Secondary and University Students' Descriptions of Quantum Uncertainty and the Wave Nature of Quantum Particles

Maria Vetleseter Bøe¹ · Susanne Viefers¹

Accepted: 25 October 2021 / Published online: 27 November 2021
© The Author(s) 2021, corrected publication 2022

Abstract

Teaching and learning of quantum physics at secondary level is an active field of research. One important challenge is finding ways to promote understanding of quantum concepts without the mathematical formalism that is embedded in quantum mechanics but unavailable on the secondary level. We investigated Norwegian secondary students' ($N=291$) descriptions of the wave nature of quantum particles and the uncertainty principle, as expressed during work with learning resources using a sociocultural approach emphasizing history, philosophy, and nature of science aspects. Responses from university students ($N=40$) given after a formalism-based course in quantum physics were included for comparison. Themes were identified using thematic analysis and analyzed from the perspective of pedagogical link-making, seeing different themes as representing different levels of explanations of the concepts (phenomenological, qualitative, mathematical). The most dominant theme in descriptions of particle wave nature was that particles exhibit wave behavior in experiments, while referring to the mathematical description of particles by wave functions was a less prominent theme, even among university students. Two uncertainty principle themes were found: uncertainty as inability to measure pairs of variables precisely, and uncertainty as innate blurriness in nature. Largely missing from descriptions of both concepts were meaningful links between different levels of explanations. Based on the results, we discuss ways forward for teaching particle wave nature and uncertainty in secondary education.

✉ Maria Vetleseter Bøe
m.v.boe@fys.uio.no

¹ Department of Physics, University of Oslo, Oslo, Norway

1 Introduction

1.1 Quantum Physics and Its Place in Education

The importance of quantum mechanics in modern science and technology can hardly be overestimated. The development of quantum theory during the first half of the twentieth century constituted a paradigm shift that fundamentally changed our understanding of nature, with stunning predictions for the microscopic world that seem totally at odds with everyday life physical intuition. New technology followed in the wake of this development and is omnipresent today: the functionality of all semiconductor-based technology, present in computers, solar cells, and LED devices, as well as lasers, superconducting devices, and many more, is based on quantum mechanical principles. The qualitative and technological development up to this point is often referred to as the first quantum revolution. At the time of writing, the *second quantum revolution* is unfolding (Durakiewicz & Greene, 2018; Touzalin et al., 2016), with major worldwide initiatives to use previously untapped quantum properties like entanglement to develop entirely new technology, including quantum sensors, quantum internet, and quantum computers. These developments add to the importance of educating young people in quantum physics (Stadermann et al., 2019), both to meet future demands of competence in the field and to introduce coming generations to our best current understanding of the universe (Kaur et al., 2020; Shabajee & Postlethwaite, 2004).

In recent years, more and more countries have added quantum physics to their curricula in upper secondary physics (Escalada et al., 2004; Hoekzema et al., 2007; Lautesse et al., 2015; Mannila et al., 2001; Mashhadi & Woolnough, 1999; Michelini et al., 2000; Müller & Wiesner, 2002; Stadermann et al., 2019). Quantum physics is among the topics physics students typically find the most interesting (Angell et al., 2004), which makes it particularly valuable for motivation and engagement (Renninger & Hidi, 2016). The counter-intuitive nature of quantum physics, with phenomena like tunneling, wave-particle duality, or entanglement, is often perceived as deeply fascinating. Such phenomena, therefore, provide a great starting point for getting students interested in physics in general and quantum physics in particular. At the same time, quantum physics can be challenging for both teachers and students. A recent review by Bouchée et al. (2021) identified four reasons for students' conceptual difficulties in quantum physics. These are struggles (a) to relate the mathematical formalism to experiences in the physical world; (b) to interpret counter-intuitive phenomena and concepts; (c) to transit from a deterministic to a probabilistic world view; and (d) to understand the limitations of language to express quantum phenomena, concepts, and objects. They also proposed two approaches for overcoming these difficulties, namely using digital materials and discussing the history and philosophy of science. The mathematical formalism of quantum mechanics is quite advanced, and is, therefore, rarely introduced before lower undergraduate education (Stadermann et al., 2019). Moreover, even presenting quantum physics in a qualitative non-formalistic way requires students to get used to a conceptually new way of thinking (Hoehn et al., 2019; Renstrøm, 2011). Secondary physics curricula cover a range of topics besides quantum physics and rarely allow students a lot of time to develop their understanding. The change from determinism to probabilities, concepts like superposition and entanglement, the uncertainty principle, and "wave-particle duality" represent breaks with classical physics and ways of thinking which frequently introduce challenges in students' reasoning about quantum phenomena (Henriksen et al., 2018; Olsen, 2002; Quadros et al., 2018; Renstrøm, 2011). It is thus

important to develop and evaluate teaching strategies that help students overcome the challenges above and develop quantum understanding. Although these issues have been discussed in physics education research for more than 20 years (see, e.g., Zollman, 1999), they are still prevalent (Bungum et al., 2018; Krijtenburg-Lewerissa et al., 2017; Bouchée et al., 2021). Among challenging topics in quantum physics is wave-particle duality, identified as a key topic by experts in a Delphi study by Krijtenburg-Lewerissa et al. (2019).

In this study, we present results from a design-based research project (Anderson & Shattuck, 2012) in upper secondary physics in Norway, developing digital learning resources in quantum physics and general relativity using a sociocultural approach emphasizing student discussions as well as history, philosophy, and nature of science (NOS) aspects (Henriksen et al., 2014). These learning resources introduce wave-particle duality including particle wave nature and uncertainty without mathematical formalism and in a short time frame. Analyzing students' written responses to questions within the resources allows us to investigate students' expressed ideas about these topics in relation to the learning environment in which they were formed. For comparison, we include responses to the same questions from students in an introductory university quantum physics course, where students were able to rely on both mathematical formalism and on a lot more time to develop their understanding. We view the students' descriptions through the lens of pedagogical link-making (Scott et al., 2011) in order to discover links or lack of links between the building blocks in the complex structure that is conceptual understanding in physics. This framework allows us to study how conceptual understanding of particle wave nature and uncertainty is challenged and/or promoted using a sociocultural approach to learning that does not rely on mathematics. For the context of the present paper, we should note that the Norwegian translation of "matter" is not used as much as "matter" is in English, and schools, textbooks, and learning resources tend to use the Norwegian translation of "particle" (partikkel) when discussing the wave nature of matter. Therefore, we use "particle wave nature" instead of the more commonly used "the wave nature of matter" or "the wave nature of quantum objects" in this study. We are, however, aware that "particle" is an ambiguous term with many meanings in science, even within physics, and that this poses challenges to teaching and learning (Bouchée et al., 2021; Bunge, 2003).

1.2 Quantum Physics Education Research

The above highlights the importance of research into teaching strategies to improve students' conceptual understanding of the quantum world at upper secondary level, and indeed this is a very active field of research. Comprehensive overviews can be found in review articles by Krijtenburg-Lewerissa et al. (2017) and Bouchée et al. (2021). Their papers review a large body of literature concerning both the learning difficulties encountered, tools to analyze students' conceptual understanding, teaching strategies that have been implemented, and ways forward for research in the field. Among the aspects studied in the literature are students' understanding of the wave nature of matter, including wave-particle duality, the double-slit experiment, Heisenberg's uncertainty principle, and the concept of wave functions (in particular the probability interpretation). A number of studies find that students' views of these issues can typically be grouped in categories of understanding ranging from misplaced classical thinking, via a mixture of classical and quantum thinking, to (quasi)quantum description (Ayene et al., 2011; Greca & Freire, 2003; Ireson, 1999, 2000; Mannila et al., 2001). In the case of the uncertainty principle, Ayene et al. (2011) found that a majority of students in their study expressed classical thinking, in the

sense of attributing uncertainty to extrinsic factors or measurement error. Another group demonstrated thinking in terms of measurement disturbance, whereas even those few who stated that uncertainty was an inherent quantum mechanical property appeared to have a rather simplistic and vague understanding. In general, it is quite common for students to attribute uncertainty to external effects, rather than it being an inherent property of quantum objects (Müller & Wiesner, 2002).

Generally speaking, misplaced classical thinking and confusion due to literal interpretation of classical metaphors (Brookes & Etkina, 2007; McKagan et al., 2008) seem to be rather common, e.g., in the way students think about the wave function (Özcan, 2011). In response to these challenges, a lot of interesting work has been done on development of teaching strategies with focus on conceptual understanding of quantum physics (Bouchée et al., 2021; Krijtenburg-Lewerissa et al., 2017). Still, more research is needed, both on student conceptions themselves and on teaching strategies, to promote conceptual understanding in particular and learning in quantum physics in general. One example of such research was conducted by Henriksen et al. (2018), investigating students' descriptions of the nature of light, arguing for a historical-philosophical perspective as an entry point for upper secondary physics students to explore the development and interpretation of quantum physical concepts. In a related study, Bungum et al. (2018) found that small-group discussions could be productive for developing secondary students' understanding of quantum concepts, specifically by helping students to articulate conceptual difficulties, deepen their understanding through exchange of views, and formulate new questions. However, they pointed out that teacher support is needed to help resolve conceptual difficulties and answer new questions that arise during discussions. In their study of student discussions in quantum physics, Hoehn et al. (2019) also found that students' tentative reasoning could be productive even though they expressed ideas that were not necessarily scientifically correct. They argued for moving beyond a binary view of learning focused on the right or wrong nature of student answers, towards looking at the process of student learning. The present paper adds to these studies by investigating Norwegian secondary students' written descriptions of quantum particle wave nature and Heisenberg's uncertainty principle after having worked with the topics through a sociocultural approach without mathematics, but using discussions and history, philosophy, and NOS aspects.

1.3 Quantum Particle Wave Nature and Heisenberg's Uncertainty Principle

Particle wave nature, and the term wave-particle duality, refers to the fact that quantum entities, such as electrons, are fundamentally different from the classical perception of particles as point-like objects or tiny balls. Rather, these quantum objects sometimes display behavior typically associated with the classical concept of particles, and sometimes resemble more what, in classical physics, is typically thought of as wave phenomena. These quantum objects have no counterpart in the classical world, which makes it hard to get an intuition for them, but in a sense they possess both "particle-like" and "wave-like" properties, and it depends on the experiment at hand which of these aspects best describes the object's behavior. A popular metaphor for this is that of the blind men and the elephant (Bohm, 1989): Several blind men decide to explore an elephant by feeling with their hands. The first touches its ear and decides an elephant is like some sort of fan. The second touches the torso and says it's like a hard wall. The third touches the tail and concludes that the elephant is some kind of rope, etc. Obviously, none of these observations give the full

picture of what an elephant really looks like—much like particle-like behavior and wave-like behavior do not give the full picture of a quantum object.

Particle wave nature has striking experimental consequences. One of its most famous manifestation is found in the double-slit experiment: electrons, atoms, and even large molecules have been shown to produce an interference pattern when sent through a double slit (this is an active field of research to this day, e.g., addressing the question how big a quantum object can get before this quantum behavior disappears and the object behaves like a classical particle) (Hornberger et al., 2012). Each individual electron (say) fired at a double slit will hit a screen behind the slits, leaving a small dot where it hit—as naively expected from a particle. However, after shooting a large number of electrons, one finds (in the idealized case) that the dots on the screen form stripes of high density as well as regions (stripes) that are *never* hit, in the shape of an interference pattern—as classically associated with wave behavior. The qualitative explanation within the Copenhagen interpretation (Faye, 2019) is that each electron is described in terms of a *probability wave* which passes through both slits, forming an interference pattern in the probability for observing the electron, such that, eventually, there will be many hits where the probability was large, and none where the probability was zero. An additional twist, often very mysterious to students at first encounter, is that the interference pattern goes away if one observes which slit the electron went through. To demystify this, one has to address how the measurement process itself alters the state of the system, a process which is often challenging to students (Huseby & Bungum, 2019; Zhu & Singh, 2012a, b). Alternatively, a beautiful explanation can be given in terms of the uncertainty relation (Feynman et al., 2003). Another fascinating phenomenon that can be ascribed to particle wave nature is tunneling—that quantum objects may pass through barriers that, classically, they do not have enough energy to pass. A common example is alpha decay, a radioactive process where helium nuclei escape from a heavier nucleus by tunneling through the potential barrier provided by the nuclear force.

In accordance with these phenomena, the equation of motion describing quantum objects like electrons has the form of a wave equation—the Schrödinger equation, whose solution is called the wave function. The wave function describes the state of the quantum particle and, according to the Born interpretation, encodes the probability of observing it in a given state—e.g., at a certain position at a given time. In other words, measurement outcomes are only predicted probabilistically. This “loss of determinism” is often troubling to students. It is thus important to emphasize that determinism is not lost altogether. The probability function following from the Schrödinger equation predicts the distribution of measurement outcomes for doing the same experiment a large number of times, with identically prepared systems, just like the many hits in the double-slit experiment eventually form interference strips; this prediction is exact and deterministic. While the actual mathematical formalism is usually introduced at undergraduate level, these basic concepts are typically conveyed in a qualitative manner to upper secondary students (Stadermann et al., 2019).

The Heisenberg uncertainty principle can be illustrated on a conceptual level as yet another consequence of particle wave nature. In its simplest form, it tells us that quantum particles cannot have a sharp position and sharp momentum (i.e., de Broglie wavelength) simultaneously—much like there is a trade-off between a well-defined position and a well-defined wavelength for any classical wave packet. However, as will be discussed in this paper, it is often difficult for students to grasp the concept of uncertainty, what it really means, and whether or not it is an inherent property of a quantum system or some sort of measurement problem (Ayene et al., 2011; Krijtenburg-Lewerissa et al., 2017). Again, it is helpful to think in terms of ensembles: the uncertainty of a given observable in a given

state corresponds to the spread in measured values if one were to perform identical measurements of this observable on a very large number of identical systems in this particular quantum state. Moreover, here, one easily touches upon interpretational issues. It is natural for learners of quantum mechanics to wonder “what, then, is the position of the electron before I measure it?” This soon leads to a discussion of the measurement process itself (Zhu & Singh, 2012a), and most commonly, this is done in the framework of the Copenhagen interpretation: before measurement takes place, the system is in some quantum state, as determined by the Schrödinger equation. This quantum state and the probabilities implied by it give a complete description of the system, in the sense that observables (say, position) do not *have* well-defined values prior to measurement. Measurement then *changes* the state of the system into one with a sharp value of the observable in question; the wave function “collapses” upon measurement. In other words, quantum mechanical measurement cannot be seen as a passive process of observing, but entails an interaction which, in fact, determines the state of the system. This also means that measurements do produce exact values, even though the *prediction* of the experimental outcome is probabilistic.

The above descriptions of quantum physics are in line with the Copenhagen interpretation. The claim of the Copenhagen interpretation that a quantum particle does not *have* a well-defined position (say) before we measure it was troubling to some of the founding fathers of quantum mechanics, and discussions on the interpretations of quantum mechanics are ongoing to this day. An attempt to get around this, originally proposed by Einstein et al. (1935), is to say that the theory must be *incomplete*—there must exist additional information, so-called “hidden variables,” in addition to the wave function, to fully characterize a quantum state. In this “realist” view, a particle would indeed have a well-defined position (say) prior to measurement, only that quantum mechanics cannot tell us. Indeed, this is a natural, naïve, gut reaction for anyone being confronted with quantum physics for the first time. However, any local hidden-variable theories, and thus naïve realism, have been proven wrong in experiments testing Bell’s inequality (Brunner et al., 2014). There exist other possible interpretations, including the Many-Worlds interpretation (Vaidman, 2018) and non-local hidden-variable theories like Bohmian mechanics (Goldstein, 2017). The Norwegian upper secondary curriculum that was taught when these data were collected does not specify which interpretation should be taught. It does, however, state that students should reflect on epistemological consequences of entanglement and uncertainty (see curriculum details below), thus prompting the introduction of philosophical perspectives on quantum physics. The university physics class in which we collected data explicitly teaches a Copenhagen interpretation of quantum physics.

1.4 Theoretical Lens

A pronounced goal of secondary quantum physics education in many countries including Norway is a qualitative conceptual understanding of particle wave nature and Heisenberg’s uncertainty principle (Stadermann et al., 2019). This article studies secondary physics students’ written responses to questions within learning resources developed to promote such conceptual understanding (see overview of learning resources below) and compare them to responses from university students given after a full semester of teaching. These resources adopt a sociocultural view on learning which entails that the use of language is a crucial feature of learning processes (Vygotsky, 1978) and that students make meaning of physics concepts through interaction with others (Mortimer & Scott, 2003). Embedded in such an approach to learning, Scott et al. (2011)

presented *pedagogical link-making* as a framework for understanding the teaching and learning of scientific conceptual knowledge. The authors presented three types of pedagogical link-making:

- 1) The first type involves *supporting knowledge building* by making links between different building blocks of knowledge, i.e., connecting the concepts *momentum* and *force* to each other. Within this type of pedagogical link-making, six approaches to knowledge building were identified. These are making links between the following:
 - Everyday and scientific ways of explaining
 - Scientific concepts
 - Scientific explanations and real world phenomena
 - Modes of representation
 - Analogous cases
 - Different scales/levels of explanations, such as the microscopic/theoretical, macroscopic/phenomenological, and symbolic/mathematical levels

The latter point is particularly relevant in the present study, which looks at how students' descriptions of the scientific concepts *particle wave nature* and *the uncertainty principle* portray links between the concepts and different scales/levels of explanations. Scott et al. (2011) describe the microscopic/theoretical level as involving "explanations based on abstract models, which include non-directly observable entities such as atoms, molecules and ions" (p. 11). The mathematical level concerns mathematical/symbolic representations that apply to both macroscopic and microscopic scales. For quantum physics, mathematical level explanations include description of even sub-atomic entities and the equations that govern their behavior. Arguably, the mathematical level can be seen as a sub-level of the microscopic/theoretical level. As students struggle to connect qualitative (non-mathematical) concepts to the mathematical descriptions of quantum physics (Krijtenburg-Lewerissa et al., 2017), it is helpful to treat qualitative and mathematical types of microscopic/theoretical level explanations distinctly. In this study, therefore, the *qualitative level* is introduced for explanations using qualitative concepts or ideas about quantum entities rather than their mathematical descriptions or experimental (phenomenological) behavior.

- 2) The second type of pedagogical link-making concerns *promoting continuity* by linking teaching and learning events across time, e.g., by explicitly reminding students of Newton's second law when introducing how it takes a force to change an object's momentum. Two approaches to promoting continuity were identified:

- Developing the scientific story
- Managing and organizing

Both of these approaches involve enabling cumulative knowledge building over time, by linking new concepts, explanations, phenomena, and classroom activities to previously addressed ones over short (micro), intermediate (meso), and extended (macro) timescales spanning from minutes to years.

- 3) The third type of pedagogical link-making involves *encouraging emotional engagement* that connects the subject matter to positive feelings, i.e., by using an experiment

that fascinates and interests students. Two kinds of approaches to promoting emotional engagement are the following:

- Generic approaches, such as calling students by their names and giving praise
- Addressing substantive content, that is, connecting the content to the individuals

Addressing substantive content can, for example, involve students to make predictions about phenomena before performing an experiment. This often makes students invest in the content in terms of curiosity or interest, especially if the experiment itself is memorable or striking, leading to their active engagement in searching for an answer or explanation afterwards.

Pedagogical link-making is “concerned with the ways in which teachers and students make connections between ideas in the ongoing meaning-making interactions of classroom teaching and learning” (Scott et al., 2011, p. 3). Thus, it allows us to investigate learning processes as well as teaching strategies. We study *students’* descriptions as signs of meaningful links that are present or not present between central building blocks of scientific knowledge in quantum physics, and how *teaching* in general and sociocultural approaches in particular can make links between these building blocks in order to promote learning. Given that secondary students have to develop their understanding without much mathematics and without much time, the framework is particularly helpful when looking for ways in which these two limitations introduce challenges for secondary students’ link-making between different kinds of knowledge, such as scientific concepts and their representations on different levels and scales, such as the phenomenological and mathematical level, and across time (continuity). Previous studies have used pedagogical link-making to investigate both university and secondary education in different subjects. Quadros et al. (2018) studied the teaching of university science professors and found that they rarely made macro links to help students build a coherent idea of the curriculum in lessons. Wood et al. (2014) drew on pedagogical link-making when developing a framework for analyzing learning during peer instruction dialogues in physics, and Mudadigwa and Msimanga (2019) used pedagogical link-making to investigate secondary chemistry teachers’ instruction about the electrolytic cell.

1.5 The ReleQuant Project and Quantum Physics in Norwegian Upper Secondary School

The present work is part of the larger ReleQuant project which develops web-based learning resources for quantum physics and general relativity in upper secondary school and studies the use of these in physics classrooms (Henriksen et al., 2014). The learning resources follow a sociocultural approach, and include historical and philosophical reflections. ReleQuant employs a design-based research methodology (DBR), which entails that the research is situated in a real educational context (the physics classroom), focused on design and testing of an intervention (web-based learning resources) in several cycles (Anderson & Shattuck, 2012). The project group collaborated with nine upper secondary schools, predominantly in the greater Oslo area. Physics teachers and students from these schools participated in classroom trials of the learning resources and in interviews. The teachers also took part in workshops and seminars to secure a close connection between the practice field and research and development. Between each iteration of classroom trials, the learning resources were revised based on analyses of student interviews and their responses to tasks in the materials, teachers’ experiences with using the materials, and input from

physics experts and science educators. In this article, we report on the last two iterations of classroom trials (2016 and 2017) and development of the module *Particles as waves* within the larger web-based learning resources *Quantum physics* (available in English from <https://www.viten.no/filarkiv/quantum-physics/>) for upper secondary school. Norwegian schools offer physics as an elective stand-alone subject only in the last two years of upper secondary education, when the students are typically 17–19 years old. The *Quantum physics* learning resources were designed to help teachers and students meet the competence aims described in the Norwegian curriculum for the last of those two years. Quantum physics is not a large part of the curriculum, and teachers typically spend 1–2 weeks of lessons on the topic. The curriculum emphasized qualitative understanding of quantum physics phenomena, and includes philosophical and epistemological reflections on quantum physics and the nature of its break with classical physics. Formalism or the Schrödinger equation is not mentioned, and quantitative- or calculations-based approaches are thus not much used. In terms of particle wave nature and uncertainty, the Norwegian upper secondary physics curriculum at the time of data collection stated that students should learn to (NDET, 2006):

- Give an account of Einstein's explanation of photoelectric effect, and give a qualitative account of how results from experiments with photoelectric effect, Compton scattering, and the wave nature of particles represent a break with classical physics
- Give an account of Heisenberg's uncertainty principle, describe the phenomenon "entangled photons," and give an account of their epistemological consequences (the official English version of the curriculum says "cognitive consequences," but "epistemological consequences" is, in our view, a better translation from the Norwegian original "erkjennelsesmessige konsekvenser")

In 2021, a new Norwegian physics curriculum was introduced (NDET, 2021). It includes open formulations that focus on the differences between quantum and classical objects, and does not explicitly mention philosophical reflections or whether quantum physics should be treated only qualitatively.

At the time of data collection, the curriculum stated that students should be able to "give an account of" Heisenberg's uncertainty principle, and (depending on how you interpret the sentence) of its epistemological consequences. It was up to teachers, authors of textbooks, designers of learning resources, and, not least, exam developers to interpret the curriculum and how it should be implemented and assessed. The ReleQuant learning resources take a conceptual approach with little mathematics. Interpretational and epistemological issues are addressed mostly indirectly in the learning resources, for example by asking students to discuss what the double-slit experiment says about what electrons really are ontologically, and reflect on what uncertainty means for what we can know about nature. However, the measurement process itself is not explicitly discussed, due to limited time available as well as the needed mathematics. As the role of measurement is at the heart of many interpretational debates in quantum physics, this potentially deprives students of a helpful tool in the epistemological reflections the curriculum asks them to make.

In line with the view on learning as a social process outlined above, the learning resources include a range of activities where students use oral and written language, often together with peers. The language-based approach also fell in line with the curriculum emphasis on qualitative understanding. Moreover, the learning resources include a range of visualizations such as animations, films, and simulations, which has been found to facilitate learning of often counter-intuitive phenomena (Kohnle et al., 2013; Krijtenburg-Lewerissa et al., 2017). Historical and philosophical aspects of quantum physics are used to promote

understanding of NOS, to allow for philosophical reflections as stated by the curriculum, and to motivate students.

The learning resource *Quantum physics* comprises the five modules *Need for a new physics*, *Light as particles*, *X-rays*, *Particles as waves*, and *Quantum physics and philosophy*. The estimated time to complete all modules is 360 min. The 45-min module *Particles as waves* addresses particle wave nature, including the electron double-slit experiment and Heisenberg's uncertainty principle.

1.5.1 The Learning Resource “Particles as Waves” in 2016 and 2017 Trials

This article uses data collected during students' work with the *Particles as waves* module in two consecutive classroom trials. Specifically, the data comprise student responses to two written tasks about particle wave nature and Heisenberg's uncertainty principle given at the end of the module in the 2016 and 2017 trials. Below follows an introduction to the content of the *Particles as waves* module that is most relevant to this article, including changes that were made between the 2016 and 2017 versions. The latest English version of the complete learning resources can be found on <https://www.viten.no/filarkiv/quantum-physics/>.

The main components of the module are the following:

- The double-slit experiment demonstrating interference in electrons. The students watch a “Dr. Quantum” film on the phenomenon (<https://www.youtube.com/watch?v=Q1YqgPatzho>) before engaging in a role-play discussion where one student [journalist] interviews another student [physics researcher] about the electron interference experiment.
- Two short films where a physics professor talks about the uncertainty principle and the wave nature of quantum particles. The first film explains the uncertainty principle using the de Broglie relation. The other film focuses on the wave nature of particles as being exhibited in observable phenomena such as interference and tunneling, and just briefly mentions that quantum particle wave nature stems from the Schrödinger equation.
- A listening exercise where uncertainty is linked to the increasing difficulty of hearing the pitch of a sound when the sound wave becomes shorter in time, thus making an analogy between a classical wave packet and quantum physics.
- A brief presentation of the time energy uncertainty principle, linking it to quantum fluctuations.
- A consolidation exercise where students first discuss a few questions about particle wave nature and the uncertainty principle in small groups, they then write down responses to the same questions, before discussing them in the whole class with the teacher.

Between the 2016 and 2017 classroom trials, changes were made to the entire web-based learning resource *Quantum physics* based on classroom observations and teachers' experiences, analyses of student responses and interviews, and input from physics experts and science/physics educators. Input from students and science educators motivated the inclusion of key sentences, which are highlighted sentences summarizing the most important content throughout the learning resources and collated at the end of each module. Preliminary analyses of students' written responses from the 2016 trial indicated that students struggled with the meaning of the uncertainty principle and how it relates to measurement (Ræder et al., 2017), prompting a change in formulation of the uncertainty principle from

“there are pairs of variables in nature that cannot be sharply determined at the same time” in 2016, to “there are pairs of variables in nature that cannot be sharp at the same time” in 2017. The word “sharp” is used here as a translation of the Norwegian word “skarp,” which means “not blurry.” In Norwegian, the uncertainty principle is called the “unsharpness relation.” The formulation that variables cannot be sharply determined/sharp “at the same time” is imprecise and might lead students to believe that the uncertainty principle can be circumvented by measuring the variables at different times. A more precise formulation would be that the variables cannot be sharply determined/sharp in the same quantum state. However, the notion of the quantum state is typically not introduced at secondary level and was, therefore, avoided.

1.5.2 Responses from Introductory University Quantum Physics Students for Comparison

Our data material includes written responses from students at the end of an introductory quantum physics course at the University of Oslo. The data allows for a comparison of upper secondary students' descriptions of quantum physics to descriptions given by university students who had been through a standard format introductory quantum physics course, where students have access to both the mathematical formalism of quantum mechanics and a lot more time (five months) for their understanding to develop. Moreover, the results formed a baseline for ongoing learning material development for the university course in question. The course is given in the second year of a bachelor program in physics and awards 10 credits (1/3 of a semester workload). Its main components are the following:

- The historical development of quantum physics including the de Broglie relation and important experimental results such as the photoelectric effect and the double-slit electron experiment
- The wave function and Born's statistical interpretation
- The Schrödinger equation and solving it for key one-dimension potentials
- The hydrogen atom and solving the Schrödinger equation in three dimensions
- Identical particles
- Spin
- Applications such as molecular vibrations and rotations

The inclusion of student responses from this course allowed for comparing ideas about particle wave nature and the uncertainty principle expressed by students who have experienced very different context for pedagogical link-making: secondary students' responses after a few lessons using a qualitative approach and university students' responses after having completed a full 10 credit course over 5 months including a lot of formalism and calculations.

1.6 Research Questions

- How do secondary and university students describe the meaning of particle wave nature and the uncertainty principle?
- Which links between different building blocks of knowledge about particle wave nature and the uncertainty principle can be found in the descriptions?

2 Methods

2.1 Data Collection

The data consist of written responses to two questions:

- 1) In the film *Particles as waves*, [physics professor] talks about particles having wave nature. What is meant by that?
- 2) What does Heisenberg's uncertainty principle say, and what does it imply about how much we can know about nature?

The responses were written by students in three respondent groups:

- a) Two hundred ten responses from 184 upper secondary physics students in project partner schools Spring 2016
- b) One hundred twenty-two responses from 107 upper secondary physics student in project partner schools Spring 2017
- c) Eighty responses from 40 students in the introductory quantum physics course given at the University of Oslo Spring 2018

For upper secondary students, these questions were included as the last part of their work with the module *Particles as waves*, and answered electronically. They were instructed to first discuss the questions in pairs, and then write down responses to them. It appears that many of the students interpreted this instruction to mean that only one of them had to write the answer down on behalf of the group. Therefore, there are fewer secondary school responses to each question than participating students. It is also possible that some of the students did not respond at all. The above are the questions as they were formulated in the 2017 version. In 2016, the questions were worded slightly differently, reflecting the different use of the term *wave properties* in the 2016 version mentioned above: (1) what is meant by particles having wave properties? Based on the way *wave properties* were used in the 2016 version, we argue that it is likely the respondents' would interpret the question similarly to the 2017 formulation. Analyses support this argument. Also, the 2016 version of question 2 was: what does Heisenberg's uncertainty principle say? What do the wave properties of particles and the uncertainty principle imply for how much we can know about nature? Arguably, the last part of the question includes more than in the 2017 version, since it asks about what the wave properties of particles implies for how much we can know about nature. This has been taken into account during data coding. Although this question was intended to prompt reflections about epistemology, such reflections were scarce.

The university students responded to a short questionnaire on paper during their break of their last regular lecture of the semester. For secondary students, discussing the questions with peers before answering was part of the task. University students were given the opportunity to discuss with each other if they wanted to. Responses, therefore, should be considered potentially resulting from small group discussions and not merely individual responses. As the university students had not seen the film that is referred to in question 1 above, this reference was omitted in their questionnaire.

2.2 Analysis

Data were analyzed thematically (Braun & Clarke, 2008) using *Atlas.ti* software. Thematic analysis was chosen because it allowed us to identify robust themes in students' descriptions across datasets and then view these themes and their connections to the learning resources and to each other from the perspective of pedagogical link-making, looking for signs of meaningful links and lack thereof within students' understanding. The unit of analysis was a response to one of the two questions. One student typically submitted two responses, one to each question. First, the 2017 upper secondary and 2018 university responses were coded primarily inductively in several rounds by both authors. Based on this initial coding, we decided upon a set of codes and themes which was then used in a new round of coding done independently by both authors across the entire dataset. The coding was subsequently validated by comparing the analyses of the two authors. Discrepancies in the coding were discussed and resolved. As the 2016 data had already been subjected to preliminary analyses by Ræder et al. (2017), these were only included in the round of coding using the final set of codes. Ræder et al. did not use the same code set. All responses coded with inductively identified themes were subjected to an additional layer of interpretational coding based on pre-defined pedagogical link-making codes. That is, the responses were interpreted as describing one or more building blocks of scientific knowledge between which pedagogical links should be made according to Scott et al. (2011). This was done by the first author and validated through discussions with the second author. Co-occurrence analyses were used to investigate how often students' descriptions included more than one theme/code. The responses with such co-occurrences were then scrutinized qualitatively to establish whether the building blocks of knowledge represented by the codes were meaningfully linked in the response, or just described alongside each other. The final sets of themes and codes are given in Table 1 with examples of responses and which data source the response is from.

3 Findings

In the following, we present our findings as responses to the research questions. Illustrative quotes from the data are used throughout. A summarizing table of code occurrences within each respondent group is given in Table 2. Co-occurrences of codes within the same responses are shown in Table 3.

3.1 Students' Descriptions of the Meaning of Particle Wave Nature

3.1.1 Theme: Wave Nature Means that Particles Exhibit Wave Behavior in Experiments

In responses to the question about what is meant by particles having wave nature, the by far most dominant theme in all respondent groups (181 occurrences, included in 87% of responses) was that *wave nature means that particles exhibit wave behavior in experiments*. In such descriptions, students linked the scientific concept of particle wave nature to a phenomenological level of explanation of the concept: experimental wave behavior. Specifically, students very often mentioned interference experiments with electrons:

Table 1 Themes and codes with illustrative quotes. There are four inductively generated themes and six theory driven codes based on pedagogical link-making. The four themes each correspond to one theory driven code, and two additional theory driven codes correspond to inductive codes that did not develop into themes

Themes	Pedagogical link-making codes		Illustrative quotes
<i>Wave nature means that particles exhibit wave behavior in experiments</i>	Phenomenological level wave nature		"Particles can in experiments behave like waves. F. ex. interference." (Secondary, 2017)
<i>Wave nature means that particles are described mathematically by wave functions and wavelengths</i>	Mathematical level wave nature		"We also talk about particles with wave nature when they have wavelength and frequency and can be described by wave equations." (Secondary, 2017)
<i>Uncertainty as innate blurriness in nature</i>	Qualitative level uncertainty		"That photons/particles/waves with well-defined momentum cannot have a well-defined position and vice versa." (Secondary, 2016)
<i>Uncertainty as inability to measure pairs of variables precisely at the same time</i>	Phenomenological level uncertainty		"One cannot determine a particle's momentum and position sharply at the same time." (Secondary, 2016)
	Qualitative level wave nature		"That they can have properties like a wave. They are not exclusively a point mass." (University, 2018)
	Mathematical level uncertainty		"[Uncertainty principle standard deviation formula]. Cannot determine position and momentum at the same time." (University, 2018)

Table 2 Occurrences of pedagogical link-making codes within the respondent groups (secondary 2016, secondary 2017, and university 2018), including the percentage of responses to the relevant question that were assigned each code. Note that some responses were coded with more than one code, resulting in a total number of occurrences that exceeds the number of responses

Pedagogical link-making codes	Occurrences					
	Secondary 2016		Secondary 2017		University 2018	
	Number of coded responses	Percentage	Number of coded responses	Percentage	Number of coded responses	Percentage
Wave nature question	106 responses		61 responses		40 responses	
Phenomenological level wave nature	98	92%	51	84%	32	80%
Mathematical level wave nature	7	7%	17	28%	9	23%
Qualitative level wave nature	8	8%	3	5%	10	25%
Uncertainty question	104 responses		61 responses		40 responses	
Phenomenological level uncertainty	77	74%	30	49%	31	78%
Mathematical level uncertainty	6	6%	5	8%	21	53%
Qualitative level uncertainty	49	47%	34	56%	9	23%
					207 responses	
					181	87%
					33	16%
					22	11%
					205 responses	
					138	67%
					32	16%
					92	45%

Table 3 Co-occurrences of codes for secondary and university responses separated (top right part of matrix) and for all responses in total (bottom left part of matrix). Number of meaningful links between mathematical and phenomenological level wave nature and qualitative and phenomenological level uncertainty in parentheses

Respondent group	Phenomeno- logical level wave nature	Mathematical level wave nature		Qualitative level wave nature		Phenomenological level uncertainty		Mathematical level uncertainty		Qualitative level uncertainty	
		Secondary	University	Secondary	University	Secondary	University	Secondary	University	Secondary	University
Phenomenological level wave nature	-	17 (3)	8 (3)	9	6	0	0	2	0	0	0
Mathematical level wave nature	25 (6)	-		5	3	2	0	0	0	3	0
Qualitative level wave nature	15	8		-		5	1	1	0	6	0
Phenomenological level uncertainty	0	2	6	6		-		8	16	24 (7)	3 (0)
Mathematical level uncer- tainty	2	0		1		24		-		2	6
Qualitative level uncertainty	0	3		6		27 (7)		8		-	

Particles behave like waves in certain experiments, for example double-slit experiments. (Secondary student, 2016)

Particles can behave like waves, since the interference pattern can demonstrate the wave pattern in particles. A particle can also enter into areas (tunneling) where it really doesn't have enough energy to be. Additionally, we say that light can behave as waves, and sometimes as particles. (Secondary student, 2017)

Particles exhibit wave nature in certain conditions, as for example when particles are shot at sufficiently narrow slits. They will then start interfering with themselves, which makes them create interference patterns. (University student, 2018)

Fifteen responses coded with this theme were also coded as qualitative level explanations of particle wave nature (Table 3). That is, alongside the reference to experimental behavior, they included some qualitative statement about what particles are like on a microscopic level. Five of these descriptions appeared to mix classical and quantum ideas about particles:

Even though particles generally don't have the same properties as waves, they can in some cases behave like waves, since they can create interference patterns in experiments. The reason for this is uncertain. (Secondary student, 2016)

"Even though particles generally don't have the same properties as waves" is a claim about what particles are and what they are not. We interpret such responses as manifestations of ideas that particles *are* something other than waves, but that they sometimes change or behave differently. This can be interpreted as an attempt to reconcile a classical understanding of the scientific concept *particle* with quantum ideas. In the Copenhagen interpretation, quantum particles would be seen as inherently having both wave and particle properties but that the experiment determines which property is exhibited. Overall, 34 of the 181 responses coded as phenomenological wave nature descriptions were found to mix classical and quantum ideas, which is low considering previous research about students' reasoning in quantum physics (e.g., Ayene et al. (2011)).

3.1.2 Theme: Particles Are Described Mathematically by Wave Functions and Wavelengths

Less prominent (33 occurrences) but robustly present in the data was the theme that wave nature of particles has to do with a mathematical wave description of quantum systems. In these responses, students linked the concept particle wave nature to its mathematical level of explanation. Seventeen (28%) 2017 secondary responses to this question included that theme, whereas only 7 responses (7%) in 2016 did the same. 2017 responses often mentioned the wave function, whereas students in 2016 used the de Broglie relation, for example:

Particles can behave like waves. We have in quantum physics a basic equation of motion (wave equation). (Secondary student, 2017)

The wavelength of a particle equals the Planck constant divided by the momentum of the particle. We have to use waves to describe how particles move. (Secondary student, 2016)

Only 9 (23%) of the university students' descriptions were coded with this theme, even though they had all spent several months solving the Schrödinger equation to obtain wave functions describing quantum particle systems.

That the position of the particle can be described by a probability wave, which says something about the probability of the particle being in that position. (University student, 2018)

Although the wave function is only mentioned and not explicitly handled by the students in any of the versions, the 2017 module introduced a key sentence where the link between the concept *particle wave nature* and its phenomenological as well as mathematical levels of explanations are explicitly made. The key sentence is “We say that particles have wave nature when they behave like waves (for example, in interference), have wave properties (wavelength and frequency) and can be described mathematically by wave functions.”

3.1.3 Very Few Responses Described Meaningful Links Between the Two Wave Nature Themes

Among the 33 responses linking wave nature to its mathematical level explanation, 25 also described the phenomenological level (experimental behavior) in the same responses (see Table 3 for overview of co-occurrences of codes). For example, one of the University students wrote:

The wave nature of a particle can among other things be seen in that you can observe an interference pattern in a double-slit experiment. Among other things a particle can be described using probability waves (which describe their state). (University student, 2018)

However, the phenomenological and mathematical levels of explanation are not meaningfully linked in the quote above. There is no formulation of, for example, how an observed interference pattern is explained by a mathematical wave description of electrons. Only six responses across all respondent groups made such meaningful links in their descriptions (Table 3), three of which were given by university students. All six descriptions link the phenomenological to the mathematical level by expressing that particle behavior in experiments can only be explained by a mathematical wave model. Three examples are:

In some methods of measurement one will observe that particles have wave nature in the sense that we measure interference. This is a result of interference in the wave function. (University student, 2018)

That particles sometimes behave in ways that are most easily described by a wave model. (University student, 2018)

That several properties of particles can only be explained by wave theory. For example interference patterns. (Secondary student, 2016)

3.2 Students Descriptions of the Uncertainty Principle

3.2.1 Theme: Uncertainty as Inability to Measure Variables Precisely at the Same Time

Among responses to the question “What does Heisenberg’s uncertainty principle say?,” the most common theme across datasets with 138 occurrences (67% of responses) was *uncertainty as an inability to measure pairs of variables precisely at the same time*. This theme represents explanations linking the concept of uncertainty to a phenomenological level explanation. A wide range of verbs were used to describe the uncertainty principle and

the relationship between variables that it concerns. In addition to the most common *measure*, verbs such as *find*, *determine*, *observe*, *know*, *estimate*, *detect*, *compute*, *have*, *define*, *exist*, and *be* were all used to describe what cannot be done by someone *to* two variables at the same time (phenomenological level) and *by* two variables themselves at the same time (qualitative level). Responses describing the uncertainty principle as something someone cannot do *to* the variables—measure, determine, observe, detect, know etc.—were coded as belonging to the inability to measure theme. Some examples are:

Heisenberg's uncertainty principle says that it is impossible to know exactly both momentum and position or energy and time at the same time. That is, you cannot measure or determine both position and momentum at the same time, just as it is impossible to measure the exact energy of a particle over a short time span. (Secondary student, 2016)

You cannot sharply find momentum and position at the same time. (Secondary student, 2016)

Seventy-seven (74%) secondary responses in 2016 and 30 (49%) secondary responses in 2017 were coded with this theme (Table 2). Thirty-one (78%) of university students described uncertainty in terms of inability to measure, often in a more elaborate way than secondary responses:

It says that one cannot at the same time determine position and momentum with a sharp value. Nor can time and energy of a particle be determined at the same time. Consequence of this is that one cannot determine the future for a particle. The quantum mechanical world behaves differently than the mechanical one we are used to. (University student, 2018)

Only a handful of responses described the uncertainty principle in terms of measurement error or measurement disturbance.

3.2.2 Theme: Uncertainty as Innate Blurriness in Nature

The other robustly prevalent theme with 92 occurrences in students' descriptions of the uncertainty principle was uncertainty as an innate blurriness in nature. These responses linked the uncertainty principle as a scientific concept to a qualitative level explanation involving what quantum systems themselves *are* and what they are not. Forty-nine (47%) secondary responses in 2016 and 34 (56%) secondary responses in 2017 were coded with this theme. Among the university students, only 9 (23%) responses were coded as qualitative level uncertainty. Typically, responses in this theme described uncertainty as pairs of variables themselves not *being* sharp or well-defined, as opposed to someone not being able to get sharp or precise measurements of those variables at the same time. Examples of responses are:

Heisenberg's uncertainty principle says that there are pairs of variables in nature that cannot be sharp at the same time. Momentum and position is such a pair. (Secondary student, 2017)

It says that there is a fundamental non-sharpness in a particle's momentum and position. Greater precision in one lowers precision in the other. (University student, 2018)

It says that a particle's position cannot be sharp if the particle's momentum is also sharp. If one increases, the other decreases. This is not caused by measurement uncertainty, but by a fundamental law of nature. (Secondary student, 2016)

Whereas all descriptions coded with this theme referred to some fundamental blurriness in quantum particles themselves, some responses also included a qualitative level explanation of what this blurriness entails. One secondary response which did is:

Heisenberg's uncertainty principle is about that a particle cannot have a well-defined position at the same time as it has well-defined momentum. This is explained from the wave nature of a particle, since a wave with a well-defined wavelength and, therefore, momentum will have a non-sharp or unclear position. (Secondary student, 2017)

One university student used the meaning of measurements in quantum physics to distinguish between the state of the system itself (qualitative level explanation) and measurements on the system (phenomenological level).

Heisenberg's uncertainty principle says something about the fundamentally non-sharp state of a system. It is tempting to interpret [uncertainty principle formula] as if the position of the particle is unknown, but this is not entirely correct. The particle does not have a position until the wave function collapses. On the contrary, the interpretation of [standard deviation in x] resembles a wave packet. (University student, 2018)

The response is in line with a Copenhagen interpretation, which is largely what is presented in the university course.

3.2.3 Mathematical Level Descriptions

Other responses just wrote down the formula for the uncertainty principle for position and momentum, especially university students who replied writing by hand where the threshold for writing a formula is presumably lower than on a computer or tablet. Responses that only comprised a formula were not assigned either the measurement or the innate blurriness theme, but were coded with *mathematical level uncertainty*. That code was also used for descriptions that included the commutation relation version of the uncertainty principle, as a formula or in words. One such example is:

[Uncertainty principle standard deviation formula]. That there are limits to how precisely we can know some things, regardless of how precise our instruments of measurement get. [General uncertainty principle commutation formula] (University student, 2018)

3.2.4 Philosophical Reflections on Uncertainty

The uncertainty question also asked what the uncertainty principle means for what we can know about nature. The idea that uncertainty means we cannot get complete information about nature was found across all datasets, in 59 responses in total (31 secondary 2016, 22 secondary 2017, and 6 university). For example:

This means that if we know for example the momentum of a particle, we can never with complete certainty know the position of the particle. Therefore, there is always some information we cannot know for sure. (Secondary student, 2017)

That means that we cannot measure one variable precisely if the other one is measured precisely. Therefore, we will never know everything about a state/particle. (University student, 2018)

These kinds of responses can be interpreted as connecting the phenomenological level explanation that uncertainty means we cannot measure pairs of variables precisely to a specific qualitative level implication about the ontology of quantum particles, i.e., that there is information about the particles' momentum and position, but that it is unavailable to us. However, the part of the question that was intended to get at students' epistemological reflections ("... and what does [the uncertainty principle] mean for what we can know about nature?") is arguably quite vague and difficult to respond to, as is illustrated by the fact that 106 (84 secondary and 22 university) responses to the uncertainty question failed to respond to that part of the question. It is also possible that the question itself is leading, suggesting the inappropriate interpretations of uncertainty that are seen in the data. Inferences based on this result should therefore be made with caution.

3.2.5 Qualitative Level Explanations of Particle Wave Nature as a Meaningful Link Between the Two Uncertainty Themes

Although 27 responses included both uncertainty as innate blurriness (qualitative level explanation) and uncertainty as inability to measure pairs of variables (phenomenological level explanation), only seven were found to meaningfully link the two levels (Table 3). All such responses came from secondary students, and they mostly made the connection using explanations of particle wave nature on a qualitative level (six co-occurrences):

[Heisenberg] meant that nature is blurry, with that he meant that there were pairs in nature that could not be determined sharply at the same time. One of these pairs are position and momentum. This comes from the fact that particles have wave properties and that waves don't have a "given" position. (Secondary student, 2016)

A more elaborate description along the same lines is:

Heisenberg's uncertainty principle says that momentum and position cannot be sharp at the same time. The explanation is that if you picture a matter wave with lots of peaks and troughs, you can determine the wavelength, but not the exact position of the wave. If the matter wave has just one peak and trough, you can determine the position of the wave more precisely, but not the wavelength. Since there is a relation between momentum and wavelength, this means you cannot determine momentum and position of a particle precisely. (Secondary student, 2017)

This way of explaining the uncertainty principle as a consequence of particle wave nature and a wave being by definition spread out in space was used in the *Particles as waves* learning resource. Also included in the learning resource was an explicit definition of a classical particle as a clearly localized object.

4 Discussion and Implications

We have presented secondary and university students' descriptions of the concepts particle wave nature and the uncertainty principle. Using pedagogical link-making as a lens, we have investigated which levels of explanations were used by the students, and if and how

these explanations were linked. Below, we discuss these results and their implications for physics education, focusing on the secondary level and the sociocultural, and history, philosophical, and NOS perspectives embedded in the learning resources in question.

4.1 Linking Levels of Explanations of Particle Wave Nature Meaningfully

By far, the most used level of explanations of particle wave nature was phenomenological, as secondary and university students alike described wave nature in terms of wave behavior in experiments. This was not surprising. Seeing an interference pattern arise from electrons being shot at a double slit is arguably as tangible as the microscopic quantum world gets for students. The counter-intuitive quantum weirdness of the experiment is likely to trigger situational interest or build on existing individual interest (Renninger & Hidi, 2016). In this way, the experiment provides pedagogical link-making by encouraging emotional engagement with the subject matter (Scott et al., 2011). The experiment is not too advanced or expensive and thus accessible to many secondary physics classrooms. Our results demonstrate that the phenomenological level of explanation of particle wave nature, for example, an electron interference experiment, is a fruitful starting point for teaching about this concept in secondary as well as university education. However, we argue in line with Scott et al. (2011) that the goal of science education should be deep learning, where student understanding of a scientific concept includes links between several levels of explanation and forms of representation. The *Particles as waves* learning resource placed by far the most emphasis on wave behavior in experiments, but it also mentioned the mathematical wave descriptions (mathematical level), and had students discuss what electron interference can and cannot tell us about the nature of electrons on an ontological, qualitative level. Still, not many secondary students included mathematical or qualitative level explanations in their descriptions of particle wave nature, and among those who did, only a handful made meaningful links between the different levels of explanations, demonstrating the challenges students have with such connections (Bouchée et al., 2021). However, those few meaningful links that were made demonstrate a possible approach to pedagogical link-making concerning particle wave nature, namely connecting the phenomenological and mathematical levels of explanation by formulating that the mathematical wave description of particles is there *because* it is able to explain what is observed in experiments. It sounds simple and obvious, but the fact that so few made that meaningful connection suggests that it was not sufficiently supported by the teachers or learning resources. One concrete example of this is the following key sentence included in the learning resource: “We say that particles have wave nature when they behave like waves (for example, in interference), have wave properties (wavelength and frequency) and can be described mathematically by wave functions.” This key sentence should be phrased differently, to include that the mathematical wave descriptions explain the experimental behavior, and that the link between them is what makes it a scientific model. Promoting such a link between observations in experiments and mathematical models brings with it an opportunity to emphasize NOS further, connecting quantum physics content knowledge to procedural and epistemic knowledge in physics more broadly as should be a goal for science education (Kind & Osborne, 2017). Arguably, secondary students do not need to know *how* electrons are described by wave functions, but they can still learn *why* this is the case, because it explains observations. This can then be connected to more general understanding of NOS, for example, through discussing the history of quantum physics as suggested by Bouchée et al. (2021).

Our data are not of a type that allows us to conclude whether secondary students' reasoning about wave-particle duality is poorer, as good, or better than that of university students. What we can say is that a full university semester of formalism-based quantum physics does not guarantee descriptions of particle wave nature that more often include different levels of explanations that are meaningfully linked. Our participating university students were taught the formalistic tools that explain *how* wave descriptions work as well as why they explain phenomena like electron interference and tunneling. They spent a few weeks early in the semester on wave-particle duality, including electron interference experiments, Compton scattering, and the photoelectric effect, and then spent months solving the Schrödinger equation for different potentials to find the wave function for electrons. Still, only 9 of our 40 university student descriptions included mathematical level explanations of particle wave nature. And none of those included meaningful links between the experimental behavior and the mathematical wave descriptions. This result signals a lack of support for pedagogical link-making for particle wave nature in the course, possibly connected to an instrumental approach to quantum mechanics that is often seen at university level, where mathematical problem solving dominates without necessarily being connected to conceptual understanding (Bouchée et al., 2021; Fraser et al., 2014; Johansson et al., 2018).

4.2 Using Particle Wave Nature to Link Levels of Explanations of Uncertainty to Each Other

When it comes to the uncertainty principle, the phenomenological level of explanation was represented in the theme *uncertainty as inability to measure pairs of variables precisely at the same time*. A majority of student descriptions included this theme, but among secondary students in particular, the qualitative level in the theme *uncertainty as innate blurriness in nature* was used in almost half of the responses. The question posed to students ("What does Heisenberg's uncertainty principle say, and what does it imply about how much we can know about nature?") to some extent asks for a phenomenological level explanation, since it refers to how much we can know about nature. This may contribute to explaining why that level is present to such an extent in secondary school even though the *Particles as waves* learning resource emphasized the qualitative level explanation. It may also reflect that a phenomenological explanation is less abstract, also for uncertainty, and/or that the textbooks used in Norwegian secondary schools in 2016 and 2017 both use a phenomenological explanation. Also for the uncertainty principle, we argue that a good conceptual understanding of the concept includes meaningful links between different levels of explanations. For example, such an explanation could formulate how uncertainty as an innate blurriness in nature would manifest itself as an inability to measure certain pairs of variables precisely at the same time. Very few secondary or university students included such a meaningful link in their descriptions, as was also the case for descriptions of particle wave nature. Although we need more comprehensive data to robustly conclude that the students' learning of these concepts has been fragmented, these results suggest that there is great potential for better pedagogical link-making and deeper learning. As we found for particle wave nature, the few instances where students did provide meaningful links between different levels of explanations offer insight into how better pedagogical link-making of the uncertainty principle can be pursued. These students explained how uncertainty as innate blurriness leads to the inability to measure certain pairs of variables precisely by using particle wave nature as the meaningful link. They in essence used the same argument as

was presented in the *Particles as waves* learning resource, relying on a qualitative level explanation of particle wave nature: when particles are represented by waves, we can see that a wave has a less well-defined position the more spread out the wave is. Since we need a certain spread to have a regular, well-defined wavelength, position and wavelength cannot be precise at the same time. From that, the uncertainty principle can be deduced using the de Broglie relation, which states that a particle's momentum is inversely proportional to its wavelength. The students then have to accept the de Broglie relation, of course, which can appear very mysterious. It can, however, be made more accessible by using the electron interference experiment again, drawing on students' emotional engagement and familiarity with this phenomenological level explanation of particle wave nature. Students can do their own experiment (or a simulation), where they can control the momentum that is given to the electrons, calculate their wavelength from measurements of the interference pattern, and find the de Broglie relation.

One of the reasons that the learning resources focused on uncertainty as innate blurriness, explained using particle wave nature on a qualitative level, was to limit the possibility of students erroneously linking uncertainty to measurement error or disturbance, as is common at secondary level (Krijtenburg-Lewerissa et al., 2017). By not using measurements in the explanation of the uncertainty principle, the approach also avoids the challenge that students often struggle to understand the role of measurement in quantum physics (Huseby & Bungum, 2019; Zhu & Singh, 2012a). An understanding of uncertainty as having inevitable consequences for measurements of certain pairs of variables, but not being *caused* by measurement problems, can be deduced from particle wave nature qualitatively, as seen above, or using quantum mechanics formalism. The latter requires an understanding of the wave function, how it describes the probability of measurements, and of how it leads to the relationship between the spread in measurements for certain pairs of variables to be as described by the uncertainty principle. The university students in our sample had worked with that formalism and with the uncertainty principle in both formalistic and qualitative ways in the course, but none of them did use the formalism to meaningfully link uncertainty as innate blurriness to its consequences for measurement. A more in-depth study of student reasoning is needed to investigate university student understanding of uncertainty, but our results and Bouchée et al. (2021) suggest that links to the qualitative level explanation could be strengthened for students to not only master the formalism, but be able to connect it to conceptual understanding.

Another advantage in deducing the uncertainty principle from particle wave nature is that it creates a meaningful link between those concepts, as opposed to them just being two separate weird things in quantum physics. However, the approach also introduces questions of interpretations of quantum physics as it requires some ontological ideas about quantum particles. To a certain extent, all physics explanations on the microscopic level are models that to larger or lesser extent reflect reality. Physicists still argue about what quantum physics should be interpreted to mean for reality, and physics educators discuss how and when students should learn about it (Bouchée et al., 2021; Bunge, 2003, 2012). Cheong and Song (2014) argued that quantum physics education should not introduce interpretations of quantum physics until after students are familiar with the formalism. Do we use particle wave nature to help students understand uncertainty as innate blurriness, and run the risk of imposing an interpretation of quantum physics in which electrons are seen as physical waves? Should we do it, but make the interpretational issues explicit and discuss them with students (Bouchée et al., 2021)? The *Quantum physics* learning resources actively used this approach and prompted students to discuss related philosophical aspects. With regard to the uncertainty principle, 59 (53 secondary and 6 university) responses in

our study described it as leading to incomplete knowledge about nature rather than as that information not existing in itself, corroborating other studies (Baily & Finkelstein, 2010). As stated in the Results section, the question given to students was vague and potentially leading, and rather than draw major conclusions about students' epistemological reflections, we will use the result to look carefully at how epistemological reflections can better be prompted in upcoming revisions of the learning resources, as part of the DBR process. However, previous research from the ReleQuant project (Henriksen et al., 2018) found that students were able to reflect on philosophical aspects related to the wave-particle duality of light, but that their descriptions rarely moved beyond naïve realistic interpretations. Our results might suggest that one possible contributor to this is the lack of mathematical tools and understanding of measurement in quantum physics, which was seen in another study from the project (Huseby & Bungum, 2019), to meaningfully connect the innate blurriness of quantum systems to the inability to measure pairs of variables within those systems. We argue, however, that the potential benefits of qualitative level explanations of particle wave nature for conceptual understanding of wave-particle duality as well as uncertainty outweigh the risks of introducing interpretational issues too soon, in particular in secondary school, where it is not possible to rely on formalistic, mathematical level explanations as an alternative. Moreover, although it is probably unrealistic for secondary students to attain full, comprehensive understanding of wave-particle duality within the limited time frame available, tentative reasoning which meaningfully links some, but not all levels of explanation, can be a valuable first step towards deeper understanding at a later stage (Bouchée et al., 2021; Hoehn et al., 2019).

4.3 Implications in the Light of Secondary Physics Curricular Development

Besides the lack of sufficient mathematics available in secondary school, a challenge for conceptual understanding in quantum physics is little time devoted to this topic in physics curricula (Stadermann et al., 2019). However, even after a whole semester of solving the Schrödinger equation, few of our university students connected particle wave nature to its mathematical wave description. This suggests that macro-level continuity has not been sufficiently promoted, as was found for university level science teaching by Quadros et al. (2018). By explicitly and repeatedly linking the solving of a wave equation to the phenomenological level of wave-particle duality and the historical development of quantum physics (Bouchée et al., 2021), the course could better support continuity by, literally, developing the scientific story, as recommended by Scott et al. (2011). In secondary school, it may be possible to compensate for little time to promote continuity *within* quantum physics, by connecting central concepts in quantum physics to the broader physics curriculum.

A new Norwegian physics curriculum was released in 2021, and together with new evidence from this study, this has prompted a revision of the learning resources used in this work. In line with other recent curriculum reforms, the new Norwegian curriculum focuses on identifying a set of core ideas in subjects and developing curricula to promote deep learning of these ideas as opposed to fragmented learning of a range of disconnected topics (NOU Harlen et al., 2010; MOE, 2015). This connects well to Scott et al. (2011) and Bouchée et al. (2021) suggestions to promote understanding by linking concepts, levels of explanations, and forms of representation meaningfully. Important questions then are: What are the core ideas in physics? What are the most central ideas in quantum physics, and how do they relate to the core ideas in physics? The new Norwegian physics curriculum presents the following as four core ideas in the

subject: *practices and ways of thinking in physics, energy and energy transfer, forces and fields, and matter, time, and space* (our translation) (NDET, 2021). It includes one standard concerning quantum physics: “Explain what separates quantum objects from classical objects, and describe situations where quantum effects are observed.” Wave-particle duality, including particle wave nature and the uncertainty principle, is at the core of what separates quantum objects from classical objects, and was identified as one of the essential topics in quantum physics by experts in a Delphi study by Krijtenburg-Lewerissa et al. (2019). Wave-particle duality also connects to several of the core ideas in the new Norwegian physics curriculum. Most notably, the core ideas *matter, time and space* and *practices and ways of thinking in physics* offer possibilities for promoting pedagogical link-making for better understanding of wave-particle duality specifically, and for other physics concepts more generally. For example, the learning resources should increase their emphasis on how both concepts and mathematical structures in quantum physics are *models* that allow us to describe and predict quantum *object*, rather than *particle* (Bouchée et al., 2021; Bunge, 2003), behavior. As with all scientific models, quantum physics models have strengths and weaknesses. These models have been, and are being, developed by physicists, and the historical approach in these learning resources illustrates these as practices and ways of thinking in physics. Treating the wave and particle descriptions of electrons as models more explicitly may help students create meaningful links rather than conflicting paradoxes, and it can strengthen understanding of models in physics more broadly.

In contrast to the previous curriculum, the new one does not include philosophical reflections or explicitly state that quantum physics should be treated qualitatively. This will most likely lead to fewer activities on philosophical reflections in the learning resources, which as a whole comprises 6 h (45 min are dedicated to particle wave nature). Together with the overall reduction of standards in the curriculum, this opens up for a more in-depth treatment of the quantum/classical distinction as described in the new standard. The upcoming revision of the learning resources will aim to exploit this to better support pedagogical link-making between the different levels of explanations of the central quantum concepts and observations. The new curriculum even opens up for a quantitative treatment, which could allow for including some mathematical descriptions of the role of measurement in quantum physics. However, the advanced nature of the mathematics involved does still, we believe, prohibit moving much beyond the probability interpretation of the wave function.

4.4 Limitations

Our findings present students’ descriptions of particle wave nature and uncertainty given at one point in time. Their descriptions may to greater or lesser extent reflect their actual ideas about these topics, which were most likely more complex and elaborate than what is expressed in written answers. A longitudinal study following students and their expressed ideas over time, and using interviews as well as coursework or tests, would provide a richer picture of students’ ideas and how understanding can be best promoted through teaching. Combining such investigations with research on how and to what extent teachers and learning resources provide pedagogical link-making would be particularly fruitful in future studies.

5 Conclusion

The present study demonstrates how students' descriptions of quantum physics concepts can be understood as belonging to different levels of explanations, which are or are not meaningfully linked to form deeper understandings of the concepts. As part of our DBR project, the results on students' descriptions of wave-particle duality and uncertainty suggest how our short, qualitative, history and NOS-focused learning resources can be improved. Although a comprehensive understanding of these concepts is unrealistic at secondary level, the study informs physics education on how teaching, through pedagogical link-making (Scott et al., 2011), can support students in overcoming their conceptual challenges (Bouchée et al., 2021) and connecting the first building blocks of engaging insights in quantum physics and its powerful implications for scientific and technological development.

Acknowledgements We would like to thank all participating teachers and students for their contribution and the Norwegian Centre for Science Education for hosting the learning resource *Quantum physics* on their digital science learning resource site www.viten.no. We are especially grateful to Øystein Sørborg at the Norwegian Centre for Science Education, for his contributions to the work both as a science educator and as a digital learning environment developer. This work has been financially supported by the Norwegian Research Council and the Olaf Thon trust.

Funding Open access funding provided by University of Oslo (incl Oslo University Hospital). This work has been financially supported by the Norwegian Research Council and the Olaf Thon trust.

Availability of Data and Material Not available due to privacy issues.

Code Availability Not applicable.

Declarations

Ethics Approval The study and its handling of data have been approved by the Norwegian Centre for Research Data.

Consent to Participate All student participants have given informed consent to participate in the study in accordance with guidelines from the Norwegian Centre for Research Data.

Consent for Publication Not applicable.

Conflict of Interest The authors declare that they have no conflict of interest.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Anderson, T., & Shattuck, J. (2012). Design-based research: A decade of progress in education research? *Educational Researcher*, 41(1), 16–25. <https://doi.org/10.3102/0013189X11428813>

- Angell, C., Guttersrud, Ø., Henriksen, E. K., & Isnes, A. (2004). Physics: Frightful, but fun. Pupils' and teachers' views of physics and physics teaching. *Science Education*, 88(5), 683–706. <https://doi.org/10.1002/sce.10141>
- Ayene, M., Kriek, J., & Damtie, B. (2011). Wave-particle duality and uncertainty principle: Phenomenographic categories of description of tertiary physics students' depictions. *Physical Review Special Topics - Physics Education Research*, 7(2), 020113. <https://doi.org/10.1103/PhysRevSTPER.7.020113>
- Baily, C., & Finkelstein, N. D. (2010). Refined characterization of student perspectives on quantum physics. *Physical Review Special Topics - Physics Education Research*, 6(2), 020113. <https://doi.org/10.1103/PhysRevSTPER.6.020113>
- Bohm, D. (1989). *Quantum Theory*. Dover Publications Inc.
- Bouchée, T., de Putter-Smits, L., Thurlings, M., & Pepin, B. (2021). Towards a better understanding of conceptual difficulties in introductory quantum physics courses. *Studies in Science Education*, 1–20. <https://doi.org/10.1080/03057267.2021.1963579>
- Braun, V., & Clarke, V. (2008). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Brookes, D. T., & Etkina, E. (2007). Using conceptual metaphor and functional grammar to explore how language used in physics affects student learning. *Physical Review Special Topics - Physics Education Research*, 3(1), 010105. <https://doi.org/10.1103/PhysRevSTPER.3.010105>
- Brunner, N., Cavalcanti, D., Pironio, S., Scarani, V., & Wehner, S. (2014). Bell nonlocality. *Reviews of Modern Physics*, 86(2), 419–478. <https://doi.org/10.1103/RevModPhys.86.419>
- Bunge, M. (2003). Twenty-five centuries of quantum physics: From Pythagoras to us, and from subjectivism to realism. *Science & Education*, 12, 445–466. <https://doi.org/10.1023/A:1025336332476>
- Bunge, M. (2012). Does quantum physics refute realism, materialism and determinism? *Science & Education*, 21, 1601–1610. https://doi.org/10.1007/978-94-007-4408-0_15
- Bungum, B., Bøe, M. V., & Henriksen, E. K. (2018). Quantum talk: How small-group discussions may enhance students' understanding in quantum physics. *Science Education*, 102(4), 856–877. <https://doi.org/10.1002/sce.21447>
- Cheong, Y. W., & Song, J. (2014). Different levels of the meaning of wave-particle duality and a suspensive perspective on the interpretation of quantum theory. *Science & Education*, 23, 1011–1030. <https://doi.org/10.1007/s11191-013-9633-2>
- Durakiewicz, T., & Greene, L. (2018). Commentary: Enabling a quantum leap. *Physics Today*, 71(9), 10–11. <https://doi.org/10.1063/PT.3.4008>
- Einstein, A., Podolsky, B., & Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47(10), 777–780. <https://doi.org/10.1103/PhysRev.47.777>
- Escalada, L. T., Rebello, N. S., & Zollman, D. A. (2004). Student explorations of quantum effects in LEDs and luminescent devices. *The Physics Teacher*, 42(3), 173–179. <https://doi.org/10.1119/1.1664385>
- Faye, J. (2019). Copenhagen interpretation of quantum mechanics. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Winter 2019 Edition).
- Feynman, R. P., Leighton, R. B., & Sands, M. (2003). *Feynman Lectures on Physics Vol. 3: Basic Books*.
- Fraser, J. M., Timan, A. L., Miller, K., Dowd, J. E., Tucker, L., & Mazur, E. (2014). Teaching and physics education research: Bridging the gap. *Reports on Progress in Physics*, 77(3), 032401. <https://doi.org/10.1088/0034-4885/77/3/032401>
- Goldstein, S. (2017). Bohmian Mechanics. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Summer 2017 Edition).
- Greca, I. M., & Freire, O. (2003). Does an emphasis on the concept of quantum states enhance students' understanding of quantum mechanics? *Science & Education*, 12(5), 541–557. <https://doi.org/10.1023/A:1025385609694>
- Harlen, W., Bell, D., Devés, R., Dyasi, H., Garza, G. F. d. l., Léna, P., et al. (2010). *Principles and big ideas of science education*. In W. Harlen (Ed.). Association for Science Education.
- Henriksen, E. K., Bungum, B., Angell, C., Tellefsen, C. W., Frågât, T., & Bøe, M. V. (2014). Relativity, quantum physics and philosophy in the upper secondary curriculum: challenges, opportunities and proposed approaches. *Physics Education*, 49(6), 678–684. <https://doi.org/10.1088/0031-9120/49/6/678>
- Henriksen, E. K., Angell, C., Vistnes, A. I., & Bungum, B. (2018). What is light? *Science & Education*, 27(1), 81–111. <https://doi.org/10.1007/s11191-018-9963-1>
- Hoehn, J. R., Gifford, J. D., & Finkelstein, N. D. (2019). Investigating the dynamics of ontological reasoning across contexts in quantum physics. *Physical Review Physics Education Research*, 15(1), 010124. <https://doi.org/10.1103/PhysRevPhysEducRes.15.010124>
- Hoekzema, D., Berg, Evd, Schooten, G., & Dijk, L. v. (2007). The particle/wave-in-a-box model in Dutch secondary schools. *Physics Education*, 42(4), 391–398. <https://doi.org/10.1088/0031-9120/42/4/009>

- Hornberger, K., Gerlich, S., Haslinger, P., Nimmrichter, S., & Arndt, M. (2012). Colloquium: Quantum interference of clusters and molecules. *Reviews of Modern Physics*, 84(1), 157–173. <https://doi.org/10.1103/RevModPhys.84.157>
- Huseby, A., & Bungum, B. (2019). Observation in quantum physics: Challenges for upper secondary physics students in discussing electrons as waves. *Physics Education*, 54(6), 065002. <https://doi.org/10.1088/1361-6552/ab3694>
- Ireson, G. (1999). A multivariate analysis of undergraduate physics students' conceptions of quantum phenomena. *European Journal of Physics*, 20(3), 193–199. <https://doi.org/10.1088/0143-0807/20/3/309>
- Ireson, G. (2000). The quantum understanding of pre-university physics students. *Physics Education*, 35(1), 15–21. <https://doi.org/10.1088/0031-9120/35/1/302>
- Johansson, A., Andersson, S., Salminen-Karlsson, M., & Elmgren, M. (2018). “Shut up and calculate”: The available discursive positions in quantum physics courses. *Cultural Studies of Science Education*, 13(1), 205–226. <https://doi.org/10.1007/s11422-016-9742-8>
- Kaur, T., Blair, D., Stannard, W., Treagust, D., Venville, G., Zadnik, M., et al. (2020). Determining the intelligibility of Einsteinian concepts with Middle School Students. *Research in Science Education*, 50(6), 2505–2532. <https://doi.org/10.1007/s11165-018-9791-y>
- Kind, P. M., & Osborne, J. (2017). Styles of scientific reasoning: A cultural rationale for science education? *Science Education*, 101(1), 8–31. <https://doi.org/10.1002/sce.21251>
- Kohnle, A., Bozhinova, I., Browne, D., Everitt, M., Fomins, A., Kok, P., et al. (2013). A new introductory quantum mechanics curriculum. *European Journal of Physics*, 35(1), 015001. <https://doi.org/10.1088/0143-0807/35/1/015001>
- Krijtenburg-Lewerissa, K., Pol, H. J., Brinkman, A., & Joolingen, W. R. (2017). Insights into teaching quantum mechanics in secondary and lower undergraduate education. *Physical Review Physics Education Research*, 13, 010109. <https://doi.org/10.1103/PhysRevPhysEducRes.13.010109>
- Krijtenburg-Lewerissa, K., Pol, H. J., Brinkman, A., & van Joolingen, W. R. (2019). Key topics for quantum mechanics at secondary schools: A Delphi study into expert opinions. *International Journal of Science Education*, 41(3), 349–366. <https://doi.org/10.1080/09500693.2018.1550273>
- Lautesse, P., Vila Valls, A., Ferlin, F., Héraud, J.-L., & Chabot, H. (2015). Teaching quantum physics in upper secondary school in France. *Science & Education*, 24(7), 937–955. <https://doi.org/10.1007/s11191-015-9755-9>
- Mannila, K., Koponen, I. T., & Niskanen, J. A. (2001). Building a picture of students' conceptions of wave- and particle-like properties of quantum entities. *European Journal of Physics*, 23(1), 45–53. <https://doi.org/10.1088/0143-0807/23/1/307>
- Mashhadi, A., & Woolnough, B. (1999). Insights into students' understanding of quantum physics: Visualizing quantum entities. *European Journal of Physics*, 20(6), 511–516. <https://doi.org/10.1088/0143-0807/20/6/317>
- McKagan, S. B., Perkins, K. K., & Wieman, C. E. (2008). Deeper look at student learning of quantum mechanics: The case of tunneling. *Physical Review Special Topics - Physics Education Research*, 4(2), 020103. <https://doi.org/10.1103/PhysRevSTPER.4.020103>
- Michellini, M., Ragazzon, R., Santi, L., & Stefanel, A. (2000). Proposal for quantum physics in secondary school. *Physics Education*, 35(6), 406–410. <https://doi.org/10.1088/0031-9120/35/6/305>
- MOE. (2015). NOU 2015:8. Fremtidens skole. Fornyelse av fag og kompetanser. [School of the future. Renewal of subjects and competencies]. Norwegian Ministry of Education.
- Mortimer, E., & Scott, P. (2003). *Meaning-making in secondary science classrooms*. Open University Press.
- Mudadigwa, B., & Msimanga, A. (2019). Exploring teacher pedagogical practices that help learners make connections during the teaching of reactions in aqueous solutions at senior secondary level. *African Journal of Research in Mathematics, Science and Technology Education*, 23(2), 332–343. <https://doi.org/10.1080/18117295.2019.1688476>
- Müller, R., & Wiesner, H. (2002). Teaching quantum mechanics on an introductory level. *American Journal of Physics*, 70(3), 200–209. <https://doi.org/10.1119/1.1435346>
- NDET. (2006). Physics - programme subject in programmes for specialization in general studies [curriculum document]. In N. D. f. E. a. Training (Ed.). Norwegian Directorate for Education and Training.
- NDET. (2021). Læreplan i fysikk [Physics curriculum]. In N. D. f. E. a. Training (Ed.). Norwegian Directorate for Education and Training.
- Olsen, R. V. (2002). Introducing quantum mechanics in the upper secondary school: A study in Norway. *International Journal of Science Education*, 24(6), 565–574. <https://doi.org/10.1080/09500690110073982>
- Özcan, Ö. (2011). Pre-service physics teachers' comprehension of wave function and operator concepts in quantum mechanics. *International Journal of Physical Sciences*, 6. <https://doi.org/10.5897/AJBM11.534>

- Quadros, A. L. D., Silva, A. S. F., & Mortimer, E. F. (2018). Pedagogical link-making in higher education science classrooms. *Química Nova*, 41(2), 227–235.
- Renninger, K., & Hidi, S. (2016). *The power of interest for motivation and engagement*. New York, N.Y.: Routledge.
- Renstrøm, R. (2011). *Kvantefysikkens utvikling: I fysikklærebøker, vitenskapshistorien og undervisning. The development of quantum physics: In physics textbooks, history of science, and teaching*. PhD thesis, University of Oslo, Oslo.
- Ræder, H., Angell, C., & Henriksen, E. K. (2017). *Uskarp forståelse: Analyse av elevsvar etter bruk av læringsressursen "Kvantefysikk" knyttet til partiklers bølgeegenskaper og uskarphetsrelasjonene. [Non-sharp understanding: Analysis of student responses after using the learning resource "Quantum physics" related to the wave nature of particles and the uncertainty principle]*. Paper presented at the Nordic Research Symposium on Science Education, Trondheim, Norway, 7–9 June 2017
- Scott, P., Mortimer, E., & Ametller, J. (2011). Pedagogical link-making: A fundamental aspect of teaching and learning scientific conceptual knowledge. *Studies in Science Education*, 47(1), 3–36. <https://doi.org/10.1080/03057267.2011.549619>
- Shabajee, P., & Postlethwaite, K. (2004). What happened to modern physics? *School Science Review*, 81(297), 51–56. arXiv:physics/0401016
- Stadermann, H. K. E., van den Berg, E., & Goedhart, M. J. (2019). Analysis of secondary school quantum physics curricula of 15 different countries: Different perspectives on a challenging topic. *Physical Review Physics Education Research*, 15(1), 010130. <https://doi.org/10.1103/PhysRevPhysEducRes.15.010130>
- Touzalin, A. d., Marcus, C., Heijman, F., Cirac, I., Murray, R., & Calarco, T. (2016). Quantum Manifesto. A New Era of Technology. QUROPE - Quantum Information Processing and Communication in Europe.
- Vaidman, L. (2018). Many-Worlds Interpretation of Quantum Mechanics. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy (Fall 2018 Edition)*.
- Vygotsky, L. (1978). Mind in society: The development of higher psychological processes. In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.). Harvard University Press.
- Wood, A. K., Galloway, R. K., Hardy, J., & Sinclair, C. M. (2014). Analyzing learning during Peer Instruction dialogues: A resource activation framework. *Physical Review Special Topics - Physics Education Research*, 10. <https://doi.org/10.1103/PhysRevSTPER.10.020107>
- Zhu, G., & Singh, C. (2012a). Improving students' understanding of quantum measurement. I. Investigation of difficulties. *Physical Review Special Topics - Physics Education Research*, 8(1), 010117. <https://doi.org/10.1103/PhysRevSTPER.8.010117>
- Zhu, G., & Singh, C. (2012b). Improving students' understanding of quantum measurement. II. Development of research-based learning tools. *Physical Review Special Topics - Physics Education Research*, 8(1), 010118. <https://doi.org/10.1103/PhysRevSTPER.8.010118>
- Zollman, D. A. NARST 1999 (1999). Research on teaching and learning quantum mechanics. In D. A. Zollman (Ed.), National Association for Research in Science Teaching.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.