivity' in science, the students tried to explain what meaning they give to this term based on their ideas and beliefs. This kind of discussion suggests that students were trying to define science by the research and the choice of the best word ('universal', 'objective', 'intersubjective') without contemplating a plurality of features that characterise it.

From the analysis of the collective discussion F1_cd, we noticed that the FRA immediately acts as an *activator*, providing students with vocabulary and ideas to tap into their personal and disciplinary knowledge and repertories. Students' attitudes suggest that they have grasped the essence of the FRA model in a very natural way, discerning the difference between characterising and defining, and recognising the multidimensional nature of science. Furthermore, the plurality of aims and values, scientific practices and methods that they highlighted, as well as the following discussion suggested to us that the FRA framework scaffolds students' reasoning. It seemed to us that the it helped students to think about how science can be characterised, allowing them to explore their experiences and knowledge.

Also in T1.1, students provided rich descriptions of science and the issue of its demarcation. Although the digital whiteboard provided a thriving ground from which to start, it seems to us that the FRA framework promotes students' change in terms of *attitude* toward NOS. In particular, we noticed that students began to articulate a discourse about NOS, embracing its multidimensional nature. Different students also highlighted the core value of the FRA framework as not defining science but characterising it. Most of the students (26 out of 33) naturally chose the FRA categories (both the cognitive-epistemic and the socio-institutional system) as cluster criteria, proving that they have clearly grasped its essence and are able to use it in an effective way to discuss NOS. The change in *attitude* (from researching the attribute to considering a plurality of features to characterise science) suggested the 'paradigm change' that FRA can naturally promote, providing a new vocabulary and lenses to rethink a personal view of science by placing it into a broader and multidimensional frame. Furthermore, the use of the FRA model in class proved very effective in spontaneously engaging students in reflections on NOS.

We believe that the introduction of the FRA without anchoring to a specific topic can only partially activate students to explore the NOS and help them to effectively unravel for example the characteristics of the cognitive-epistemic nucleus.

8 FRA as a Lens to Investigate the Relationship Between Physics and Mathematics in the Historical Case

8.1 Description of the Dataset

For the third part of the study, we analyse the T3 essays produced by the students at the end of the module, in which they were asked to discuss the educational potential of introducing the parabola and parabolic motion from a historical and interdisciplinary points of view, the aspects that characterise physics and mathematics as disciplines, and the idea of interdisciplinarity that emerges from this case (see Table 2). In particular, one essay question was the following: 'What aspects and characteristics of physics and mathematics as disciplines can be brought to light if this "historical case" is analysed by means of FRA?'. Through this question, the students were asked to autonomously apply the FRA wheel to discuss the module contribution in stressing the disciplinary identities and their dialogue. The answers to this question represent the dataset of this study.

8.2 Goals and Methods of the Analysis

The aim of this study is to investigate in what ways the FRA wheel was applied by the students as a vocabulary source to name and characterise the disciplines in an interdisciplinary context. During the data immersion stage, we observed three main ways to address the question, and we selected three answers that were particularly clear expressions of such ways. We thus focused on these answers to elaborate on the three main categories, and then used them to re-analyse the other answers. This second round allowed us to recognise that all the other answers represented either hybrid positions (a combination of the three main

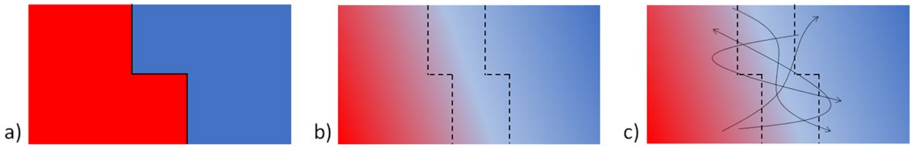


Fig. 4 Schematic representation of the three roles of the FRA wheel to characterise disciplinary territories. **a** FRA is used to delimit physics with respect to mathematics and vice versa (domain-specific orientation); **b** FRA is used to recognise both differences and similarities within and across the two disciplines (both domain-specific and domain-general orientation); **c** FRA is used to characterise the boundary mechanisms

stances) or positions that did not contribute significantly to outlining theoretically meaningful categories because they were similar to the three selected or because the argument was not so focused on the role of FRA and the boundary objects. Thus, during the analysis, our initial empirical approach moved toward a more theoretical one, since the analysis allowed us to unwrap unexpected relations between the boundary framework and the FRA wheel. As a preview of the results, we can say that the students used the FRA wheel in three different ways.

The first use focuses mainly on identifying what distinguishes the disciplines. This emerged in the predominance of words like ‘distinguish’, ‘separate’, ‘divide’ and ‘demarcate’ and in the emphasis on ‘differences’ and ‘distinctions’ between physics and mathematics. This use emphasises the domain-specific orientation of the FRA wheel. In contrast, we also observed two ways oriented to a search for similarities and intersections and to characterising the boundary zone. Here, the students are more focused on describing how disciplines ‘intertwine’; ‘can be unified’; ‘are blended’; and showed ‘similarities’, ‘connections’ and ‘integrations’. However, the second student incorporates an inner feature of the FRA framework: she individuated key characteristics of physics and mathematics that emerged from the historical episodes (domain-specific approach) as well as the commonalities (general domain), the resemblances that make physics and mathematics a ‘family’. The third use represents a theoretical innovation of the FRA since it unfolds a new application of this tool: its application as a boundary mechanism and use in reshaping the boundary territory, showing how characteristics are exchanged between mathematics and physics, returning dynamics to the FRA wheel.

It emerged, from students’ use, that the framework has the same form of ambiguity of the boundary object and boundary crossing: it can be used to separate and to connect, as well as to reflect, on the mechanisms of discipline changes in an interdisciplinary context.

As already introduced, after the bottom-up recognition of these approaches, three cases appeared particularly clear to building possible markers to characterise the different kinds of attitudes. The nature of the emerging disciplinary boundaries and mechanisms implemented to move from one territory to the other strongly differ across the three attitudes. The different attitudes we pointed out are represented in Fig. 4. The first case (Fig. 4a) is characterised by the use of FRA to delimit physics with respect to mathematics and vice versa (domain-specific approach); aims, values, practices and other elements of the FRA wheel are functional to the construction of a boundary that separates two disciplinary territories. The second case (Fig. 4b) reveals the natural use of the FRA, both domain-specific and general domain, demarcating a boundary zone in which disciplines share characteristics and dialogue. In the final case (Fig. 4c), FRA was used to characterise boundary crossing mechanisms, to recognise in the historical case how the boundary territories between physics and mathematics changed, and their interplay, as well as re-characterising the disciplinary identities.

8.3 Data Analysis and Results

The essays that we selected as case studies to show the three FRA uses were written by female students and we evaluated all of them as very good examples of analyses of the historical case. All three students used the FRA wheel in an extremely rich and articulated way, showing a significant appropriation of the framework. Their comparable level of appropriation highlights a plurality of ways in which the same analytical tool was applied by the students.

8.4 FRA as a Way to Recognise Disciplines' Identities [S29]

The first case study that we analyse is that of S29. She begins her essay by presenting the historical case of the parabola and parabolic motion while highlighting, as discussed by the teachers in the course, the different forms of interplay between physics and mathematics that occurred. Indeed, she overtly commences her analysis by stating that the historical case sheds light on the forms of interdisciplinarity that are mostly neglected nowadays in textbooks:

> "*Placing parabolic motion in a historical context highlights how historically mathematics and physics were much more intertwined than they are today in textbooks, providing, with the work "Discourses and mathematical demonstrations around two new sciences" by Galilei, an example of interdisciplinarity. In fact, the text respects the characteristics of an interdisciplinary approach, highlighting the references to the disciplines, but at the same time explaining and motivating their intertwining.*"

However, when she moves to the task 'What aspects and characteristics of physics and mathematics as disciplines can be brought to light if this historical case is analysed through FRA?', S29 changes her approach. The references to interdisciplinarity, disciplinary contaminations and intertwining disappear and the elements of the FRA wheel (specifically, aims, cognitive and epistemic practices, methodological rules, scientific knowledge and socio-institutional system) are used to establish a boundary between the disciplinary territories of physics and mathematics. She explicitly explains this change of approach as being motivated by reference to the FRA:

> "*Although Galilei's text is a valid example of interdisciplinarity, the references to the disciplines are evident and appear in an epistemologically significant way. In particular, referring to the framework of the Family Resemblance Approach, and also considering the analysis of the textbooks, the following characteristic aspects of the two disciplines emerged.*"

After this sentence, the student writes a bullet list of characteristics of physics, according to the FRA elements, followed by another list for mathematics. Also graphically, she carefully emphasises the distinction between the two disciplines, constructing a clear-cut boundary between two territories, as represented in Fig. 4a:

> "*Physics*:
>
> - *Aims: Criterion of simplicity that manifests itself through the separation of the variables, that is, the independence of the motions; intelligibility, as we speak of common experiences.*
> - *Practices: Experiment (epistemic practice) and dialogue (cognitive practice).*
> - *Methodological rules: Construction of models starting from the observation of phenomena.*
> - *Scientific knowledge: Mathematics provides logical and consistent explanations to develop understanding.*[1]
> - *Socio-institutional system: The dissemination of concepts that revolutionize the vision of the world inevitably leads to a cultural and social revolution.*
>
> *Mathematics*:
>
> - *Aims: Rationality and consistency of axiomatic-deductive reasoning and objective value and truth of proof.*
> - *Methodological rules: Model building with a solid logical argument.*
> - *Socio-institutional system: A rationally structured argument has a social utility in that it produces responses to human needs and guarantees the equality of intellectual authority.*"

We can conclude that, for S29, FRA offered a bunch of effective words to characterise the two disciplines and to separate the contribution of mathematics from that of physics. In this use of the FRA, the student leveraged the possible domain-specific orientation of the framework.

8.5 FRA Used to Recognise Both Differences and Similarities Within and Across the Disciplines [S18]

The second case study is provided by S18. In her essays, she searches for both the domain-specific aspects of mathematics and physics and their connections and 'contact points' in the territory where they are 'inseparable'. From the analysis of the historical case, she recognised it as a 'blurred and not very clear-cut' interdisciplinary space.

> "*The birth of conics and of motions are highly interdisciplinary events, presenting blurred and not very clear-cut boundaries between mathematics and physics* [...]."

The metaphor of the blurred boundary territory individuated by S18 is what led us to develop the representation in Fig. 4b. In describing the 'inseparability of disciplines' in the

---

[1] Here, the student is referring to the structure of Euclidean proof used by Galileo to prove that the trajectory of a projectile is a parabola.

historical case, S18 explicitly refers to the FRA model ('If we put ourselves in a FRA perspective'). Starting from an assumption that the 'inseparability of the disciplines' emerged in the historical case, in the following section of the essay, the student uses both the FRA domain-specific approach to point out the elements that characterise each discipline (mathematics in green, physics in red) and the domain-general approach to highlight the family resemblance aspects (in blue):

> *In the inseparability of the disciplines that have emerged in these historical cases, we find some of their constituent and most significant characteristics. If we put ourselves in a FRA perspective, we would immediately notice rigour as the basis of knowledge and value and an objective of mathematics, and mathematics itself as an element of knowledge for physics* [...]. *Again, in the physical field, various experimental practices are presented: sensory observation (e.g., the thrown ball seen by Guidobaldo) must generate hypotheses (e.g., the catenary as a curve) that allow other experiments (e.g., the ball soiled with ink) and lead to a model that must always maintain a logical and sensible structure (we can consider it as a value and aim shared by the disciplines) in order to explain its functioning. Another physical value is reproducibility, in this case, understood as validation of the method and the object of knowledge analysed (e.g., the "wondrous way" is actually wondrous for this reason). The* [historical] *evolution* [...] *shows us various common aspects of mathematics and physics: they outline a method of non-static hypothetical-deductive approach, both for the kind of activity to carry out one's studies, both with regard to work ethics and the certification/validation of one's work.*

In this case, we found it noteworthy that the *blurred* transition between the disciplinary domains does not create a *blurry* characterisation of interdisciplinarity. On the contrary, for S18, FRA allows us to characterise the disciplines and to identify the examples of characteristics that resemble mathematics and physics, highlighting occasions when disciplinary contamination occurs.

8.6 FRA Used to Characterise the Boundary Mechanisms [S20]

For the third case study, we selected S20. The student addresses the task from a very personal position. She started by stressing the relevance of an interdisciplinary approach today, since emerging interdisciplinary fields call for an overthrow of strongly-rooted cultural dichotomies:

> "*The disciplinary division between mathematics and physics embodies one of the deepest and most deeply rooted dichotomies of our culture, namely* [the dichotomy] *between theory and experimentation, between abstract knowledge and real concrete knowledge of the world. It brings with it, in this accelerated, complex, uncertain present, characterised by the fusion of knowledge, where disciplines are called to merge, intersect, and reflect each other (climatology, AI, big data science, ...), prejudices of form and thought which, perhaps, we no longer need.*"

For S20, one way that can help students and teachers to change their ways in order to characterise the disciplinary identities consists in reflecting on their nature before and after an 'interdisciplinary approach', exploring the 'boundary territory' through a 'good compass' and the historical narratives:

> "*It is interesting to see how the identity characterisations of the two disciplines* [mathematics and physics] *change before and after the interdisciplinary approach, therefore, after having explored that boundary territory with a good compass and through narratives coming from the 'great history of physics and mathematics', that were able, first of all, to break down the dichotomic prejudice.*"

The boundary territory is conceived as a 'geographical boundary', an 'area of disciplinary intersection' characterised by a spatial zone that 'separates' and, at the same time, 'unites two cultures':

> "*The area of disciplinary intersection is conceived as a geographical boundary[...] which separates, but also unites, two cultures that face each other.*"

The boundary territory can be investigated and re-shaped through the four boundary crossing mechanisms re-analysed with the FRA wheel to position the role of disciplinary identities. The first mechanism used to shape the boundary zone is *identification*, explored through the FRA wheel:

> "*There are four mechanisms of boundary investigation and crossing identified by Akkerman and Bakker. The first is disciplinary identification (identification) with which the specificities of the disciplines are highlighted. For this purpose, the FRA wheel (Family Resemblance Approach) is chosen as the theoretical framework, a tool that helps and guides us in defining both the epistemic heart of the two disciplines divided into aims and values, practices, methods, and methodological rules and knowledge (core), and the relationship of the community of reference with society, their being part of society (first circle), and their more general relationships with citizenship and the economic and political decision-making powers that govern it (second circle).*"

Here, the FRA has the role of highlighting the disciplinary specificities (domain-specific approach) but is, at the same time, a mechanism for crossing the boundaries because:

> "*The trend is no longer to separate and distinguish but to unite.*"

Thus, she highlights also the domain-general orientation of the framework. Yet in this case, it is not only a matter of recognising resemblances; the FRA (in green) is used to characterise the crossing mechanism and new boundary territory, shedding light on the interplay between mathematics and physics in the specific case of discovery of the parabolic shape of the projectile trajectory ('passing through the experiences of Guidobaldo and Galileo'). In S20's words:

> "[We have realised] *how much mathematics begins to characterise physics (reflection). We talk about proof and mathematical models in methods, we talk about identifying basic assumptions in practices (coordination), and we talk about the basic assumption in itself in knowledge. [...] Obviously, the methods and practices of generating proofs of agreement between theory and the world remain strong, translated into characterisation through the value of universality, Mathematics, with proof, with models, thus becomes an argumentative structure that keeps reasoning under control in a physical problem.*"

Using the FRA to characterise a mechanism of boundary crossing, S20 not only identifies the common characteristics that physics and mathematics began to display in the specific historical case, but also dynamically shapes and characterises a new boundary territory, as well as new identity specificities of the disciplines.

In history, a process of reflection started breaking down the boundary and opening up a common zone; then, boundary objects like proof, models and theorems (recognisable as typical methodological elements or basic assumptions in mathematics) became boundary objects that, through a coordination process, were used to ground also physics:

> "[...] *The experience of IDENTITIES project*, [...] *has therefore brought back to light* [...] *an implicit foundation of physics, which had become so implicit that we forget that sometimes mathematics is more "physical"* [i.e., there is much more mathematics in the foundations of physics] *than it may seem*."

Thanks to the boundary crossing identified by S20 in the historical cases, it is also possible to 'bring the two disciplines closer together' and 'breaking down the dichotomous prejudice'.

> "*The placement of the study of motions and conics in the "great history of physics and mathematics" has certainly led to bringing the two disciplines closer together by breaking down the dichotomous prejudice from which we started: Therefore, a greater awareness has grown that mathematics and physics are close and that the aims and values of the one also guide the other*."

The sense of the argument is, in the end, re-stated by the student: the process of characterising the disciplines and their identity is needed to pursue the very goal, which is to recognise and unpack the boundary crossing mechanisms and break down the dichotomous prejudice.

The overall reasoning gives back a very articulated relationship between the two frameworks, where the FRA wheel allows to characterise the dynamic process of boundary-making and un-making. *Identification*, characterized by the FRA wheel, allows the dichotomous prejudice to be broken down, and stereotyped ways to separate the two disciplines to be replaced with an epistemological grounded process of *disciplinarizazion:* the disciplinary identities are established through a contrastive approach. This combination of the other mechanisms prevents the new boundary to become a barrier, inasmuch as they foster boundary permeability and boundary crossing.

8.7 Partial Discussion

In this analysis, we pointed out how students used FRA (Erduran & Dagher, 2014a; Irzik & Nola, 2011) to investigate the relationship between physics and mathematics in historical cases. From the students' essays, we identify three main and different uses of the FRA framework to characterise the disciplines in an interdisciplinary context. In the three approaches, the FRA model was re-conceptualised as a process that 'entails a questioning of the core identity of each of the intersecting sites' (Akkerman & Bakker, 2011, p. 142), which lead students to define 'one practice in light of another, delineating how it differs from the other practice'.

S29 uses FRA as a way to recognise disciplinary identities and create a clear-cut demarcation line (Fig. 4a). The student in fact distinguishes between physics and mathematics recognising different identity aspects in terms of the categories of the cognitive-epistemic and socio-institutional systems. This use of the FRA recalls the possible domain-specific orientation of the framework.

S18 uses FRA to identify differences and similarities within and across the disciplines (Fig. 4b). She used the categories both to recognise the disciplines' specificities and to highlight the resemblance in the historical case. She describes the operation of crossing boundaries as a process of blurring to ‘encompass multiplicities from an overarching perspective and trace them back to the same shape’. This use recalls the domain-specific and domain-general orientation of the FRA.

S20 uses the FRA to characterise the boundary crossing mechanisms (Fig. 4c). Analysing the dialogue between physics and mathematics in the specific historical case, she breaks down a deep cultural ‘dichotomy’ between the two disciplines, and then reshapes and characterises a new boundary described as a space in which the disciplines can merge, intersect and reflect each other. In this way, she sheds light on how, in the interdisciplinary case, the dialogue between the two disciplines and the boundary crossing led to a change in the disciplinary identities and their epistemologies. The last case represents the theoretical contribution of our work which emerged from the interaction of the students with the tools. The application of the FRA *per se* can involve questioning the characteristics of one discipline over another, namely looking for domain-specific and domain-general characteristics and building family resemblance imply the comparison between at least two disciplines. Nevertheless, the comparison does not imply a dialogue or interplay. The use of the FRA in an interdisciplinary context triggered the three main kinds of attitudes and uses. The first one, by deepening the epistemic cores and leveraging on the domain-specific approach, suggests the need to maintain a clear-cut separation between the two disciplines and to remain in their own comfort zone. The second one, which embodies the intrinsic ambiguity of the FRA (both domain-specific and domain-general orientation), in the interdisciplinary context blended the clear-cut distinction drawing a new boundary space of dialogue between the two disciplines. Finally, the third, using the FRA to characterise the boundary crossing, breaks down the boundary, and then shapes and characterises a new area in which the disciplines are changed thanks to their dialogue and interaction. The last two cases suggest a different attitude from the first: not keeping the disciplines separate but building a boundary space that they are able to inhabit.

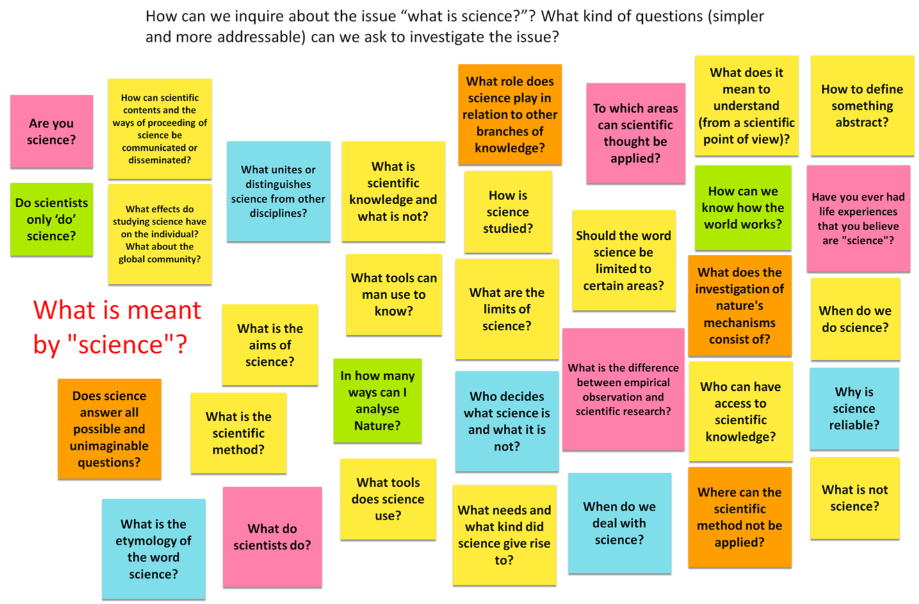
In the last case, for S20, the FRA becomes a way to investigate the disciplines in their dialogue by acquiring a dynamic characteristic.

9 Conclusions

In this contribution, we discussed the potential of the FRA model as a teaching and learning tool to investigate the identity aspects that characterise physics and mathematics as disciplines and the roles that disciplines can bring into play in an interdisciplinary context.

The study was carried out within the IDENTITIES project and involved a group of student teachers at the University of Bologna.

According to our research questions, we aimed to explore (i) whether and how the FRA could be a teaching support and provide an effective and rich vocabulary to broaden prospective teachers’ ideas on disciplinary identities and (ii) how the FRA wheel had the potential to help prospective teachers to unpack, make visible and keep under control the process of crossing disciplinary boundaries. The preliminary results of the first analysis showed that the framework was immediately familiar to the prospective teachers and provided a vocabulary and ideas allowing them to tap into their personal and disciplinary knowledge. Furthermore, the FRA framework triggered a process of students’ change in terms of *attitude* toward the nature of the disciplines, encouraging them to consider and embrace the complex and multidimensional characterisation of the scientific disciplines. The second study pointed out three cases in which students associated the FRA model with clear roles to characterise the disciplines in an interdisciplinary context. In this phase, an



How can we inquire about the issue "what is science?"? What kind of questions (simpler and more addressable) can we ask to investigate the issue?

What is meant by "science"?

Are you science?

Do scientists only 'do' science?

How can scientific contents and the ways of proceeding of science be communicated or disseminated?

What effects do studying science have on the individual? What about the global community?

Does science answer all possible and unimaginable questions?

What is the etymology of the word science?

What unites or distinguishes science from other disciplines?

What is the aims of science?

What is the scientific method?

What do scientists do?

What is scientific knowledge and what is not?

What tools can man use to know?

In how many ways can I analyse Nature?

What tools does science use?

What role does science play in relation to other branches of knowledge?

How is science studied?

What are the limits of science?

Who decides what science is and what it is not?

What needs and what kind did science give rise to?

To which areas can scientific thought be applied?

Should the word science be limited to certain areas?

What is the difference between empirical observation and scientific research?

When do we deal with science?

What does it mean to understand (from a scientific point of view)?

How can we know how the world works?

What does the investigation of nature's mechanisms consist of?

Who can have access to scientific knowledge?

Where can the scientific method not be applied?

How to define something abstract?

Have you ever had life experiences that you believe are "science"?

When do we do science?

Why is science reliable?

What is not science?

Fig. 5 Digital whiteboard: How can we inquire about the issue 'what is science?'? What kind of questions (simpler and more addressable) can we ask to investigate the issue?

important role was played also by the framework of boundaries (Akkerman & Bakker, 2011). Through a bottom-up approach, we recognised three attitudes that differ from each other, according to the nature of the emerging disciplinary boundaries and the mechanisms put into play to shift from one territory to another. The first case used the FRA framework and the domain-specific orientation to delimit physics with respect to mathematics and vice versa, clearly cutting the disciplinary worlds. In the second case, the FRA was used in both domain-specific and domain-general orientation to overcome the physics-mathematics dichotomies and the boundaries themselves, focusing on identity aspects and shedding light on the resemblances and connections between the two disciplines. In the final case, the FRA framework was used to illuminate the boundary crossing mechanisms between disciplines in the historical case. In this case, the approach not only supported the construction of the boundary zone between mathematics and physics but also paved the way for crossing the boundary and re-characterising the disciplinary identities in the light of the interplay that took place, adding a dynamic component to the framework.

The results of our part studies, all together, are an exploitation of the FRA positioning in the debate on NOS by searching for 'characterisations' of science, instead of following a 'definitory approach' to NOS. The results show to what extent this feature of the FRA has the potential to contribute, beyond the debate on NOS in STEM education, also to the debate on interdisciplinarity. If treated together with the boundary meta-theory of Akkerman and Bakker, FRA appears to be very effective in respecting the epistemological complexity of the issue and providing a vocabulary to reason on the disciplinary identities and their dialogue. In this sense, the networking of FRA and the boundary meta-theory are able to outline a third way that combines both the need to stress disciplinary identities and to cross the boundaries among them. Thanks to both its theoretical framework and empirical results, the study represents an important achievement of the IDENTITIES project, by providing an important empirical base for a model of interdisciplinarity for prospective teachers education.

Annex. What is science? (clustering whiteboard)

In Fig. 5, we report the digital whiteboard obtained in the B2 activity. Students were asked to unpack the far-reaching question 'what is meant by "science"'?, trying to reformulate the question and touching on a variety of dimensions that characterise science: the *societal* dimension, recalling the figure of the scientist and the impact of science on the individuals; the *epistemological* aspect, referring to the limits on knowledge in sciences, to its methods, tools, and practices, to what is science and what is not; the *cognitive* dimension, concerning the link between personal life experiences and science, and the accessibility to scientific knowledge.

Author Contribution All the authors of the paper contributed to the paper and the implementation of the module. Laura Branchetti and Paola Fantini had a crucial role in the design and implementation of the module on parabola and parabolic motion. Rachele Casarotto, Martina Caramaschi and Eleonora Barelli participated in the data analysis. Sara Satanassi participated in the data collection, guided the data processing and analysis and guided the paper writing to which all the authors contributed. Olivia Levrini coordinated the design and implementation of the module, the data collection and the data analysis.

Funding Open access funding provided by Alma Mater Studiorum - Università di Bologna within the CRUI-CARE Agreement. The article reports a study carried out within the IDENTITIES Project. This project has been funded with the support of the European Union and the Italian National Agency within the framework of the Erasmus + Programme (Grant Agreement n°2019–1-IT02-KA203-063184).

Declarations

Consent for Publication The publication has been approved by all co-authors.

Disclaimer The European Commission's support for the production of this publication does not constitute an endorsement of the contents, which reflect the views only of the authors; the Commission cannot be held responsible for any use which may be made of the information contained therein.

References

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169.

Alvargonzález, D. (2011). Multidisciplinarity, interdisciplinarity, transdisciplinarity, and the sciences. *International Studies in the Philosophy of Science, 25*(4), 387–403.

Anfara, V. A., Jr., Brown, K. M., & Mangione, T. L. (2002). Qualitative analysis on stage: Making the research process more public. *Educational Researcher, 31*(7), 28–38.

Bagaglini, V., Branchetti, L., Gombi, A., Levrini, O., Satanassi, S., & Viale, M. (2021). Il ruolo del testo nell'interdisciplinarità tra matematica, fisica ed educazione linguistica: il tema del moto parabolico tra testi storici e manuali di fisica per la scuola secondaria. *Italiano a Scuola*, *3*, 133–184.

Barelli, E., Barquero, B., Romero, O., Aguada, M.R., Giménez, J., Pipitone, C., Sala-Sebastià, G., Nipyrakis, A., Kokolaki, A., Metaxas, I., Michailidi, E., Stavrou, D., Bartzia, I., Lodi, M., Sbaraglia, M., Modeste, S., Martini, S., Durand-Guerrier, V., Bagaglini, V., Satanassi, S., Fantini, P., Kapon, S., Branchetti, L., & Levrini, O. (2022). Disciplinary identities in interdisciplinary topics: challenges and opportunities for teacher education. In G.S. Carvalho, A.S. Afonso & Z. Anastácio (Eds.), *Fostering scientific citizenship in an uncertain world (Proceedings of ESERA 2021), Part 13* (co-ed. M. Evagorou & M.R. Jimenez Liso), (pp. 934–943). Braga: CIEC, University of Minho.

Bernstein, B. (2018). On the classification and framing of educational knowledge. In M. F. D. Young (Ed.), *Knowledge and control.* Collier-Macmillman.

Boland, R. J., & Tenkasi, R. V. (1995). Perspective making and perspective taking in communities of knowing. *Organization Science, 6*, 350–372.

Branchetti, L., Cattabriga, A., & Levrini, O. (2019). Interplay between mathematics and physics to catch the nature of a scientific breakthrough: The case of the blackbody. *Physical Review Physics Education Research, 15*(2), 020130.

Branchetti, L., & Levrini, O. (2019). Disciplines and interdisciplinarity in STEM education to foster scientifc authenticity and develop epistemic skill. In S. Kapon (Chair) & S. Erduran (Discussant), Crossing boundaries – Examining and problematizing interdisciplinarity in science education, invited symposium held at the *European Science Education Research Association (ESERA) Biannual Conference*.

Brown, V.A., Deane, P.M., Harris, J.A., Russell, J.Y. (2010). Towards a sustainable future. In Brown, V.A., Harris, J.A., Russell J.Y. (Eds.), *Tackling wicked problems: Through the transdisciplinary imagination*. 1630. Earthscan

Caramaschi, M., Cullinane, A., Levrini, O., & Erduran, S. (2022). Mapping the nature of science in the Italian physics curriculum: From missing links to opportunities for reform. *International Journal of Science Education, 44*(1), 115–135.

Cerreta, P. (2019). Guidobaldo, Galileo e l'esperimento della pallina tinta d'inchiostro. *Giornale di Fisica*, 167–186.

Cheung, K.K.C., & Erduran, S. (2022). A systematic review of research on family resemblance approach to nature of science in science education. *Science & Education*, 1–37.

Collins, A., & Ferguson, W. (1993). Epistemic forms and epistemic games: Structures and strategies to guide inquiry. *Educational Psychologist, 28*(1), 25–42.

Cullinane, A. (2018). Incorporating nature of science into initial science teacher education. *Unpublished PhD dissertation. University of Limerick, Ireland.*

Cutnell, J.D., Johnson, W. K., Young, D., Stadler, S. (2015). I problemi della fisica. Meccanica e termodinamica. translation by Romeni C., Zanichelli, pp 11−28.

Dagher, Z.R. (2020). Balancing the epistemic and social realms of science to promote nature of science for social justice. In *Nature of Science for Social Justice* (pp. 41–58). Springer.

Damerow, P., Freudenthal, G., McLaughlin, P., & Renn, J. (1992). Concept and inference: Descartes and Beeckman on the fall of bodies. In *Exploring the limits of preclassical mechanics* (pp. 8–67). Springer.

Del Centina, A. (2016). On Kepler's system of conics in *Astronomiae pars optica*. *Archive for History of Exact Sciences, 70*, 567–589.

Engeström, Y., Engeström, R., & Kärkkäinen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and Instruction*.

Erduran, S., & Dagher, Z. (2014a). *Reconceptualizing the nature of science for science education: Scientific knowledge, practices and other family categories*. Springer.

Erduran, S., & Dagher, Z. R. (2014b). Regaining focus in Irish junior cycle science: Potential new directions for curriculum and assessment on nature of science. *Irish Educational Studies, 33*(4), 335–350.

Erduran, S., & Kaya, E. (2018). Drawing nature of science in pre-service science teacher education: Epistemic insight through visual representations. *Research in Science Education, 48*(6), 1133–1149.

Erduran, S., Kaya, E., & Avraamidou, L. (2020). Does research on nature of science and social justice intersect? Exploring theoretical and practical convergence for science education. In *Nature of science for social justice* (pp. 97–113). Springer.

Galilei, G. (1638). Two new sciences (S. Drake, Trans.). *Madison: University of Wisconsin Press. (Original work published 1638).*

Gibbons, M. (1998). Higher education relevance in the 21st century.

Gilbert, J. K., & Zylbersztajn, A. (1985). A conceptual framework for science education: The case study of force and movement. *The European Journal of Science Education, 7*(2), 107–120.

Hermans, H., & Hermans-Konopka, A. (2010). *Dialogical self theory: Positioning and counter-positioning in a globalizing society*. Cambridge University Press.

Hjorth, P., & Bagheri, A. (2006). Navigating towards sustainable development: A system dynamics approach. *Futures, 38*(1), 74–92.

Irzik, G., & Nola, R. (2011). A family resemblance approach to the nature of science. *Science & Education, 20*, 591–607.

Irzik, G., & Nola, R. (2014). New directions for nature of science research. In M. Matthews (Ed.), *International handbook of research in history, philosophy and science teaching* (pp. 999–1021). Springer.

Kapon, S., Erduran, S. (2021). Crossing boundaries – Examining and problematizing interdisciplinarity in science education. In Levrini, O., Tasquier, G., Amin, T.G., Branchetti, L., Levin, M. (Eds.), *Engaging with Contemporary Challenges through Science Education Research. Contributions from Science Education Research*, vol 9. Springer. https://doi.org/10.1007/978-3-030-74490-8_21

Karam, R. (2015). Introduction of the thematic issue on the interplay of physics and mathematics. *Science & Education, 24*(5), 487–494.

Kaya, E., & Erduran, S. (2016). From FRA to RFN, or how the family resemblance approach can be transformed for science curriculum analysis on nature of science. *Science & Education, 25*(9–10), 1115–1133.

Kaya, E., Erduran, S., Akgun, S., & Aksoz, B. (2017). Öğretmen Eğitiminde Bilimin Doğası: Bütünsel Bir Yaklaşım. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 11*(2), 464–501.

Kaya, S., Erduran, S., Birdthistle, N., & McCormack, O. (2018). Looking at the social aspects of nature of science in science education through a new lens: The role of economics and entrepreneurship. *Science & Education, 27*(5–6), 457–478.

Kelly, R., & Erduran, S. (2018). Understanding aims and values of science: Developments in the junior cycle specifications on nature of science and pre-service science teachers' views in Ireland. *Irish Educational Studies*, 1–28

Klein, J. T. (2010). A taxonomy of interdisciplinarity. *The Oxford Handbook of Interdisciplinarity, 15*(6), 15.

Klein, J.T. (2004). Interdisciplinarity and complexity: An evolving relationship. *Structure*, *6*(1–2), 2–10.

Koyré, A. (1939). *Études Galiléennes. 1. A l'aube de la science classique*. Hermann.

Koyré, A. (1965). Newton and Descartes. *his Newtonian Studies, London, Chapman and Hall*, 53–114

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

Lélé, S., & Norgaard, R. B. (2005). Practicing interdisciplinarity. *BioScience, 55*(11), 967–975.

McDonald, C. V., & Abd-El-Khalick, F. (2017). *Representations of nature of science in school science textbooks*. Taylor & Francis.

Morgan, B. (2004). Teacher identity as pedagogy: Towards a field-internal conceptualisation in bilingual and second language education. *International Journal of Bilingual Education and Bilingualism, 7*(2–3), 172–188.

Nyboer, E. A., Reid, A. J., Jeanson, A. L., Kelly, R., Mackay, M., House, J., ... & Cooke, S. J. (2022). Goals, challenges, and next steps in transdisciplinary fisheries research: Perspectives and experiences from early-career researchers. *Reviews in Fish Biology and Fisheries*, 1–26.

Palonen, T., Boshuizen, H., & Lehtinen, E. (2014). How expertise is created in emerging professional fields. In *Promoting, assessing, recognizing and certifying lifelong learning* (pp. 131–149). Springer.

Park, W., Yang, S., & Song, J. (2020a). Eliciting students' understanding of nature of science with text-based tasks: Insights from new Korean high school textbooks. *International Journal of Science Education, 42*(3), 426–450.

Park, W., Wu, J. Y., & Erduran, S. (2020b). The nature of STEM disciplines in the science education standards documents from the USA, Korea and Taiwan focusing on disciplinary aims, values and practices. *Science & Education, 29*(4), 899–927.

Pharo, E. J., Davison, A., Warr, K., Nursey-Bray, M., Beswick, K., Wapstra, E., & Jones, C. (2012). Can teacher collaboration overcome barriers to interdisciplinary learning in a disciplinary university? A case study using climate change. *Teaching in Higher Education, 17*(5), 497–507.

Pharo, E., Davison, A., McGregor, H., Warr, K., & Brown, P. (2014). Using communities of practice to enhance interdisciplinary teaching: Lessons from four Australian institutions. *Higher Education Research & Development, 33*(2), 341–354.

Redish, E.F., & Bing, T.J. (2009). Using math in physics: Warrants and epistemological frames. In D. Raine, C. Hurkett & L. Rogers (Eds.), *Physics community and cooperation, Vol. 2. GIREP-EPEC & PHEC 2009 international conference*, University of Leicester.

Redish, E. F., & Kuo, E. (2015). Language of physics, language of math: Disciplinary culture and dynamic epistemology. *Science & Education, 24*(5), 561–590.

Redish, E.F. (2006). Problem solving and the use of math in physics courses. *arXiv preprint physics/0608268*.

Renn, J., Damerow, P., Rieger, S., & Giulini, D. (2001). Hunting the white elephant: When and how did Galileo discover the law of fall? *Science in Context, 14*(s1), 29.

Ryu, M., Mentzer, N., & Knobloch, N. (2019). Preservice teachers' experiences of STEM integration: Challenges and implications for integrated STEM teacher preparation. *International Journal of Technology and Design Education, 29*(3), 493–512.

Samson, G. (2014). From writing to doing: The challenges of implementing integration (and interdisciplinarity) in the teaching of mathematics, sciences, and technology. *Canadian Journal of Science, Mathematics and Technology Education, 14*(4), 346–358.

Schmitz, C.L., Stinson, C.H., James, C.D. (2010). Community and environmental sustainability: Collaboration and interdisciplinary education. Critical Social Work 11: 3. http://www.uwindsor.ca/criticalsocialwork/. Accessed 23 Mar 2011.

Sillitoe, P. (2004). Interdisciplinary experiences: Working with indigenous knowledge in development. *Interdisciplinary Science Reviews, 29*(1), 6–23.

Suchman, L. (1994). Working relations of technology production and use. *Computer Supported Cooperative Work*, *2*, 21e39.

Tuminaro, J., & Redish, E. F. (2007). Elements of a cognitive model of physics problem solving: Epistemic games. *Physical Review Special Topics-Physics Education Research, 3*(2), 020101.

Tzanakis, C., & Thomaidis, Y. (2000). Integrating the close historial development of mathematics and physics in mathematics education: Some methodological and epistemological remarks. *For the Learning of Mathematics, 20*(1), 44–55.

Tzanakis, C. (2016). Mathematics & physics: An innermost relationship. Didactical implications for their teaching & learning. In *History and Pedagogy of Mathematics*.

Uhden, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012). Modelling mathematical reasoning in physics education. *Science & Education, 21*(4), 485–506.

Walker, J.S. (2017). Physics (5th ed.). *Pearson Education*, Inc.

Williams, J., & Roth, W.M. (2019). Theoretical perspectives on interdisciplinary mathematics education. In *Interdisciplinary Mathematics Education* (pp. 13–34). Springer.

Williams, J. (2019). Becoming un-disciplined with science and mathematics. *Selected Works*, 16.

Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics, 83*(2), 267–284.

Winsløw, C., Barquero, B., De Vleeschouwer, M., & Hardy, N. (2014). An institutional approach to university mathematics education: From dual vector spaces to questioning the world. *Research in Mathematics Education, 16*(2), 95–111.

Yeh, Y. F., Erduran, S., & Hsu, Y. S. (2019). Investigating coherence about nature of science in science curriculum documents: Taiwan as a case study. *Science & Education, 28*, 291–310

Ziman, J. M. (1991). *Reliable knowledge: An exploration of the grounds for belief in science*. Cambridge University Press.

The work described in the article is original. It has not been published before and is not under consideration for publication anywhere else. .

One of the co-author, Olivia Levrini, is Associate Editor of the journal and she does not take part in any way in the review process.