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ORIGINAL RESEARCH

The diversification benefits of cryptocurrency factor portfolios: Are they there?

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Abstract

We investigate the out-of-sample diversification benefits of cryptocurrencies from a generalised perspective, a *cryptocurrency-factor* level, with traditional and machine-learning-enhanced asset allocation strategies. The cryptocurrency factor portfolios are formed in an analogous way to equity anomalies by using more than 2000 cryptocurrencies. The findings indicate that a stock–bond portfolio incorporating size- and momentum-based cryptocurrency factors can achieve statistically significant out-of-sample diversification benefits for investors with different risk preferences. Additionally, machine-learning-enhanced asset allocation strategies can boost the traditional approaches by enriching (shrinking) the distributions of weights allocated to potentially effective cryptocurrency factors. Our findings are robust to (i) the inclusion of transaction costs, (ii) an alternative benchmark portfolio, and (iii) a rolling-window estimation scheme.

Keywords Cryptocurrency factors · Portfolio optimisation · Diversification benefits · Machine learning

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1 Introduction

About 400 years ago, a saying in Don Quixote (Cervantes 2016) proposed a rule for risk management: "It is the part of a wise man to keep himself today for tomorrow, and not venture all his eggs in one basket". This adage, emphasising the wisdom of diversification, has since become deeply embedded in modern societal thought. Following the pioneering work of Markowitz (1952), academics and practitioners endeavour to discover new asset classes that can diversify the risks within a traditional stock–bond portfolio. Extensive literature documents the diversification benefits of alternative assets, such as commodities (Daskalaki and Skiadopoulos 2011; Bessler and Wolff 2015) and real estate (Chun et al. 2004; Huang and Zhong 2013). Cryptocurrencies, a decentralised product of blockchain technology (Biais et al. 2019; Lee 2020), have drawn attention from both researchers and investors and constitute a considerable market size.[1] Thus, recent studies have investigated cryptocurrencies from the perspectives of asset pricing (see, e.g., Liu and Tsyvinski 2020; Koutmos and Payne 2021; Liu et al. 2022; Han et al. 2023) and diversification benefits (Platanakis et al. 2018; Platanakis and Urquhart 2020; Huang et al. 2022). However, among these diversification-focused studies, most primarily concentrate on the most prominent cryptocurrencies, such as Bitcoin and Ethereum, with only a few considering a broader array of other cryptocurrencies.

We examine the diversification with over 2,000 cryptocurrencies at a *cryptocurrency-factor* level. Earlier studies (Eun et al. 2010; Koedijk et al. 2016) show that a traditional benchmark portfolio[2] can benefit from the inclusion of equity factor portfolios mainly because these factor portfolios have remarkably lower correlations with a stock–bond portfolio than other asset categories. In this study, we propose a portfolio diversification strategy that involves size, momentum, volume, and volatility factor mimicking cryptocurrency factor portfolios, and we assess the performance of boosted portfolios on an out-of-sample basis across various portfolio optimisation techniques. The rationale for our research is two-fold.

First, we study the diversification benefits from a generalised perspective, using over 2000 cryptocurrencies to construct four grand groups of cryptocurrency factors to evaluate the values contributed by the factors formed rigorously. The existing literature concentrates on popular cryptocurrencies through dynamic conditional correlations (Tzouvanas et al. 2020), the stochastic spanning approach (Anyfantaki et al. 2021), algorithmic trading strategies (Cohen 2021, 2023), cryptocurrency market index (Hachicha and Hachicha 2021), crash risks (Koutmos and Wei 2023), and the portfolio optimisation framework (Platanakis and Urquhart 2020), making the sample size relatively small. We collect as many cryptocurrencies as possible and then seek a manner to integrate the information inherent to gathered cryptocurrencies without losing generality. Inspired by the cornerstone of factor investing by Fama and French (1992, 1993), who construct factor portfolios with substantial abnormal returns based on a range of state variables (e.g., market, size, and value), we form 28 cryptocurrency factors in a similar approach to Liu et al. (2022) and Han et al. (2023). Factor-based portfolios are more effective in diversifying idiosyncratic risks than existing asset classes due to (i) factors' abilities to explain the expected returns, (ii) factors' well-diversified premium, and (iii) factors' reduced risks in the poor market conditions

[1] See https://cryptoslate.com/coins, the global market capitalisation of cryptocurrencies is near $1 trillion as of September 2022.

[2] We construct the benchmark portfolio with proxies for equities, bonds, and risk-free rates.

contrasted with other sole asset classes (Eun et al. 2010; Koedijk et al. 2016; Dichtl et al. 2021). Thus, we are motivated to evaluate the diversification benefits of cryptocurrency factors rather than individual crypto assets.

Second, we enhance the performance of optimised portfolios by combining the usage of traditional portfolio optimisation framework and machine learning, to mitigate the poor out-of-sample portfolio performance caused by estimation errors (DeMiguel et al. 2009; Bielstein and Hanauer 2019; Platanakis and Urquhart 2020; Kan et al. 2022). Since cryptocurrency factors are highly volatile (see Table 3) compared to traditional assets, and given the speculative nature associated with cryptocurrencies, mitigating the influence of estimation errors on cryptocurrency factors becomes increasingly gruelling during the portfolio optimisation procedure. To alleviate the uncertainty attributable to input parameters, we leverage the power of machine learning to predict cryptocurrency factor's one-period-ahead expected returns. Specifically, we employ the combination elastic net (C-ENet) approach, which is a linear machine learning method that combines the process of feature selection and predictive regression and is suitable to handle the time-varying properties of asset returns such as equity index and anomalies (Rapach and Zhou 2020; Dong et al. 2022). The reason we focus on expected returns of factors and other benchmarks rather than covariance matrices is that (i) expected returns on cryptocurrency factors are deemed significantly more pronounced than covariance matrices in the optimisation (Chopra and Ziemba 1993); and (ii) we aim to control the effects of the benchmark portfolio (stock–bond portfolios) during the optimisation, ensuring that the discovered diversification benefits are produced by including cryptocurrency factors rather than more accurate forecasts of benchmark assets.

In summary, the objective of this paper is to analyse the diversification benefits of the bulk of available cryptocurrencies on a factor level via a practical approach. Additionally, from the perspective of forecasting, we endeavour to reduce the estimation errors of expected returns during the out-of-sample asset allocation procedure. To achieve this, we collect over 2000 cryptocurrencies and form four grand categories of cryptocurrency factors based on state variables identified in seminal asset pricing literature[3] (e.g., size, momentum, volume, and volatility), which are further divided into 28 individual factors in a standard manner. Subsequently, we investigate the out-of-sample diversification benefits from the inclusion of one cryptocurrency factor each time to a stock–bond benchmark portfolio for investors with different risk-aversion levels via a range of known asset allocation strategies such as naïve allocation ($\frac{1}{N}$ to each asset), traditional mean-variance (Markowitz 1952), Bayes-Stein (Jorion 1985, 1986), Black-Litterman (Black and Litterman 1992), and those variants with new estimated expected returns produced by C-ENet under an expanding-window scheme. We use these asset allocation strategies because we intend to thoroughly examine the diversification benefits with popular portfolio optimisation tools and ensure that our findings are solid for any selected asset allocation approaches.

We emphasise that the purpose of this study is not to compare the performances of different portfolio optimisation approaches but to provide a broader horizon. We compare the out-of-sample performances of 28 factors to the stock–bond portfolio benchmark by using (i) the Sharpe ratio (SR) and (ii) the certainty-equivalent return (CER) with statistical tests, which ascertains economic and statistical significance (DeMiguel et al. 2009). To the best of our knowledge, we are the first to evaluate the diversification benefits of cryptocurrency factors, which are formed from the bulk of cryptocurrencies rather than only a few, via

[3] See, for instance, Fama and French (1992, 1993, 2008, 2015, 2018), Liu et al. (2022), Han et al. (2023), among others.

asset pricing methods, and we improve the existing asset allocation strategies by incorporating the forecasted returns produced by C-ENet.

Our contribution is twofold. First, we enrich the literature on the diversification benefits of cryptocurrencies from a more general perspective than current studies. Our study shows that incorporating size and momentum cryptocurrency factors to the stock–bond portfolio outperforms the combinations of a benchmark portfolio and other groups of cryptocurrency factors (e.g., volume and volatility) across all selected portfolio optimisation techniques for aggressive, moderate, and conservative investors. In particular, incorporating the factor portfolio constructed on market capitalisation—*MARCAP*—exhibits the most vital diversification benefits, evidenced by the statistically highest performance metrics. Similarly, the *RMOM3* factor, a cryptocurrency factor formed on past three-week risk-adjusted momentum (e.g., Sharpe ratio), exhibits solid outperformance. Thus, we argue that the superior risk-return trade-off generated by two significant factors (e.g., *MARCAP* and *RMOM3*) is the primary source of diversification benefits for most investors, and these two factors also play a crucial role in cryptocurrency asset pricing models, as documented in the literature (Liu et al. 2022; Han et al. 2023; Koutmos 2023).

Our findings imply that investors may include size and momentum to mimic cryptocurrency factors in their stock–bond portfolios to gain benefits. Furthermore, we find a negative relationship between the diversification benefits of cryptocurrencies and investor risk-aversion levels—risk-seeking (risk-averse) investors enjoy more (less) diversification benefits when adding cryptocurrency factors to a stock–bond portfolio. An explanation is that investors with high risk-aversion levels will allocate less weight to highly volatile assets, namely cryptocurrency factors, which is indicated by the risk-aversion coefficient (λ) inherent to the utility functions and the estimators of selected shrinkage methods such as the Bayes-Stein and the Black-Litterman. Our core findings are robust to (i) the inclusion of transaction costs, (ii) an alternative benchmark portfolio, and (iii) a rolling-window estimation approach.

Second, we combine forecast inputs with prevalent asset allocation strategies to tackle estimation errors during the portfolio optimisation procedure. We are the first to apply combination forecasting (C-ENet) to the Bayes-Stein and the Black-Littterman shrinkage estimators. The advantage of mixing the C-ENet and shrinkage methods is that C-ENet improves the efficiency of shrinkage methods by inputting a less biased return prediction. In addition, C-ENet does not neglect the information in predictors caused by over-shrinking compared to other linear prediction methods (e.g., lasso, elastic net, etc.), which guarantees the maximum efficiency of selected predictors. Our empirical results demonstrate that, on average, the asset allocation strategies adopting forecasted expected returns have an approximate 4% higher out-of-sample performance than strategies that ignore the contribution of machine-learning techniques on predicting returns, especially for the momentum factors. We suggest that the improvement from including C-ENet on the out-of-sample portfolio performance is due to (i) the adaptive selection of effective predictors in dynamic market states and (ii) the supremacy of combining univariate predictions. The values added by employing combination forecasting are also robust to various robustness tests.

Our study relates to the literature on equity and cryptocurrency anomalies. Extensive studies devoted to the search for equity anomalies that either provide explanatory power to asset pricing models or have abnormal returns, such as the Fama-French three-factor model (Fama and French 1993), Carhart four-factor model (Carhart 1997), the Hou-Xue-Zhang four-factor model (Hou et al. 2015), and the Fama-French five-factor model (Fama and French 2015). The common feature of previous studies is that they start by identifying factors with (statistically) abnormal returns and then construct asset pricing models. Similarly, Liu et al. (2022) propose that cryptocurrency factors such as market, size, and momentum effectively explain abnormal returns, and develop a crypto three-factor model following a formal asset pricing procedure. In contrast, our paper is motivated by the abnormal returns on cryptocurrency factors, and starts with forming cryptocurrency factors, then evaluates the diversification benefits of these factors. Our paper is also related to the literature on the diversification benefits of cryptocurrencies. Seminal studies focus mainly on Bitcoin and propose its diversification benefits in a domestic context (Briere et al. 2015; Guesmi et al. 2019; Platanakis and Urquhart 2020) and international context (Dyhrberg 2016; Kajtazi and Moro 2019). Nevertheless, most preceding studies concentrate on individual cryptocurrencies and evaluate in-sample performance with few asset allocation techniques for a relatively short sample range. We are motivated by the issues of limited sample size and optimisation approaches in the existing literature and aim to evaluate the out-of-sample diversification benefits of a vast range of cryptocurrencies at the factor-portfolio level, using various asset allocation strategies augmented by machine learning.

The rest of the paper is organised as follows. Section 2 discusses the methodologies of portfolio optimisation, machine learning forecasting, and their combined usage, as well as the out-of-sample performance metrics. Section 3 demonstrates the formation of cryptocurrency factors and summary statistics. Section 4 discusses the empirical results and the mechanisms of significant cryptocurrency factors. Section 6 re-examines the core findings through robustness tests. Lastly, Sect. 7 summarises the paper and points out the future research directions.

2 Methodology

We evaluate the diversification benefits of 28 cryptocurrency factors based on different categories (e.g., size, momentum, volume, and volatility) via various out-of-sample performance metrics of portfolios created with equities, bonds, risk-free assets, and one cryptocurrency factor, in comparison with the benchmark portfolio constructed with stocks, bonds, and risk-free assets. In light of seminal studies developing various portfolio optimisation approaches, we rigorously choose seven techniques with constraints among well-known methods, which are also employed in practice by fund managers. As one challenging task in portfolio optimisation task is the existence of estimation errors in the input parameters (DeMiguel et al. 2009), we employ shrinkage methods, the Bayes-Stein approach (Jorion 1986) and Black-Litterman model (Black and Litterman 1992), and their variants with forecasted returns of cryptocurrency factors through combination elastic net (C-ENet) Dong et al. (2022), to avoid the negative effects of estimation errors. The selected asset allocation strategies have different philosophies that lead to varying routes for assigning weights to assets, and we evaluate each factor across all chosen approaches to acquire robust outcomes.

2.1 Traditional portfolio optimisation techniques

We now briefly discuss the prominent theories regarding asset allocation: Markowitz mean-variance, Bayes-Stein, and Black-Litterman, and their advanced applications in portfolio optimisation with machine learning.

2.1.1 Markowitz mean-variance optimisation framework

The Markowitz mean-variance approach (Markowitz 1952) is the cornerstone of modern portfolio theory, and is still applied by both academics and practitioners after more than seventy years. The mean-variance approach (MV) provides an optimal asset allocation strategy that relies on the expected returns and the covariance matrix, by maximizing the quadratic utility function of investors with respect to risky assets' weights. To illustrate, we denote $\boldsymbol{x}$ as the N-dimensional vector of portfolio weights being allocated to N risky assets, and $1 - \mathbf{1}_N^T \boldsymbol{x}$ as the weight being assigned to a risk-free asset, where $\mathbf{1}$ is the N-dimensional vector of ones (DeMiguel et al. 2009). We calculate the relative weight vector for risky-only assets as follows:

$$\mathrm{w}_t = \frac{\boldsymbol{x}_t}{\left|\mathbf{1}_N^{\mathrm{T}} \boldsymbol{x}_t\right|}. \tag{1}$$

Further, $\boldsymbol{\mu}$ represents the N-dimensional expected excess returns on risky assets, and $\boldsymbol{\Sigma}$ is the $N \times N$ covariance matrix of returns. Thus, the MV optimisation problem is shown as follows:

$$\begin{aligned} &\max_{\boldsymbol{x}} \boldsymbol{x}^{\mathrm{T}} \boldsymbol{\mu} - \frac{\lambda}{2} \boldsymbol{x}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{x}, \\ &\text{s.t. } x_i \geq 0, \ \forall\, i, \end{aligned} \tag{2}$$

where λ is the risk aversion coefficient that represents the investors' risk levels, and we set $\lambda = 1, 3, 5$ to represent investors with different risk preferences. We also impose non-leverage and non-negative constraints ($x_i \geq 0$) on asset weights to avoid extreme influences caused by borrowing and short selling (e.g., impractically allocating significantly negative weights to some assets). The non-negative constraints can help MV produce a covariance matrix that has similar effects to a shrinkage form of that, to tackle estimation errors during the out-of-sample phase (Jagannathan and Ma 2003). Nevertheless, the classic MV framework ignores the impact of estimation errors, and takes the sample mean ($\hat{\boldsymbol{\mu}}$) and sample covariance ($\hat{\boldsymbol{\Sigma}}$) of the portfolio as inputs, resulting in poor out-of-sample performance over time (DeMiguel et al. 2009). To resolve the unstable performance of the MV model, this brings us to another strand of literature relating to improving the estimates of sample mean and covariance.

2.1.2 The Bayes-Stein approach

The purpose of the Bayes-Stein model (BS) is to address the estimation error by developing a new set of mean and covariance estimates (e.g., $\boldsymbol{\mu}_{BS}$ and $\boldsymbol{\Sigma}_{BS}$) that can be used within the MV framework (Jorion 1986). This technique has been widely applied (see Board and Sutcliffe 1994; Garlappi et al. 2007; DeMiguel et al. 2009; Platanakis et al. 2021; Huang et al. 2022; Gounopoulos et al. 2022). The Bayes-Stein model weakens the influences of estimation risk by pushing the expected returns ($\boldsymbol{\mu}$) to the global mean (μ_G) to minimize the impact of estimation errors. The Bayes-Stein shrinkage method calculates the shrunk mean return vector ($\boldsymbol{\mu}_{BS}$) and covariance matrix ($\boldsymbol{\Sigma}_{BS}$) as follows:

$$\boldsymbol{\mu}_{BS} = (1 - g)\boldsymbol{\mu} + g\mu_G \mathbf{1}, \tag{3}$$

where g $(0 \leq g \leq 1)$ is the shrinkage factor and is computed as (Jorion 1985):

$$g = \frac{N+2}{(N+2) + T(\boldsymbol{\mu} - \mu_G \mathbf{1})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - \mu_G \mathbf{1})}, \tag{4}$$

where N is the number of risky assets, T is the in-sample estimation period. Moreover, the covariance matrix ($\boldsymbol{\Sigma}_{BS}$) under Bayes-Stein framework is given by:

$$\boldsymbol{\Sigma}_{BS} = (\frac{T+\varphi+1}{T+\varphi})\boldsymbol{\Sigma} + \frac{\varphi}{T(T+\varphi+1)} \frac{\mathbf{1}\mathbf{1}^{\mathrm{T}}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{1}}, \tag{5}$$

where φ denotes the prior precision:

$$\varphi = \frac{N+2}{(\boldsymbol{\mu} - \mu_G \mathbf{1})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \mu_G \mathbf{1})}. \tag{6}$$

With the Bayes-Stein shrinkage estimates of mean returns ($\boldsymbol{\mu}_{BS}$) and covariance matrix ($\boldsymbol{\Sigma}_{BS}$), we can apply MV approach to solve the optimisation task with the same constraints illustrated in Eq. 2.

2.1.3 The Black-Litterman model

The Black-Litterman model (BL) handles the negative effect of estimation errors during the portfolio construction process by involving investors' views on asset returns and introducing a reference portfolio for obtaining the neutral (or implied) returns (Black and Litterman 1992). The model has been applied in both academia and industry (see., Da Silva et al. 2009; Bessler and Wolff 2015; Chen and Lim 2020; Platanakis and Urquhart 2020; Platanakis et al. 2021).

According to the Black and Litterman (1992), the column vector of implied excess returns ($\boldsymbol{H}$) is computed as:

$$\boldsymbol{H} = \lambda \boldsymbol{\Sigma} \boldsymbol{x}^{\text{Reference}}, \tag{7}$$

where $\boldsymbol{x}^{\text{Reference}}$ is a column vector that consists of the weights of the reference portfolio. We set $\boldsymbol{x}^{\text{Reference}}$ to the equally weighted portfolio (i.e., $\boldsymbol{x}^{\text{Reference}} = \frac{1}{N}\mathbf{1}$) in accordance with Platanakis and Sutcliffe (2017), Platanakis et al. (2018), and Platanakis and Urquhart (2020), which assume that the estimates of asset returns have a large portion of estimation error. The Black-Litterman approach calculates the posterior estimate of mean returns ($\boldsymbol{\mu}_{BL}$) as:

$$\boldsymbol{\mu}_{BL} = [(c\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}]^{-1}[(c\boldsymbol{\Sigma})^{-1}\boldsymbol{H} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{Q}], \tag{8}$$

where $\boldsymbol{P}$ is a diagonal matrix with ones in its leading diagonal and zeros elsewhere, $\boldsymbol{Q}$ represents the column vector comprising the investors' views (e.g., subjective returns), and c denotes the reliability coefficient for the vector of implied excess returns ($\boldsymbol{H}$). To apply in practice, we set $c = 0.1625$ and use the mean returns to approximate the investors' subjective returns ($\boldsymbol{Q}$), which is in line with Platanakis and Urquhart (2020). $\boldsymbol{\Omega}$ is regarded as the level of confidence in investors' views (Chen and Lim 2020), and is calculated as:

$$\boldsymbol{\Omega} = \frac{1}{\delta}\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^{\mathrm{T}}, \tag{9}$$

where we set $\frac{1}{\delta} = 1$ following Meucci (2010) and Platanakis and Urquhart (2020). Subsequently, we calculate the posterior covariance of the Black-Litterman model ($\boldsymbol{\Sigma}_{BL}$) as follows (Satchell and Scowcroft 2000; Platanakis and Urquhart 2020):

$$\boldsymbol{\Sigma}_{\mathrm{BL}} = \boldsymbol{\Sigma} + \left[(c\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}. \tag{10}$$

We apply the MV technique (see Eq. 2) to solve the assets allocation task by using the Black-Litterman estimates ($\boldsymbol{\mu}_{BL}$ and $\boldsymbol{\Sigma}_{\mathrm{BL}}$) as inputs.

2.1.4 Naïve portfolio allocation

The naïve asset allocation rule (EW or 1/N) is an effective and simple asset allocation strategy that assigns equal weights ($W_{1/N} = \frac{1}{N}$) to the N risky assets (DeMiguel et al. 2009). We use EW as a parameter-free allocation strategy to take into account the non-technical investors' practical asset allocation actions.

2.2 Novel portfolio optimisation techniques with machine learning

The development of the Bayes-Stein and the Black-Litterman techniques helps to mitigate estimation risks, and has facilitated practical portfolio optimisation. Our aim has been to alleviate further the negative effects of estimation errors in the input parameters by using a machine learning technique—combination elastic net (Rapach and Zhou 2020)—to predict returns with greater accuracy. We focus on forecasting returns rather than covariances because return prediction plays a more critical role in the out-of-sample performance of portfolio optimisation techniques (Chopra and Ziemba 1993). In this section, we introduce advanced machine learning techniques suitable to generate estimates of expected returns for the MV framework in a linear way (e.g., combination elastic net).

2.2.1 Forecasting of expected returns

Standard ordinary least squares regression (OLS) is prone to overfitting for in-sample data, producing deficient out-of-sample predictions (Huang et al. 2022). To solve this problem, Tibshirani (1996) develops a Lasso shrinkage regression model that shrinks certain slope coefficients towards zero so as to improve the model's interpretation and execute sparse estimation. However, a defect of Lasso is that it tends to choose one feature arbitrarily from a set of highly correlated features and shrinks the other features' coefficients. Thus, Zou and Hastie (2005) refine the technique by introducing a combination of Lasso (l_1 norm) and Ridge (l_2 norm) in a new model, elastic net (ENet), which employs the aggregated effect of both techniques. Since our aim is to use ENet to forecast the future returns of cryptocurrency factors with current predictors, we have the following objective function of ENet:

$$\min_{\alpha,\beta_1,\ldots,\beta_n} \left\{ \sum_{t=1}^{T-1} \left(r_{t+1} - \beta_0 - \sum_{i=1}^{n} \beta_i x_{i,t} \right)^2 + \gamma \sum_{i=1}^{n} \left[\frac{1}{2}(1-\delta)\beta_i^2 + \delta \left| \beta_i \right| \right] \right\}, \tag{11}$$

where T is the sample size, r_{t+1} is the asset's expected return at $t+1$, β_i is the i^{th} predictor's loading, γ ($\gamma \geq 0$) is a hyperparameter that controls the extent of shrinkage of regularized term, and δ ($0 \leq \delta \leq 1$) is another hyperparameter that fuses the effects of Lasso and

Ridge components in the penalty term (Rapach and Zhou 2020). In particular, we use the Akaike information criterion (AIC), which compares the model performance and tackles overfitting and underfitting issues, then decide the optimal γ value. To facilitate the out-of-sample performance, Rapach et al. (2010) propose the concept of 'forecast combination' that aggregates the predictive information from numerous predictors to avoid overfitting, which exploits the diversification of forecasts under different forecasting states. With the increases in computational power available, academics have started to combine machine learning techniques to reach more accurate forecasts, and the combination of elastic net (C-ENet) has been developed and applied to finance (Dong et al. 2022).

C-ENet is based on the sense that combining the individual forecasts of single predictors is better than the forecast from putting all predictors in one regression. To illustrate, Welch and Goyal (2008) evaluate the forecasting performances of a large number of effective predictors suggested by the literature, and find that the predictions produced by a multivariate regression cannot beat the naïve sample mean of individual forecasts during the out-of-sample phase. One possible reason is that the performance of those predictors varies with time, whereby favourable and poor performances alternate (Rapach et al. 2010). Subsequently, Rapach et al. (2010) resolve such issues by combining the forecasts of individual predictors, which provides considerable out-of-sample performance. Rapach and Zhou (2020) employ the concept of 'combination' to ENet, and show the effectiveness of C-ENet. Before illustrating the C-ENet, we will define simple predictive regression as it is the basis of C-ENet forecasting.

A common form of stock index univariate predictive regression is shown as (Rapach and Zhou 2020):

$$r_t = \alpha + \beta x_{j,t-1} + \varepsilon_t, \tag{12}$$

where r_t is the stock index excess return at t, $x_{j,t-1}$ is the predictor j at $t-1$, and ε_t is the error term. To apply Eq. 12 for the out-of-sample prediction, we have the following form:

$$\hat{r}^{(j)}_{t+1|t} = \hat{\alpha}^{(j)}_{1:t} + \hat{\beta}^{(j)}_{1:t} x_{j,t}, \tag{13}$$

where $\hat{\alpha}^{(j)}_{1:t}$ and $\hat{\beta}^{(j)}_{1:t}$ are the OLS estimates of α and β using the first to t^{th} sample. We refer to Eq. 13 as estimated univariate predictive regression.

C-ENet comprises three steps, and we denote t_1 as the size of the in-sample period. *Step I*, we calculate the recursive estimated univariate predictive regression over the holdout out-of-sample phase:

$$\hat{r}^{(j)}_{s|s-1} = \hat{\alpha}^{(j)}_{1:s-1} + \hat{\beta}^{(j)}_{1:s-1} x_{j,s-1}, \tag{14}$$

for $s = t_1 + 1, \dots, t$ and $j = 1, \dots, J$. The holdout out-of-sample period is the period that lies between the in-sample period and the out-of-sample period, and the purpose of this interval is to train the C-ENet. We use lagged one-week returns to lagged five-week returns as predictors to apply predictive regression.

Step II, we use the forecasted returns ($\hat{r}_{s|s-1}$) from Eq. 14 as predictors, and run the Granger and Ramanathan (1984) regression to obtain the estimates of parameters through ENet during the holdout out-of-sample phase:

$$r_s = \eta + \sum_{j=1}^{J} \theta_j \hat{r}^{(j)}_{s|s-1} + \varepsilon_s. \tag{15}$$

We denote $\mathcal{J}_t \subseteq \{1, \ldots, J\}$ as the index subset of the whole univariate predictive regression predictions filtered by ENet, where we follow Rapach and Zhou (2020) to impose non-negative restrictions on θ_j ($\theta_j \geq 0$) to ensure the economic significance that forecasted returns ($\hat{r}^{(j)}_{s|s-1}$) are positively correlated to the practical returns (r_s).

Step III, we estimate C-ENet forecasts by exploiting the simple average, with both the index subset acquired from *step II* and the one-period-ahead return forecasting produced by the estimated univariate predictive regression (e.g., $\hat{r}^{(j)}_{t+1|t}$ in Eq. 13) as inputs:

$$\hat{r}^{\text{C-ENet}}_{t+1|t} = \frac{1}{\left|\mathcal{J}_t\right|} \sum_{j \in \mathcal{J}_t} \hat{r}^{(j)}_{t+1|t}, \tag{16}$$

where $\left|\mathcal{J}_t\right|$ is the cardinality of $\mathcal{J}_t$, and *step III* is being applied after the holdout out-of-sample period.

We next gather the forecasted returns ($\hat{r}^{\text{C-ENet}}_{t+1|t}$) by C-ENet to replace the prevailing expected returns when we apply the MV, Bayes-Stein, and Black-Litterman techniques when solving the optimal asset allocation tasks. Since we aim to evaluate the diversification benefits of cryptocurrency factors and forecasting studies on benchmark assets are in the literature (Neely et al. 2014; Huang et al. 2014; Rapach et al. 2013), we focus on forecasting cryptocurrency factors rather than the other benchmark assets.

2.2.2 Markowitz mean-variance optimisation framework incorporating machine learning

For the MV approach, we replace the N-dimensional expected excess returns on risky assets ($\boldsymbol{\mu}$) with a new excess return vector ($\boldsymbol{\mu}_{ml}$) produced by C-ENet. Specifically, we use prevailing expected returns to approximate the future returns on benchmark assets but replace the prevailing expected returns of cryptocurrency factors with the predicted returns generated by C-ENet. Subsequently, we optimize Eq. 2 to estimate the optimal weight vector $\boldsymbol{x}$ with the input parameter $\boldsymbol{\mu}_{ml}$. We refer to the mean-variance approach with machine learning as MV-ML.

2.2.3 The Bayes-Stein approach with machine learning

Recall that the Bayes-Stein Shrunk expected return vector ($\boldsymbol{\mu}_{BS}$) is a weighted sum of prevailing expected returns ($\boldsymbol{\mu}$) and global mean return (μ_G). We change all prevailing expected returns ($\boldsymbol{\mu}$) to $\boldsymbol{\mu}_{ml}$ from Eqs. 3 to 6, to construct a new set of inputs $\boldsymbol{\mu}_{BS-ML}$ and $\boldsymbol{\Sigma}_{BS-ML}$ for MV optimisation. We refer to Bayes-Stein with machine learning as BS-ML.

2.2.4 The Black-Litterman model with machine learning

The Black-Litterman model reduces estimation errors by including investors' subjective returns ($\boldsymbol{Q}$) during the portfolio formation period. To apply machine learning technique, we replace the $\boldsymbol{Q}$ with $\boldsymbol{\mu}_{ml}$ in Eq. 8, to formulate a new machine learning based forecast $\boldsymbol{\mu}_{BL-ML}$, while the $\boldsymbol{\Sigma}_{BL}$ remains the same. We then use the new pair of inputs to apply MV optimisation. We refer to Black-Litterman with machine learning as BL-ML.

Fig. 1 Procedure of applying machine learning to traditional approaches. This figure depicts the process of replacing sample mean with forecasted returns produced by machine learning techniques when optimising the portfolio weights

Step 1: Mean return forecasting for *t+1* via C-ENet at *t*

↓

Step 2: Calculation of input parameters with forecasted returns for MV, BS, and BL

↓

Step 3: Solve optimisation tasks to obtain optimal weights for *t+1*

2.2.5 The procedure of applying machine learning based portfolio optimisation techniques

To visualise the combination of traditional asset allocation approaches and machine learning, we document the process in Fig. 1. First, we predict the returns for cryptocurrency factor portfolios at $t + 1$ based on the information up to time t by employing C-ENet on lagged predictors. Second, we replace the parameter $\boldsymbol{\mu}$ with $\boldsymbol{\mu}_{ml}$ and calculate the expected returns for MV, BS, and BL to form a set of new inputs for mean-variance optimisation. Lastly, we construct portfolios for $t+1$ with optimal weights estimated at t and repeat these three steps until the end of the entire sample.

2.3 Performance metrics

We use the Sharpe ratio (SR) and certainty-equivalent return (CER) to evaluate the out-of-sample diversification benefits of cryptocurrency factor portfolios. We employ an expanding window scheme to implement several asset allocation strategies.[4] To illustrate, assuming there is a T-week-long dataset for cryptocurrency factors, we denote M as the length of the initial in-sample period.[5] Hence, for the first out-of-sample estimation week t ($t = M + 1$), we employ the in-sample data (M weeks) to estimate the parameters being used in different aforementioned portfolio optimisation techniques. As a result, we can get the optimal portfolio weights with respect to different strategies for week t, thereby obtaining the portfolio return in week t. Subsequently, we continue the expanding-window approach by enlarging the in-sample estimation period by one more week ($M_{new} \leftarrow M + 1$), and estimate related portfolio parameters to conduct asset allocation in the following week t ($t = M_{new} + 1$). This procedure is repeated until the bottom of the entire sample.

We compute two risk-adjusted metrics, Sharpe ratio ($\widehat{SR}$) and certainty-equivalent return ($\widehat{CER}$), for the out-of-sample returns produced by each asset allocation strategy k based on cryptocurrency factors. $\widehat{SR}_k$ is defined as the expected return of out-of-sample excess returns ($\hat{\mu}_k$) divided by the out-of-sample standard deviation ($\hat{\sigma}_k$):

[4] The results by using the rolling window approach are reported in robustness checks.

[5] We set M to 100 (almost 25% of the dataset) for each cryptocurrency factor because (i) using at least 100 observations is a necessary manner to maintain the training process of selected machine learning techniques (Dong et al. 2022), and (ii) the maximum data range for each cryptocurrency factor is 392, whereby we need a relatively large test set to investigate the out-of-sample performance of enhanced portfolios.

Table 1 Summary of selected asset allocation methods

Number	Model	Abbreviations
1	Equally weighted	EW
2	Mean-variance with short sale constraints	MV
3	Bayes-Stein with short sale constraints	BS
4	Black-Litterman with short sale constraints	BL
5	Mean-variance with machine learning and short sale constraints	MV-ML
6	Bayes-Stein with machine learning and short sale constraints	BS-ML
7	Black-Litterman with machine learning and short sale constraints	BL-ML

This table reports the chosen asset allocation techniques and their abbreviations in this paper

$$\widehat{SR}_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k}. \tag{17}$$

Moreover, we also assess whether the out-of-sample Sharpe ratios of the enhanced portfolios with cryptocurrency factors statistically outperform the benchmark portfolio by evaluating the test statistics proposed by Jobson and Korkie (1981).[6] On the other hand, $\widehat{CER}$ represents the riskless return that an investor is willing to take by forgoing the returns on risky assets:

$$\widehat{CER}_k = \hat{\mu}_k - \frac{\lambda}{2}\hat{\sigma}_k^2. \tag{18}$$

We follow the design of DeMiguel et al. (2009) to evaluate the statistical significance of $\widehat{CER}_k$ compared to the benchmark.[7]

Lastly, we summarise the asset allocation strategies we use and provide their abbreviations in Table 1.

[6] Particularly, we denote i and n as i^{th} cryptocurrency factor and the benchmark, respectively, with their sample means ($\hat{\mu}_i$ and $\hat{\mu}_n$), sample standard deviations ($\hat{\sigma}_i$ and $\hat{\sigma}_n$), and covariance ($\hat{\sigma}_{i,n}^2$).

$$\hat{z}_{SR} = \frac{\hat{\sigma}_n\hat{\mu}_i - \hat{\sigma}_i\hat{\mu}_n}{\sqrt{\vartheta}}, \quad \text{where } \vartheta = \frac{1}{T-M}\left(2\hat{\sigma}_i^2\hat{\sigma}_n^2 - 2\hat{\sigma}_i\hat{\sigma}_n\hat{\sigma}_{i,n} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_n^2 + \frac{1}{2}\hat{\mu}_n^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_n}{\hat{\sigma}_i\hat{\sigma}_n}\hat{\sigma}_{i,n}^2\right).$$

[7] We define $v = (\mu_i, \mu_n, \sigma_i^2, \sigma_n^2)$ as a vector, and compute the test statistics for $\widehat{CER}_k$ as follows:

$$\hat{z}_{CER} = f(v)/\Phi \quad \text{where } f(v) = \left(\mu_i - \frac{\lambda}{2}\sigma_i^2\right) - \left(\mu_n - \frac{\lambda}{2}\sigma_n^2\right), \text{ and } \Phi = f'(v)^{\mathrm{T}}\begin{pmatrix} \sigma_i^2 & \sigma_{i,n} & 0 & 0 \\ \sigma_{in} & \sigma_n^2 & 0 & 0 \\ 0 & 0 & 2\sigma_i^4 & 2\sigma_{i,n}^2 \\ 0 & 0 & 2\sigma_{i,n}^2 & 2\sigma_n^4 \end{pmatrix} f'(v)$$

3 Data and cryptocurrency factors

To form the cryptocurrency factors, we follow the empirical design of Liu et al. (2022) and Han et al. (2023) and consider a large number of cryptocurrency data from Coingecko.com, which is a comprehensive platform providing cryptocurrency-related information such as prices, volumes, and market capitalizations. The cryptocurrency data comprises over 2,000 cryptocurrencies from January 2014 to June 2021. Given that our dataset has 392 (weekly) observations over the period January 2014 to June 2021, we treat the first 100 observations as the in-sample data, then the second 100 observations are regarded as holdout out-of-sample data for forecasting purposes, and the remaining observations are the out-of-sample test set to evaluate the risk-adjusted performances under the expanding-window scheme. For the components of the benchmark portfolio, we collect the data of S &P 500 and 10-year T-bond from CRSP to approximate the returns on equities and bonds, respectively. The proxy of risk-free rates (i.e., one-month T-bill) are gathered from Kenneth French website.[8]

We divide the cryptocurrency factors into four categories—size, momentum, volume, and volatility—in an analogous way to the literature on the equity market, and the four large categories are further divided into 28 cryptocurrency factors (Han et al. 2023) based on distinct features and holding periods, in order to reveal hidden insights from cryptocurrencies.[9] In addition, we choose 28 cryptocurrency factors analogously to equity factors (Chen and Lim 2020), which are based only on price and market information as these are the only public data for cryptocurrencies. Moreover, the selected 28 cryptocurrency factors are proven to represent most characteristics of the cryptocurrency market (Liu et al. 2022). We document the detailed definition and abbreviation of each cryptocurrency factor in Table 2.

To establish 28 cryptocurrency factors used in this study, we sort all cryptocurrencies into quintiles in ascending order based on the factors considered in each week t, and we next track the returns on the factors in the following week $(t + 1)$. Subsequently, we form the returns on quintile portfolios at $t + 1$ by calculating the market-capitalization-weighted return of all available cryptocurrencies at $t + 1$, and repeat this procedure until the end of the entire sample to form the complete quintile portfolios. The excess returns of

[8] see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[9] Unlike studies investigating ordinary asset classes (e.g., equities, mutual funds, etc.), one common fact for cryptocurrency research is that cryptocurrencies have a relatively short sample period in which abundant infant cryptocurrencies emerge frequently. Given the unique characteristics of cryptocurrencies, we observe that the aggregated market share of prevalent cryptocurrencies such as Bitcoin and Ethereum dropped from roughly 90% in January 2014 to 63% in June 2021 (see, e.g., CoinMarketCap). Consequently, the dominance of the preceding two coins facilitates the literature that induces stylised facts for the overall cryptocurrency market via popular cryptocurrencies at the initial development stage(see, e.g., Gandal et al. 2018; Borri 2019; Liu and Tsyvinski 2020). On the contrary, recent studies proceed to establish stylised facts by focusing on an enormous amount of cryptocurrencies because the newly emerged cryptocurrencies are shaking up the dominance of giant coins such as Bitcoin and Ethereum (Liu et al. 2022; Han et al. 2023; Cai and Zhao 2024). To this end, we argue that dissimilar starting dates for cryptocurrencies are trivial to discover the hidden patterns in the cryptocurrency market since the rapid iteration (e.g., the frequent occurrence of infant and defunct cryptocurrencies) is a plain feature, and we might face survivorship bias if we do not consider the small-weight coins. More importantly, the negative mean return and skewness of the AGE factor portfolio in Table 3 also indicate that the portfolios formed on age cannot generate statistically significant returns. Thus, dissimilar start/end dates for cryptocurrencies are insignificant.

Table 2 The cryptocurrency factors

Category	Factor used	Definition
Size	MARCAP	Log last day market capitalization in the portfolio formation week
Size	LPRC	Log last day price in the portfolio formation week
Size	Age	The number of existent weeks that listed on Coinmarketcap.com
Momentum	MOM1	One-week momentum
Momentum	MOM2	Two-week momentum
Momentum	MOM3	Three-week momentum
Momentum	MOM4	Four-week momentum
Momentum	MOM8	Eight-week momentum
Momentum	MOM26	Twenty-six-week momentum
Momentum	RMOM1	One-week risk-adjusted momentum based on the Sharpe ratio
Momentum	RMOM2	Two-week risk-adjusted momentum based on the Sharpe ratio
Momentum	RMOM3	Three-week risk-adjusted momentum based on the Sharpe ratio
Momentum	RMOM4	Four-week risk-adjusted momentum based on the Sharpe ratio
Momentum	RMOM8	Eight-week risk-adjusted momentum based on the Sharpe ratio
Momentum	RMOM26	Twenty-six-week risk-adjusted momentum based on the Sharpe ratio
Volume	VOL	Log average daily volume in the portfolio formation week
Volume	VOLPRC	Log average daily volume times price in the portfolio formation week
Volume	VOLSCALE	Log average daily volume times price then divided by market capitalization in the portfolio formation week
Volatility	RETVOL	The standard deviation of daily returns in the portfolio formation week
Volatility	RETSKEW	The skewness of daily returns in the portfolio formation week
Volatility	RETKURT	The kurtosis of daily returns in the portfolio formation week
Volatility	MAXRET	The maximum daily return of the portfolio formation week
Volatility	STDPRCVOL	Log standard deviation of dollar volume in the portfolio formation week
Volatility	MEANABS	The mean absolute daily return divided by dollar volume in the portfolio formation week
Volatility	BETA	The regression coefficient of $\beta_{MKT,i}$ in the one-factor model, $R_i - R_f = \alpha^i + \beta_{MKT,i}MKT + \epsilon_i$.The model is estimated using the daily returns of the previous 365 days before formation week
Volatility	$BETA^2$	Beta squared

Table 2 (continued)

Category	Factor used	Definition
Volatility	IDIOVOL	The IDIOVOL volatility is measured as the standard deviation of the residuals after estimating the one-factor model, $R_i - R_f = \alpha^i + \beta_{MKT,i}MKT + \epsilon_i$. The model is estimated using the daily returns of the previous 365 days before the formation week.
Volatility	DELAY	The improvement of R^2 in $R_i - R_f = \alpha^i + \beta_{MKT,i}MKT + \beta_{MKT_{-1},i}MKT_{-1} + \beta_{MKT_{-2},i}MKT_{-2} + \epsilon_i$compared to a regression that only uses MKT, where MKT_{-1} and MKT_{-2} are lagged one- and two-day market index returns. The model is estimated using the daily returns of the previous 365 days before formation week.

We replicate the creation of cryptocurrency factors in the studies of Liu et al. (2022) and Han et al. (2023), and report the categories, abbreviations, and definitions for cryptocurrency factors in this table. For instance, the procedure for creating a portfolio based on MARCAP is that each week all cryptocurrencies were sorted into quintiles using market capitalization, then we track the returns in the following week. All the portfolios were rebalanced weekly

Table 3 Summary statistics

Benchmark and factors	Mean	Median	Std	Skewness	Kurtosis	J-B test	*p* value
S &P 500	0.0025	0.0042	0.0236	−0.644	13.7708	1921.937	0.001
One-month T-bill	0.0001	0.0001	0.0002	0.8154	2.1456	55.3662	0.001
Ten-year T-bond	0.0008	0.0009	0.0082	0.5974	8.6441	543.6354	0.001
MARCAP	−0.0269	0.0213	0.1046	0.8481	6.3198	226.43	0.001
LPRC	−0.0215	−0.0016	0.1675	3.3295	23.5690	7615.15	0.001
AGE	−0.0272	0.0008	0.2193	−2.6346	16.8386	3572.28	0.001
MOM1	0.0359	0.0226	0.1876	1.6170	15.5703	2744.66	0.001
MOM2	0.0349	0.0246	0.1790	1.3031	13.8838	2035.29	0.001
MOM3	0.0262	0.0143	0.2063	1.1433	18.3854	3921.41	0.001
MOM4	0.0137	0.0067	0.1477	0.8768	7.3223	351.75	0.001
MOM8	−0.0072	−0.0060	0.1628	1.7656	17.9600	3780.36	0.001
MOM26	−0.0177	−0.0069	0.1768	−1.7661	18.7137	3955.79	0.001
RMOM1	0.0268	0.0133	0.1521	1.8871	20.4177	5174.58	0.001
RMOM2	0.0264	0.0254	0.1667	0.8223	22.7316	6337.96	0.001
RMOM3	0.0302	0.0273	0.1780	2.8670	27.9396	10614.22	0.001
RMOM4	0.0228	0.0158	0.1424	0.4663	7.4890	339.83	0.001
RMOM8	0.0035	0.0096	0.1425	0.4462	7.6440	357.82	0.001
RMOM26	−0.0110	−0.0036	0.1530	−0.4496	9.7310	703.26	0.001
VOL	−0.0016	0.0040	0.2256	−9.5698	154.7759	381261.46	0.001
VOLPRC	−0.0236	0.0063	0.1605	3.7847	28.7442	11731.03	0.001
VOLSCALE	−0.0117	−0.0025	0.1412	2.7424	18.8200	4567.44	0.001
RETVOL	0.0014	−0.0107	0.2007	1.6729	18.0143	3854.99	0.001
RETSKEW	0.0121	0.0050	0.1527	1.7954	20.9129	5437.60	0.001
RETKURT	0.0036	0.0044	0.1270	0.2689	7.1715	288.21	0.001
MAXRET	0.0201	0.0078	0.1464	0.9446	5.8347	189.06	0.001
STDPRCVOL	−0.0222	0.0049	0.1587	3.7806	29.2807	12183.67	0.001
MEANABS	0.0174	−0.0055	0.1845	4.2588	30.2452	13275.29	0.001
BETA	−0.0029	0.0013	0.1492	0.1955	13.8392	1661.68	0.001
BETA2	−0.0014	0.0019	0.1296	−0.3705	20.3684	4268.71	0.001
IDIOVOL	−0.0037	0.0123	0.1298	−0.8429	21.3632	4803.19	0.001
DELAY	−0.0115	0.0169	0.1239	0.2481	8.3735	411.33	0.001

This table reports the summary statistics, such as mean, median, standard deviation, skewness, kurtosis, and Jarque-Bera test with its corresponding *p* value, for benchmarks and each of the 28 cryptocurrency factors

cryptocurrency factors are the differences between the fifth and first quintiles, constructed via buying the fifth quintile and shorting the first quintile (e.g., fifth minus first quintile). For the case that the difference between two quintile portfolios is negative, we simply take reverse action (e.g., first minus fifth quintile), as we value the abnormal returns generated by quintile portfolios. To gain better insights into the dataset, we demonstrate the summary statistics for the cryptocurrency factors and benchmark assets in Table 3.

To digest the general characteristics of the 28 cryptocurrency factor portfolios, we calculate and present the expected value of the first four moments—the mean, standard deviation, skewness, and kurtosis—as well as the median, which can deliver robust central tendency when underlying assets are highly volatile, across all cryptocurrency factor portfolios. In particular, we use absolute values to measure the expected values for the first four moments because the 28 factor portfolios may offset each other's effect, so the trends of moments would be underestimated. We identify that the expected mean, median, standard deviation, skewness, and kurtosis values are 0.0166, 0.0101, 0.1613, 1.8821, and 21.8664, respectively, which are several orders of magnitude larger than those of traditional assets such as stocks, bonds, and risk-free assets. Hence, the high returns and risks of cryptocurrency factor portfolios motivate us to assess their alternative usages, including the possibility of providing diversification benefits, among others.

4 Empirical results

In this section, we discuss the out-of-sample diversification benefits by adding cryptocurrency factors to the benchmark portfolio (e.g., equities, bonds, and risk-free assets) via various asset allocation approaches with different investors' risk preferences. For each cryptocurrency factor, we calculate the Sharpe ratio and certainty-equivalent return of the enhanced portfolios with significance level for both traditional and novel asset allocation strategies listed in Table 1, across three levels of risk preference (e.g., $\lambda = 1, 3, 5$). Although numerous studies (e.g., Bessler and Wolff 2015; Platanakis and Urquhart 2020) assert that the optimal λ values for conservative, moderate and aggressive investors are 10, 5, and 2, respectively, from the perspective of assessing the diversification benefits of commodities, we shorten the interval from [2, 10] to [1, 5] by assuming that investors who participate in cryptocurrency markets are relatively more risk-seekers than investors who take other asset classes into account. We document the empirical results for $\lambda = 1, 3, 5$, mimicking the aggressive, moderate and conservative investors (Huang et al. 2022), in Table 4, 5 and 6, respectively. In each table, the bold row illustrates the risk-adjusted returns for the benchmark portfolio, and other rows represent the cryptocurrency factor being evaluated in terms of its diversification benefits, while the columns illustrate the performances across different asset allocation techniques. We detail the corresponding *p*-values and their calculation in the Appendix for brevity. Further, to illustrate the diversification benefits of adding cryptocurrency factors, we graphically display the dynamic out-of-sample metrics across the entire investment horizon in Figs. 2 and 3.

4.1 Out-of-sample metrics for aggressive investors

Table 4 depicts the out-of-sample performance grouped by SRs (Panel A) and CERs (Panel B) for the benchmark portfolio and enhanced portfolios with cryptocurrency factors for aggressive investors ($\lambda = 1$), where the first row shows the risk-adjusted returns for benchmark portfolio and the rest are those of cryptocurrency factors. Panel A reports the Sharpe ratios. For the traditional portfolio approaches, we find that five portfolios incorporating cryptocurrency factors outperform the benchmark portfolio when employing EW, while the number of statistical outperformance when using MV, BS, and BL ranges from 7 to 10. Regarding the novel portfolio approaches, the lower bound of the outperformed portfolios

Table 4 Out-of-sample risk-adjusted metrics for $\lambda = 1$

	Panel A: SR							Panel B: CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Bench	**0.0831**	**0.0912**	**0.0622**	**0.0730**	**0.0912**	**0.0622**	**0.0730**	**0.0008**	**0.0022**	**0.0013**	**0.0015**	**0.0022**	**0.0013**	**0.0015**
MARCAP	0.4113***	0.3732**	0.3732**	0.4055***	0.3666**	0.3645**	0.39***	0.0076**	0.0257***	0.0257***	0.0148***	0.0251***	0.0249***	0.0152***
LPRC	0.2009	0.1957	0.1892*	0.1932*	0.223*	0.1932	0.1989*	0.0044	0.009**	0.0065**	0.0062**	0.0126**	0.0091**	0.0068**
AGE	0.0251	0.0154	0.0377	0.0422	0.0523	0.0823	0.0485	0.0003	−0.0008	0.0007	0.0009	0.0013	0.0021	0.0011
MOM1	0.233*	0.2250	0.2311*	0.2306**	0.2395*	0.2419*	0.235**	0.0059*	0.0157**	0.0129**	0.0089**	0.0166**	0.0135***	0.009**
MOM2	0.2515*	0.2334	0.2332*	0.2519**	0.2422*	0.244*	0.2539**	0.0068*	0.0187**	0.0152**	0.0103***	0.0193**	0.0169**	0.0106***
MOM3	0.1708	0.1733	0.1761	0.1728	0.1836	0.1775*	0.1753	0.0041	0.0075*	0.0059*	0.0058*	0.0074*	0.0056**	0.0058*
MOM4	0.1471	0.1393	0.1380	0.1493	0.1561	0.1564	0.1524	0.0034	0.0054	0.0041	0.0048	0.0060	0.0046	0.0049
MOM8	−0.0123	0.102***	0.0784	0.0241	0.102***	0.077*	0.0277	−0.0008	0.0025***	0.0017*	0.0002	0.0025***	0.0016*	0.0004
MOM26	−0.0181	0.1056***	0.0991**	0.0339	0.1056***	0.0831*	0.0352	−0.0019	0.0027***	0.0024**	0.0006	0.0027***	0.0019**	0.0006
RMOM1	0.1986	0.1869	0.1839	0.1928	0.1822	0.1824	0.1924*	0.0048	0.0131*	0.0103*	0.0073**	0.0122*	0.0096*	0.0072**
RMOM2	0.2465*	0.2303	0.2457*	0.2492**	0.2360	0.2545*	0.2527**	0.0056	0.0136**	0.0105**	0.0083**	0.0149**	0.0121**	0.0085**
RMOM3	0.2459**	0.2377*	0.238**	0.2372**	0.2607**	0.2641**	0.245**	0.0062**	0.0155**	0.0119***	0.009***	0.0163***	0.0127***	0.0091***
RMOM4	0.2095	0.1903	0.1890	0.2065*	0.1869	0.1926	0.2075*	0.0049	0.0126*	0.0099*	0.0075**	0.0117*	0.0097**	0.0075**
RMOM8	0.1110	0.0959	0.0629*	0.1156	0.1316	0.0827	0.1196	0.0029	0.0024	0.0012	0.0037	0.0035	0.0017	0.0039
RMOM26	0.0634	0.1056***	0.0817*	0.0791	0.1056***	0.1102	0.0839	0.0014	0.0027***	0.0019**	0.0019	0.0027***	0.0024	0.0022
VOL	0.0711	0.0912***	0.0587	0.0894	0.0958	0.0648*	0.0898	0.0014	0.0022***	0.0011	0.0022	0.0023	0.0013*	0.0023
VOLPRC	0.1706	0.1571	0.1760	0.1768	0.1957	0.1913	0.1720	0.0034	0.0075	0.0064	0.0052	0.0096*	0.0081*	0.0052
VOLSCALE	0.0363	0.0274	0.0538	0.0526	0.1299	0.1105	0.0616	0.0005	0.0003	0.0011	0.0011	0.0047	0.0032	0.0014
RETVOL	−0.1182	0.0190	0.0279	−0.0760	0.0654	0.0456	−0.0711	−0.0037	0.0001	0.0004	−0.0032	0.0015	0.0008	−0.0029
RETSKEW	0.1094	0.1002	0.1123	0.1194	0.0942	0.0953	0.1173	0.0020	0.0031	0.0027	0.0031	0.0026	0.0022	0.0030
RETKURT	0.0957	0.0859	0.0631*	0.1060	0.0611	0.0506	0.1023	0.0017	0.0020	0.0013	0.0026	0.0013	0.0010	0.0025
MAXRET	0.1157	0.0976	0.1038	0.1186	0.1288	0.1434	0.1264	0.0028	0.0047	0.0043	0.0042	0.0070	0.0064	0.0044
STD-PRCVOL	0.1783	0.1682	0.1840	0.1815	0.2394*	0.2394*	0.2041*	0.0035	0.0077	0.0063*	0.0053	0.0118**	0.0102**	0.0062
MEANABS	−0.0658	−0.0740	−0.0330	−0.0390	−0.0717	−0.0380	−0.0405	−0.0018	−0.0050	−0.0017	−0.0019	−0.0037	−0.0018	−0.0018

Table 4 (continued)

	Panel A: SR							Panel B: CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
BETA	−0.0183	0.0802	0.0827	0.0255	0.1210	0.0666	0.0211	−0.0007	0.0019	0.0014*	0.0003	0.0038	0.0014	0.0002
BETA^2	−0.0002	0.1103***	0.0893	0.0425	0.1518	0.0995	0.0481	−0.0003	0.0028***	0.0015*	0.0008	0.0036	0.0017*	0.0010
IDIOVOL	−0.0165	0.0246	0.0617	0.0154	0.0930	0.0840	0.0233	−0.0008	0.0003	0.0010	−0.0001	0.0023	0.0015*	0.0002
DELAY	0.0328	−0.0070	0.0301	0.0474	0.0213	0.0382	0.0534	0.0005	−0.0031	0.0004	0.0011	−0.0006	0.0007	0.0013

This table reports the out-of-sample Sharpe ratios and certainty-equivalent returns for both the benchmark portfolio and portfolios including cryptocurrency factors at risk aversion level $\lambda = 1$. *, **, and *** represent that the portfolio including a corresponding cryptocurrency factor outperforms the benchmark portfolio with the significance levels of 10%, 5%, and 1%, respectively

Table 5 Out-of-sample risk-adjusted metrics for $\lambda = 3$

	Panel A: SR							Panel B: CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Bench	**0.0831**	**0.0811**	**0.0066**	**0.0697**	**0.0811**	**0.0066**	**0.0697**	**0.0007**	**0.0011**	**−0.0003**	**0.0007**	**0.0011**	**−0.0003**	**0.0007**
MARCAP	0.4113***	0.3827***	0.3845***	0.3999***	0.3475**	0.3447**	0.3952***	0.0072***	0.0155***	0.0134***	0.0096***	0.0154***	0.0137***	0.0098***
LPRC	0.2009	0.171*	0.1281	0.1908	0.1802	0.1398	0.1945	0.0038	0.0038*	0.0022	0.0041	0.0047*	0.0029	0.0043
AGE	0.0251	0.0659	0.0545	0.0292	0.0868	0.0559	0.0314	−0.0004	0.0007	0.0005	−0.0005	0.0013	0.0005	−0.0004
MOM1	0.233*	0.2245**	0.2011**	0.2246*	0.2326**	0.2114**	0.2275*	0.0052*	0.0063**	0.0049**	0.0058**	0.0066**	0.0052**	0.0058**
MOM2	0.2515*	0.2448**	0.2271**	0.2453**	0.247**	0.2326**	0.2457**	0.006*	0.0075**	0.0061**	0.0067**	0.0079**	0.0066**	0.0068**
MOM3	0.1708	0.1561*	0.1191	0.1642	0.1557*	0.1113	0.1671	0.0035	0.0033*	0.0021	0.0036	0.0033*	0.0018	0.0037
MOM4	0.1471	0.1359	0.0922	0.1425	0.1447	0.1010	0.1437	0.0028	0.0027	0.0013	0.0029	0.0030	0.0015	0.0030
MOM8	−0.0123	0.0917**	0.0302**	−0.0001	0.0917**	0.0331**	0.0012	−0.0017	0.0014**	0.0001*	−0.0014	0.0014**	0.0002*	−0.0014
MOM26	−0.0181	0.0868**	0.0579**	−0.0048	0.0868**	0.0519*	−0.0036	−0.0041	0.0013**	0.0006*	−0.0028	0.0013**	0.0004	−0.0029
RMOM1	0.1986	0.1857	0.1631	0.1900	0.1830	0.1601	0.1898	0.0042	0.005*	0.0037	0.0046	0.0048*	0.0035	0.0045
RMOM2	0.2465*	0.2374**	0.1994**	0.2441*	0.2495**	0.2155**	0.2456*	0.0051*	0.0058**	0.0041**	0.0057*	0.0064**	0.0047**	0.0058*
RMOM3	0.2459**	0.222**	0.1879**	0.2334**	0.2381***	0.2167**	0.238**	0.0055**	0.006**	0.0043**	0.006**	0.0064***	0.0049**	0.0061**
RMOM4	0.2095	0.1988*	0.1745	0.1990	0.1991*	0.1787*	0.1997	0.0042	0.0053*	0.004*	0.0048*	0.0053*	0.0041*	0.0048*
RMOM8	0.1110	0.0903*	0.0171**	0.1113	0.1055	0.0267**	0.1125	0.0020	0.0013*	−0.0001*	0.0020	0.0018	0.0001	0.0021
RMOM26	0.0634	0.0868**	0.0389**	0.0699	0.0868**	0.0839	0.0707	0.0006	0.0013**	0.0002*	0.0008	0.0013**	0.0008	0.0008
VOL	0.0711	0.0810	0.0013	0.0744	0.0841*	0.0041	0.0745	0.0008	0.0011	−0.0003*	0.0009	0.0012*	−0.0003*	0.0009
VOLPRC	0.1706	0.1732	0.1441	0.1676	0.1523	0.0953	0.1678	0.0029	0.0037	0.0026	0.0033	0.0034	0.0015	0.0033
VOLSCALE	0.0363	0.0689	0.0263**	0.0382	0.0888	0.0247*	0.0430	0.0001	0.0008	0*	0.0001	0.0013	−0.0002	0.0002
RETVOL	−0.1182	0.0540	−0.0174	−0.1056	0.0710	−0.0091	−0.1045	−0.0044	0.0004	−0.0006	−0.0046	0.0008	−0.0005	−0.0044
RETSKEW	0.1094	0.1145	0.0686	0.1085	0.1039	0.0535*	0.1085	0.0016	0.0020	0.0008	0.0017	0.0017	0.0005	0.0017
RETKURT	0.0957	0.0837	0.0094**	0.0950	0.0765	0.0095**	0.0935	0.0013	0.0012	−0.0002*	0.0014	0.0010	−0.0002*	0.0014
MAXRET	0.1157	0.1189	0.0998	0.1116	0.1408	0.1188	0.1149	0.0020	0.0024	0.0016	0.0021	0.0032	0.0022	0.0022
STDPRCVOL	0.1783	0.1736*	0.1403	0.1732	0.2215**	0.1718	0.1848	0.0030	0.0037*	0.0025	0.0034	0.0057**	0.0037	0.0037
MEANABS	−0.0658	0.0075	−0.0023	−0.0580	0.0094	−0.0224	−0.0592	−0.0023	−0.0010	−0.0007	−0.0026	−0.0008	−0.0009	−0.0026
BETA	−0.0183	0.0984**	0.0467*	−0.0014	0.1340	0.0292**	−0.0035	−0.0012	0.0015**	0.0003*	−0.0010	0.0024	0.0001*	−0.0010

Table 5 (continued)

	Panel A: SR							Panel B: CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
$BETA^2$	−0.0002	0.1068**	0.0497*	0.0163	0.1240	0.0542*	0.0183	−0.0008	0.0018**	0.0003	−0.0005	0.0022	0.0004	−0.0004
IDIOVOL	−0.0165	0.0758	0.032*	−0.0042	0.1056*	0.043*	−0.0014	−0.0015	0.0010	0.0002*	−0.0014	0.0017*	0.0003*	−0.0013
DELAY	0.0328	0.0605	0.0494*	0.0395	0.0753	0.0478*	0.0415	−0.0002	0.0003	0.0004*	−0.0002	0.0009	0.0004*	−0.0001

This table reports the out-of-sample Sharpe ratios and certainty-equivalent returns for both the benchmark portfolio and portfolios including cryptocurrency factors at risk neutral level $\lambda = 3$. *, **, and *** representthat the portfolio including a corresponding cryptocurrency factor outperforms the benchmark portfolio with the significance levels of 10%, 5%, and 1%, respectively

Table 6 Out-of-sample risk-adjusted metrics for $\lambda = 5$

	Panel A: SR							Panel B: CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Bench	**0.0831**	**0.0584**	**−0.0036**	**0.0736**	**0.0584**	**−0.0036**	**0.0736**	**0.0006**	**0.0000**	**−0.0003**	**0.0005**	**0.0000**	**−0.0003**	**0.0005**
MARCAP	0.4113***	0.3954***	0.3725***	0.404***	0.3696***	0.3567***	0.4006***	0.0069***	0.0103***	0.0087***	0.0081***	0.0103***	0.0091***	0.0082***
LPRC	0.2009	0.1337	0.0976	0.1949	0.1550	0.1179	0.1972	0.0033	0.0018	0.0009	0.0034	0.0024	0.0014	0.0035
AGE	0.0251	0.0571	0.027**	0.0277	0.0690	0.0371*	0.0290	−0.0011	0.0000	−0.0002	−0.0013	0.0003	0.0000	−0.0012
MOM1	0.233*	0.1886**	0.1608	0.2278*	0.1965**	0.1779*	0.2297*	0.0045*	0.0034**	0.0024	0.0046*	0.0036**	0.0028*	0.0047*
MOM2	0.2515*	0.2151**	0.1855*	0.248**	0.2212**	0.1875*	0.2482**	0.0052*	0.0043**	0.0031*	0.0054**	0.0045**	0.0033*	0.0054**
MOM3	0.1708	0.1224	0.0795	0.1677	0.1216	0.0782	0.1695	0.0028	0.0015	0.0006	0.0028	0.0015	0.0006	0.0028
MOM4	0.1471	0.1098	0.0596	0.1453	0.1153	0.0670	0.1461	0.0021	0.0012	0.0003	0.0021	0.0013	0.0004	0.0021
MOM8	−0.0123	0.0705**	0.029**	−0.0041	0.0705**	0.0329**	−0.0034	−0.0026	0.0003	0.0001	−0.0024	0.0003	0.0001	−0.0024
MOM26	−0.0181	0.0621**	0.0432**	−0.0100	0.0621**	0.0498**	−0.0092	−0.0062	0.0002	0.0001	−0.0052	0.0002	0.0002	−0.0053
RMOM1	0.1986	0.1583	0.1296	0.1929	0.1543	0.1257	0.1929	0.0035	0.0025	0.0016	0.0035	0.0024	0.0015	0.0035
RMOM2	0.2465*	0.1935**	0.1574	0.2464*	0.2059**	0.1768*	0.2473*	0.0045*	0.0033**	0.0020	0.0048*	0.0036**	0.0024	0.0048*
RMOM3	0.2459**	0.1808**	0.1414	0.2389**	0.1916**	0.1734*	0.2419**	0.0048**	0.0031**	0.0019	0.0049**	0.0034**	0.0025*	0.005**
RMOM4	0.2095	0.1734*	0.1395	0.2038	0.1734*	0.1377	0.2043	0.0036	0.003*	0.0019	0.0038	0.0029*	0.0018	0.0038
RMOM8	0.1110	0.0695**	0.017**	0.1122	0.0813*	0.0266**	0.1129	0.0011	0.0003	0.0000	0.0010	0.0005	0.0000	0.0010
RMOM26	0.0634	0.0621**	0.037**	0.0684	0.0621**	0.0839	0.0689	−0.0002	0.0002	0.0001	0.0000	0.0002	0.0005	−0.0001
VOL	0.0711	0.0584	−0.0080	0.0732	0.0609**	−0.0052	0.0732	0.0002	0.0000	−0.0003	0.0003	0.0001	−0.0002	0.0002
VOLPRC	0.1706	0.1412	0.1054	0.1692	0.1232	0.0816	0.1694	0.0025	0.0020	0.0011	0.0026	0.0015	0.0006	0.0026
VOLSCALE	0.0363	0.0545	0.0097**	0.0373	0.0637	0.0045*	0.0404	−0.0002	−0.0001	−0.0002	−0.0004	0.0001	−0.0005	−0.0003
RETVOL	−0.1182	0.0375	−0.0280	−0.1110	0.0492	−0.0183	−0.1105	−0.0052	−0.0005	−0.0005	−0.0054	−0.0002	−0.0004	−0.0053
RETSKEW	0.1094	0.0874	0.0417*	0.1089	0.0796	0.0385**	0.1090	0.0012	0.0007	0.0001	0.0012	0.0005	0.0001	0.0012
RETKURT	0.0957	0.0623**	0.0002***	0.0954	0.0579	0.0009***	0.0944	0.0009	0.0001	−0.0002	0.0009	0.0000	−0.0002	0.0009
MAXRET	0.1157	0.1036	0.0703	0.1130	0.1185	0.0895	0.1152	0.0013	0.0009	0.0004	0.0011	0.0013	0.0008	0.0012
STDPRCVOL	0.1783	0.1388	0.1022	0.1756	0.1763*	0.1501	0.1832	0.0026	0.0019	0.0010	0.0027	0.0029*	0.0020	0.0029
MEANABS	−0.0658	0.0103	−0.0229	−0.0613	0.0112	−0.0265	−0.0622	−0.0028	−0.0011	−0.0008	−0.0031	−0.0010	−0.0007	−0.0031
BETA	−0.0183	0.0867**	0.0467*	−0.0070	0.1014	0.0311**	−0.0084	−0.0017	0.0007	0.0002	−0.0016	0.0010	0.0001	−0.0017

Table 6 (continued)

	Panel A: SR							Panel B: CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
$BETA^2$	−0.0002	0.0905**	0.0498*	0.0108	0.1046	0.0542*	0.0121	−0.0013	0.0008	0.0002	−0.0011	0.0011	0.0002	−0.0011
IDIOVOL	−0.0165	0.0693**	0.032*	−0.0082	0.0901*	0.043*	−0.0065	−0.0022	0.0004*	0.0001	−0.0022	0.0008	0.0002	−0.0022
DELAY	0.0328	0.0683	0.0356*	0.0378	0.0793	0.0465*	0.0391	−0.0008	0.0001	0.0000	−0.0010	0.0005	0.0002	−0.0009

This table reports the out-of-sample Sharpe ratios and certainty-equivalent returns for both the benchmark portfolio and portfolios including cryptocurrency factors at risk seeking level $\lambda = 5$. *, **, and *** represent that the portfolio including a corresponding cryptocurrency factor outperforms the benchmark portfolio with the significance levels of 10%, 5%, and 1%, respectively

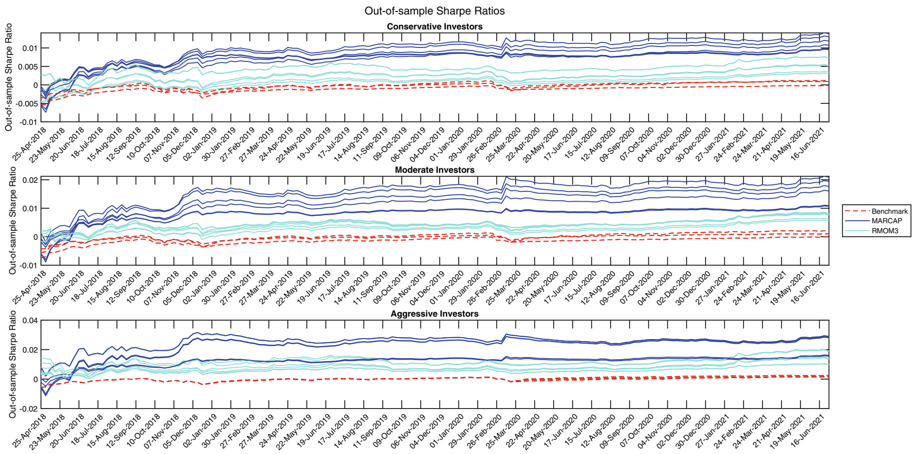

Fig. 2 SRs across the investment horizon Notes. This figure shows the out-of-sample Sharpe ratios of (i) a benchmark portfolio incorporating *MARCAP* (blue), (ii) a benchmark portfolio incorporating *RMOM3*(cyan), and (iii) the *benchmark* (red) portfolio evaluated by both traditional and novel portfolio optimisation techniques for conservative (the top subplot), moderate (the middle subplot), and aggressive (the bottom subplot) across Q3 2020 to Q1 2021, respectively. We do not distinguish the asset allocation techniques for each subplot because the aim of this figure is to show the outperformance of an enhance stock–bond portfolio relative to the benchmark portfolio (Color figure online)

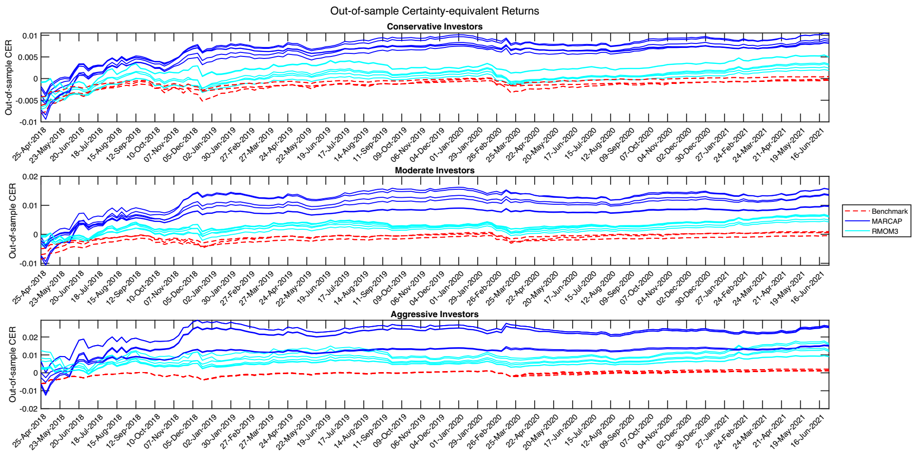

Fig. 3 CERs across the investment horizon Notes. This figure shows the out-of-sample certainty equivalent returns of (i) a benchmark portfolio incorporating *MARCAP* (blue), (ii) a benchmark portfolio incorporating *RMOM3*(cyan), and (iii) the *benchmark* (red) portfolio evaluated by both traditional and novel portfolio optimisation techniques for conservative (the top subplot), moderate (the middle subplot), and aggressive (the bottom subplot) across Q3 2020 to Q1 2021, respectively. We do not distinguish the asset allocation techniques for each subplot because the aim of this figure is to show the outperformance of an enhance stock–bond portfolio relative to the benchmark portfolio (Color figure online)

adding cryptocurrency factors increases to 9, indicating that machine learning can reduce the negative effects of estimation errors.

Panel B demonstrates the CERs for risk-seeking investors. We observe four portfolios incorporating size (e.g., *MARCAP*) and momentum (e.g., *MOM1-2* and *RMOM3*) related factors that have significantly outperformed CERs for any given portfolio management approaches but EW. Also, the CER of each outperformed portfolio incorporating cryptocurrency factors that apply the MV framework and its variants beat the EW strategy, whereas the SR metric does not show similar properties. We argue that the different nature of SR and CER leads to conflicts. This is mainly because CER considers the risk preference of investors, while SR ignores them. Moreover, both SRs and CERs of the EW strategy only generate the least number of statistically significant portfolios comprising cryptocurrency factors (e.g., five and four, respectively) across all chosen strategies, uncovering the inefficiency of the naïve diversification rule regarding cryptocurrencies.

We find that portfolios involving cryptocurrency factors constructed on a size (*MARCAP*) and a momentum factor (*RMOM3*) statistically surpass the benchmark portfolio under two out-of-sample metrics. To generalize our findings, we then select the portfolios that statistically outperform the benchmark through traditional and novel approaches regarding SR and CER metrics for further robustness tests (e.g., evaluating the diversification benefits with transaction costs, alternative benchmark portfolio, rolling-window strategy), because we aim to ensure that diversification benefits of enhanced portfolios are solid, and can provide practical investment implications to investors.

4.2 Out-of-sample metrics for moderate investors

We present out-of-sample risk-adjusted metrics for moderate investors ($\lambda = 3$) in Table 5, where SRs and CERs are in Panel A and B, respectively. We first interpret the results of SRs for moderate investors, where strategies such as EW, BL, and BL-ML produce the least number (five) of statistically outperformed enhanced portfolios. By contrast, strategies including MV, BS, and their variants can generate more outperformed portfolios than aggressive investors, with quantities of 15, 15, 13, and 16, respectively. Furthermore, we find five portfolios, incorporating cryptocurrency factors based on size and momentum groups, can consistently outperform the benchmark portfolio across all selected techniques. We note that more risk-seeking investors can enjoy more diversification benefits offered by cryptocurrency factors because the number of dominant portfolios increased by three for moderate investors compared to aggressive investors in terms of SRs.

Alternatively, the empirical results of CERs documented in Panel B of Table 5 exhibit a comparable trend with SRs for moderate investors. For instance, the EW strategy is still the worst among all selected strategies, with five statistically outperformed portfolios. Likewise, the number of statistically dominant portfolios of BL and BL-ML increases by one, contrary to that of SRs. However, other strategies such as MV, BS, and their variants can provide over 15 statistically dominant portfolios. Lastly, we find five dominant portfolios across both SR and CER, and they are *MARCAP*, *MOM1-2* and *RMOM2-3*. To ensure the generalization of our findings, we adopt the intersection of outcomes for aggressive and moderate investors–(*MARCAP*) and (*RMOM3*)–for further robustness tests.

We argue that EW is incompetent in forming dominant portfolios when taking statistical significance into account, whereas BL and its variant become less pronounced for more risk-averse investors. To illustrate, the incompetence of EW is consistent with aggressive investors, while the performance of BL and BL-ML for moderate investors (e.g., around five) is significantly different from that of aggressive investors (e.g., nine). We emphasise the core reason causing BL to perform poorly is that BL involves a column vector of implied returns (see Eq. 7), which involves the specification of the risk preference. The inflated λ values affect the weights assigned to cryptocurrency factor portfolios during the optimisation procedure, making BL less efficient for more risk-averse investors.

4.3 Out-of-sample metrics for conservative investors

We document out-of-sample empirical results for conservative investors (e.g., $\lambda = 5$) in Table 6. We note that the diversification benefits of cryptocurrency factors for conservative investors are less pronounced than for aggressive and moderate investors. Specifically, the EW strategy for SR and CER performs similarly as in the cases of aggressive and moderate investors we examined previously. However, strategies other than EW behave differently for conservative investors. For instance, the numbers of statistically significant SRs across MV, BS, BL, and their variants are 14, 14, 5, 13, 16, and 5, respectively, whereas the numbers of statistical CERs are 7, 2, 5, 7, 4, and 5, respectively. The sharp decline of CERs indicates that the utility of diversification benefits provided by cryptocurrency factor portfolios for conservative investors are less than those of aggressive and moderate investors. Further, we find two dominant portfolios comprising cryptocurrency factors (e.g., *MARCAP* and *MOM2*) from the perspective of SR, and only one (e.g., MARCAP) dominant factor portfolio in terms of CERs. On the other hand, we argue that machine-learning-based approaches can enhance the performance of existing shrinkage methods such as the BS approach—the number of dominant portfolios increased from 14 to 16 dominant portfolios after applying machine learning for SRs and from two to four for those of CERs.

4.4 Dynamic out-of-sample metrics

Our out-of-sample period encompasses a phase where cryptocurrencies experienced extreme upward movements (from Q3 2020 to Q1 2021) compared to traditional markets. To address concerns that these surges might skew our results, we visually present the investment results throughout the entire out-of-sample horizon. For simplicity, we showcase the dynamic SRs (Fig. 2) and CERs (Fig. 3) of two efficient portfolios that incorporate cryptocurrency factors, namely *MARCAP* and *RMOM3*. Evidently, for all types of investors, these two portfolios containing cryptocurrency factors demonstrate superior performance compared to benchmarks during both regular and extreme upward periods, regardless of the asset allocation strategies.

5 Mechanisms of significant cryptocurrency factors for all investors

So far, we have identified two efficient portfolios incorporating cryptocurrency factors (e.g., *MARCAP* and *RMOM3*) that considerably boost the performance of the stock–bond benchmark portfolio under all selected asset allocation strategies. In this section, we delve into the mechanisms inherent in successful cryptocurrency factors.

Size- and momentum-based cryptocurrency factors, such as *MARCAP* and *RMOM3*, are the most versatile hedging assets amid the 28 factors, whereas volume- and volatility-related cryptocurrency factors fail to deliver statistically significant diversification benefits across all selected approaches. For instance, the volume-based factors offer negligible diversification benefits, indicating that the volume-related factors can survive 27% of all strategies for conservative, moderate, and aggressive investors regarding SRs. At the same time, CERs illustrate similar proportions of statistically significant diversification benefits only for aggressive and moderate investors. On the other hand, the volatility-related factors are inefficient for aggressive investors but beneficial to moderate and conservative investors, as noted by the leap in the number of significant cryptocurrency factors measured by SRs. We argue that the incompetence of volume- and volatility-based factors is attributable to their low expected returns and high standard deviations, which cannot provide added values and enhance the risk-adjusted performance of the benchmark portfolio. Therefore, we assert that volume- and volatility-based cryptocurrency factors are not favourable features to investors seeking abnormal returns or diversification benefits.

Conversely, size- and momentum-based cryptocurrency factors demonstrate superiority in diversifying the risks exemplified by higher returns for the same level of risk compared to other groups. To illustrate, the number of significant factors formed on size- and momentum- groups for aggressive, moderate, and conservative investors are two, five, and two in terms of SRs, whereas the quantities of CERs are four, five, and one, respectively. Among them, the two significant cryptocurrency factors (*MARCAP* and *RMOM3*) are the best-performed diversifiers. Particularly, *MARCAP* has been proven effective for all three classes of investors, and *RMOM3* are valuable to aggressive and moderate investors, ascertaining that both factors aid benchmark portfolios in achieving significantly higher risk-adjusted returns. Also, the average SRs for *MARCAP* at different risk levels ($\lambda = 1, 3, 5$) is 0.3833, the highest and almost four times the benchmark portfolio, while CERs have similar results.

Among the effective cryptocurrency factors, all three types of investors prefer the cryptocurrency factor based on market capitalisation (*MARCAP*) to that formed on three-week risk-adjusted momentum (*RMOM3*). Table 7 illustrates the weights allocated to these two factors across different portfolio techniques during the out-of-sample evaluation phase. The weights of significant cryptocurrency factors exhibit a gradually decreasing trend along with increasing risk aversion coefficients (λ). For instance, the mean weight of *MARCAP* evaluated by the MV approach is 100% for aggressive investors, then it falls to 73% for moderate investors, eventually dropping to 46% for conservative investors. The same patterns are also identified for *RMOM3* via all selected asset allocation approaches. Therefore, this finding strengthens the viewpoint that risk-seeking investors benefit more from incorporating cryptocurrency factors into a portfolio than risk-averse investors. On the other hand, we find the weights of cryptocurrency factors evaluated by the BL and BL-ML are much less than those evaluated by the MV, BS, and their variants. We argue that the BL considers more risk preferences than the BS, whereas the MV considers the least risk preferences. These findings may be caused by the fact that the BS and BL involve shrinkage when estimating the expected returns and then input to the mean-variance framework, where the sample-based MV approach takes the historical mean returns as input to optimise the weights.

Thereafter, we assess whether the weight allocations to cryptocurrency factors correlate positively with outperformance. We use box plots to depict and compare the differences in asset weights between each cryptocurrency factor across six selected techniques in Figs. 4, 5, and 6 for investors with risk aversions of 1, 3, and 5, respectively.[10] Each figure consists of two columns, where the left and right columns represent the traditional (e.g., MV,

[10] We exclude the distribution of the EW approach because we aim to discover the dynamics of weights assigned to each factor.

Table 7 Distributions of weights allocated to significant cryptocurrency factors

		Traditional approaches			Variant approaches		
		MV (%)	BS (%)	BL (%)	MV-ML (%)	BS-ML (%)	BL-ML (%)
Panel A: Aggressive investors							
MARCAP	Mean	100	100	53	98	98	59
	Std. Dev	0	1	7	12	12	8
	Max	100	100	70	100	100	80
	Min	100	89	43	0	0	6
RMOM3	Mean	82	50	36	81	58	37
	Std. Dev	9	11	2	20	26	5
	Max	100	93	44	100	100	52
	Min	62	34	33	0	0	8
Panel B: Moderate investors							
MARCAP	Mean	73	56	36	85	71	37
	Std. Dev	16	18	3	17	19	3
	Max	100	100	43	100	100	46
	Min	47	31	31	0	0	19
RMOM3	Mean	29	17	29	32	21	29
	Std.Dev	4	4	1	12	11	2
	Max	48	33	31	71	59	35
	Min	21	12	28	0	0	20
Panel C: Conservative investors							
MARCAP	Mean	46	36	31	55	45	33
	Std. Dev	12	12	2	13	13	2
	Max	73	65	36	89	81	38
	Min	29	19	29	0	0	21
RMOM3	Mean	18	10	27	20	13	28
	Std. Dev	3	2	0	8	7	1
	Max	30	20	29	44	37	31
	Min	13	7	27	0	0	22

This table reports the out-of-sample distributions of weights allocated to two significant factors—*MARCAP* and *RMOM3*— for aggressive, moderate, and conservative investors across six different techniques, respectively. We omit the distributions of equal-weighted portfolios because we aim to evaluate the dynamics of weights assigned to each significant factor

BS, and BL)and novel approaches (i.e., MV-ML, BS-ML, and BL-ML), respectively. We find no direct positive relationship between the weights and outperformance. For instance, aggressive investors tend to allocate more weights to cryptocurrency factors with higher expected returns, such as *MARCAP*, *MOM1-2*, *RMOM1-3*, to gain diversification benefits (see Fig. 4). However, except for *MARCAP* and *RMOM3*, not all diversification benefits added by the aforementioned factors are statistically significant across all selected asset allocation techniques (e.g., *MOM1-2* and *RMOM1-2* via the MV approach, etc.). Likewise, although cryptocurrency factors based on *DELAY* and *MAXRET* are allocated with massive weights by aggressive investors, enhanced portfolios incorporating these two cryptocurrency factors can barely provide statistically significant outperformance. Therefore, we

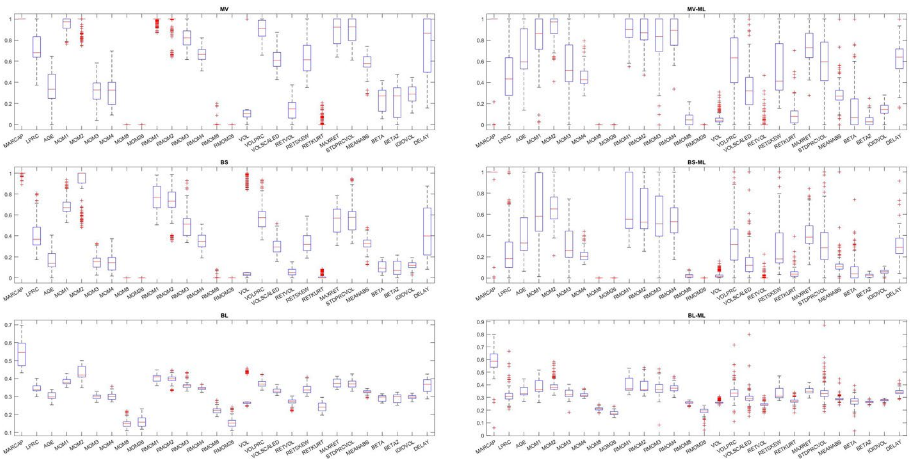

Fig. 4 Distributions of weights allocated to cryptocurrency factors for $\boldsymbol{\lambda = 1}$ Notes. This figure employs the box plot to illustrate the distributions of weights assigned to each cryptocurrency factor portfolio in enhanced portfolios (e.g., combining a stock–bond portfolio and a cryptocurrency factor portfolio) via the mean-variance approach (top left), Bayes-Stein approach (middle left), Black-Litterman (bottom left), and their variants on the symmetrical right-hand side during the out-of-sample period for aggressive investors. The red line within each box represents the central tendency (median) of the weights allocated to a cryptocurrency factor portfolio, and the red plus sign represents the outliers (Color figure online)

argue that weights assigned to cryptocurrency factors are not the only determinant leading to diversification benefits.

We then investigate whether the core findings of diversification benefits for aggressive investors are true for moderate (see Fig. 5) and conservative investors (see Fig. 6). *MARCAP* remains the strongest factor with the most allocated weight by conservative investors. Conversely, the cryptocurrency factor formed on one-week risk-adjusted momentum (*RMOM1*) obtains the most weights compared to its peers (e.g., *RMOM2-4*). Still, a benchmark incorporating *RMOM1* fails to generate any outperformance across all techniques. Additionally, moderate investors assign extensive weights on a cryptocurrency factor based on trading volume, *VOL*, while a benchmark portfolio that includes *VOL* can only produce one significant outperformance (under the MV-ML approach). Thus, we emphasise that weights and outperformance have no direct positive relationship for all types of investors.

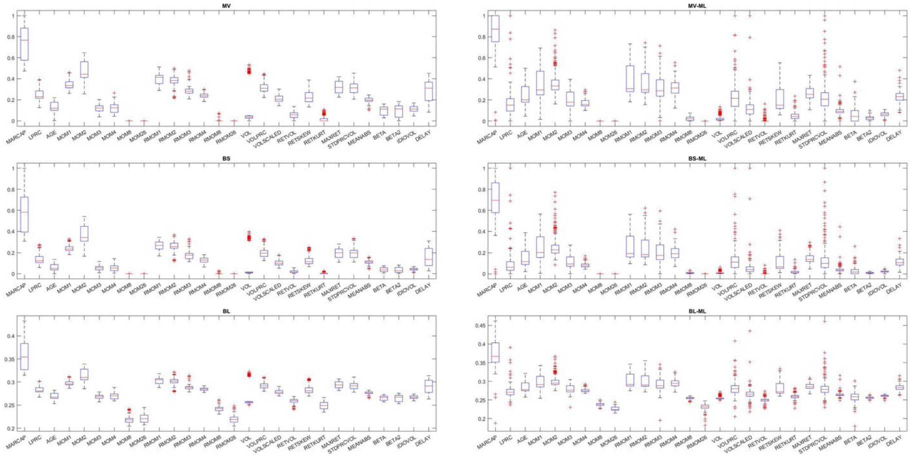

Fig. 5 Distributions of weights allocated to cryptocurrency factors for $\boldsymbol{\lambda = 3}$ Notes. This figure employs the box plot to illustrate the distributions of weights assigned to each cryptocurrency factor portfolio in enhanced portfolios (e.g., combining a stock–bond portfolio and a cryptocurrency factor portfolio) via the mean-variance approach (top left), Bayes-Stein approach (middle left), Black-Litterman (bottom left), and their variants on the symmetrical right-hand side during the out-of-sample period for moderate investors. The red line within each box represents the central tendency (median) of the weights allocated to a cryptocurrency factor portfolio, and the red plus sign represents the outliers (Color figure online)

Moreover, we note that, on average, the weights allocated to all cryptocurrency factors for moderate investors are lower than those of aggressive investors, and conservative investors have the slightest intention to invest in cryptocurrency factors.[11]

Nevertheless, we find that machine learning techniques have limited effects on forecasting size factors (e.g., *MARCAP*) but can considerably reduce the estimation errors of momentum factors (e.g., *RMOM3*). This phenomenon is evidenced by the improvement of both out-of-sample metrics, around 4%, for the MV-ML, BS-ML, and BL-ML compared to the traditional MV, BS, and BL for all three types of investors. We attribute the strength of ML-based portfolio techniques to effective input features (e.g., lagged returns) for cryptocurrency factors being predicted because lagged returns are more prominent for the momentum-based factor *RMOM3* rather than for *MARCAP*.

[11] We assert that, in general, the weights assigned to effective cryptocurrency factor portfolios (*MARCAP* and *RMOM3*) decline as the investors' risk tolerance (λ) decreases. Mathematically, λ is the hyperparameter of the utility function (see Eq. 2), the larger λ value the more emphasis is placed on minimising the risks relative to maximising the expected return. As the riskiness of cryptocurrency factor portfolios is an order of magnitude larger than traditional assets (see Table 3), the weights in cryptocurrency factor portfolios appear to have the most considerable impact compared to other underlying assets of a boosted portfolio, but this phenomenon is less vital in poorly performed cryptocurrency factor portfolios. For instance, the average weights allocated to *MOM8* - a cryptocurrency factor portfolio formed on eight-week momentum - range from around 6% for aggressive investors and 6.3% for conservative investors. By contrast, the average weights in *MARCAP* illustrate a substantial difference of approximately 40 percentage points between aggressive and conservative investors. We urge that the negligible difference in the weights in *MOM8* is primarily attributable to its negative expected return, as shown in Table 3, because the portfolio optimisation approaches tend to concentrate on the assets that can deliver better returns among the investment pool (Chopra and Ziemba 1993). In our case, *MOM8* cannot outperform the equities and bonds of the benchmark portfolio, and we conclude that poorly performed cryptocurrency factor portfolios are inclined to show sporadic movement in the weights when λ values change. Hence, a qualified cryptocurrency factor offering diversification benefits should at least provide reasonable rewards for bearing the extensive risks.

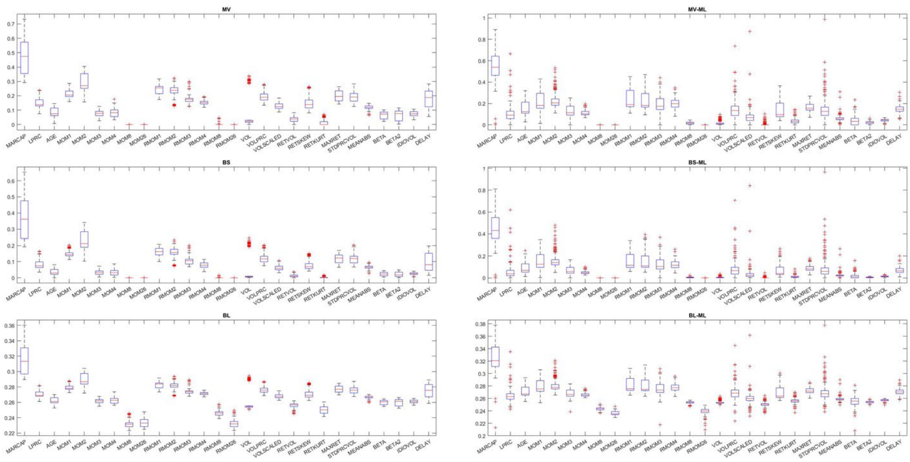

Fig. 6 Distributions of weights allocated to cryptocurrency factors for $\lambda = 5$ Notes. This figure employs the box plot to illustrate the distributions of weights assigned to each cryptocurrency factor portfolio in enhanced portfolios (e.g., combining a stock–bond portfolio and a cryptocurrency factor portfolio) via the mean-variance approach (top left), Bayes-Stein approach (middle left), Black-Litterman (bottom left), and their variants on the symmetrical right-hand side during the out-of-sample period for conservative investors. The red line within each box represents the central tendency (median) of the weights allocated to a cryptocurrency factor portfolio, and the red plus sign represents the outliers (Color figure online)

On the other hand, ML-based portfolio models can enrich the distributions (e.g., interquartile range) of potentially effective cryptocurrency factors and reduce the extreme distributions of potentially poor cryptocurrency factors. Figure 4 shows that the distributions of potentially effective cryptocurrency factors, such as *MOM1-2* and *RMOM1*, are improved by machine learning methods, which increase the values added by these cryptocurrency factors for aggressive investors. Specifically, the number of significant outperformance produced by a benchmark portfolio incorporating *MOM1-2* is two (excluding EW) before applying the machine learning, and it soars to three after C-ENet is leveraged. Conversely, machine learning techniques can shrink the central tendency (i.e., median) of the potentially poor cryptocurrency factors to suffer less from estimation risks. To illustrate, a benchmark portfolio with *STDPRCVOL* cannot produce any significant outperformance, but the use of C-ENet improves the enhanced portfolio to deliver significant benefits under all novel approaches. Moreover, ML is also favourable to moderate (i.e., improved significant metrics of *IDIOVOL* in Fig. 5) and conservative investors (e.g., boosted significant performance of *IDIOVOL* and *DELAY* in Fig. 6). To summarise, we propose that machine learning techniques can alleviate the negative effects of estimation errors by enriching (reducing) the distributions of potentially effective (poor) cryptocurrency factors. Size- and momentum-related factors are effective and suitable to diversify the traditional benchmark portfolios for aggressive and moderate investors in practice.

For further robustness tests, we choose the two portfolios that can beat the benchmark for aggressive and moderate investors as the final statistically outperformed factor portfolios—portfolios comprising *MARCAP* and *RMOM3*. We pick these two portfolios because (i) they provide statistically significant diversification benefits for both aggressive and moderate investors, and (ii) conservative investors may not choose to invest in cryptocurrency factor portfolios as a diversification tool since cryptocurrencies have an order of magnitude of higher volatility than other traditional assets such as stocks, bonds, and commodities. Hence, we will focus on these two specific cryptocurrency factor portfolios hereafter.

6 Robustness tests

In this section, we conduct a series of robustness tests to validate the diversification benefits of certain cryptocurrency factors (e.g., *MARCAP*and *RMOM3*) identified in previous sections, including (i) considering transaction costs when measuring performance, (ii) testing alternative benchmark portfolios, and (iii) applying a rolling estimation window scheme. Similar to Tables 4, 5, and 6, the bold rows report the risk-adjusted returns for the benchmark portfolio with different risk preferences.

6.1 Transaction costs

So far, our empirical results suggest that *MARCAP* and *RMOM3* can deliver substantial diversification benefits. However, considering transaction costs is crucial. These costs might offset the potential value gained by adding cryptocurrency factors to the benchmark portfolio, rendering trading less profitable. Thus, we re-examine the diversification benefits of the two dominant cryptocurrency factors by subtracting the transaction costs from the portfolio excess returns at time t. The transaction costs, denoted by TC_t, are given by:

$$\mathrm{TC}_t = \sum_{i=1}^{N} \mathrm{T}_i\left(\left|\mathrm{x}_{i,t} - \mathrm{x}^{+}_{i,t-1}\right|\right), \tag{19}$$

where $\mathrm{x}^{+}_{i,t-1}$ denotes the weight of the i^{th} asset at the end of time $t-1$ (i.e., considering the weight changes caused by the price movement just before the rebalancing); T_i represents the proportionate transaction costs of trading US equities with 50 basis points, bonds and risk-free rates with 17 basis points, and cryptocurrency factors with 50 basis points (DeMiguel et al. 2009; Platanakis and Urquhart 2020).

We present the results of the out-of-sample risk-adjusted metrics inclusive of transaction costs for the benchmark portfolio and the two portfolios with cryptocurrency factors in Table 8, whereby we find these two cryptocurrency factors can still boost the benchmark portfolio's performance remarkably. Similar to the core findings, the enhanced portfolio incorporating *MARCAP* factor can still statistically outperform the benchmark portfolio after considering transaction costs for investors with risk aversion levels at $\lambda = 1, 3, 5$, with an average SR of 0.3665 and an average CER of 0.0131. Meanwhile, *RMOM3* exhibits similar diversification benefits to aggressive and moderate investors except for conservative investors indicated by four insignificant out-of-sample metrics (e.g., portfolio performance with BS, BS-ML, and their variants). We argue that, on average, the relatively low SR (0.1917) and CER (0.0061) compared to size-related factors cannot compensate for the risks taken by conservative investors, so that *RMOM3* cannot boost the benchmark's performance significantly. The empirical results considering transaction costs are in accordance with our previous findings, verifying the solid diversification benefits of *MARCAP* and *RMOM3*.

Table 8 Out-of-sample risk-adjusted performance including transaction costs

	SR							CER						
	Traditional Approaches				Variant Approaches			Traditional Approaches				Variant Approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Panel A: Aggressive investors														
Bench	**0.0831**	**0.0912**	**0.0622**	**0.0730**	**0.0912**	**0.0622**	**0.0730**	**0.0008**	**0.0022**	**0.0013**	**0.0015**	**0.0022**	**0.0013**	**0.0015**
MARCAP	0.3599***	0.3732**	0.3732**	0.4027***	0.3638**	0.3614**	0.381***	0.0066**	0.0257***	0.0257***	0.0147***	0.0249***	0.0246***	0.0148***
RMOM3	0.2098*	0.2357*	0.2347*	0.2328**	0.2374*	0.234*	0.2376**	0.0053*	0.0154**	0.0117***	0.0088***	0.0146**	0.0111**	0.0088***
Panel B: Moderate investors														
Bench	**0.0831**	**0.0811**	**0.0066**	**0.0697**	**0.0811**	**0.0066**	**0.0697**	**0.0007**	**0.0011**	**−0.0003**	**0.0007**	**0.0011**	**−0.0003**	**0.0007**
MARCAP	0.3599***	0.3806***	0.3819***	0.3786***	0.3322**	0.3266**	0.3758***	0.0063**	0.0154***	0.0133***	0.009***	0.0145***	0.0128***	0.0092***
RMOM3	0.2098*	0.2178**	0.1794*	0.1974*	0.2199**	0.1927**	0.2016*	0.0045*	0.0059**	0.004*	0.0048*	0.0058**	0.0043**	0.0049*
Panel C: Conservative investors														
Bench	**0.0831**	**0.0584**	**−0.0036**	**0.0736**	**0.0584**	**−0.0036**	**0.0736**	**0.0006**	**0.0000**	**−0.0003**	**0.0005**	**0.0000**	**−0.0003**	**0.0005**
MARCAP	0.3599***	0.3921***	0.3678***	0.3668***	0.3546***	0.3396***	0.3652***	0.0059**	0.0102***	0.0085***	0.0072***	0.0098***	0.0085***	0.0073***
RMOM3	0.2098*	0.1702*	0.0817	0.1873	0.1723*	0.0974	0.1904	0.0038*	0.0028*	0.0006	0.0034	0.0028*	0.0009	0.0035

This table reports the out-of-sample Share ratios (SR) and certainty-equivalent returns (CER) for benchmark portfolios and portfolios including superior cryptocurrency factors (*MARCAP* and *RMOM3* inclusive of transaction costs) at three risk levels—$\lambda = 1, 3,$ and 5. *, **, and *** represent that the portfolio including a corresponding cryptocurrency factor outperforms the benchmark portfolio with the significance levels of 10%, 5%, and 1%, respectively

Table 9 Out-of-sample risk-adjusted performance for the multi-asset benchmark

	SR							CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Panel A: Aggressive investors														
Mult_ Bench	**0.0702**	**0.0526**	**0.0595**	**0.0596**	**0.0526**	**0.0595**	**0.0596**	**0.0012**	**0.0011**	**0.0013**	**0.0013**	**0.0011**	**0.0013**	**0.0013**
MARCAP	l0.3334***	l0.3732**	l0.3733**	l0.3794***	l0.3666**	l0.3643**	l0.3674***	0.0057**	0.0257***	0.0257***	0.0118***	0.0251***	0.0248***	0.0123***
RMOM3	0.2109**	0.2324*	0.2267*	0.2107**	0.256**	0.2545**	0.2193**	0.0048*	0.015**	0.0109**	0.0068**	0.0159***	0.0118***	0.007**
Panel B: Moderate investors														
Mult_ Bench	**0.0702**	**0.0557**	**0.0188**	**0.0578**	**0.0557**	**0.0188**	**0.0578**	**0.0008**	**0.0004**	**−0.0003**	**0.0005**	**0.0004**	**−0.0003**	**0.0005**
MARCAP	0.3334***	0.3763***	0.3743***	0.3308***	0.3422**	0.3326**	0.3308***	0.0054**	0.015***	0.0124***	0.007***	0.0151***	0.0127***	0.0073***
RMOM3	0.2109**	0.2044**	0.1715*	0.1978**	0.222**	0.1943**	0.2027**	0.0042*	0.0054**	0.0038*	0.0043**	0.0058**	0.0043**	0.0044**
Panel C: Conservative investors														
Mult_ Bench	**0.0702**	**0.0390**	**−0.0010**	**0.0622**	**0.0390**	**−0.0010**	**0.0622**	**0.0004**	**−0.0005**	**−0.0005**	**0.0002**	**−0.0005**	**−0.0005**	**0.0002**
MARCAP	0.3334***	0.3789***	0.3504***	0.3332***	0.3539***	0.3403***	0.3332***	0.0051***	0.0097***	0.0078***	0.0058***	0.0096***	0.0083***	0.0059***
RMOM3	0.2109**	0.1639*	0.1246	0.2046**	0.176*	0.1441	0.2079**	0.0036**	0.0026*	0.0015	0.0036**	0.003**	0.0019	0.0036**

This table reports the out-of-sample Share ratios (SR) and certainty-equivalent returns (CER) for multi-asset benchmark portfolios and portfolios incorporating superior cryptocurrency factors (*MARCAP* and *RMOM3*) at three risk levels—$\lambda = 1, 3,$ and 5. *, **, and *** represent that the portfolio including a corresponding cryptocurrency factor outperforms the multi-asset benchmark portfolio with the significance levels of 10%, 5%, and 1%, respectively

6.2 Alternative benchmark portfolios

Heretofore, we have investigated whether cryptocurrency factor portfolios enrich a stock–bond portfolio with diversification benefits. Nevertheless, abundant studies emphasise the diversification benefits generated by commodities (Bessler and Wolff 2015; Gao and Nardari 2018) and real estate (Huang and Zhong 2013; Lu et al. 2013) to boost the performance of a stock–bond portfolio. Therefore, one concern regarding our main empirical findings is whether the diversification benefits remain significant after including commodities and real estate in the benchmark portfolio. To address this concern, we re-evaluate the diversification benefits of the two significant cryptocurrency factors—*MARCAP* and *RMOM3*—via a new benchmark portfolio incorporating a commodity index and real estate into our initial stock–bond portfolio (namely multi-asset benchmark), and ascertain whether adding two significant cryptocurrency factors to the multi-asset benchmark can generate further values for investors. We adopt the most widespread commodity index, e.g., the GSCI index, to approximate commodities, and choose the Ziman REIT index as a proxy for real estate (Bessler and Wolff 2015; Platanakis and Urquhart 2020; Huang et al. 2022).

We document the out-of-sample metrics for the multi-asset benchmark and enhanced portfolios in Table 9. Similar to our previous findings, *MARCAP* still delivers enormous diversification benefits when added to our multi-asset benchmark portfolio, indicated by considerably higher average SRs (0.3524) and CERs (0.0123) in comparison to those of the multi-asset benchmark portfolio (e.g., average SRs of 0.0485 and CERs of 0.0005) for aggressive, moderate, and conservative investors. Similarly, multi-asset benchmark portfolios incorporating *RMOM3* produce statistically significant diversification benefits, namely average SRs of 0.2022 and CERs of 0.0059, for almost all types of investors, except for the BS approach and its variant for conservative investors. Thus, we argue that the diversification benefits of the two influential cryptocurrency factors are also robust to an alternative benchmark portfolio, i.e., a multi-asset portfolio with commodities and real estate.

6.3 A rolling-window approach

To alleviate concerns over whether our empirical findings are subject to an expanding estimation window approach, we replicate our primary empirical analysis and re-assess the diversification benefits of the two significant cryptocurrency factors using a rolling window approach. As mentioned in Sect. 2.3, we employ an initial window with a length of 100 (observations), which is a sufficient sample size by convention, to ensure the training quality of selected ML techniques via an expanding-window scheme (Dong et al. 2022). Therefore, to produce comparable outcomes, we set the length of the rolling-window approach to the same window length (100) as that of our previous expanding-window approach and re-investigate whether the diversification benefits are still persistent in the rolling-window scheme.

We depict the out-of-sample metrics for our benchmark and enhanced portfolios in Table 10. Our core findings evaluated formerly by an expanding-window technique are also robust to the rolling-window estimation. To illustrate, a portfolio incorporating *MARCAP* produces statistically significant SRs and CERs across aggressive, moderate, and conservative investors under all asset allocation strategies, with average SRs of 0.3744 and CERs of 0.0141. Likewise, in most cases, *RMOM3* can enhance the benchmark portfolio except

Table 10 Out-of-sample risk-adjusted performance under rolling-window estimation

	SR							CER						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Panel A: Aggressive investors														
Bench	**0.0831**	**0.0912**	**0.0276**	**0.0634**	**0.0912**	**0.0276**	**0.0634**	**0.0008**	**0.0022**	**0.0004**	**0.0013**	**0.0022**	**0.0004**	**0.0013**
MARCAP	0.4113***	0.3732**	0.3726**	0.3927***	0.3666**	0.3651**	0.375***	0.0076**	0.0257***	0.0256***	0.0158***	0.0251***	0.025***	0.0167***
RMOM3	0.2459**	0.2274*	0.2131*	0.2316**	0.2548*	0.2524**	0.2448**	0.0062**	0.0154**	0.0101**	0.0089***	0.0185***	0.0154***	0.0099***
Panel B: Moderate investors														
Bench	**0.0831**	**0.0736**	**−0.0199**	**0.0622**	**0.0736**	**−0.0199**	**0.0622**	**0.0007**	**0.0009**	**−0.0006**	**0.0006**	**0.0009**	**−0.0006**	**0.0006**
MARCAP	0.4113***	0.3576**	0.3543***	0.3916***	0.3489**	0.328**	0.3886***	0.0072***	0.0163***	0.014***	0.01***	0.0171***	0.0148***	0.0103***
RMOM3	0.2459**	0.2114**	0.1507	0.227**	0.2398**	0.2254**	0.2345**	0.0055**	0.0058**	0.0029	0.0059**	0.0076**	0.0058**	0.0062**
Panel C: Conservative investors														
Bench	**0.0831**	**0.0402**	**−0.0471**	**0.0712**	**0.0402**	**−0.0471**	**0.0712**	**0.0006**	**−0.0004**	**−0.0009**	**0.0005**	**−0.0004**	**−0.0009**	**0.0005**
MARCAP	0.4113***	0.3737***	0.3518***	0.3993***	0.3506***	0.3427***	0.3957***	0.0069***	0.0109***	0.009***	0.0083***	0.011***	0.0098***	0.0085***
RMOM3	0.2459**	0.1684*	0.1066	0.2349**	0.2076**	0.1777	0.2397**	0.0048**	0.0028*	0.0011	0.0048**	0.0041**	0.0028	0.005**

This table reports the out-of-sample Share ratios (SR) and certainty-equivalent returns (CER) for stock–bond benchmark portfolios and portfolios incorporating superior cryptocurrency factors (*MARCAP* and *RMOM3*) at three risk levels—$\lambda = 1, 3,$ and 5. *, **, and *** represent that the portfolio including a corresponding cryptocurrency factor outperforms the stock–bond benchmark with the significance levels of 10%, 5%, and 1%, respectively for the BS method and its variant employed by moderate and conservative investors. Specifically, *RMOM3* fails to deliver statistically significant performances by the BS method for moderate investors. In contrast, BS with machine learning, BS-ML, can improve the benchmark portfolio dramatically with statistically significant SRs (0.2254) and CERs (0.0058), as shown in Panel B of Table 10. In addition, for *RMOM3*, we emphasise that ML can also boost the performance of the traditional MV approach by an average of 16% for SR and 32% for investors with different risk preferences, even under a rolling-window scheme. Furthermore, *RMOM3* can provide statistically significant diversification benefits when measured by SRs and CERs with almost all portfolio optimisation techniques but BS and BS-ML for conservative investors. One possible reason is that the value added by *RMOM3* cannot compensate the risks taken by conservative investors. Thus the BS model allocates more weights to μ_G (e.g., the global mean) to minimise the risks during the optimisation process.

7 Conclusion

To sum up, this paper evaluates the diversification benefits of numerous cryptocurrency factor portfolios when added to a stock–bond benchmark portfolio by dissecting a range of asset allocation strategies for investors with three different levels of risk preference. Our paper is the first to assess the out-of-sample diversification benefits of cryptocurrency factors in a portfolio management framework with machine learning. Therefore, our work contributes to (i) the literature on cryptocurrencies and portfolio management and (ii) retail and institutional investors.

We learn that out of the 28 cryptocurrency factors used in this study, two cryptocurrency factors formed on size (e.g., *MARCAP*) and momentum (*RMOM3*) can deliver statistically significant diversification benefits when added to a stock–bond portfolio favoured by most types of investors under two different risk-adjusted metrics. Mainly, *MARCAP* adds an average of over 400% to the SR and CER of the benchmark portfolio evaluated by all asset allocation strategies at all risk aversion levels. On the other hand, *RMOM3* can substantially improve the benchmark portfolio performance for aggressive and moderate investors across all selected portfolio techniques, except for the case of conservative investors evaluated by the BS approach. Even though *RMOM3* fails to generate statistically significant outputs for conservative investors through all portfolio optimisation methods, we assert that conservative investors have a low probability of participating in the crypto market trading. Thus, we focus more on the significant cryptocurrency factors for both aggressive and moderate investors. To this strand, we find that investors with low risk-aversion levels (e.g., aggressive) can enjoy more benefits than highly risk-averse investors.

To validate the robustness of our core findings, we re-investigate the diversification benefits of the two prominent cryptocurrency factors in various ways. First, our core findings are robust when including transaction costs. The post-cost diversification benefits highlight the superiority of including cryptocurrency factors in a stock–bond portfolio and provide investors with solid implications for practical investment. Second, our empirical findings are robust to an alternative benchmark (e.g., the multi-asset benchmark)—cryptocurrency factors are proven to augment the diversification benefits beyond commodities and real estate. Lastly, the empirical results are robust to both expanding- and rolling-window schemes. Therefore, our analysis proposes that the inclusion of cryptocurrency factors in a stock–bond portfolio is favourable to investors.

Our core results demonstrate the diversification benefits offered by cryptocurrency factors on an out-of-sample basis. Nevertheless, we implement a limited number of portfolio optimisation and machine learning techniques, and our tests are still based on historical expected returns, forecasted returns, and historical return covariance matrices. Thus, these input parameters may generate large estimation errors when the market is volatile. We suggest that future research focus on effectively mitigating the estimation errors or optimising the asset weights directly.

Moreover, future researchers should explore other high-quality cryptocurrency data except for price and develop dynamic risk models that not only consider historical data but also adapt to the inherent volatility of cryptocurrency markets, allowing for real-time risk assessments. Additionally, delving into behavioural and macroeconomic factors that exert genuine influence on cryptocurrency markets is imperative for a comprehensive understanding. For instance, examining how investor sentiment, social media dynamics, and broader economic forces shape market trends can enhance risk management strategies. Meanwhile, regulators can play a crucial role by considering implementing standardised reporting and risk Conflict of interest requirements specific to cryptocurrencies, ensuring transparency and aiding investors in making informed decisions. This strengthening market surveillance is essential for detecting and preventing market manipulation or fraudulent activities. Furthermore, fostering global regulatory cooperation can lead to harmonised frameworks, providing a consistent regulatory environment for market participants worldwide. This collaborative effort between researchers and regulators is crucial for creating a more transparent, informed, and resilient cryptocurrency market, ultimately mitigating risks, promoting responsible investment practices in cryptocurrencies, and making investors readily integrate cryptocurrencies into their portfolios from a risk management perspective.

Appendix 1: *p* values for empirical results

In this section, we present the p values for out-of-sample metrics of Tables 4, 5, 6 and 8, 9, 10 in Table 11, 12, 13, 14, 15 and 16. Specifically, we estimate the test statistics for the out-of-sample Sharpe ratio (DeMiguel et al. 2009) as follows:

$$\begin{aligned}\hat{z}_{SR} &= \frac{\hat{\sigma}_n\hat{\mu}_i - \hat{\sigma}_i\hat{\mu}_n}{\sqrt{\vartheta}},\\ \text{where } \vartheta &= \frac{1}{T-M}\left(2\hat{\sigma}_i^2\hat{\sigma}_n^2 - 2\hat{\sigma}_i\hat{\sigma}_n\hat{\sigma}_{i,n} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_n^2 + \frac{1}{2}\hat{\mu}_n^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_n}{\hat{\sigma}_i\hat{\sigma}_n}\hat{\sigma}_{i,n}^2\right)\end{aligned} \tag{20}$$

where T is the size of the sample, M denotes the length of the in-sample estimation period, $\hat{\mu}_i$ and $\hat{\mu}_n$ are the sample means for portfolios incorporating cryptocurrency factors and the benchmark, $\hat{\sigma}_i$ and $\hat{\sigma}_n$ denote the sample standard deviations, and $\hat{\sigma}_{i,n}^2$ is the sample covariance. We then compared the test statistics, $\hat{z}_{SR}$, with critical values to examine the significance of the results.

On the other hand, we evaluate the test statistics of certainty-equivalent return (CEQ) in a similar vein. We define $v = (\mu_i, \mu_n, \sigma_i^2, \sigma_n^2)$ as a vector, and compute the test statistics for $\widehat{CEQ}_k$ as follows (DeMiguel et al. 2009):

Table 11 p values for out-of-sample metrics for aggressive investors

	Panel A: SR							Panel B: CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
MARCAP	0.0091	0.0400	0.0373	0.0081	0.0192	0.0309	0.0058	0.0442	0.0009	0.0093	0.0040	0.0005	0.0034	0.0074
LPRC	0.3650	0.2375	0.0249	0.0452	0.0228	0.5022	0.0986	0.4973	0.0272	0.0343	0.0447	0.0027	0.0152	0.0023
AGE	0.1150	0.3678	0.1435	0.5010	0.5946	0.1335	0.5697	0.1977	0.4601	0.4609	0.5389	0.3912	0.1353	0.5614
MOM1	0.0018	0.4419	0.0784	0.0267	0.0885	0.0899	0.0313	0.0800	0.0143	0.0272	0.0492	0.0358	0.0084	0.0217
MOM2	0.0138	0.2089	0.0182	0.0021	0.0107	0.0616	0.0470	0.0471	0.0280	0.0135	0.0075	0.0252	0.0323	0.0031
MOM3	0.2772	0.3053	0.5922	0.5728	0.4383	0.0988	0.4834	0.1694	0.0476	0.0362	0.0788	0.0780	0.0334	0.0134
MOM4	0.2683	0.4312	0.2221	0.2478	0.4401	0.3639	0.3058	0.1108	0.3799	0.2504	0.5697	0.5905	0.2433	0.5004
MOM8	0.4013	0.0075	0.3918	0.3759	0.0058	0.0512	0.1413	0.5481	0.0060	0.0884	0.5719	0.0055	0.0728	0.3884
MOM26	0.4598	0.0100	0.0177	0.5856	0.0035	0.0887	0.3273	0.1129	0.0045	0.0323	0.3606	0.0037	0.0469	0.5148
RMOM1	0.3067	0.2089	0.1628	0.2545	0.4631	0.4914	0.0694	0.5245	0.0373	0.0593	0.0436	0.0934	0.0668	0.0103
RMOM2	0.0010	0.5216	0.0922	0.0385	0.1213	0.0378	0.0352	0.4269	0.0036	0.0203	0.0333	0.0467	0.0405	0.0242
RMOM3	0.0365	0.0224	0.0135	0.0337	0.0239	0.0312	0.0118	0.0378	0.0209	0.0097	0.0099	0.0086	0.0039	0.0045
RMOM4	0.1886	0.5148	0.4835	0.0934	0.1539	0.1911	0.0099	0.2233	0.0784	0.0883	0.0457	0.0558	0.0299	0.0074
RMOM8	0.3449	0.1966	0.0896	0.1495	0.1221	0.3786	0.4862	0.5499	0.3252	0.2028	0.5498	0.4813	0.5412	0.2425
RMOM26	0.2560	0.0018	0.0339	0.2051	0.0051	0.5532	0.4145	0.4366	0.0066	0.0061	0.3037	0.0028	0.4583	0.2417
VOL	0.1508	0.0039	0.1273	0.3506	0.3159	0.0998	0.5058	0.5481	0.0083	0.2950	0.3490	0.4474	0.0834	0.4048
VOLPRC	0.3428	0.5472	0.1688	0.2950	0.5637	0.5587	0.4568	0.3874	0.2630	0.3282	0.4569	0.0884	0.0721	0.1093
VOLSCALE	0.4092	0.2716	0.5680	0.1624	0.4653	0.4232	0.5166	0.4374	0.3193	0.3189	0.1585	0.5073	0.2624	0.2231
RETVOL	0.2991	0.4749	0.5176	0.2612	0.3761	0.5896	0.3747	0.2714	0.2878	0.3733	0.3810	0.2979	0.2991	0.3577
RETSKEW	0.2652	0.4097	0.2803	0.4783	0.3070	0.3462	0.4474	0.4288	0.5755	0.4612	0.3000	0.5159	0.1672	0.1302
RETKURT	0.5864	0.2639	0.0838	0.4695	0.5771	0.1160	0.2784	0.1421	0.1819	0.2621	0.2509	0.1058	0.3700	0.1477
MAXRET	0.4313	0.2408	0.2152	0.4556	0.4123	0.3953	0.4302	0.1733	0.4156	0.5297	0.5871	0.3854	0.5984	0.3768
STDPRCVOL	0.1238	0.2744	0.3257	0.2205	0.0715	0.0856	0.0282	0.3577	0.2653	0.0430	0.3459	0.0036	0.0444	0.1323
MEANABS	0.4655	0.1689	0.5184	0.1693	0.3941	0.2831	0.5034	0.3181	0.5133	0.2973	0.4067	0.5093	0.5431	0.5656

Table 11 (continued)

	Panel A: SR							Panel B: CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
BETA	0.3519	0.3448	0.5385	0.2766	0.3247	0.5818	0.1211	0.1954	0.2293	0.0898	0.3967	0.3519	0.4064	0.5097
$BETA^2$	0.5865	0.0019	0.4336	0.3932	0.4376	0.2805	0.4101	0.3659	0.0020	0.0454	0.3140	0.5830	0.0620	0.4477
IDIOVOL	0.5056	0.1096	0.1419	0.5874	0.4257	0.2156	0.3017	0.4601	0.2734	0.3585	0.3783	0.1782	0.0562	0.4474
DELAY	0.1610	0.2342	0.2289	0.2658	0.1761	0.2740	0.1608	0.3132	0.5181	0.4657	0.2800	0.3271	0.2932	0.4878

This table reports the out-of-sample Sharpe ratios (SRs) and certainty-equivalent returns (CEQs) of Table 4 for both the benchmark portfolio and portfolios including cryptocurrency factors at risk aversion level $\lambda = 1$

Table 12 *p* values for out-of-sample metrics for moderate investors

	Panel A: SR							Panel B: CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
MARCAP	0.0073	0.0043	0.0069	0.0095	0.0392	0.0353	0.0011	0.0073	0.0037	0.0058	0.0012	0.0006	0.0098	0.0028
LPRC	0.2950	0.0591	0.3297	0.1252	0.2143	0.5171	0.1078	0.3975	0.0962	0.1929	0.1965	0.0342	0.5664	0.2953
AGE	0.5319	0.1390	0.4345	0.3501	0.2090	0.3858	0.1611	0.2366	0.1760	0.2986	0.2874	0.1656	0.3175	0.1458
MOM1	0.0671	0.0300	0.0028	0.0056	0.0076	0.0010	0.0435	0.0615	0.0005	0.0287	0.0395	0.0118	0.0224	0.0285
MOM2	0.0832	0.0309	0.0260	0.0432	0.0049	0.0454	0.0054	0.0061	0.0248	0.0321	0.0111	0.0419	0.0486	0.0423
MOM3	0.3585	0.0143	0.3797	0.1023	0.0767	0.5244	0.5584	0.3530	0.0279	0.4733	0.2185	0.0957	0.4101	0.4001
MOM4	0.5935	0.3526	0.2357	0.1504	0.3539	0.3928	0.4814	0.1863	0.1452	0.2276	0.5293	0.5555	0.4498	0.4626
MOM8	0.1415	0.0331	0.0258	0.1855	0.0469	0.0295	0.3203	0.2149	0.0288	0.0811	0.3019	0.0494	0.0090	0.2605
MOM26	0.5710	0.0328	0.0226	0.5198	0.0266	0.0554	0.4400	0.3557	0.0030	0.0726	0.3783	0.0265	0.5150	0.5294
RMOM1	0.2836	0.2196	0.3895	0.5334	0.3034	0.1563	0.3219	0.4945	0.0318	0.3261	0.4761	0.0110	0.1549	0.2349
RMOM2	0.0300	0.0201	0.0417	0.0404	0.0195	0.0180	0.0140	0.0525	0.0486	0.0355	0.0312	0.0146	0.0425	0.0912
RMOM3	0.0130	0.0043	0.0215	0.0129	0.0030	0.0212	0.0060	0.0320	0.0128	0.0044	0.0419	0.0058	0.0474	0.0031
RMOM4	0.3475	0.0706	0.2218	0.4925	0.0074	0.0394	0.1017	0.3923	0.0285	0.0828	0.0191	0.0443	0.0393	0.0827
RMOM8	0.2103	0.0001	0.0095	0.1712	0.2340	0.0087	0.1693	0.4384	0.0208	0.0318	0.1669	0.4357	0.3855	0.1849
RMOM26	0.3994	0.0451	0.0470	0.2106	0.0241	0.2880	0.3619	0.1738	0.0238	0.0908	0.3761	0.0016	0.1269	0.5025
VOL	0.2324	0.1342	0.3182	0.1869	0.0026	0.5773	0.3153	0.3257	0.2913	0.0790	0.2821	0.0532	0.0712	0.5357
VOLPRC	0.5808	0.4812	0.1037	0.4400	0.4530	0.4226	0.3762	0.2643	0.4251	0.5874	0.1380	0.3935	0.3069	0.2546
VOLSCALE	0.2091	0.4862	0.0114	0.2854	0.5455	0.0856	0.3012	0.2319	0.4794	0.0995	0.1933	0.4906	0.1979	0.5962
RETVOL	0.2590	0.4043	0.5551	0.5545	0.3958	0.2663	0.5265	0.5011	0.3121	0.4644	0.3492	0.5045	0.2783	0.1366
RETSKEW	0.3212	0.5522	0.1166	0.3662	0.4582	0.0179	0.2683	0.3955	0.5551	0.1969	0.3162	0.4746	0.1196	0.5732
RETKURT	0.1939	0.2610	0.0202	0.3743	0.1244	0.0276	0.2374	0.4818	0.3794	0.0184	0.3490	0.3589	0.0994	0.5274
MAXRET	0.2208	0.2216	0.1771	0.5782	0.5678	0.5094	0.4641	0.5812	0.4395	0.3018	0.5675	0.3397	0.2159	0.2981
STDPRCVOL	0.1879	0.0360	0.1944	0.1006	0.0158	0.4498	0.4126	0.4525	0.0559	0.4783	0.5977	0.0481	0.3675	0.5819
MEANABS	0.3715	0.3195	0.2437	0.3508	0.4808	0.4812	0.3880	0.1578	0.1257	0.2522	0.3901	0.3655	0.5506	0.3703

Table 12 (continued)

	Panel A: SR							Panel B: CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
BETA	0.4738	0.0323	0.0123	0.3522	0.2736	0.0046	0.1739	0.3160	0.0271	0.0712	0.1083	0.5005	0.0143	0.3392
$BETA^2$	0.1991	0.0336	0.0432	0.4472	0.2284	0.0010	0.3661	0.2284	0.0185	0.4309	0.1848	0.2394	0.1991	0.1975
IDIOVOL	0.2397	0.5731	0.0906	0.2963	0.0025	0.0671	0.5186	0.2634	0.5402	0.0471	0.3020	0.0179	0.0969	0.3037
DELAY	0.5857	0.1285	0.0450	0.3912	0.4433	0.0719	0.4250	0.5222	0.4077	0.0377	0.5386	0.4924	0.0465	0.5070

This table reports the out-of-sample Sharpe ratios (SRs) and certainty-equivalent returns (CEQs) of Table 5 for both the benchmark portfolio and portfolios including cryptocurrency factors at risk aversion level $\lambda = 3$

Table 13 p values for out-of-sample metrics for conservative investors

	Panel A: SR							Panel B: CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
MARCAP	0.0090	0.0043	0.0033	0.0060	0.0090	0.0070	0.0038	0.0017	0.0050	0.0100	0.0036	0.0005	0.0021	0.0040
LPRC	0.4675	0.5771	0.3714	0.3701	0.2556	0.1356	0.1910	0.2668	0.2148	0.5681	0.4416	0.5811	0.3190	0.5702
AGE	0.1465	0.3317	0.0005	0.5575	0.4214	0.0001	0.1152	0.1029	0.4052	0.5005	0.2165	0.5662	0.4816	0.5132
MOM1	0.0208	0.0227	0.1636	0.0009	0.0364	0.0354	0.0780	0.0573	0.0396	0.2645	0.0223	0.0156	0.0585	0.0830
MOM2	0.0437	0.0218	0.0049	0.0025	0.0046	0.0594	0.0121	0.0290	0.0201	0.0862	0.0307	0.0496	0.0204	0.0414
MOM3	0.5207	0.5286	0.5818	0.3444	0.2102	0.2131	0.3684	0.4379	0.2245	0.3379	0.2995	0.3997	0.5003	0.1525
MOM4	0.4811	0.2738	0.3306	0.4197	0.5587	0.1808	0.4578	0.5107	0.5205	0.2773	0.3150	0.3861	0.4504	0.4712
MOM8	0.3889	0.0217	0.0442	0.2965	0.0089	0.0317	0.4120	0.4789	0.2946	0.3147	0.5782	0.3865	0.5249	0.2382
MOM26	0.2640	0.0401	0.0500	0.5905	0.0064	0.0116	0.1118	0.4112	0.3942	0.5817	0.1430	0.3502	0.3608	0.1451
RMOM1	0.4037	0.1554	0.3037	0.5420	0.3741	0.2845	0.2042	0.5523	0.5422	0.3195	0.4909	0.1742	0.4099	0.2303
RMOM2	0.0441	0.0478	0.1620	0.0471	0.0428	0.0043	0.0692	0.0446	0.0422	0.1981	0.0304	0.0242	0.2689	0.0798
RMOM3	0.0489	0.0142	0.1669	0.0343	0.0455	0.0611	0.0450	0.0494	0.0080	0.2184	0.0351	0.0188	0.0974	0.0486
RMOM4	0.1967	0.0754	0.2731	0.3093	0.0156	0.5095	0.4125	0.4218	0.0860	0.3009	0.4160	0.0985	0.3797	0.5668
RMOM8	0.4693	0.0403	0.0034	0.5754	0.0498	0.0378	0.4712	0.4602	0.3420	0.4195	0.5438	0.1994	0.2977	0.5961
RMOM26	0.5156	0.0078	0.0229	0.4091	0.0466	0.5175	0.5477	0.3012	0.4294	0.5507	0.5977	0.4266	0.1542	0.1181
VOL	0.3913	0.3914	0.5275	0.1174	0.0443	0.3039	0.1182	0.4090	0.3836	0.5810	0.4731	0.4313	0.3617	0.2299
VOLPRC	0.4731	0.1774	0.1720	0.4030	0.2272	0.2621	0.3009	0.5810	0.3701	0.1151	0.4482	0.3599	0.1295	0.5450
VOLSCALE	0.3032	0.2931	0.0305	0.1834	0.1940	0.0095	0.2616	0.2651	0.2149	0.1570	0.2555	0.2142	0.4260	0.1331
RETVOL	0.4848	0.2171	0.4702	0.4464	0.5120	0.5140	0.2467	0.2377	0.2409	0.5400	0.3222	0.4780	0.4016	0.4916
RETSKEW	0.2547	0.3615	0.0325	0.5159	0.5051	0.0278	0.2315	0.1570	0.5893	0.5243	0.1253	0.3331	0.2628	0.4151
RETKURT	0.4403	0.0117	0.0046	0.2923	0.3693	0.0099	0.4776	0.2151	0.3899	0.4016	0.3999	0.3242	0.1177	0.3569
MAXRET	0.5902	0.2174	0.3643	0.1257	0.4784	0.4010	0.5286	0.3039	0.1540	0.3299	0.3254	0.3756	0.5027	0.4504
STDPRCVOL	0.5941	0.5647	0.3048	0.1002	0.0541	0.2039	0.2096	0.5361	0.1261	0.2098	0.3298	0.0959	0.4950	0.3259
MEANABS	0.2629	0.1480	0.4738	0.4743	0.3716	0.2691	0.5162	0.2667	0.1295	0.4705	0.3534	0.2000	0.3136	0.1843

Table 13 (continued)

	Panel A: SR							Panel B: CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
BETA	0.3763	0.0479	0.0893	0.2783	0.3732	0.0173	0.4114	0.4758	0.2842	0.5709	0.1086	0.5145	0.4133	0.3694
$BETA^2$	0.4983	0.0373	0.0126	0.5112	0.1126	0.0414	0.4657	0.4253	0.4633	0.1472	0.5388	0.1072	0.2472	0.1900
IDIOVOL	0.4907	0.0184	0.0745	0.5461	0.0243	0.0130	0.2125	0.5631	0.0068	0.3905	0.4186	0.4256	0.5323	0.1280
DELAY	0.2750	0.2435	0.0927	0.1257	0.3963	0.0163	0.5192	0.5084	0.3645	0.4472	0.2062	0.3716	0.4513	0.5782

This table reports the out-of-sample Sharpe ratios (SRs) and certainty-equivalent returns(CEQs) of Table 6 for both the benchmark portfolio and portfolios including crypto-currency factors at risk aversion level $\lambda = 5$

Table 14 *p* values for out-of-sample metrics inclusive of transaction costs

	SR							CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Panel A: Aggressive investors														
MARCAP	0.0044	0.0043	0.0029	0.0063	0.0398	0.0346	0.0035	0.0473	0.0052	0.0095	0.0007	0.0021	0.0078	0.0091
RMOM3	0.0783	0.0296	0.0152	0.0424	0.0785	0.0271	0.0114	0.0321	0.0415	0.0082	0.0057	0.0286	0.0143	0.0070
Panel B: Moderate investors														
MARCAP	0.0080	0.0044	0.0045	0.0047	0.0140	0.0338	0.0090	0.0454	0.0075	0.0026	0.0069	0.0013	0.0012	0.0019
RMOM3	0.0146	0.0293	0.0073	0.0822	0.0361	0.0463	0.0493	0.0655	0.0445	0.0539	0.0282	0.0488	0.0018	0.0326
Panel C: Conservative investors														
MARCAP	0.0097	0.0037	0.0031	0.0012	0.0092	0.0014	0.0033	0.0449	0.0050	0.0062	0.0058	0.0070	0.0003	0.0053
RMOM3	0.0032	0.0827	0.2700	0.5234	0.0246	0.3907	0.5688	0.0048	0.0054	0.1103	0.4407	0.0599	0.1570	0.4981

This table reports the post-transaction-cost out-of-sample Sharpe ratios (SRs) and certainty-equivalent returns (CEQs) of Table 8 for both the benchmark portfolio and portfolios including outperformed cryptocurrency factors at risk aversion level $\lambda = 1, 3, 5$

Table 15 *p* values for out-of-sample metrics for multi-asset benchmarks

	SR							CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Panel A: Aggressive investors														
MARCAP	0.0062	0.0035	0.0035	0.0014	0.0394	0.0046	0.0024	0.0122	0.0010	0.0086	0.0070	0.0073	0.0065	0.0052
RMOM3	0.0163	0.0662	0.0118	0.0074	0.0010	0.0482	0.0485	0.0124	0.0234	0.0328	0.0145	0.0075	0.0056	0.0214
Panel B: Moderate investors														
MARCAP	0.0027	0.0075	0.0090	0.0073	0.0203	0.0469	0.0026	0.0267	0.0095	0.0027	0.0025	0.0093	0.0007	0.0030
RMOM3	0.0296	0.0102	0.0636	0.0399	0.0251	0.0325	0.0398	0.0233	0.0300	0.0112	0.0258	0.0419	0.0460	0.0249
Panel C: Conservative investors														
MARCAP	0.0028	0.0065	0.0092	0.0051	0.0097	0.0020	0.0011	0.0030	0.0040	0.0042	0.0031	0.0069	0.0009	0.0040
RMOM3	0.0148	0.0306	0.1528	0.0297	0.0283	0.1776	0.0000	0.0142	0.0551	0.5355	0.0021	0.0452	0.1655	0.0417

This table reports the out-of-sample Sharpe ratios (SRs) and certainty-equivalent returns (CEQs) of Table 9 for both the multi-asset benchmark portfolio and portfolios including outperformed cryptocurrency factors at risk aversion level $\lambda = 1, 3, 5$

Table 16 *p* values forout-of-sample metrics via the rolling-window scheme

	SR							CEQ						
	Traditional approaches				Variant approaches			Traditional approaches				Variant approaches		
	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML	EW	MV	BS	BL	MV-ML	BS-ML	BL-ML
Panel A: Aggressive investors														
MARCAP	0.0080	0.0459	0.0069	0.0050	0.0202	0.0087	0.0058	0.0303	0.0021	0.0052	0.0099	0.0049	0.0069	0.0041
RMOM3	0.0017	0.0293	0.0801	0.0173	0.0083	0.0256	0.0183	0.0370	0.0262	0.0402	0.0082	0.0019	0.0012	0.0082
Panel B: Moderate investors														
MARCAP	0.0064	0.0008	0.0090	0.0052	0.0272	0.0303	0.0076	0.0086	0.0038	0.0008	0.0073	0.0033	0.0084	0.0037
RMOM3	0.0414	0.0088	0.1648	0.0440	0.0022	0.0343	0.0367	0.0219	0.0190	0.5898	0.0199	0.0220	0.0078	0.0163
Panel C: Conservative investors														
MARCAP	0.0031	0.0089	0.0025	0.0031	0.0041	0.0071	0.0014	0.0087	0.0008	0.0046	0.0003	0.0075	0.0070	0.0021
RMOM3	0.0340	0.0557	0.5253	0.0279	0.0451	0.3098	0.0179	0.0244	0.0256	0.5646	0.0233	0.0127	0.3156	0.0351

This table reports the out-of-sample Sharpe ratios (SRs) and certainty-equivalent returns (CEQs), under a rolling-window scheme, of Table 10 for both the benchmark portfolio and portfolios including outperformed cryptocurrency factors at risk aversion level $\lambda = 1, 3, 5$

$$\hat{z}_{CEQ} = f(v)/\Phi,$$
$$\text{where } f(v) = \left(\mu_i - \frac{\lambda}{2}\sigma_i^2\right) - \left(\mu_n - \frac{\lambda}{2}\sigma_n^2\right),$$
$$\text{and } \Phi = f'(v)^{\mathrm{T}} \begin{pmatrix} \sigma_i^2 & \sigma_{i,n} & 0 & 0 \\ \sigma_{in} & \sigma_n^2 & 0 & 0 \\ 0 & 0 & 2\sigma_i^4 & 2\sigma_{i,n}^2 \\ 0 & 0 & 2\sigma_{i,n}^2 & 2\sigma_n^4 \end{pmatrix} f'(v)$$

we next use these test statistics to determine the significant levels of out-of-sample CEQs.

Declarations

Conflict of interest No potential Conflict of interest was reported by the author(s).

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