

Models as Relational Categories

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Abstract Model-based learning (MBL) has an established position within science education. It has been found to enhance conceptual understanding and provide a way for engaging students in authentic scientific activity. Despite ample research, few studies have examined the cognitive processes regarding learning scientific concepts within MBL. On the other hand, recent research within cognitive science has examined the learning of so-called relational categories. Relational categories are categories whose membership is determined on the basis of the common relational structure. In this theoretical paper, I argue that viewing models as relational categories provides a well-motivated cognitive basis for MBL. I discuss the different roles of models and modeling within MBL (using ready-made models, constructive modeling, and generative modeling) and discern the related cognitive aspects brought forward by the reinterpretation of models as relational categories. I will argue that relational knowledge is vital in learning novel models and in the transfer of learning. Moreover, relational knowledge underlies the coherent, hierarchical knowledge of experts. Lastly, I will examine how the format of external representations may affect the learning of models and the relevant relations. The nature of the learning mechanisms underlying students' mental representations of models is an interesting open question to be examined. Furthermore, the ways in which the expert-like knowledge develops and how to best support it is in need of more research. The discussion and conceptualization of models as relational categories allows discerning students' mental representations of models in terms of evolving relational structures in greater detail than previously done.

Keywords Models · Modelling · Model-based learning · Concept learning · Cognitive processes · Relational categories · Inquiry

1 Introduction

Model-based learning (henceforth, MBL) has gained an established status in science education during the last three decades. One of the most important goals of MBL is enhancing students'

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conceptual understanding and facilitating conceptual change (Campbell et al. 2015; Coll and Lajium 2011). MBL also helps to engage students in epistemically “authentic” scientific activities (Gilbert 2004; Koponen 2007; Lehrer and Schauble 2015). Indeed, models and modeling capture much of what is essential in constructing, communicating, and accepting scientific knowledge (Frigg and Hartmann 2017; Giere 1988; Gilbert 2004; Koponen 2007).

Within science education and philosophy of science, models are often viewed as representations of some aspects of the world (Frigg and Hartmann 2017; Oh and Oh 2011). In addition, models can be seen as representation of some aspects of theory or that theories are applied through models¹ (Bailer-Jones 2009, pp. 126–152, 209; Frigg and Hartmann 2017). In these respects, models have an important role in exploring theories and constructing the meaning of concepts of the theory (cf. Nersessian 2008, p. 46).

Regarding learning with models, two broad approaches towards using models can be distinguished: using ready-made models (e.g., analogies or visualizations) to introduce more complex concepts and asking students to construct models (Amin et al. 2014; Gilbert and Justi 2016, p. 29–30). While in these approaches models are typically taken to be representations of some aspects of the world, they differ with respect to the relation of models and theory and whether the approach emphasizes the end product or the process of modeling (cf. Gilbert and Justi 2016, p. 29–30; Koponen 2007). As a result, the cognitive processes related to learning might differ.

From the point of view of learning and instruction, the key question is why models are central for learning scientific concepts. Answers arguing for the centrality of models in science and/or equating models with scientific knowledge are inadequate since they identify external reasons for the importance of models instead of cognitive reasons based on the psychology of learning. Some studies have examined the cognitive processes² underlying MBL (see, Clement 1989; Perkins and Grotzer 2005; Kokkonen and Mäntylä 2017; Nersessian 1995), but only a few have attempted to develop a cognitively justified approach to understanding concept learning within MBL, despite concept learning being among the most important aims of MBL (Louca and Zacharia 2012). Moreover, constructing, using, and learning from models is affected by the modeling tool (and/or representational format) used. In other words, different ways of representing information may affect what is learned from the representation, but this aspect is not particularly well understood (Louca and Zacharia 2012).

Interestingly, recent research on concept learning within cognitive science has geared towards learning of so-called relational categories, which are categories whose membership is determined on the basis of a shared relational structure (Goldwater and Schalk 2016). Goldwater and Schalk (2016) have proposed that this might provide an interesting interdisciplinary link between science education research (especially conceptual change) and cognitive science. They proposed that the fundamental link between them is that relational concepts are central in learning science, as reasoning about such mundane topics as density, for example, requires quite sophisticated relational knowledge let alone more complex concepts found in physics.

In this theoretical paper, I examine the role of models in concept learning from the viewpoint of relational knowledge and argue that models can be conceptualized as relational categories. By doing so, I also aim to shed light on the nature of knowledge and the cognitive processes involved in modeling and learning with models. I will mainly focus on physics,

¹ That is, theories are not about concrete phenomena (i.e., they do not represent) but models are models of concrete phenomena and thereby facilitate access to the phenomena and tell us how the theory is to be applied in a specific situation (Bailer-Jones 2009, p. 126–152, 209)

² A cognitive process is here understood as a series of operations that access and use knowledge stored in the memory (Machery 2009, p. 9).

although the results are widely generalizable. In physics, concepts are embedded in laws and/or models, which involve two or more concepts and some information about their interdependencies. Consequently, concept learning entails learning such models and the relevant relations.

First, I discuss how concepts are understood in cognitive science. Second, I review the different approaches towards models and modeling in science education and examine them in the light of the proposed conceptualization of seeing models as relational categories. Third, I discuss the role of different representational means in facilitating learning and point out the implications for research and practice. The proposed conceptualization provides a cognitively well-motivated basis for the science-based views of models often used in MBL. Also, by approaching MBL and its benefits by viewing models as relational structures or complex arrangements thereof opens up an interesting possibility for bridging research on MBL with cognitive science.

2 Relational Concepts

2.1 Concepts and Categories

Within cognitive science, concepts have been described as constituents of thoughts, bodies of knowledge, mental representations, and knowledge structures. Machery (2009, p. 12) has proposed the following definition, which captures much of what psychologists mean by the term concept:

A concept of x is a body of knowledge, stored in the long-term memory and that is used by default in the process underlying most, if not all, higher cognitive competences when these processes result in judgements about x .

Despite the overarching definition offered above, much of the psychological literature of concepts focuses on categorization.³ In addition to categorization, concepts are considered to act as components of thoughts acting as filters between us and the world (Goldstone and Kersten 2003). In other words, we access the world via concepts, which provide diagnostic and often informative parsing of the world. Concepts enable inductive generalizations—once an entity is categorized as a dog we know that it probably barks and has four legs. Moreover, the generative nature of creative thought is typically attributed to concepts: concepts can be combined to form new ones that can be readily comprehensible on the basis of the parent concepts (although certainly not always). This is typically connected to the *systematicity* of conceptual thought. That is, there are some regularities in how concepts' meanings form when concepts are combined; shaped by some emergent properties and real-world plausibility (Goldstone and Kersten 2003).

While the terms “concept” and “category” are sometimes used synonymously, they are distinguished here. A concept refers to the mental representations we have about the entities in the world, whereas category means the groups of the entities themselves. The concept of “dog” means the mental representation(s) we have about dogs, while the category “dog” means the entities in the world categorized as dogs (Goldstone and Kersten 2003).

³ Categorization is here understood as a judgment that an item belongs to a certain class (Machery 2009, p. 153). Many cognitive acts can be understood as categorization: recognizing objects as triangles or systems as instances of Newton's second law (Goldstone and Kersten 2003).

2.2 Relational Concepts

One focus of interest in psychological research on concepts is how the concepts are mentally represented. The representational format of concepts has important implications on how the concepts are learned. Cognitive psychology has largely focused in studying so-called *feature-based categories*, which are represented by sets of features describing the average or particular members (as in *prototype* or *exemplar theories*, respectively). For example, dogs are animals that “bark,” “have fur,” “four legs,” and so forth. Feature-based theories of concepts might be useful for education, as in the case of children learning the basic level biological categories (Sakamoto and Love 2010). Their usefulness is, however, limited, as learning science requires grasping complex combinations of concepts often couched in mathematical formalism (Goldwater and Schalk 2016). However, recent research has explored a more abstract basis for categorization, namely, *relations*.

The relational categories framework is based on the idea that our understanding of such simple concepts such as *father* stems from the relation(s) fathers bear to their offspring (for example, begetting) (Gentner 2005). In short, a relational category is a category whose membership is determined by a common relational structure (in contrast to common features) (Gentner and Kurtz 2005, p. 151). That is, fathers of different species are grouped together despite lacking perceptual similarity. If we consider a more abstract case such as “central force system,” it might be even more evident that relations form the basis of our understanding. In the case of Earth and Moon, for example, we may say that the Moon is related to the Earth by the virtue of revolving around it. We may also construct a physics model of the situation, which can be represented by mathematical functions (Gentner 2005).

The relational categories can be further divided into *role-* and *schema-governed categories* (Gentner 2005; Goldwater et al. 2011; Goldwater and Schalk 2016). In essence, schemas denote whole relational systems, whereas role-governed categories share a common role within such system (Earth occupies a role category in a schema category central force system). In addition, relations can be divided into first- and higher-order relations. First-order relations relate objects (i.e., feature-based categories), whereas second- and higher-order relations relate relations (Gentner 1983). However, feature-based and relational categories can become connected, as the different roles in a particular schema are typically filled with a member of a feature-based category (Goldwater et al. 2011).

It has been argued that the different categories (featural and relational) are cognitively different in that they entail different cognitive mechanisms to be learned, reasoned with, and encoded (Goldwater and Schalk 2016). Moreover, relational knowledge is fundamental to much of our higher cognitive competences as it plays a vital role in, for example, categorization, analogies, explanation, concept learning, reasoning, and problem solving (Halford et al. 2010). One crucial property of relational representations underlying these competences is that relations preserve structural mappings, which allows, for example, making transitive inferences (Halford et al. 2010). *Structurally consistent* relational representations allow the inference “A is left of C” from the premises “A is left of B” and “B is left of C”. These kinds of representations allow making connections between perceptually dissimilar items—hence, they enable relative independence from similarity of content and abstraction (Halford et al. 2010). Hence, relational representations allow us to make inferences that go beyond the information given and also beyond experience.

Relational representation also underlies the ability for analogical reasoning, which has an important role in, for example, concept learning and reasoning. For example, analogies enable

us to gain insight of unfamiliar phenomena by virtue of comparison to previously known examples. Famous examples include the water pipe analogy for electric circuits and the solar system—atom analogy. While the benefits of using analogues in education have been known for some time (Duit 1991), it also has a vital role in scientific innovation (Dunbar 1997; Dunbar and Blanchette 2001; Nersessian 2008, pp. 131–135, 196–197). One mechanism through which analogies work is *structural alignment*: identifying the underlying relational pattern in phenomena and matching it with the novel phenomena under study (Gentner 1983, 1989, 2005). A crucial thing to notice is that the entities involved in analogical mapping need not to share resemblances—they are mapped by the virtue of their similar roles and relational structure (Gentner 2005).

Relations also provide the cohering nature of knowledge via linking the category features—that is, features are not arbitrary but are often causally connected (birds are able to fly *because they have wings*) (Murphy and Medin 1985). Categories whose features are connected in the light of prior knowledge are typically referred to as *coherent categories*. Learning of novel categories is faster when they are coherent. Importantly, many categories are defined only by systems of relations that link features without specifying them; it suffices that the category members satisfy the relational structure (Rehder and Ross 2001). This kind of category membership amounts to a more abstract basis for categorization; the category members can be devoid of any perceptual similarities.

3 Models as Relational Categories

Models encompass many of the essential aspects of creating, communicating, and accepting scientific knowledge (Frigg and Hartmann 2017; Giere 1988). Regarding the learning of scientific concepts, the roles of models in constructing the meaning and content of concepts are of central importance. Another central question is how the knowledge contained in the models gets represented and learned. Towards this end, I bridge the cognitive view of concepts with MBL by briefly discussing the nature of models and arguing that models can be seen as relational categories.

Within science education and philosophy of science, models are commonly discussed as being representations (Frigg and Hartmann 2017; Oh and Oh 2011). Here, this broad characterization is adopted. More specifically, models are here understood as *external* representations. I use the term external here to distinguish models as used in science from mental models and other internal (i.e., cognitive) representations. Sure enough, mental models can represent scientific models or play a part in model construction⁴ (cf. Nersessian 2008).

Moreover, I will focus my attention on conceptual models represented via various representational means (mathematics, diagrams, graphs, and so forth). Conceptual models in physics are models which give interpretive descriptions of phenomena in terms of the abstract concepts and ideas of physics typically employing mathematical formalism (Bailer-Jones 2009, pp. 1–4, 185–188). In physics, models encompass descriptions of the variables, and relations between the variables. The variables, on the other hand, serve to represent objects, states of a system and their developments, or interactions (e.g., forces) (see, e.g., Wells et al.

⁴ To be sure, internal representations (e.g., mental models or concepts) are not straightforwardly turned into external models, as this is a complex process involving making use of various representational means, past research, and (possibly) interacting with others.

1995). Regarding the learning of such models, an important question is how models and the information incorporated in them gets represented and learned. To this end, I argue that groups of related models can be conceptualized as relational categories, in direct correspondence to the section 2.2.

For example, a “central force system” can be thought as a category, which includes the phenomena that satisfy the relevant relations (Gentner and Kurtz 2005). In addition, as Frigg (2010; see also, Nersessian 2015) has noted, models can become objects of study in their own right, and therefore, we could think of a category that consists of the models (instead of the phenomena) that describe a central force system. One justification for these claims comes from the semantic view of theories, which identifies theories as consisting of classes of structures (or classes of models) (Lorenzano 2013). These classes are identified on basis of principles or laws—that is, on the basis of relations (Lorenzano 2013; see also, Giere 1988, 1994). For example, according to Giere (1988, p. 82), Newtonian models are constructed by combining Newton’s laws with various force functions.

The semantic view, particularly Giere’s version of it, has been influential in science education (Koponen 2007; Koponen and Tala 2014). Recently, in contrast to the semantic view, models have been described as being autonomous from theories rather than constitutive of them (Morrison and Morgan 1999; see also, Frigg and Hartmann 2017). In this view, modeling proceeds, for example, via *model template*, which is “an abstract conceptual idea embedded into a mathematical form or method” (Knuuttila and Loettgers 2014, p. 298). In this respect, models of this kind do not fall into categories based on laws or principles associated with a specific theory but are based on some more general templates that, nevertheless, are relational by the virtue of being instantiations of some mathematical structure. These templates then may make us sensitive to perceiving certain patterns across different empirical systems (Knuuttila and Loettgers 2014).

In addition to the apparent differences regarding the relation between models and theory, the above views have also differing, even conflicting, views about the underlying epistemic aspects of modeling—for example, why and how are models able to represent the world. I, however, am mostly concerned about the structure and representational means associated with models. The differences between the above views are reflected in the different approaches within MBL, which emphasize different aspects of models and modeling, partly depending on the perceived goals of the different approaches (to be clarified in detail later). With respect to learning, it is important to discern these differences as the learning processes and the practical implications may differ.

Regarding the structure of models, there are various kinds of relations embedded in models. While for example, Ohm’s law, $U = RI$, typically has a causal reading (voltage causes current), Kirchhoff’s current law, $\sum I_k = 0$, may be best described as a constraining equation. There are also various relational patterns, which refer to the ways in which the variables are linked to each other (such as linear causality, common cause, feedback loops, and cyclic causalities) (for examples, see Kokkonen and Mäntylä 2017; Perkins and Grotzer 2005; Rottman et al. 2012).

It should be noted, however, that viewing models as relational categories does not mean seeing models as mere structures or relations. As pointed out in section 2.2, relational and featural representations may become connected, as members of feature-based categories typically fill the roles within a certain relational schema (Goldwater and Gentner 2015). This is perhaps reflected in Frigg’s (2010) note of how scientists often think of models as if they were physical things. Scientists might, for example, describe a central force system as

consisting of two objects with size, shape, and other perceptual features in addition to the relational information.

To be sure, there are differences in the categories typically discussed within cognitive science compared to scientific models. While cognitive science has examined fairly simple categories, models in science may include complex relations that span across multiple levels of hierarchies. Moreover, models themselves may become connected forming coherent, hierarchical systems of knowledge (as explained in more detail later on) (see, also, Giere 1988, p. 82–86). However, it is precisely the nature of relational representations that underlies these kinds of features associated with models. The relations provide coherence to the individual models by connecting the variables together and to the conceptual structure via the interrelations between the models.

4 Models in Science Education

Regarding the educational implications of seeing models as relational categories, it is important to look at what role relational knowledge plays in learning complex scientific knowledge. Insofar as models are at the core of learning this knowledge, it is important to examine the different approaches towards learning about models and modeling within MBL. This is because, in the different approaches, models play different roles and thus different aspects of the knowledge are emphasized. Also, the learning goals (targeted knowledge and processes) differ. Consequently, the cognitive processes and representations might differ.

4.1 Approaches Towards MBL

Within science education, models and modeling emerged as a novel area of investigation in the 1980's (Amin et al. 2014, p. 64; Gilbert and Justi 2016, p. 29). MBL recognizes different approaches of learning with models and modeling (for reviews see, e.g., Gilbert and Justi 2016; Oh and Oh 2011). For example, already in the 1980's, two distinct traits emerged: use of models as a way to introduce scientific ideas and modeling as a way to build knowledge and understanding (Amin et al. 2014; Gilbert and Justi 2016, p. 29). Reflecting on the literature and the discussion in the previous section, three broad groups arise as candidates for reinterpretation from the point of view of relational categories: *Using ready-made models*, *constructive modeling*, and *generative modeling*.

In “using ready-made models,” models are often used to introduce, demonstrate, and explain complex ideas, phenomena, and processes to students. Different visualizations, use of analogies, concrete material models are familiar examples of this kind of model use (Campbell et al. 2015; Coll et al. 2005; Justi and Gilbert 2003; Oh and Oh 2011). The goal is explicitly learning a target model (Justi and Gilbert 2003). This category overlaps with “learning curricular models” and “learning to use models” suggested by Gilbert and Justi (2016, p. 61) and with “exploratory modeling” described by Oh and Oh (2011). One of the common features in these approaches is using a model provided by the teacher and emphasizing that students are not involved in modeling per se (i.e., constructing a model).

While “using ready-made models” is an inherently teacher-centered approach, the goal of “constructive modeling” is to make students construct models often in inquiry-based contexts. Thus, models are used to generate hypotheses and explanations, as well as design experiments and gain understanding. Importantly, models are seen as parts of a larger explanatory

framework, which provides the outline for the model construction—thus, models are seen as subordinated by theories (Hestenes 1992; Windschitl et al. 2008). Also, models are typically evaluated by experimentation and subsequently revised (see, e.g., Hestenes 1992).

Model construction is also the target of “generative modeling,” but in contrast to “constructive modeling,” it is acknowledged that in science, models are seldom straightforwardly derived from any existing theory (Koponen and Tala 2014). In this kind of approach, modeling can be viewed as an epistemic practice for gaining knowledge, and models as tools for knowledge construction. Hence, models are given more autonomous status both in theory formation and in inquiry in general (see, e.g., Koponen and Tala 2014; Lehrer and Schauble 2015; Passmore et al. 2009). This is an interesting viewpoint as it assigns models a central role in the creative process of generating scientific knowledge (Koponen and Tala 2014).

It should be noted that using ready-made models and model construction approaches embraces different learning goals (Justi and Gilbert 2002) and differs with respect to the complexity of the reasoning targeted (Amin et al. 2014). While the goal of using ready-made models might be enhancing conceptual understanding within a timeframe of a single lesson (or less), model construction approaches often try to teach students also about science and/or how to do science via interventions spanning multiple lessons or even years (Amin et al. 2014).

“Constructive modeling” and “generative modeling” can be contrasted with “learning the reconstruction of a model” and “learning to construct a model *de novo*”, respectively (as suggested by Gilbert and Justi 2016, p. 61). In Gilbert and Justi’s categorization, the distinction is made based on whether the model to be constructed is known to the students beforehand or not (Gilbert and Justi 2016, p. 61). However, the categorization here is partly motivated by the relation between models and theory. While “constructive modeling” sees models as subordinated by theories (cf. Giere 1988), “generative modeling” sees them more as autonomous tools for investigation (cf. Morrison and Morgan 1999). When interpreted from the point of view of relational categories, these different roles of models also reflect different learning goals and different aspects of conceptual knowledge associated with models. While in the former the target is often expert-like knowledge structure (cf. Hestenes 1992), the latter emphasizes the process of constructing and justifying knowledge (cf. Koponen and Tala 2014). In what follows I discuss the three categories in more detail and give illustrative examples of them. Moreover, I examine the cognitive aspects related to each in the light of the relational categories framework.

4.2 Using Ready-Made Models

4.2.1 Ready-Made Models in Science Education

Using analogies provides a familiar example of using models to introduce or demonstrate a complex idea, concept, or phenomenon. The assumption underlying learning with analogies is that it helps students understand a novel domain by showing how it is similar to some already known domain—or as Duit (1991, p. 651) put it: “to make the unfamiliar familiar”. Furthermore, analogies are also seen as important “tools of discovery” (Harrison and Treagust 1993, p. 1291), as scientists frequently use analogies in developing their models (Coll et al. 2005).

Analogies are essentially seen as a way to enhance the learning and understanding of the target domain. For example, Harrison and Treagust (1993; Treagust et al. 1996) taught refraction by using an analogy between two wheels rolling from paper to carpet and light traveling from glass to air. Clement (1993) on the other hand described the use of *bridging*

analogies, which are structured intermediate analogies. Bridging analogies are used to create a chain of structured analogies in order to scaffold students into grasping the validity of the original analogy, which they might otherwise fail to see (Clement 1993).

In addition to analogies, different visualizations, concrete objects, and sufficiently simplified teaching models are frequently employed. For example, Smith et al. (1997) used a variety of models and simulations to engage students in explicit reasoning about weight and density, which are known obstacles for students. They provided students with various conceptual models, for example, the dots-per-box model, intended to offer them visual external representations of physics concepts with shared relational structure (Smith et al. 1997). In a related study, Wisner and Amin (2001) represented heating as the number of E's entering an object from a heat source and temperature as the number of E's per molecule—thus explicating the distinction between heat (the total number of E's) and temperature (the E's per molecule).

Frederiksen et al. (1999) introduced students to successive models at different levels of abstractions in the context of electric circuits. They devised a series of models with “derivational linkages,” which means that the models are related pairwise at lower and higher levels of abstractions. For example, students are first introduced to a qualitative particle model of electricity, which is developed into aggregate model based on simple flow equation (Frederiksen et al. 1999). Consequently, this is succeeded by a fully fledged algebraic model.

4.2.2 Cognitive Aspects of Using Ready-Made Models

Importantly, in the above examples, grasping the relational structure of the models is key. Dots-per-box, E's-per-molecule, and rate of electrons are relational constructs and learning them requires grasping this relational structure. Analogies should also be understood as structural similarities. That is, analogies are not based on perceptual similarity but on relational correspondence between the target and the source (Gentner 1983). For example, in Clement's (1993) examples, the table and the spring do not share perceptual similarities but both *push up on* something, that is, exert an upward force. Simply put, the situations are structurally similar. Research on how people understand relations and causality have been an important focus in cognitive science. Much of this research has focused on analogies and analogical reasoning, as it is an important process for acquiring relational concepts. Especially, recent research on concept learning has also come to concern relational categories.

Within cognitive science, research on how people understand analogies led to the development of the *structural alignment* framework according to which the underlying characteristic of analogs is the alignment of the relational structure (Gentner and Markman 1997; see also, Gentner 1983, 1989, 2005). That is, in the above example, spring maps to table and hand maps to book and these are tied together by the relation *push up on*. Structural mapping does not concern only on analogical reasoning but also it is seen as an important mechanism for learning relational knowledge more generally.

Importantly, not all features or relations are transferred across the analogy; a relation belonging to an interconnected system of relations is more likely to be imported (Gentner 1983). That is, relations that belong together are more likely to be mapped than separate ones. But, in addition, *the systematicity principle* favors nested relational structures, structures wherein lower-order relations are governed by higher-order ones (Gentner 1983; Gentner and Markman 1997; Goldwater and Schalk 2016).

Research has also pointed out several potential pitfalls in using analogies (Clement 2013; Duit 1991). For example, students might not have sufficient knowledge about the

source domain in order to draw the intended inferences. The source may also be “too far” from the target for students to grasp the mapping between them (Clement 2013, p. 425). Moreover, students’ prior knowledge may interfere with the process and prevent students from understanding the analogy. Students might, for example, transfer nonessential properties to the target domain (e.g., the appearance of the compared cases) instead of the deep, relational features (Richland and Simms 2015). Harrison and Treagust (1993) suggested that teachers should use analogies that are familiar, and that shared as well as unshared attributes of the source and target should be explicitly identified. The bridging analogies strategy discussed by Clement are intended to provide students an initial, physical intuition and then gradually build the mapping between the source and the target while retaining the relational structure (Clement 2013). In other words, the students were helped to create a physical intuition of the source (which was not readily available) and then to transfer this into the target step by step.

While in science education analogies are seen as a way to enhance learning of the unfamiliar target domain, the structural alignment framework also emphasizes that analogical comparison and analogical reasoning are essential in order “to promote *general* causal abstractions [emphasis added]”, which are fundamental for the transfer of knowledge across domains (Goldwater and Gentner 2015, p. 138). It is argued that while other processes exist for acquiring causal and/or relational knowledge in specific domains, transfer performance hinges on acquiring abstract, general relational representations. It is proposed that analogical comparison across relationally similar cases enables this. It should be noted, however, that comparison across cases is most beneficial when the source and target are accurately represented (Goldwater and Gentner 2015)—that is, abstracting the higher-order relations depends on accurately representing the lower-order details. This might be related to how providing students with “anchoring intuitions” enriches students’ representations of the targets thereby enhancing analogical transfer (Clement 2013).

Moreover, it has been suggested that science teaching enhances finding relational commonalities across domains and this is based on the ability to perceive the commonalities⁵ (Goldwater and Gentner 2015; Rottman et al. 2012). Thus, finding the relational commonalities is not just about being more knowledgeable on some domain or across multiple domains but also about possessing general causal patterns and being inclined to look for these patterns. Therefore, acquiring the patterns and finding commonalities enhances understanding on each domain and enables novel insight.

4.3 Constructive Modeling

4.3.1 Constructive Modeling in Science Education

The science education studies presented above exemplify the benefits of using models to support the learning of abstract science ideas. Some have, however, pointed out that approaches similar to those presented above make use of ready-made models (Amin et al. 2014; Jonassen et al. 2005; Justi and Gilbert 2002; Oh and Oh 2011). Jonassen et al. (2005) argued that while, for example, intelligent tutoring systems and microworlds allow learners to interact

⁵ Rottman et al. (2012) suggested that science students had acquired more experience of comparing across different domains. This multidisciplinary experience may then lead to extraction of general causal patterns.

with them, they are used only for infer the propositions built in the systems and for confirming hypotheses.⁶

In contrast, certain approaches take model construction as an explicit target for instruction (Halloun 2007; Hestenes 1992; Passmore et al. 2009; Windschitl et al. 2008). Often, modeling is embedded in inquiry activities, which relies on cyclical development and testing of models. Typically, inquiry starts with a question or a problem posed in some phenomenological context. Then a model is constructed in order to generate hypotheses and/or predictions about experiments, whose purpose is to validate or test the model (Louca and Zacharia 2012). In model-based inquiry approaches, the structure of scientific knowledge is emphasized so as to underscore the models' role as parts of larger, more comprehensive frameworks (i.e., theories) (Windschitl et al. 2008; see also, Passmore et al. 2009). Indeed, Windschitl et al. (2008) noted that too often naive discovery is employed wherein questions are often "arbitrary" and hypotheses "poorly informed guesses" (p. 946).

In Hestenes's and Halloun's approaches, the focus is on learning a selected number of basic models in order to acquire "the essential factual and procedural knowledge" (Hestenes 1987; see also, Halloun 2007). The structure of physics knowledge is then seen to comprise of these families of models together with the theory and principles for relating the models and constructing them from the theory (Halloun 1996; Hestenes 1992; Wells et al. 1995). Acquiring and mastering the basic models is seen as essential in the development of scientific understanding, as they help to develop a meaningful understanding of the concepts and lay the ground for developing more complex models (Halloun 1996).⁷

The "families" of such models constitute what Halloun (2007, p. 664) called a "middle-out theory structure" much in the vein of Giere's (1988, 1994) conception of the structure of scientific theories. According to the model-based view of science, scientific theories are equated with populations of models along with hypotheses linking the models with the observable world. The population of models on the other hand consists of related families of models, which are typically constructed (in Newtonian mechanics) by combining Newton's laws with various force functions (Giere 1988, p. 82). Therefore, the models are related to each other via sharing mutual concepts and/or relations. For example, Coulomb's law uses the concepts of force and charge, whereas the concept of electric field relates force, charge, and Coulomb's law. Furthermore, certain models are derived from other, more general ones by adding a specific force function (such as a damping force in the case of a simple linear oscillator). This creates hierarchical relations between models (in contrast to the interdependencies brought by mutual concepts and/or relations). In this way, models are central for introducing structure for the whole knowledge system⁸ (Nousiainen and Koponen 2010).

⁶ Of course, it can be argued that what differs between modeling and model use approaches is the intended scope and the complexity as well as the scope of the targeted knowledge (cf. Amin et al. 2014).

⁷ Giere (1988, p. 75) notes that Kuhn (2012/1962, p. 186-190) was right to emphasize exemplars in the training of scientists. Through exemplars, scientists learn to interpret mathematical symbolism and to identify cases to which the exemplars fit (Giere 1988, p. 75; see also, Kuhn 2012/1962, p. 186-190).

⁸ Creation of new concepts and new relations (i.e., laws) can be supported through operationalization (i.e., making concept measurable through preexisting concepts) and subsequent experimentations. The results of the experiments are presented through models, which then serve to introduce these new concepts and relations to the preexisting structure.

4.3.2 Cognitive Aspects of Constructive Modeling

In constructive modeling approaches, models are seen as parts of a larger framework, and this underscores the structure, coherence, and interconnected nature of scientific knowledge. These kinds of approaches have had good results with respect to both conceptual knowledge and general problem solving (Taconis et al. 2001; Wells et al. 1995).

The hierarchical and interconnected nature of physics knowledge is reflected in the studies of experts' and novices' representation of physics knowledge. Several studies have found that experts categorize physics problems according to the underlying principles, whereas novices are sensitive to the superficial surface features (such as the different objects present in the task) (Chi et al. 1981; Savelsbergh et al. 2011; Snyder 2000). What is more, Snyder (2000) showed that experts, intermediates, and novices all represented problems at multiple levels of abstraction, but that intermediates' and experts' representations were more principle-based than those of novices who were sensitive to the concrete features of the problems. Also, only experts and intermediates could represent the problems at a high level of abstraction based on physics principles. Rather similarly, another study showed that experts, in contrast to novices, could form more coherent, multi-level representations of physics problems (Savelsbergh et al. 2002). Furthermore, and perhaps more importantly, while some novices possessed relevant abstract knowledge, they did not connect to the concrete features of the problems (Savelsbergh et al. 2002).

The above problems are related to the fact that recognizing a phenomenon or a problem as an instance of Newton's second law, for example, is not straightforward. Typically, recognition of the deep features of a problem requires a lot of elaborations (or "construction rules") in order to build a meaningful physics representation (Savelsbergh et al. 2011). Indeed, Hestenes (1987) argued that experts spend considerable amount of time in constructing problem representations to which specific principles can then be applied. Which features of the problem are relevant is a real insight issue, and without any scaffold students are likely to attend to irrelevant things.

As argued above, the ability to form complex interconnections and abstract, hierarchical representation is a hallmark of relational representations (Halford et al. 2010). More specifically, relations cohere individual concepts and categories such as central force systems, but additionally, relations cohere entire conceptual systems (as exemplified by, for example, experts' representations of physics problems). In essence, specific instantiations of certain physics laws (such as different force laws) cohere features within individual categories (that is, instantiations of the same problem type) and relations among different categories or other knowledge structures provide global coherence, that is characteristic of expert knowledge.

To the extent that relations and abstract general principles underlie the deep, coherent structure of expert knowledge, the question is how such knowledge is formed. There is some evidence that abstracting the principles through analogical comparison together with specific instructional methods such as self-explanation, scaffolding, and studying worked examples would be beneficial (Richey and Nokes-Malach 2015). Especially in mathematics education, research has begun to examine the benefit of *case comparisons*, in which students are asked to compare, for example, two different problem solution procedures (Alfieri et al. 2013). Comparisons may lead to reduced, more abstract, and principle-based representation of the common features of the cases, which might lead to better transfer performance (Alfieri et al. 2013). The implementation of such methods within MBL has not been examined, but it would be an interesting path for future research. For example, how finding commonalities between

novel cases and the basic models described by Halloun (1996) and Hestenes (1992) could help develop the deep, structured knowledge?

It has also been suggested that teachers' demonstration of the modeling of problem solving processes or letting the students compare worked out solutions leads to learning of deep, structured knowledge (Richey and Nokes-Malach 2015; Rittle-Johnson and Star 2007; Savelsbergh et al. 2011). It may also be that the sufficiently simplified epistemology of, for example, Hestenes's and Halloun's approaches as well as their clear and comprehensive rules that lead from the theoretical principles to model construction contribute to their effectiveness by scaffolding the teachers and students alike. However, more research is needed about the development of coherent knowledge systems and how interrelated concepts cohere or fail to cohere across contexts (see, also, Goldwater and Schalk 2016).

4.4 Generative Modeling

4.4.1 Generative Modeling in Science Education

Hestenes's approach, for example, emphasizes the mathematization of physics and the subordination of models to theories (Koponen and Tala 2014). Such an approach "clearly constitutes an authentic way of modelling in physics" when a well-developed theory is used as a framework for making predictions (Koponen and Tala 2014, p. 1149; see also Koponen 2007). Moreover, as experiments are typically for validating the models, the relation between them is that of "verificatory justification" (Koponen and Mäntylä 2006, p. 32). In certain approaches, however, models have a more autonomous role with regard to the theory, in that the theory has only a guiding role in how models are actually constructed (Koponen and Tala 2014). Models are primarily seen as purposeful representations created for making sense of the world and as forms of explanations for questions arising in some social context. Hence, models are seen as tools for thinking, reasoning, and creating scientific knowledge. Lehrer and Schauble (2015) also argued for viewing modeling as a practice rather than merely a tool for making predictions. This view highlights the role of models in knowledge generation, as the models are subject to open-ended revision influenced not only by new phenomena but also by, for example, new instrumentation.

In *generative modeling*, the modeling activities are embedded in dynamic simulations, which in turn are framed in terms of difference or differential equations or, for example, agent-based simulations (Koponen and Tala 2014). Importantly, attention needs to be paid on how certain model-level relations and dependencies produce observable behaviors—in particular, what kind of motion arises, for example, from linear or nonlinear restoring forces and their combinations and how does damping affect the phenomena (Koponen and Tala 2014). Simulations have a key role in "running the models" in the virtual world, as they allow us to explore the dynamical behavior of the models as well as learn and gain knowledge from the models (Koponen and Tala 2014).

In the elementary school context, Lehrer and Schauble (2015) have noted that inquiry and external representations are intertwined as evolving external representations can act as resources and facilitate different inferences and bring about novel questions (Lehrer and Schauble 2015). Even simple models, such as scale models, are important in communicating what is being modeled (Lehrer et al. 2001; Lehrer and Schauble 2015). As students become more knowledgeable and competent in using different kinds of models and acquire a richer

repertoire of representations, there is a shift towards more abstract models (Lehrer et al. 2001; Lehrer and Schauble 2015).

Here, the choice and understanding of the external representational format is of central importance. For example, the simulations are based on computational templates, which are mathematical structures such as formulas (Koponen and Tala 2014).⁹ These structures are familiar frameworks that are suitably decontextualized so that they can be flexibly transferred and applied across contexts. The template determines which aspects of the target system can be represented and thus strongly influences what can be learned from the model (Koponen and Tala 2014). Hence, mathematics is not seen as mere computation but as a representational means facilitating conceptual development. For example, children were able to differentiate between weight and density—a difficult topic—after being introduced to the mathematical concepts of volume, measure, and similarity (Lehrer et al. 2001; Lehrer and Schauble 2015; see also, Amin et al. 2014). Hence, it would be “preferable to learn more about how these forms of understanding [mathematics and physics] can mutually bootstrap each other” (Lehrer et al. 2001, p. 71).

Viewing models as somewhat autonomous tools for thinking expands the notion of models and their role in science and science education. When allowing models a relative autonomy from theories, we may ignore, for example, some causal factors in order to explore certain others and in doing to render the models “unrealistic.” Indeed, within philosophy of science, it has been argued that modeling often does not strive for realistic representations of its targets; it suffices that they are representative (Morrison and Morgan 1999). Some argue that this is precisely the aspect of modeling that makes models work and allows them to be conveniently mathematically represented (Humphreys 2004, p. 84–86, 116–124; Knuuttila and Boon 2011; Knuuttila 2011; Koponen 2007). Moreover, this is what allows us to learn from the models (Knuuttila 2011; Knuuttila and Boon 2011). Interestingly, as a consequence, the abstractions may make the results derived from the models intractable. That is, we may not know which assumptions are responsible for the results and, furthermore, the results of a certain model may depend partly on the chosen representations (Knuuttila 2011).

4.4.2 Cognitive Aspects of Generative Modeling

Seen from the relational categories framework, the important aspects in the above kind of modeling are the roles of generic relational mental representations and external representational formats. As noted earlier, understanding of the generic causal relations is of central importance in making connections across contexts—that is, noticing the similarities between development economic pricing bubbles and melting of polar ice caps, for example (both are “positive feedback systems”) (Goldwater and Gentner 2015; Nersessian 1995).

Within science education, students’ “repertoire” of different causal modeling styles has been recognized as one major hurdle in acquiring science concepts and models and consequently in using these model types in novel situations (Perkins and Grotzer 2005). That is, students might find modeling with constraint equations familiar after being exposed to modeling dynamic systems with Newton’s laws or electric circuits with Ohm’s law (Perkins and Grotzer 2005). Similarly, Collins (2011) has identified various *epistemic forms*, which denote structures and functional or causal relationships. For example, constraint equations are a specific type of

⁹ In this, they follow Humphreys (2004, p. 60–67) who introduces the concept of conceptual template on which computational models are built.

epistemic form. Epistemic forms are kinds of generic relational patterns that guide the inquiry process and consequently concept formation as well (Collins 2011). Initially, however, students are largely unaware of sophisticated modeling styles (and/or epistemic forms) and are inclined towards linear and/or sequential causality (for example, A causes B, which causes C and so forth) instead of more complex patterns such as constraint-based interactions (Kokkonen and Nousiainen 2016; Kokkonen and Mäntylä 2017; Perkins and Grotzer 2005).

Goldwater and Gentner (2015) argued that while cognitive science has found several experiences that lead people to infer causal structures, gaining abstract, generic knowledge about the relations requires both exposure to particular causal phenomena and applying this knowledge across domains. Perkins and Grotzer (2005) explicitly introduced students to different types of causal models through discussion and application in different contexts (electricity, density, natural selection and ecosystems). They concluded that students who were exposed to discussions about different causalities outperformed the control group. Importantly, the discussions were framed in the context of a particular science topic and did not involve presenting students with generic accounts of causality. Evidence from the development of children's models and representational capabilities also suggests that they first need to acquire sufficient knowledge of the domain and the target phenomena (for example through concrete models) and then progress to more decontextualized, abstract models (Lehrer and Schauble 2015).

The notes on external representational formats and templates suggest that not only do students need to possess abstract, general causal knowledge but they also need to have sufficient representational capabilities in order to engage in advanced modeling practices. Rather obviously, they need to have sufficient knowledge of computational methods and/or sufficient mathematical proficiency in order to generate computational and/or mathematical models. However, besides viewing mathematics as mere computation for deriving results, there is evidence that suggests (along with the preceding discussion) that mathematical and other means of external representations deeply interact with the relational reasoning processes. As well as having some interesting cognitive implications, this underlines the epistemological arguments offered above.

At the very least, different external representational formats (graphs, symbolic representations, mathematical equations) can support or hinder relational reasoning and the acquirement of relational knowledge. For example, the use of graphs instead of tables leads to better and more flexible performance when interpreting data—but may also induce interpretive bias (Braithwaite and Goldstone 2013). In the context of problem solving, students' performance depends on the representational format (Kohl and Finkelstein 2006) and their strategy choice (De Cock 2012). Furthermore, students' ability to master different representations also affects their learning of specific concepts (Nieminen et al. 2012). On a more radical note, Zhang (1997) has argued in favor of *representational determinism* according to which the representational forms define what information can be perceived and which cognitive processes and structures activated. In short, the representations could determine which cognitive biases get activated and thus affect reasoning and learning.

5 Discussion

5.1 The Role of Generic Relational Knowledge

In the preceding sections, I offered three broad roles for models and modeling within science education. The importance of generic relational knowledge arose as a common theme from the

reinterpretation of the three roles. Across all three roles, one vital aspect of learning with and about models is grasping the relational knowledge. Recent research suggests that sensitivity to the relational patterns is an important factor in learning and this can be enhanced, for example, through comparing and aligning different cases, which leads to abstracted relational knowledge (Goldwater and Gentner 2015).

It seems that specific training on, for example, science enhances sensitivity for looking for relational patterns (see, Goldwater and Gentner 2015). One interesting interpretation of this is that people with certain background are more prepared for certain types of learning tasks even if the tasks are not within their area of expertise. It has been suggested that people's *interpretive knowledge* strongly affects their ability to learn from different kinds of situations (Schwartz et al. 2005). Interpretive knowledge guides how people frame the problem and this has major implications for subsequent thinking and cognitive processing (Schwartz et al. 2005). This suggests that learners' knowledge of different kinds of model systems, for example, affects how they are going to frame a novel problem and what kind of conceptualizations and inferences they can make—as suggested also by others (see, Perkins and Grotzer 2005). As suggested above, mathematics, computational methods, and instrumentation are intertwined with concept formation and modeling. Therefore, they can also be interpreted as parts of what make up interpretive knowledge and affect how people will frame novel problems. It would be an interesting line of research for the future to investigate how, for example, mathematical proficiency, exactly affects concept formation and learning and how it prepares people for future learning.

5.2 Difficulty of Relational Mapping

Some of the problems in learning relational knowledge may be attributed to the complexity of the mapping process, which leads to high cognitive processing demand (Richland and Simms 2015). Indeed, while relational mapping is important, it is also highly demanding and requires lot of cognitive capacity. For example, it simply takes more time to process relational matches than featural (objects that share literal similarity) (Goldstone and Medin 1994). Likewise, processing relations require a higher toll on the working memory, in accordance with the cognitive load theory (Goldwater and Schalk 2016).

The relational complexity theory assesses the relational complexity of representations as the number of entities that are related, and maintains that our working memory capacity limits our processing to a quaternary relation in parallel (Halford et al. 2007; Halford et al. 2010). More complex structures can be handled by processing them sequentially or by chunking (which renders some of the relations inaccessible). In addition to the number of relations, the relational structure itself might have an effect on the learnability. Category learning research suggests that chain-like structures are easier to learn, as are those where relations operate on the same object (Corral and Jones 2014).

5.3 Mechanisms of Change

Regarding the development of relational knowledge, research suggests that it is a slow and gradual process. Despite grounding my arguments in category learning research, I argue that learning models or physics concepts are not based on simple category assignment. Instead, it is likely to involve various related mechanisms such as analogical mapping, *theory revision*, and *theory redescription* (Halford et al. 2010). Theory revision is a process of changing the

relevant relational structure. It can happen when learners evaluate their existing relational knowledge against evidence and change it in response to error (Dixon and Kelley 2007). Within cognitive science, the theory revision process has been demonstrated across a wide variety of domains. There is evidence that the revision of the relational structure can happen, for example, via *refinement*, that is, by stripping irrelevant information, or via *elaboration*, which amounts to adding information to the representation (Corral and Jones 2014). Theory redescription on the other hand capitalizes on successful performance in contrast to revision, which happens in response to errors. Redescription means consolidating existing relations into new (higher-order) representations via abstraction (Dixon and Kelley 2007).

Research within science education has examined a similar kind of mechanisms although discussion between the disciplines (science education and cognitive science) has been limited. For example, Kokkonen and Mäntylä (2017) proposed model change, refinement, and elaboration as the possible change mechanisms related to student's explanation models. Elaboration refers to adding relations to the existing structure, while refinement means simplifying the structure via conglomerating different models into one. Similarly, Clement and Steinberg (2002) described the evolution of learner's mental models as a small stepwise revision induced by discrepant events.

5.4 Mental Modeling

The notion of mental models has been very influential in educational research investigating the learning of scientific concepts. While there is no consensus or rigorous definition about mental models, they are typically considered to contain structural and/or causal information about the target being modeled (Markman 1999, pp. 248–276; Nersessian 2013). Often, mental models are used to describe the mental representations that people construct and use when they are reasoning about the external world (Chi 2013; Vosniadou 1994). Hence, they serve as the vehicles for making inferences about phenomena. Moreover, mental models enable us to make predictions about the system and reason about, for example, unexpected events. Consequently, relational representations are at the core of the notion of mental models, insofar as the mental models involve using relational information.

Nersessian has extensively discussed the notion of mental modeling with regard to model-based reasoning, conceptual change, and conceptual innovation in science (Nersessian 1995, 2008, 2013, 2015). Indeed, the generative modeling account (see, Koponen and Tala 2014) described above takes its inspiration from Nersessian's (1995) conception of the modeling process. Also, many accounts of conceptual change rely on notions of mental models and model-based reasoning (Chi 2013; Vosniadou 1994). Broadly speaking, model-based reasoning is reasoning in which inferences are made by means of constructing, evaluating, and applying models. According to Nersessian, mental modeling is a central cognitive process underlying model-based reasoning.

In Nersessian's account, models are developed in an iterative cycle wherein models are constructed, evaluated, and adapted. During this process, models become targets of investigation in their own right as they serve as "interim interpretations of the target problem" (Nersessian 2015). Mental models enable inferences through simulation processes and interactions with external representations. Interestingly, Nersessian (2015, p. 459) suggested that simulations can be "off-loaded" to external representations (e.g., computational models). This reiterates the point that the choice of external representations and the ability to use them strongly influence the outcomes of modeling and possibly also the concept learning process (Lehrer and Schauble 2015). While the idea that our environment influences our cognitive

processes is trivial, the extent and way in which it does so is an interesting question for future research especially within MBL, as modeling relies on constructing and reasoning with external representations.

The above discussion suggests that the relation between internal and external representations and modeling is coupled and that the external representations serve as off-loading the simulative reasoning thus supplementing the internal representations. This also implies that the affordances that the different external representations (e.g., mathematics and simulations) should be given more attention both in research and in teaching.

6 Conclusions

Models are central in learning the concepts of physics as they encompass the concepts themselves and, importantly, the relations among them. The key question is how the learners mentally represent the conceptual information incorporated in the models. Towards this end, I conceptualized models as relational categories, which are categories whose membership is determined by a common relational structure. This conceptualization has interesting implications for both teaching and research. Specifically, it offers a cognitive basis for exploring the cognitive processes related to learning concepts, which has been an understudied aspect of MBL.

Firstly, the relational categories framework underlines the importance of relational knowledge in learning the concepts. For example, analogies are often used in science education in teaching complex concepts to students. Analogies are based on the structural similarity of the target and the source, which is seen as a way to enhance the learning of an unfamiliar target. Seen from the viewpoint of relational categories, the key benefit of analogical comparison is promoting general casual abstractions. These can, in turn, further enhance learning by affecting the sensitivity to detect relational patterns in novel situations. Sensitivity for relational knowledge is important to both learning specific models and transferring knowledge to novel contexts.

In pedagogical approaches that ask students to construct models, learning and using relational knowledge is also vital. Construction of models and using models to generating prediction and hypotheses requires grasping the underlying relational patterns of the target phenomena. Also, in certain approaches, the goal is in learning expert-like knowledge structure, which is coherent and involves representations on multiple levels of hierarchy. Relational knowledge provides the cohering nature of experts' knowledge, as it provides the backbone for hierarchical principle-based knowledge structures. This is exemplified in many studies about experts' physics knowledge and here slightly reinterpreted from the point of view of relational categories framework. In the future, important open questions concern the best ways to support the formation of such knowledge.

Some recent pedagogical approaches emphasize the relative autonomy of models, i.e., models are not directly derived from any existing theory. Instead, relational and mathematical structures are, for example, often transferred from other domains in order to generate knowledge. Seen from the relational categories framework, the ability to detect relational patterns in novel empirical phenomena is vital. This requires a developed repertoire of modeling styles, for example, expertise from modeling with constraint equations. Furthermore, the chosen representational format plays an important role, as it may deeply affect what kind of inferences can be made and what is learned through constructing the models. This emphasizes, for example, that mathematics should not be seen as mere computation but as part and parcel of the concept formation process.

In the preceding sections, MBL has been conceptualized as the development of relational categories and interconnected, hierarchical systems of knowledge. In science education research, models have previously been approached from the point of view of science—as representations of phenomena. However, in this article, models are reinterpreted as relational categories, which allows for novel conceptualization of the learning process associated with models and modeling based on cognitive science. The discussion and conceptualization of models as relational categories allows discerning students' mental representations of models in terms of evolving relational structures in greater detail than previously done.

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