Robust risk choice under high-water mark contract

Congming Mu[1] · Jingzhou Yan[2] · Jinqiang Yang[3,4]

Abstract

By assuming that a risk-neutral hedge fund manager has ambiguous beliefs about the return process of risky asset, we study his robust risk choice under the high-water mark. The results show that without management fees, ambiguity aversion induces the manager to take more risk as the fund is close to the termination but take less risk as the fund approaches the high-water mark. With management fees, ambiguity aversion increases the induced risk aversion and moderates the manager's incentive to take risk, predicting that it is the manager with higher rates of management fees that reduces the risky asset holdings more when he becomes less confident and/or more pessimistic about the future returns. The model implies that managers' ambiguity aversion is a possible factor explaining hedge fund activities in stock markets during the financial crisis of 2007–2009 and in US Treasury markets during COVID-19 crisis. Finally, taxation is taken into account.

Keywords Hedge funds · Ambiguity aversion · High-water mark · Robust risk taking

JEL Classification G11 · G23

✉ Jingzhou Yan
  yanjingzhou036@163.com

  Congming Mu
  mucongming@sina.com

  Jinqiang Yang
  yang.jinqiang@mail.shufe.edu.cn

[1] College of Finance and Statistics, Hunan University, Changsha 410006, China

[2] School of Economics, Sichuan University, Chengdu 610041, China

[3] School of Finance, Shanghai University of Finance and Economics, Shanghai 200433, China

[4] Shanghai Institute of International Finance and Economics, Shanghai 200433, China

1 Introduction

Over the last several decades, the hedge fund industry has grown at a very fast pace and achieved a great size over four trillion dollars under management.[1] Hedge funds play several great roles in financial markets, such as supplying liquidly, eliminating market mispricings, even potentially disrupting financial markets, etc (see, Agarwal and Ren 2023). Differing from mutual funds, hedge funds are very flexible with respect to the types of securities they hold and the type of positions they take.[2] Thus, hedge funds have strong incentives to construct highly dynamic, complex trading strategies, thereby exposing their portfolios to a plethora of economic risk factors. In addition, capacity of the hedge fund managers or performance is often reflected by 'alpha', which is the intercept in the regression of the fund's excess return on the excess return of some passive benchmarks guided by factor pricing models in the spirit of Ross (1976) Arbitrage Pricing Theory. However, Vrontos et al. (2008) point out that while many factors that are priced in hedge fund investments have been identified, the issue of identifying the "correct" set of factors remains open; and is usually referred to as "model uncertainty".[3]

Recently, Heaton (2019) contributes one of the reasons why hedge fund activism has mostly disappointed to that hedge activists are likely to be too pessimistic since the competition for activist targets likely results in the most pessimistic—and therefore most incorrect—activist being the one to appear at a given firm. In particular, Nishimura and Ozaki (2017) show that seemingly irrational, excessively pessimistic behavior can be in fact "rational" in the sense that it is consistent with axioms that are "reasonable," under fundamental (Knightian) uncertainty about the future. Along with the findings of Heaton (2019), the arguments of Nishimura and Ozaki (2017) imply that hedge fund managers to some extent face a amount of model uncertainty. Importantly, Argyropoulos et al. (2022) point out that there is significant time variation in the set of risk factors and their respective loadings which in turn introduces severe model risk in any attempt to model and forecast hedge fund returns.[4] Accordingly, we introduce model uncertainty into the risk choice model of Drechsler (2014) by assuming that the fund managers are concerned about potential model misspecification and investigate the fund managers' robust risk taking. Specifically, a fund manager compensated by a high-water mark (hereafter, HWM) contract can freely trade a riskless asset and a risky asset, but has ambiguous beliefs about the return process of risky asset and accordingly cares about robust risk choice.[5]

---

[1] See https://www.barclayhedge.com/solutions/assets-under-management/hedge-fund-assets-under-management/.

[2] Jiang et al. (2022) show that style-shifting is a dynamic strategy commonly employed by hedge fund managers and Sun et al. (2022) identify the direct impact of fund style drift on the risk of stock price collapse.

[3] Cvitanic et al. (2003) adopt 5 different asset pricing models to compute a fund abnormal return and use the dispersion in alphas across models as a measure of model uncertainty.

[4] By analyzing the strictest policy responses to the coronavirus pandemic from January 2020 to May 2022 belong to Italy, China, Hong Kong, Greece, Austria, Peru, Singapore and Malaysia, Owjimehr and Hasanzadeh Dastfroosh (2022) find that the policy reduces uncertainty in the stock market only in Malaysia and Singapore. This implies that many market participants face great uncertainty.

[5] While we include ambiguity about the expected return of risky asset but make no assumptions regarding the origin of this ambiguity. It can arise from a lack of statistical evidence, a lack of theoretical evidence, and so on. In practice, the parameters of the asset returns distribution, such as the expected return, are largely unknown and difficult to estimate, imposing a considerable amount of model uncertainty on market participants.

   To maintain the analytical tractability as in Drechsler (2014), we adopt the homothetic robustness methodology developed by Maenhout (2004, 2006) but conduct a necessary modification. This is due to the following facts. On the one hand, there are two state variables, the fund wealth and the HWM. On the other hand, the manager has the option to shut down any further risk taking and only receives the management fees or the outside option upon the exogenous random termination. This component can be independent of the fund wealth or the ratio of fund wealth to HWM. Thus, the homothetic robustness methodology requires the manager value function net of the discounted value of the future payments that would accrue to the manager if he were to shut down any further risk taking. With this modification, our extension maintains the analytical tractability and provides some important implications in a parsimonious manner.

   The analytical solution shows that ambiguity aversion influences the fund manager's risk choice through two channels, a direct channel and an indirect channel. The direct effect enhances the (induced) effective risk aversion in the way proposed by Maenhout (2004, 2006) but the indirect effect embodies in modifying the sensitivity of induced risk aversion to the ratio of fund wealth to HWM. Their interactions can generate a variety of risk taking, thereby, to some extent, helping to explain the observed cross-sectional variation in hedge fund managers' portfolio decisions documented by Sias et al. (2016), and providing theoretical support for the evidence of DeVault and Sias (2017) that hedge fund managers' psychological characteristics influence their portfolio choice.

   In the scenario without management fees such that maximizing the present value of performance fees is the main incentive for the fund manager, our findings show that as the fund closes to the termination, the manager's ambiguity aversion reduces his effective risk aversion and hence encourages him to take more risk. This implies that ambiguity aversion can increase the manager's propensity to shift risk in response to poor performance. Nevertheless, as the fund approaches the HWM, ambiguity aversion increases the induced risk aversion and accordingly induces the fund manager to take less risk. This is partially consistent with the empirical finding of Kouwenberg and Ziemba (2007) that returns of hedge funds with incentive fees are not significantly more risky than the returns without such compensation contract. However, given a reasonable level of management fees (i.e., 2%), the effects of ambiguity aversion following poor performance diminish in that ambiguity aversion increases the induced risk aversion and dampens the manager's risk choice, thereby moderating his incentive to take risk. Along with the fact that economic turndown can lead the manager to become less confident and/or more pessimistic about the future returns of assets under management,[6] this may predict that it is the hedge fund manager with higher rates of management fees that reduces his holdings more. Moreover, this implies that manager's ambiguity aversion may be a factor that contributes to the average decline in hedge fund equity holdings during the selloff quarters in the financial crisis of 2007–2009 (see, Ben-David et al. 2012) and that induces the hedge funds to exit from US Treasury (hereafter, UST) markets with 20% higher cash holdings during COVID-19 crisis despite low contemporaneous outflows (see, Kruttli et al. 2021).[7] Finally, we extend

---

[6] Nishimura and Ozaki (2017) point out that faced with the real possibility of surprise events, human emotions often swing between optimism and pessimism.

[7] Ben-David et al. (2012) report that redemptions and leverage explain about 80% of the decline in average hedge fund equity holdings (that is, 9.5% of the 12% average decline in equity holdings). By analyzing hedge fund-creditor borrowing data, Kruttli et al. (2021) find that the step back in hedge fund UST activity is primarily driven by fund-specific liquidity management and that hedge funds exit the turmoil caused by the COVID-19 pandemic with 20% higher cash holdings despite low contemporaneous outflows. Yang et al. (2023) find that the COVID-19 pandemic generates a significant negative impact on firm value.

our model by taking taxation into account. The extended model shows that it is the trade-off between the after tax performance fees and management fees that further influences the manager's risk-taking behavior.

Our paper theoretically contributes to the literature about the HWM contract. For instance, Goetzmann et al. (2003) derive the present value of management compensation and investors' claim under the HWM contract but assume an exogenous level of risk choice. Differently, we look for the fund manager's optimal risk-taking activity. By numerically solving a risk-averse hedge fund manager's investment strategy in a discrete-time model, Hodder and Jackwerth (2007) study the effect of fund manager's horizon on risk taking. By contrast, we focus on the effect of fund manager's ambiguity aversion on his risk choice with the assumption that the fund manager is risk neutral. By integrating performance fees with other fundamental factors (i.e., termination, outside payoff, investors' redemption), Lan et al. (2013), Buraschi et al. (2014) and Drechsler (2014) investigate how the interaction of these factors affects fund manager's optimal risk taking. Based on their research, we further incorporate the fund manager's ambiguity aversion and give our focus on the induced effect on risk choice. Recently, Guasoni and Obłój (2016) pay attention to the incentives of performance fees under the HWM contract and find that for a manager with risk aversion less than one, the HWM contract decreases risk taking. Our findings show that given the HWM contract, the fund manger's ambiguity aversion also can decrease his risk taking, which is different from the effect arising from weak investor protection in that Bian et al. (2022) find that weak investor protection generates ambiguous effects on dynamic leverage choice. Ambiguity aversion also makes our paper differ from He and Kou (2018), Zhao et al. (2018) and Mu et al. (2021). He and Kou (2018) propose an analytical framework to compute and compare two managerial incentive schemes for hedge funds by assuming cumulative prospect theory (CPT) preferences for the manager, Zhao et al. (2018) focus on the stochastic market conditions and Mu et al. (2021) take jump risk into account.

Our work is also related to the research on robust portfolio choice. Maenhout (2004) studies the robust dynamic investment and consumption strategies of risk-averse investors. Maenhout (2006) analyzes the optimal intertemporal portfolio problem of an ambiguity-averse investor with a mean-reverting risk premium. In these studies, robustness amounts to an increase in effective risk aversion and accordingly gives rise to a reduction in the optimal stock investment. By assuming that stock returns are predictable by an observable and an unobservable predictor, Branger et al. (2013) analyze the robust optimal investment strategy with ambiguity aversion under learning and jump-diffusion risk and find that ambiguity aversion can impact the level and structure of the optimal portfolio choice. Branger and Larsen (2013) solve the portfolio planning problem of an ambiguity-averse investor when the stock follows a jump-diffusion process. Based on the assumption that stock price follows a stochastic volatility jump-diffusion process, Escobar et al. (2015) derive the optimal portfolio for an ambiguity-averse investor who has access to stock and derivatives markets. Differently, our focus is on the hedge fund managers' risk choice with the HWM compensation contract rather than individuals' portfolio choice.

The remainder of the paper is structured as follows. Section 2 lays out the model. Section 3 presents the solution to the model for robust risk taking. The model implications are provided in Sect. 5 and an extension with taxation is presented in Sect. 6. Section 7 concludes. All proofs are collected in the Appendix.

2 Model setup

2.1 Investment opportunities with ambiguous beliefs

A risk-neutral fund manager but with ambiguity aversion (discussed below) has access to two financial assets: one is riskless and provides a fixed return rate r, and the other is risky and its price S_t evolves according to

$$\frac{dS_t}{S_t} = \mu dt + \sigma d\mathcal{B}_t, \quad S_0 > 0, \tag{1}$$

where $\mu(> r)$ and $\sigma(> 0)$ are two constants, and $\mathcal{B}_t$ is a one-dimensional standard Brownian motion defined on a filtered probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t \geq 0}, \mathbb{P})$. $\{\mathscr{F}_t\}_{t \geq 0}$ denotes the augmented filtration generated by $\mathcal{B}$. The manager can freely borrow and lend the risk-free asset.

Differing from Drechsler ([2014](#)) and based on the aforementioned discussion that the fund manager to some extent faces a amount of model uncertainty about the return-generating process, we assume that the manager faces model uncertainty in the sense that he believes that there may be multiple distributions surrounding the true distribution of the return provided by the financial markets. Specifically, the fund manager views the probability measure $\mathbb{P}$ as the reference model, but does not trust this model. To protect himself from model misspecifications, he takes into account of alternative models. Let $\mathbb{P}^u$ denote the probability measure of the alternative model. The change of measure from the reference measure $\mathbb{P}$ to an alternative measure $\mathbb{P}^u$ can be defined by Radon-Nikodym derivative as

$$\frac{d\mathbb{P}^u}{d\mathbb{P}} | \mathscr{F}_t = Z_t, \tag{2}$$

where $Z_t = \exp\left(-\int_0^t u_s d\mathcal{B}_s - \frac{1}{2}\int_0^t u_s^2 ds\right)$ is a $(\mathbb{P}, \mathscr{F})$–martingale with $Z_0 = 1$.[8] $\{u_t\}_{t \geq 0}$ is a real-valued process satisfying $\int_0^t u_s^2 ds < \infty$ for all $t > 0$. By Girsanov's Theorem, it follows that

$$d\mathcal{B}_t^u = d\mathcal{B}_t + u_t dt, \tag{3}$$

where $\mathcal{B}^u$ is a standard Brownian motion under $\mathbb{P}^u$, under which, the price of risky asset follows

$$dS_t = (\mu - \sigma u_t)S_t dt + \sigma S_t d\mathcal{B}_t^u, \tag{4}$$

where $\mu - \sigma u$ represents the "adjusted mean return" and captures the fact that the expected return is hard to estimated with any precision.

2.2 Wealth processes and high-water marks

The high-water mark (i.e., HWM), denoted by H_t, is referred to the highest peak that the value of assets under management has ever reached. Mathematically, it satisfies $H_t = \max_{0 \leq s \leq t} W_s$, where W denotes the fund's wealth (or value) and will be defined below.

---

[8] For the technical details, please refer to Hansen et al. ([2006](#)), and others.

Similar to Goetzmann et al. ([2003](#)) and Drechsler ([2014](#)), several adjustments in HWM should be taken into account. First, the HWM itself adjusts up at the interest rate r, implying the manager can not earn performance fees for earning the interest rate on the fund. Second, withdrawals from outside investors are allowed and occur at a continuous but constant rate ϕ.[9] A final outflow from the HWM is the management fees paid to the manager at a rate m. With these specifications, the HWM acts as follows. If the fund is not reaching a new higher level, that is $W_t < H_t$, H_t evolves deterministically following $dH_t = (r - \phi - m)H_t dt$. By contrast, when the fund is reaching a new higher mark, from $W_t = H_t$ to $W_t = H_t + dH_t^\epsilon$, the manager receives a performance fee $k\,dH_t^\epsilon$, the wealth in the fund decreases by $k\,dH_t^\epsilon$ and the HWM becomes $H + dH_t^\epsilon$. Mathematically, the HWM evolves according to

$$dH_t = (r - \phi - m)H_t dt + dH_t^\epsilon. \tag{5}$$

The ambiguity-averse fund manager invests a fraction π_t of fund wealth W_t in the risky asset at time t and the remaining wealth $(1 - \pi_t)W_t$ in the risk-free asset. Then, the fund wealth W_t evolves according to

$$\frac{dW_t}{W_t} = rdt + \pi_t(\mu - \sigma u_t - r)dt + \pi_t \sigma d\mathcal{B}_t^u - (\phi + m)dt - kdH_t^\epsilon / W_t. \tag{6}$$

Analogous to to Lan et al. ([2013](#)) and Drechsler ([2014](#)), the homogeneity in H_t of the manager's problem still holds. For convenience, we focus on the ratio of fund wealth W_t to HWM H_t and define

$$X_t = \frac{W_t}{H_t}. \tag{7}$$

According to Ito's Lemma, the dynamics for this ratio satisfy

$$dX_t = X_t \pi_t(\mu - r - \sigma u_t)dt + X_t \pi_t \sigma d\mathcal{B}_t^u - (1 + k)dH_t^\epsilon / H_t. \tag{8}$$

Following Drechsler ([2014](#)), we take two kinds of management termination into account. One refers to an exogenous random termination captured by a Poisson process with intensity λ and the other is due to that fund wealth drops to a certain proportion C of the HWM (i.e., $X_t = C < 1$). Upon either termination at time t, the fund manager obtains a payoff $\underline{V}_t$. To capture the plausibility that the outside option for the manager in a large hedge fund is larger than that in a small hedge fund, we assume that the outside option is proportional to the manager's value function at the HWM ($X_t = 1$). Thus, the outside payoff $\underline{V}_t$ can be defined as

$$\underline{V}_t = gV(X_t = 1, H_t), \quad 0 \le g < 1, \tag{9}$$

where $V(X_t, H_t)$ represents the manager's value function for given X_t and H_t, and g measures the fraction that the manager can receive upon termination.

---

[9] We make no any assumptions regarding outside investors' characteristics, such as risk aversion, ambiguity aversion and others,so we take a constant withdrawal rate.

2.3 Optimization problem with robustness

Under the reference probability measure $\mathbb{P}$, the risk-neutral manager maximizes the expected net present value of his payoff including performance fees, management fees and the outside option conditional on termination, given by

$$V(X_t, H_t) = \sup_{\pi_s} \mathbb{E}_t^{\mathbb{P}} \left[\int_t^\tau e^{-\rho(s-t)}(mW_s ds + kdH_t^\varepsilon) + e^{-\rho(\tau-t)}\underline{V_\tau} \right], \tag{10}$$

where ρ is the manager's subjective discount factor, $\tau = \inf\left\{ t : \frac{W_t}{H_t} = C \right\}$ represents the stopping time that the fund will be terminated the first time either X_t drops to C or the exogenous random termination triggers, or $\tau = \infty$ if it is never terminated.

Following Anderson et al. (2003), Hansen et al. (2006), and Hansen and Sargent (2012), we use discounted relative entropy to measure the discrepancy between the reference measure $\mathbb{P}$ and alternative measure $\mathbb{P}^u$ defined by

$$\rho\mathbb{E}^{\mathbb{P}} \left[\int_0^\infty e^{-\rho t} Z_t \ln Z_t dt \right] = \frac{1}{2}\mathbb{E}^{\mathbb{P}^u} \left[\int_0^\infty e^{-\rho t} u_t^2 dt \right]. \tag{11}$$

Then the optimization problem for the fund manager with uncertainty about the return process satisfies

$$\begin{aligned} V(X_t, H_t) = \sup_{\pi_s} \inf_u \mathbb{E}_t^{\mathbb{P}^u} &\left[\int_t^\tau e^{-\rho(s-t)}(mW_s ds + kdH_t^\varepsilon) + e^{-\rho(\tau-t)}\underline{V_\tau} \right] \\ &+ \frac{1}{2}\mathbb{E}^{\mathbb{P}^u} \left[\int_t^\tau \frac{e^{-\rho(s-t)} u_s^2}{\Psi(X_s, H_s)} ds \right]. \end{aligned} \tag{12}$$

In the terminology of robustness control, the last term in (12) is used to penalize belief distortions from the reference probability, but the denominator $\Psi(X, H) \geq 0$ comes from homothetic robustness methodology proposed by Maenhout (2004) and is used to preserve state independence and analytical tractability of the risk choice problem.[10] Thus, the larger $\Psi(X, H)$ is, the more ambiguity averse the fund manager is and the less faith the fund manager has in the validity of the reference model.

3 Model solution

According to the stochastic dynamic programming, the value function V given by (12) satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

---

[10] In the model of Anderson et al. (2003), the entropy penalty (11) is weighted by a constant (i.e., $1/\hat{\theta}$), which is state independent. Replacing $\hat{\theta}$ by a state-dependent one, Maenhout (2004) proposes the homothetic robustness to ensure state independence and analytical tractability of the portfolio selection problem. In this paper, there are two state variables, X and H.

$$(\rho + \lambda)V(X, H) = mXH + \lambda\underline{V} + \sup_{\pi} \inf_{u} \left\{ V_X(X\pi_t(\mu - r - \sigma u)) + \frac{1}{2}V_{XX}X^2\pi^2\sigma^2 \right.$$
$$\left. + \frac{u^2}{2\Psi(X, H)} \right\} + V_H H(r - \phi - m) + \left(k - \frac{1+k}{H}V_X + V_H \right)dH^\epsilon. \tag{13}$$

Recall that the last term in (8), $(1 + k)dH^\epsilon/H$, is zero almost everywhere, except at the boundary $\{X_t = 1\}$, where the HWM is reset such that $dH = dH^\epsilon$. So the last term in (13) is nonzero only at the boundary $\{X = 1\}$. Based on this observation, we first focus on the case with $C < X < 1$, where the value function V satisfies the following HJB equation:

$$(\rho + \lambda)V = mXH_t + \lambda\underline{V} + \sup_{\pi} \inf_{u} \left\{ V_X(X\pi(\mu - r - \sigma u)) + \frac{1}{2}V_{XX}X^2\pi^2\sigma^2 \right.$$
$$\left. + \frac{u^2}{2\Psi(X, H)} \right\} + V_H H(r - \phi - m) \quad \text{for} \quad C < X < 1. \tag{14}$$

Second, with $X = 1$, we have the following boundary condition

$$k - \frac{1+k}{H}V_X + V_H = 0 \quad \text{for} \quad X = 1. \tag{15}$$

The right-hand side of (14) is convex in u. Solving the inf part yields

$$u^* = \sigma\pi\Psi(X, H)XV_X. \tag{16}$$

Substituting u^* back into the HJB equation (14) yields

$$(\rho + \lambda)V = mXH + \lambda\underline{V} + V_H H(r - \phi - m)$$
$$+ \sup_{\pi} \left\{ V_X X\pi(\mu - r) - \frac{1}{2}\Psi(X, H)V_X^2 X^2\pi^2\sigma^2 + \frac{1}{2}V_{XX}X^2\pi^2\sigma^2 \right\}. \tag{17}$$

To guarantee the strict concavity of the value function $V(X, H)$, that is $V_{XX} < 0$, we assume $0 < \theta < \frac{\omega}{\rho+\lambda-r+\phi}$, where $\omega = 0.5(\mu - r)^2/\sigma^2$.[11] Based on this restriction, taking the first-order condition with respect to π, we can get the optimal investment decisions

$$\pi^* = -\frac{\mu - r}{\sigma^2} \frac{V_X}{X(V_{XX} - \Psi(X, H)V_X^2)}. \tag{18}$$

Substituting (9) and (18) back into Eq. (17) yields

$$0 = -(\rho + \lambda)V + \lambda gV(X = 1, H) + mXH - \frac{\omega V_X^2}{V_{XX} - \Psi(X, H)V_X^2} + V_H H(r - \phi - m), \tag{19}$$

where the state dependent scaler $\Psi(X, H)$ measuring the strength of the preference for robustness only emerges in the fourth term of (19). Therefore, the solution depends on the form of $\Psi(X, H)$. Following the homothetic robustness methodology proposed by Maenhout (2004), we first provide a conjecture about the scaler $\Psi(X, H)$. With above assumptions, there are two state variable, X and H. Moreover, with the HWM compensation, the

---

[11] In fact, the restriction $-1 < \theta < \frac{\omega}{\rho+\lambda-r+\phi}$ also can guarantee the concavity of the value function V. Here, we just focus on the positive ambiguity aversion, that is $\frac{\omega}{\rho+\lambda-r+\phi} > \theta > 0$.

fund manager's value includes the present value of management fees, incentive fees (if any) and the outside option. However, the manager has the option to shut down any further risk taking and only receives the management fees or the outside option upon the exogenous random termination. It is obvious that this component can be independent of the fund wealth, W, or the ratio of fund wealth to HWM, X. Thus, it is the manager's value net of the this part that depends on the two state variables and accordingly can assure the desired homotheticity property. As a result, we take

$$\Psi(X, H) = \frac{\theta}{aV(X, H) - bH}, \tag{20}$$

where θ captures the strength of the preference for robustness, a and b are two variables to be determined, and the term bH is used to adjust the non-full state dependence due to the payoff that would accrue to the manager under risk-shutdown with the homogeneity in H.

3.1 The case without management fees

In this subsection, we first derive the analytical solution for the case without management fees, that is $m = 0$, and formalize the results in the following proposition.

Proposition 1 *Assume that $0 < \theta < \frac{\omega}{\rho + \lambda - r + \phi}$ and $\beta > 0$ (defined below). Then the solution of Eq. (19) subject to boundary condition (15) is given by*

$$V(X, H) = \beta H \left[\left(\frac{X - D_0}{D_1} \right)^{\eta} + D_2 \right], \tag{21}$$

the fund manager's homothetic robust risk taking satisfies

$$\pi^* = \frac{\mu - r}{\sigma^2} \frac{1}{(1 - \eta + \theta)\left(\frac{X}{X - D_0} \right)}, \tag{22}$$

where

$$\eta = (1 + \theta) \frac{\rho + \lambda - r + \phi}{\rho + \lambda - r + \phi + \omega}, \tag{23}$$

$$\beta = \frac{k}{\frac{(1 + k)\eta}{D_1} \left(\frac{1 - D_0}{D_1} \right)^{\eta - 1} - 1}, \tag{24}$$

$$D_2 = \frac{\lambda g}{\rho + \lambda - r + \phi}, \tag{25}$$

and, if $g \geq D_2$,

$$D_1 = \frac{C - 1}{(g - D_2)^{\frac{1}{\eta}} - (1 - D_2)^{\frac{1}{\eta}}}, \tag{26}$$

$$D_0 = C - \frac{C-1}{(g-D_2)^{\frac{1}{\eta}} - (1-D_2)^{\frac{1}{\eta}}}(g-D_2)^{\frac{1}{\eta}}, \tag{27}$$

otherwise, if $g < D_2$,

$$D_0 = C, \tag{28}$$

$$D_1 = \frac{1-C}{(1-D_2)^{\frac{1}{\eta}}}. \tag{29}$$

Moreover, the worst-case measure given by (16), u^*, *satisfies*

$$u^* = \frac{\theta}{1+\theta}\frac{\mu-r}{\sigma}\frac{\rho+\lambda-r+\phi+\omega}{\omega}, \tag{30}$$

and the parameters a and b in the scaler function $\Psi(X, H)$ are given by

$$a = \eta, \tag{31}$$

$$b = \eta\beta D_2. \tag{32}$$

Equation (30) shows that the worst-case measure varies with the extent to which the fund manager is ambiguous about the return process of risky asset. Along with (4), this implies that the fund manager adjusts the perceived return according to his ambiguity aversion and that as the fund manager concerns more about the model misspecification, he perceives a lower expected excess return. Given $\omega/(\rho+\lambda-r+\phi) > \theta$, we can verify that from the perspective of robust fund manager, the expected return provided by the risky asset in the worst case is not less than the interest rate in that $\mu - \sigma u^* > r$.

In fact, adjusting the expected return downward is one channel via which ambiguity aversion influences the fund manager's risk choice. However, the HWM contract induces the risk-neutral manager to be risk averse, hence concerning about the model uncertainty has alternative effects on the manager's risk taking. To see this, we define the induced effective risk aversion denoted by *ERA(X)* as follows:

$$ERA(X) = \underbrace{(1-\eta+\theta)}_{\text{direct effect}} \times \underbrace{\left(\frac{X}{X-D_0}\right)}_{\text{indirect effect}}. \tag{33}$$

In order to inspect the effects of ambiguity aversion on the manager's optimal risk taking, we first focus on the direct effect (i.e., $1-\eta+\theta$), which represents a positive effect of ambiguity aversion on the induced risk aversion and explicitly shows how ambiguity aversion can increase the effective risk aversion in line with the argument of Maenhout (2004, 2006). Moreover, this part has some intuition about the way via which ambiguity aversion affects the induced effective risk aversion. To this end, we rearrange the direct effect as follows:

$$1-\eta+\theta = \frac{(1+\theta)\omega}{\rho+\lambda-r+\phi+\omega}, \tag{34}$$

where the numerator shows that the direct effect depends on the interactions between the uncertainty aversion and the Sharpe-ratio of risky asset (note that $\omega = 0.5(\mu - r)^2/\sigma^2$). This is intuitive in that ambiguity aversion makes the fund manager be uncertain about the expected excess return of risky asset. If the fund manager is more ambiguous about the model capturing the risky asset return, the induced direct effect on the effective risk aversion becomes stronger.

Next, we focus on the second part of (33), which characterizes the indirect effect of ambiguity aversion on the induced effective risk aversion along with the ratio of fund wealth to HWM. Indeed, the dependence on this ratio means that the induced risk aversion varies with the fund performance. The constant D_0 in the denominator implies that the fund manager has different risk-taking dynamics following poor performance (also see Drechsler 2014). Specifically, all else being equal, if $D_0 > 0$, the manager becomes more risk averse and prefers to *derisk* as the ratio of fund asset to HWM falls; and if $D_0 < 0$, the manager becomes less risk averse and prefers to *gamble* as the fund approaches the termination point. It is worth pointing out that in the gambling case, as the ratio of fund wealth to HWM falls, the leverage implied by the model is very high, which is inconsistent with reported leverage of equity funds by Ang et al. (2011).[12] Therefore, in the following analysis, we focus on the case with $D_0 > 0$.

Moreover, the presence of θ in Eqs. (23) and (27) implies that ambiguity aversion can change the endogenous risk attitude by modifying D_0. Indeed, differing from the direct effect, the indirect effect is negative in that the second part of (33) decreases in the extent to which the fund manager is ambiguous about the model.[13] As a result, the net effect generated by ambiguity aversion on the induced risk aversion depends on the relative weight between the direct effect and indirect effect, where the latter depends on the ratio of fund asset to HWM. Therefore, ambiguity aversion complicates the fund manager's risk-taking behavior, in contrast to the Merton portfolio problem in which ambiguity aversion uniformly dampens investors' portfolio choice (see, Maenhout 2004).

Finally, note that Drechsler (2014) refers to the term $\beta D_2 H$ in (21) as the risk-shutdown payoff in that under risk shutdown by setting $\pi_t \equiv 0$, neither the termination boundary nor the HWM is ever reached such that the manager only receives a payment conditional on the exogenous liquidation with a probability $\lambda \Delta t$ over time period Δt. Therefore, $\beta D_2 H$ represents the discounted value of the payment upon the exogenous liquidation. Moreover, it is independent of the ratio of fund wealth to HWM, confirming the validity of argument about Eq. (20) in conjunction with (32) by assuming there is not management fee (i.e., $m = 0$).

3.2 The case with management fees

Next, we take management fees into account. Since the general Hamilton-Jacobi-Bellman equation (19) contains management fees, which make it impossible to derive the closed-form solution. For the purpose to get the analytic solution, we follow Drechsler (2014) to adopt the approximate management fee payoff $m_H H_t$ instead of $m W_t$, where m_H is the approximate management fee payout rate. That is, the payout to fund manager is a constant

---

[12] Ang et al. (2011) report the average gross leverage across all hedge funds is 2.1. In the gambling case, Drechsler (2014) shows the leverage up to 25–45, and our numerical solution shows the leverage up to 5–10.

[13] In Appendix 2, we prove that $\frac{dD_0}{d\theta} < 0$.

fraction of HWM instead of fund wealth. With the approximated management fees, the optimal problem given by (17) becomes

$$
\begin{aligned}
(\rho + \lambda)V = {}& m_H H + \lambda \underline{V} + \sup_{\pi} \left\{ V_X X \pi (\mu - r) - \frac{1}{2}\Psi(X,H)V_X^2 X^2 \pi^2 \sigma^2 \right. \\
& \left. + \frac{1}{2}V_{XX}X^2\pi^2\sigma^2 \right\} + V_H H(r - \phi - m), \quad \text{for} \quad C \le X < 1,
\end{aligned}
\tag{35}
$$

with the following boundary condition

$$
k - \frac{1+k}{H}V_X + V_H = 0 \quad \text{for} \quad X = 1.
\tag{36}
$$

The following proposition provides the solutions for the optimal investment strategy and the corresponding value function under the approximate management fees.

Proposition 2 *If* $0 < \theta < \frac{\omega}{\rho + \lambda - r + \phi + m}$ *and* $\beta > 0$, *the solution* $V(X, H)$ *to Eq.* (35) *with boundary condition* (36) *is also given by* (21) *and the optimal risk choice* π^* *chosen by the fund manager with ambiguity aversion is given by* (22) *in Proposition* 1 *but with the alternative* η *and* D_2, *respectively, given by*

$$
\eta = (1 + \theta)\frac{\rho + \lambda - r + \phi + m}{\rho + \lambda - r + \phi + \omega + m},
\tag{37}
$$

$$
D_2 = \frac{\lambda g + m_H \beta^{-1}}{\rho + \lambda - r + \phi + m}.
\tag{38}
$$

For this case, the worst-case measure, u^*, *has the following form*

$$
u^* = \frac{\theta}{1+\theta}\frac{\mu - r}{\sigma}\frac{\rho + \lambda - r + \phi + m + \omega}{\omega},
\tag{39}
$$

and the parameters a and b in (20) *are also given by* (31) *and* (32), *respectively.*

Similar to Proposition 1, ambiguity aversion imposes direct effect and indirect effect on the risk choice with management fees via η as discussed above. Moreover, $m_H \beta^{-1}$ in the numerator of Eq. (38) represents the discounted value of the management fees that would accrue to the manager under risk-shutdown, which implies the validity of aforementioned argument about Eq. (20).

4 The Probability of reaching boundary

note that besides of management fees (if any), the manager receives the payout when either termination occurs or the HWM is reached. Therefore, along with the weight of management fees (if any) in the total payoffs, the expected waiting time to obtain the next payout influences the manager's risk taking. In order to focus on how the manager's ambiguous beliefs about the return of risky asset affect the expected waiting time to receive payout, we define $\tau_{HWM} = \inf\{s : X_s = H_t \text{ and } X_s > C, s > t\}$ and $\tau_C = \inf\{s : X_s = C \text{ and } X_s < H_t, s > t\}$. Then τ_{HWM} represents the first time that the fund wealth reaches the HWM before termination and τ_C is the first time that termination

occurs before the HWM is reached. Let $\zeta = \rho + \lambda - r + \phi + m$. Thus, for any time t, the value function satisfies

$$V(X_t, H_t) = \max_{\{\pi_s : s \geq t\}} \left\{ \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_{HWM}-t)} \right] V(1, H_t) + \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_C-t)} \right] V(C, H_t) \right\}.$$

Given the value function at the termination and at the moment the fund wealth reaches the HWM, $V(C, H_t)$ and $V(1, H_t)$, respectively, the optimal risk taking satisfies

$$
\begin{aligned}
\pi^* \in \operatorname*{arg\,max}_{\{\pi_s : s \geq t\}} & \left\{ \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_{HWM}-t)} \right] + \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_C-t)} \right] \frac{V(C, H_t)}{V(1, H_t)} \right\} \\
= \operatorname*{arg\,max}_{\{\pi_s : s \geq t\}} & \left\{ \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_{HWM}-t)} | \tau_{HWM} < \infty \right] P(\tau_{HWM} < \infty) \right. \\
& \left. + \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_C-t)} | \tau_{HWM} < \infty \right] \left(1 - P(\tau_{HWM} < \infty) \right) \frac{V(C, H_t)}{V(1, H_t)} \right\}.
\end{aligned}
\tag{40}
$$

In the above equation, $P(\tau_{HWM} < \infty)$ represents the probability of reaching the HWM prior to termination; $\mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_{HWM}-t)} | \tau_{HWM} < \infty \right]$ represents the present value of one dollar to be received at the first time that the HWM is reached prior to termination; and $\mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_C-t)} | \tau_C < \infty \right]$ is the value of one dollar paid at the moment termination occurs before the HWM is reached. In the following proportion, we provide the corresponding solutions.

Proposition 3 *At any time t with $\eta > 0.5$, given the ratio of fund wealth to HWM, $X (C \leq X \leq 1)$, the probability of reaching the HWM prior to termination, $P(X) = P(\tau_{HWM} < \infty)$, is given by*

$$P(X) = -\frac{(C - D_0)^{2\eta-1}}{(1 - D_0)^{2\eta-1} - (C - D_0)^{2\eta-1}} + \frac{1}{(1 - D_0)^{2\eta-1} - (C - D_0)^{2\eta-1}} (X - D_0)^{2\eta-1},$$
$$\tag{41}$$

where D_0 and η are given by Proposition 2. In addition, the present value of one dollar to be received at the first time the HWM is reached prior to termination, defined by $V_{HNC}(X) = \mathbb{E}_t^{\mathbb{P}^u} \left[e^{-\zeta(\tau_{HWM}-t)} | \tau_{HWM} < \infty \right]$, satisfies

$$V_{HNC}(X) = \frac{V^H(X)}{P(X)},
\tag{42}$$

where

$$
\begin{aligned}
V^H(X) = & -\frac{(C - D_0)^{\alpha_2}}{(C - D_0)^{\alpha_1}(1 - D_0)^{\alpha_2} - (1 - D_0)^{\alpha_1}(C - D_0)^{\alpha_2}} (X_t - D_0)^{\alpha_1} \\
& + \frac{(C - D_0)^{\alpha_1}}{(C - D_0)^{\alpha_1}(1 - D_0)^{\alpha_2} - (1 - D_0)^{\alpha_1}(C - D_0)^{\alpha_2}} (X_t - D_0)^{\alpha_2},
\end{aligned}
$$

with

$$\alpha_{1,2} = \frac{2\eta - 1}{2} \pm \frac{\sqrt{(\omega(1 - 2\eta))^2 + 4\omega(1 - \eta + \theta)^2 \zeta}}{2\omega}.$$

Moreover, the present value of one dollar paid at the first passage time termination occurs before the HWM is reached, defined by $V_{CNH}(X) = \mathbb{E}_t^{\mathbb{P}^\mu}\left[e^{-\zeta(\tau_C - t)} | \tau_C < \infty\right]$, *satisfies*

$$V_{CNH}(X) = \frac{V^C(X)}{1 - P(X)}, \tag{43}$$

where $V^C(X)$ *is given by*

$$V^C(X) = \frac{(1 - D_0)^{\alpha_2}}{(C - D_0)^{\alpha_1}(1 - D_0)^{\alpha_2} - (1 - D_0)^{\alpha_1}(C - D_0)^{\alpha_2}}(X_t - D_0)^{\alpha_1}$$
$$- \frac{(1 - D_0)^{\alpha_1}}{(C - D_0)^{\alpha_1}(1 - D_0)^{\alpha_2} - (1 - D_0)^{\alpha_1}(C - D_0)^{\alpha_2}}(X_t - D_0)^{\alpha_2}.$$

Based on the notations in Proposition 3, Eq. (40) can be represented as

$$\pi^* = \underset{\{\pi\}}{\arg\max} \left\{ V_{HNC}(X)P(X) + V_{CNH}(X)\left(1 - P(X)\right)\frac{V(C, H)}{V(1, H)} \right\}, \tag{44}$$

which implies that the manager should trade off the effect of desiring robust risk taking on the expected benefit when the HWM is reached against the effect on the expected value when termination occurs if implementing the risk-taking strategy.

5 Model implications

In this section, we focus on the implications of ambiguity aversion by first considering the case without management fees and then the case with management fees. The key parameters are collected in Table 1, whose values are consistent with those in Drechsler (2014) except for the exogenous termination intensity λ.[14] Moreover, we choose the commonly used two-twenty compensation contract for the case with management fees by $m = m_H = 2\%$ and $\kappa = 20\%$.

5.1 Without management fees

With the parameter values in Table 1, the assumptions in Proposition 1 require the level of ambiguity aversion to satisfy $\theta < 0.88$. So we take two different values, $\theta = 0.2$ and $\theta = 0.4$, to illustrate the effects induced by ambiguity aversion. Given these values, we can get $D_0(\theta = 0.0) = 0.432$, $D_0(\theta = 0.2) = 0.399$ and $D_0(\theta = 0.4) = 0.361$, which correspond to the scenario in which the manager reduces risk taking as the ratio of fund wealth to HWM falls.

Note that without management fees, what the manager can receive is either the performance fees when the HWM is reached, called *option effect at HWM*, or the outside option when termination triggers, referred to as *option effect at termination*. Therefore, the trade-off between the *option effect at HWM* and the *option effect at termination* is critical for the manager's risk choice. Panels A and B in Fig. 1 show that as the fund closes to the HWM (i.e., $X \to 1$), ambiguity aversion increases the induced risk aversion and hence discourages the fund manager's risk taking, *ceteris paribus*. By contrast,

---

[14] Drechsler (2014) sets $\lambda = 0$. To ensure that $\eta > 0.5$ in Proposition 3 holds, we choose $\lambda = 0.01$.

Table 1 Parameter values for the baseline model

Parameter	Symbol	Value	Parameter	Symbol	Value
Termination point	C	0.50	Outside option	g	0.35
Withdrawal rate	ϕ	0.05	Discount rate	ρ	0.03
Termination intensity	λ	0.01	Risk-free rate	r	0.01
Expected return of risky asset	μ	0.07	Volatility of risky asset	σ	0.16
Performance fee	k	0.20	Management fee	$m(= m_H)$	0.02

following poor performance (i.e., $X \to C$), ambiguity aversion reduces the fund manager's effective risk aversion and encourages him to take more risk, *ceteris paribus*.

As shown by Panels C and E of Fig. 1, from the perspective of the fund manager with robustness concern, not only the probability that the HWM will be reached prior to termination, but also the corresponding option value, decreases in ambiguity aversion, *ceteris paribus*. Therefore, the manage who cares more about the robust risk taking has no incentives to take more risk but to reduce risk taking as the fund approaches the HWM, implying that the incentive to elevate risk can be moderated by his ambiguity aversion. This is consistent with the argument that individual investors' ambiguity aversion increases the effective risk aversion and reduces their portfolio risk exposure (for instance, see Maenhout 2004, 2006). In a mean-variance model with ambiguity risk, Maccheroni et al. (2013) find that greater ambiguity aversion can reduce optimal exposure to ambiguity risk. But in our model, the induced risk aversion makes the attitude toward ambiguity entangle with the attitude to volatility risk and subsequently exhibits an increasing pattern.[15]

However, Panels D and F of Fig. 1 show that all else being equal, as ambiguity aversion increases, the fund manager perceives not only a higher probability that he can receive the outside option but also a larger value of one dollar received upon termination before receiving performance fees. As a result, along with the higher probability to receive the outside option, the larger option value induces the fund manager to take more risk following poor performance. This is consistent with existing literature. For instance, Carpenter (2000) shows that the risk-averse manager increases the risk as the fund value approaches zero. Hodder and Jackwerth (2007) find that the hedge fund manager gambles when the fund is close to termination. Moreover, this means that the ambiguity-averse fund manager has more aggressive "tournament behavior", which is in some sense analogous to the impact of loss aversion in that the fund manager with loss aversion increases risk taking as the fund value drops (see, Kouwenberg and Ziemba 2007).

5.2 With Management fees

Now, we turn to the scenario with management fees and show how management fees change the way in which ambiguity aversion influences the fund manager's risk choice by distorting the trade-off between the probability to receive a payment and the present value of a payoff received conditional on either termination or that the HWM is reached. Note

---

[15] If separating model uncertainty from volatility, the former can not be reduced to volatility risk, see (Maccheroni et al. 2013). By assuming the presence of idiosyncratic risk, Luo et al. (2022) find that increasing ambiguity aversion against the idiosyncratic risk raises the fund manager's private risk taking.
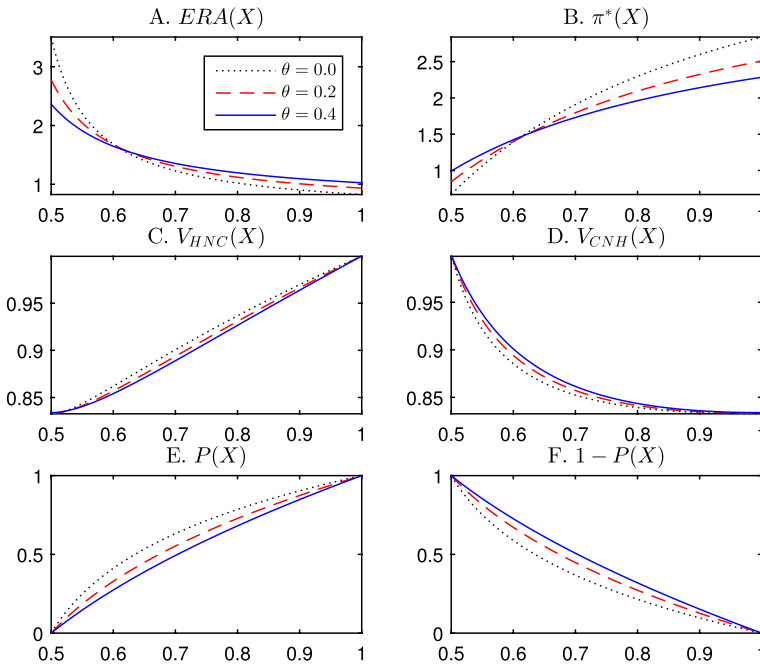
Fig. 1 Induced effects by the fund manager's ambiguity aversion for the case without management fees (i.e., $m = m_H = 0$)

that management fees impose additional influences on the trade-off through two opposing effects.[16] The first one is the "payoff effect" in that management fees directly accrue to the manager's utility. The second one is the "withdrawal effect" in that management fees are paid out and hence reduce the growth rate of asset in the fund as acting like withdrawals from the fund by outside investors.

Given the level of management fees, $m = m_H = 2\%$, the conditions of Proposition 2 require $\theta < 0.70$, upon which, we take two different values, $\theta = 0.2$ and $\theta = 0.4$, to show the effects of ambiguity aversion on the effective risk aversion and corresponding risk choice. Analogously, these values imply $D_0(\theta = 0.0) = 0.450$, $D_0(\theta = 0.2) = 0.465$ and $D_0(\theta = 0.4) = 0.495$, which correspond to the situation in which the manager reduces risk taking as the ratio of fund wealth to HWM falls.

By listing the probability that the fund manager receives performance fees prior to termination for different levels of ambiguity aversion and management fees, Table 2 shows that increasing the degree of ambiguity aversion amplifies the extent to which management fees influence the probability to receive performance fees before termination.[17] This is due to that receiving management fees greatly distorts the relative weight between the *option effect at termination* and *option effect at HWM* by introducing two opposing effects, *payoff effect* and *withdrawal effect*. In the ambiguity-neutral case (i.e., $\theta = 0.0$), management fees decrease the probability to receive performance fees prior to termination, but

---

[16] Drechsler (2014) also discusses these two opposing effects arising from management fees.

[17] In a similar way, we can illustrate how ambiguity aversion changes the extent to which management fees influence the probability to receive outside option before receiving performance fees.

Table 2 Probability to receive performance fees prior to termination

Ambiguity aversion	Management fees	$P(0.55)$	$P(0.6)$	$P(0.7)$	$P(0.8)$	$P(0.9)$	$P(0.95)$
$\theta = 0.0$	$m = m_H = 0.0\%$	0.2483	0.4113	0.6319	0.7852	0.9033	0.9539
	$m = m_H = 2.0\%$	0.2469	0.4060	0.6235	0.7779	0.8992	0.9517
$\theta = 0.2$	$m = m_H = 0.0\%$	0.1844	0.3286	0.5527	0.7279	0.8738	0.9390
	$m = m_H = 2.0\%$	0.2167	0.3593	0.5794	0.7407	0.8792	0.9415
$\theta = 0.4$	$m = m_H = 0.0\%$	0.1462	0.2737	0.4929	0.6809	0.8481	0.9257
	$m = m_H = 2.0\%$	0.1981	0.3287	0.5353	0.7074	0.8602	0.9315

the magnitude is very small. The reason for this is as follows. The probability for the manager to receive performance fees prior to termination is the result of his optimal risk taking. In the vicinity of the termination, the risk of losing the stream of management fees becomes important such that the payoff effect dominates the withdrawal effect. By contrast, as the fund approaches the HWM, the withdrawal effect dominates the payoff effect in the vicinity of the HWM. By dynamically balancing these two effects, the optimal risk-taking actions impose little influence on the probability to receive performance fees prior to termination. However, the manager's ambiguity aversion not only reverses this trend in that all else being equal, management fees increase the probability to receive performance fees prior to termination, but also amplifies the extent to which management fees change the probability. The reasoning is as follows. With ambiguity aversion, the fund manager perceives a lower expect return of the risky asset (see, Eq. 4) and accordingly a lower expected growth rate of assets under management (see, Eq. 6), which leads the fund manager to perceive a higher likelihood that the wealth will touch the termination point. This implies that when making risk choice, the manager has to not only dynamically balance the payoff effect and withdrawal effect, but also concern about and offset the perceived adverse effect. As a result, the optimal risk choice increases the probability for the manager to receive performance fees prior to termination, *ceteris paribus*.

With the optimal risk taking π^* given by (22) and the worst-case measure u^* given by (39), the fund wealth dynamic process (6) can be written as

$$\frac{dW_t}{W_t} = rdt + \pi_t^*(\mu - r)dt + \pi_t^* \sigma d\mathcal{B}_t^u - (\phi + m + \sigma u^* \pi_t^*)dt - kdH_t^\varepsilon / W_t. \qquad (45)$$

Recall that the robust fund manager optimizes against the worst-case model, this acts like benefits that accrue to the manager's utility. Thus, $\sigma u^* \pi_t^*$ in the fourth term of equation (45) increases the payoff effect of management fees. Thus, all else being equal, as the fund manager becomes more ambiguity averse, the extent to which the payoff effect dominates the withdrawal effect increases so that the robust manager take less risk as shown by Fig. 2.

Specifically, Panel A of Fig. 2 shows that in the ambiguity-neutral case (i.e., $\theta = 0.0$), as the fund approaches the HWM, management fees generate the withdrawal effect by reducing the growth rate of the wealth within the hedge fund. To offset this effect, the manager prefers to take more risk. By contrast, as the fund closes to the termination point, the payoff effect dominates so that the manger reduces risk taking, thereby reducing the likelihood that the fund will hit the termination point. Panels B and C illustrate that as the fund manager becomes more ambiguity averse, the payoff effect becomes dominant. To increase the probability to acquire management fees, the ambiguity-averse manager gradually reduces
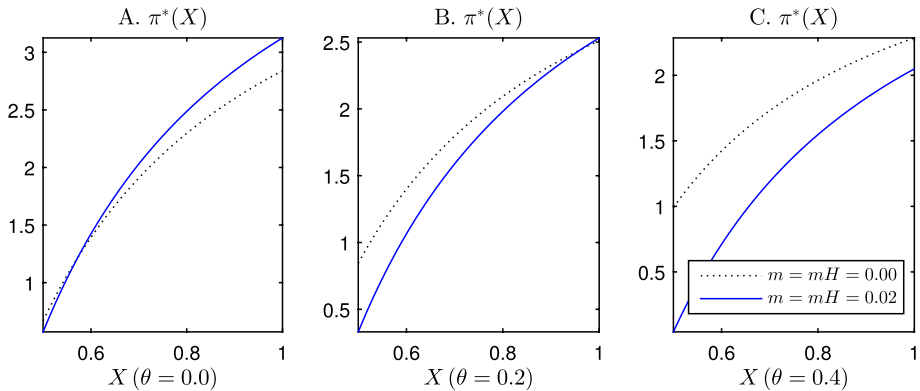
Fig. 2 Interactive effects of management fees and ambiguity aversion on the fund manager's risk taking

his risk choice, *ceteris paribus*. In particular, the comparison between Penal A and Panel C shows that management fees can enhance the effect of ambiguity aversion on the manager's risk taking by inducing him to reduce holdings. If economic turndown can lead the fund manager to become less confident and/or more pessimistic about the future returns of stocks, this may predict that it is the fund manager with higher rates of management fee that reduces the holdings more. Meanwhile, this also implies that fund managers ambiguity aversion may explain a part in the average decline in hedge fund equity holdings during the selloff quarters documented by Ben-David et al. (2012).

Differing from Figs. 1, 3 describes the effects of interactions between ambiguity aversion and management fees (i.e., $m_H = m = 0.02$). Panel A shows that with management fees, the induced risk aversion increases in ambiguity aversion. In particular, the extent to which ambiguity aversion increases the induced risk aversion becomes more significant as the fund closes to the termination point. Panel B shows that with management fees, the robust manager prefers to take less risk. These confirm the aforementioned argument that as the manager becomes more ambiguity averse, the perceived growth rate of asset within the fund becomes lower, which increases the likelihood of termination so that the risk of losing the stream of management fees becomes increasingly important, thereby increasing the payoff effect. To maintain the benefit from management fees, the ambiguity-averse manager chooses to take less risk, even at the expense of performance fees by reducing the probability to receive performance fees before termination (see, the 2nd row, 4th row and 6th row in Table 2), *ceteris paribus*. This is also consistent with the argument of Lan et al. (2013) that management fees contribute significantly to total compensation, quantitatively.

Relative to Panel C of Fig. 1, Penal C in Fig. 3 shows that management fees amplify the extent to which increasing ambiguity aversion reduces the *option effect at HWM*. The reasons are twofold. On the one hand, receiving the stream of management fees is important than receiving occasional performance fees due to the fact that management fees contribute significantly to total compensation. On the other hand, management fees increase the difficulty to receive performance fees due to the withdrawal effect. As a result, from the viewpoint of fund manager, as the fund wealth increases, the value of option at HWM falls and accordingly discourages his risk taking. Panel D of Fig. 3 shows that management fees reverse the *option effect at termination* relative to Panel D of Fig. 1. Without management fees, *option effect at termination* captures the
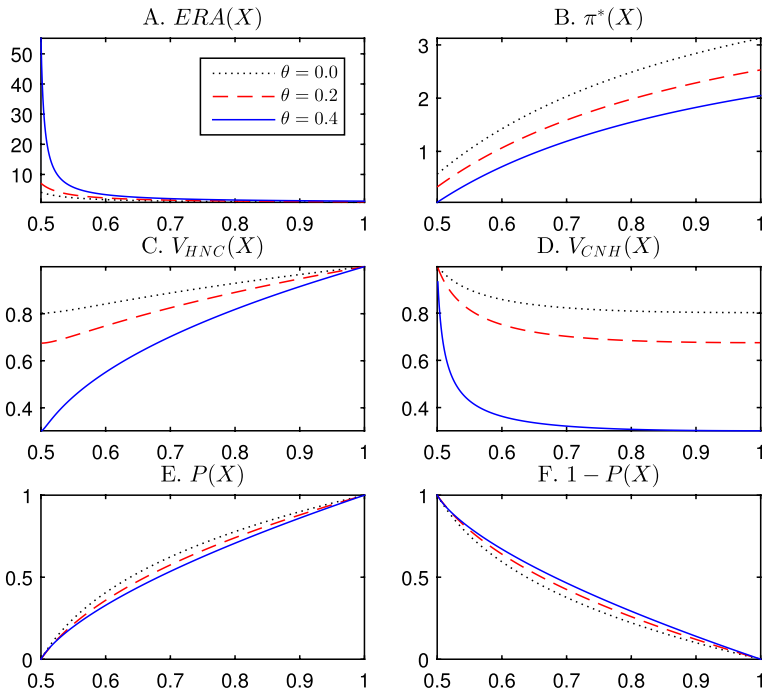
Fig. 3 Induced effects by the fund manager's ambiguity aversion for the case with management fees (i.e., $m_H = m = 0.02$)

manager's trade-off between the probability to receive the outside option and the value of the received outside option. However, management fees break down this trade-off in that *payoff effect* reduces the relative weight of outside option by allowing the manager to acquire the steam of management fees. As the fund manager becomes more ambiguity averse, the option value at termination decreases, which induces the manager to take less risk as the fund approaches the termination point.

Most recently, by analyzing hedge fund-creditor borrowing data during the COVID-19 crisis, Kruttli et al. (2021) find that hedge fund's Treasury market activity was primarily driven by fund-specific liquidity management rather than dealer regulatory constraints. Hedge funds exited the turmoil with 20% higher cash holdings despite low contemporaneous outflows. In fact, our paper may to some extent explain their empirical findings. Figure 4 illustrates the interaction between withdrawal rate and ambiguity aversion on the optimal risk choice. Without concern about ambiguity aversion, an increase in the withdrawal rate induces the manger to take more risk as documented by Drechsler (2014) (i.e., the comparison between dashed line and dotted line). However, all else being equal, ambiguity aversion induces the manager to take less risk. Thus, if the unprecedented UST market turmoil caused by COVID-19 crisis leads fund mangers to generate pessimistic views about the market, ambiguity aversion is a possible factor to drive this fund-specific liquidity management.

Fig. 4 Interactive effects of withdrawal rate and ambiguity aversion on the fund manager's risk taking with management fees (i.e., $m_H = m = 0.02$)
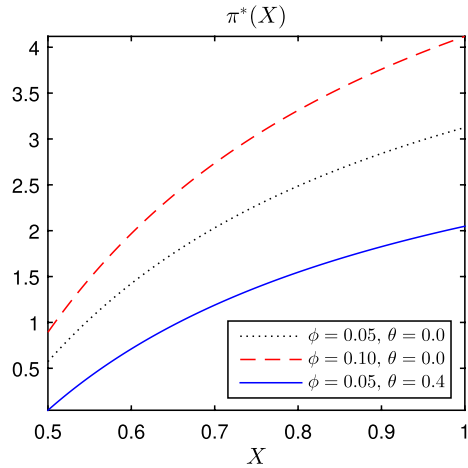
6 An extension with taxation

In this section, we investigate the effect of taxation by extending the above baseline model. Under current tax rules, the performance fee is treated as capital gains and is generally taxed at the capital gains rate and the management fee is taxed at ordinary income rates. Let τ^k denote the rate taxed at performance fees and τ^m the rate taxed at management fees. Thus, after tax, the fund manger receives a stream of management fee, $(1 - \tau^m)m$, and a performance fee $(1 - \tau^k)k$ if the HWM is reached. Moreover, the introduction of tax can not modify the HWM dynamics (5) and the fund wealth dynamics (6) as well as the dynamics for the ratio of wealth to HWM given by (8), but changes the fund manager's total utility. As a result, given taxation at both management fees and performance bonuses, the objective of the manager with ambiguity aversion is given by

$$V(X_t, H_t) = \sup_{\pi_s} \mathbb{E}_t^{\mathbb{P}} \left[\int_t^\tau e^{-\rho(s-t)} (m(1 - \tau^m)W_s ds + k(1 - \tau^k)dH_t^\epsilon) + e^{-\rho(\tau-t)} \underline{V_\tau} \right], \quad (46)$$

subject to the HWM dynamics (5) and the dynamics for the ratio of wealth to HWM (8).

Similar to the aforementioned discussion and computation, we can get the solutions for this problem and formalize the results as one proposition as follows. The proof parallels to the proof of Proposition 2.

Proposition 4 *If $0 < \theta < \frac{\omega}{\rho + \lambda - r + \phi + m}$ and $\beta > 0$, the solution $V(X, H)$ to the risk choice problem for the fund manager facing taxation given by (46) has the same form of (21) and the optimal risk choice π^* also has the same form of (22) in Proposition 2 but with the alternative β and D_2, respectively, given by*

$$\beta = \frac{k(1 - \tau^k)}{\frac{(1+k)\eta}{D_1}\left(\frac{1-D_0}{D_1}\right)^{\eta-1} - 1}, \quad (47)$$

$$D_2 = \frac{\lambda g + m_H \beta^{-1}(1 - \tau^m)}{\rho + \lambda - r + \phi + m}. \tag{48}$$

We first consider the case without management fees. If $m = m_H = 0$, we can find that D_2 given by (48) is equal to its counterpart in Proposition 1, and that only the rate taxed at performance fee emerges in the risk choice problem. Recall that in Proposition 1, while β depends on D_2 via D_0 and D_1, these three parameters (i.e., D_0, D_1 and D_2) are independent of β. Meanwhile, the induced effective risk aversion (i.e., $ERA(X) = (1 - \eta + \theta)X/(X - D_0)$ given by 33) only depends on D_0 rather than β. This means that without management fees, the rate taxed at performance fee can not change the fund manager's risk attitude and hence his risk-taking activity, but can influence the fund manager's utility, $V(X, H)/H$, via β. This may be due to that taxation of performance fees not only reduces what the manager can receive at the HWM, but also decreases the outside option value at termination because we assume that the value at termination is proportional to the manager's value function at the HWM. This equivalent reduction to some extent does not change the relative weight between *option effect at termination* and *option effect at HWM*.

Now we turn to the case with management fees. Plugging (47) into (48) yields the expression of D_2 with respect to D_0 and D_1.[18] In particular, the expression of D_2 includes a term, $m_H \times \frac{1 - \tau^m}{1 - \tau^k}$. This implies that it is the ration, $\frac{1 - \tau^m}{1 - \tau^k}$, that endogenously changes D_0 via the algebraic equation system composed of D_0, D_1 and D_2. Along with the induced effective risk aversion (i.e., $ERA(X) = (1 - \eta + \theta)X/(X - D_0)$), we can find it also is the ration, $\frac{1 - \tau^m}{1 - \tau^k}$, that influences the fund manager's effective risk aversion and subsequently his risk choice. The intuition is as follows. All else being equal, if the rate taxed at performance bonus is greater than that taxed at management fee $\left(\text{such that } \frac{1 - \tau^m}{1 - \tau^k} > 1\right)$, the tax structure increases the importance for the manager to retain the stream of management fees, thereby increasing the payoff effect of management fees and inducing the manager to take less risk. If the tax structure consists of a lower rate taxed at performance fee and a higher rate taxed at management fee such that $\frac{1 - \tau^m}{1 - \tau^k} < 1$, the opposite holds, *ceteris paribus*.

To numerically illustrate the effects induced by taxation, we take a 10% tax rate at the management fee and a 35% rate at the performance bonus such that $\frac{1 - \tau^m}{1 - \tau^k} > 1$ and show our results in Fig. 5. Firstly, Panels A and C show that the dotted line and dash-dotted line coincide, confirming our argument that without management fees, taxation (at performance fee) does not change the manager's risk taking, *ceteris paribus*. The dashed line lies over the solid line, implying that with management fees, a lower rate taxed at management fee than the rate taxed at performance fee (e.g., $0.10 = \tau^m < 0.35 = \tau^k$) induces the manager to take less risk by increasing management fees' payoff effect, thereby supporting the aforementioned argument. Secondly, Panels B and D illustrate that all else being equal, the value function (i.e., $V(X, H)/H$) with tax is smaller that its counterpart without tax. This is intuitive in that a tax on either performance fee or management fee or both reduces what the manager can receive and accordingly reduces his total benefit.

---

[18] Plugging (47) into (48) yields the expression for D_2:

$$D_2 = \frac{\lambda g + \frac{\frac{(1+k)\eta}{D_1}\left(\frac{1 - D_0}{D_1}\right)^{\eta - 1} - 1}{k} \times m_H \times \frac{1 - \tau^m}{1 - \tau^k}}{\rho + \lambda - r + \phi + m}.$$

Along with D_0 and D_1 given by Proposition 1, this implies that D_0, D_1 and D_2 construct an equation system.

7 Conclusion

This paper introduces model uncertainty into the risk choice model of Drechsler (2014) by assuming that a risk-neutral manager is concerned about potential model misspecification and investigates the manager's robust risk taking. The results show that ambiguity aversion generates not only a direct effect but also an indirect effect on the manager's risk choice. Their interactions generate several interesting and important implications for hedge funds' risk-taking behavior. In the scenario without management fees, ambiguity aversion reduces the manager's effective risk aversion and hence encourages him to take more risk as the fund is close to the termination; but as the fund approaches the HWM, ambiguity aversion increases the induced risk aversion and accordingly induces the manager to take less risk. With management fees, the effects of ambiguity aversion following poor performance diminish in that ambiguity aversion increases the induced risk aversion and moderates the manager's risk taking. This predicts that it is the fund manager with higher management fee rates that reduces the risky asset holdings more when economic turndown leads him to become less confident and/or more pessimistic about the future returns. Thus, our model implies that manager's ambiguity aversion is a possible factor that explains hedge fund activities in stock markets during the financial crisis of 2007–2009 and UST markets during COVID-19 crisis. Finally, we extend our model to consider taxation. The results show that it is the trade-off between the after tax performance fees and management fees that further impacts the manager's risk choice.

Appendix

1 Proof of Proposition 2

We prove Proposition 2 because Proposition 1 is a special case with $m_H = m = 0$.

Suppose that the value function V is of the conjectured form in Eq. (21). By calculating the partial derivative, we have

$$V_X = \frac{\beta H_t \eta}{D_1}\left(\frac{X_t - D_0}{D_1}\right)^{\eta-1}, \quad V_{XX} = \left(\frac{X_t - D_0}{D_1}\right)^{\eta-2}\frac{\beta H_t \eta(\eta-1)}{D_1^2} \quad \text{and}$$

$$V_H = \beta\left((\frac{X_t - D_0}{D_1})^{\eta} + D_2\right).$$

Substituting V_X and V_H into the boundary condition (36) gives

$$\beta = \frac{k}{\frac{(1+k)\eta}{D_1}\left(\frac{1-D_0}{D_1}\right)^{\eta-1} - 1}.$$

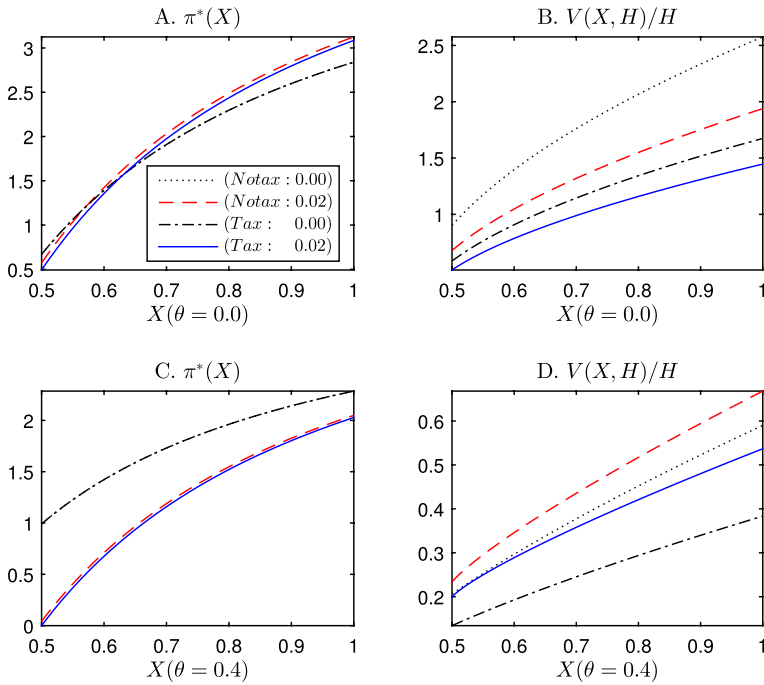Substituting $\Psi(X, H)$ given by (20) back into (19), we have

Fig. 5 Comparative statics with taxation. The first argument in the brackets indicates whether taxation exists or not and the second argument represents the level of management fees

$$0 = \left[-(\rho + \lambda) - \omega \cfrac{\eta}{\eta - 1 - \cfrac{\theta}{a\beta H \left(\frac{X_t - D_0}{D_1} \right)^\eta + a\beta D_2 H - bH} \eta \beta H \left(\frac{X_t - D_0}{D_1} \right)^\eta} + r - \phi - m \right]$$

$$\beta H \left(\frac{X_t - D_0}{D_1} \right)^\eta$$

$$+ [m_H + \beta(r - \phi - m)D_2 - \beta(\rho + \lambda)D_2 + \beta\lambda g]H. \tag{49}$$

In order to obtain a closed-form solution, we let $a = \eta$ and $b = a\beta D_2$, then we get

$$0 = \left[-(\rho + \lambda) - \omega\eta \frac{1}{\eta - \theta - 1} + r - \phi - m \right] \beta \left(\frac{X_t - D_0}{D_1} \right)^\eta$$

$$+ m_H + \beta(r - \phi - m)D_2 - \beta(\rho + \lambda)D_2 + \beta\lambda g. \tag{50}$$

Because Eq. (50) must hold for all X_t, so collecting all constant terms and equating their sum to zero, we get

$$D_2 = \frac{\lambda g + m_H \beta^{-1}}{\rho + \lambda - r + \phi + m}.$$

Meanwhile, letting the coefficient of the term including $(\frac{X_t - D_0}{D_1})^\eta$ be zero, we obtain

$$\eta = (1 + \theta)\frac{\rho + \lambda - r + \phi + m}{\rho + \lambda - r + \phi + \omega + m}. \tag{51}$$

With some calculations, we have

$$\frac{V_X}{V_{XX} - \Psi V_X^2} = \frac{X_t - D_0}{\eta - \theta - 1}. \tag{52}$$

Substituting (52) into (18) yields

$$\pi^* = \frac{\mu - r}{\sigma^2}\frac{1}{(1 - \eta + \theta)\left(\frac{X_t}{X_t - D_0}\right)}. \tag{53}$$

Let $\eta_1 = \frac{\rho + \lambda - r + \phi + m}{\rho + \lambda - r + \phi + \omega + m}$, because $\mu - r > 0$, then $\omega > 0$ and hence $0 < \eta_1 < 1$. Assume $0 < \theta < \frac{\omega}{\rho + \lambda - r + \phi + m}$, thus, $V_{XX} < 0$, it shows that V is a concave function. In order to obtain the values for D_0 and D_1, we resort to the two boundary conditions. The first boundary condition at the HWM is

$$\left(\frac{1 - D_0}{D_1}\right)^{\eta} + D_2 = 1. \tag{54}$$

There are two cases for the second boundary condition, depending on either that the fund manager continues to take risk at the termination boundary (case 1), or that he reduces the optimal risk choice to zero at this point (case 2).

Case 1 By the second boundary condition, i.e. $V(C, H_t) = \underline{V}_t = gV(X_t = 1, H_t)$, we have

$$\left(\frac{C - D_0}{D_1}\right)^{\eta} + D_2 = g. \tag{55}$$

A system of simultaneous equations (54) and (55) are obtained. If $g \geq D_2$, the solutions satisfy

$$D_1 = \frac{C - 1}{(g - D_2)^{\frac{1}{\eta}} - (1 - D_2)^{\frac{1}{\eta}}}, \tag{56}$$

$$D_0 = C - \frac{C - 1}{(g - D_2)^{\frac{1}{\eta}} - (1 - D_2)^{\frac{1}{\eta}}}(g - D_2)^{\frac{1}{\eta}}. \tag{57}$$

Case 2 If the fund manager continues to take risk at the termination boundary, then we have $\pi^*(C) = 0$. Substituting $X_t = C$ into (53) yields $D_0 = C$. Along with Eq. (55) and $D_0 = C$, if $g < D_2$, we have

$$D_1 = \frac{1 - C}{(1 - D_2)^{\frac{1}{\eta}}}. \tag{58}$$

2 Proof for $\frac{dD_0}{d\theta} < 0$

Let

$$h = \frac{\rho + \lambda - r + \phi}{\rho + \lambda - r + \phi + \omega}. \tag{59}$$

Note that $C - 1 < 0$ and $D_2 \le g < 1$, then it has $0 < \frac{g - D_2}{1 - D_2} < 1$ and $\ln\left(\frac{g - D_2}{1 - D_2}\right) < 0$. Taking the partial derivative with respect to θ yields

$$\frac{d}{d\theta}(g - D_2)^{\frac{1}{\eta}} = -(g - D_2)^{\frac{1}{\eta}} \frac{1}{\eta^2} h \ln(g - D_2), \tag{60}$$

$$\frac{d}{d\theta}(1 - D_2)^{\frac{1}{\eta}} = -(1 - D_2)^{\frac{1}{\eta}} \frac{1}{\eta^2} h \ln(1 - D_2). \tag{61}$$

Thus, it has

$$\frac{dD_0}{d\theta} = \frac{(1 - C)h}{\eta^2} \frac{(g - D_2)^{\frac{1}{\eta}}}{(1 - D_2)^{\frac{1}{\eta}}} \frac{\ln(\frac{g - D_2}{1 - D_2})}{(\ln(\frac{g - D_2}{1 - D_2}) - 1)^2} < 0. \tag{62}$$

3 Derivation of probability

Firstly, we solve for the probability of reaching the HWM prior to termination under the worst-case measure u^* and the optimal risk choice π. Let $P(X_t) = \mathbb{P}^u(\tau_{HWM} < \infty)$. Then, it has

$$P(X_t) = \mathbb{E}^{\mathbb{P}^u}[\mathbb{1}_{\tau_{HWM} < \infty} | X = X_t].$$

Mathematically, $P(X)$ satisfies the following ordinary differential equation

$$P_X X_t \pi_t (\mu - r - \sigma u_t) + \frac{1}{2} P_{XX} X_t^2 \pi_t^2 \sigma^2 = 0, \tag{63}$$

with boundary conditions

$$P(C) = 0, \quad P(1) = 1.$$

Substituting the worst-case density generator u^* and the optimal risk choice policy π^* into (63), and simplifying the results, we have

$$P_X(1 - \eta)(X_t - D_0) + \frac{1}{2} P_{XX}(X_t - D_0)^2 = 0, \tag{64}$$

The general solution to this is

$$P = P_1(X_t - D_0)^{\alpha_1} + P_2(X_t - D_0)^{\alpha_2}.$$

Substituting the general solution into the ODE (64), then the constants α_1 and α_2 are solved as

$$\alpha_1 = 2\eta - 1, \quad \alpha_2 = 0.$$

Substituting the two boundary conditions into the general solution, we can derive the solutions to the constants P_1 and P_2. Further, substituting P_1, P_2, α_1 and α_2 into the general solution, we have

$$P = -\frac{(C - D_0)^{2\eta - 1}}{(1 - D_0)^{2\eta - 1} - (C - D_0)^{2\eta - 1}} + \frac{1}{(1 - D_0)^{2\eta - 1} - (C - D_0)^{2\eta - 1}}(X_t - D_0)^{2\eta - 1}.$$

According to the standard arguments about ordinary differential equation, the expression (41) can be derived. Note that the certainty-equivalent expected waiting time satisfies

$$\mathbb{E}_t^{\mathbb{P}^u}[e^{-\zeta(\tau_{HWM} - t)}|\tau_{HWM} < \infty] = \frac{\mathbb{E}_t^{\mathbb{P}^u}[e^{-\zeta(\tau_{HWM} - t)}]}{P(X)},$$

therefor, it only needs to compute $\mathbb{E}_t^{\mathbb{P}^u}[e^{-\zeta(\tau_{HWM} - t)}]$. Let $V^H(X) = \mathbb{E}_t^{\mathbb{P}^u}[e^{-\zeta(\tau_{HWM} - t)}]$. Similarly, it satisfies the following ordinary differential equation

$$\zeta V^H = V_X^H X \pi_t (\mu - r - \sigma u_t) + \frac{1}{2} V_{XX}^H X_t^2 \pi_t^2 \sigma^2 \tag{65}$$

subject to the boundary conditions

$$V^H(C) = 0, \quad V^H(1) = 1.$$

Moreover, substituting the worst-case density generator u^* and the optimal risk choice policy π^* into (65), we get

$$\zeta V^H = \frac{2\omega(1 - \eta)}{1 - \eta + \theta}(X_t - D_0)V_X^H + \frac{\omega}{(1 - \eta + \theta)^2}(X_t - D_0)^2 V_{XX}^H = 0.$$

The general solution to this is

$$V^H = V_1^H(X_t - D_0)^{\alpha_1} + V_2^H(X_t - D_0)^{\alpha_2}.$$

Substituting the general solution into the above ODE, we can get a quadratic equation

$$-\zeta + \frac{\omega(1 - 2\eta)}{(1 - \eta + \theta)^2}\alpha_1 + \frac{\omega}{(1 - \eta + \theta)^2}\alpha_1^2 = 0.$$

Solving the quadratic equation, we get the solution for the constants α_1 and α_2:

$$\alpha_{1,2} = \frac{1}{2}(2\eta - 1) \pm \frac{1}{2\omega}\sqrt{(\omega(1 - 2\eta))^2 + 4\omega(1 - \eta + \theta)^2\zeta}.$$

Substituting the two boundary conditions into the general solution, we can derive the solutions to the constants V_1^H and V_2^H. Further, substituting V_1^H, V_2^H, α_1 and α_2 into the general solution, we have

$$V^H = -\frac{(C - D_0)^{\alpha_2}}{(C - D_0)^{\alpha_1}(1 - D_0)^{\alpha_2} - (1 - D_0)^{\alpha_1}(C - D_0)^{\alpha_2}}(X_t - D_0)^{\alpha_1}$$
$$+ \frac{(C - D_0)^{\alpha_1}}{(C - D_0)^{\alpha_1}(1 - D_0)^{\alpha_2} - (1 - D_0)^{\alpha_1}(C - D_0)^{\alpha_2}}(X_t - D_0)^{\alpha_2},$$

Similarly, let $P^C(X) = \mathbb{E}_t^{\mathbb{P}^u}[e^{-\zeta(\tau_C - t)}]$. Then we can get its HJB equation with the boundary conditions given by $P^C(C) = 1$, $P^C(1) = 0$, from which we can get the solution for P^C.

References

Agarwal V, Ren H (2023) Hedge funds: performance, risk management, and impact on asset markets. In: Oxford research encyclopedia of economics and finance

Anderson EW, Hansen LP, Sargent TJ (2003) A quartet of semigroups for model specification, robustness, prices of risk, and model detection. J Eur Econ Assoc 1(1):68–123

Ang A, Gorovyy S, Van Inwegen GB (2011) Hedge fund leverage. J Financ Econ 102(1):102–126

Argyropoulos C, Panopoulou E, Voukelatos N, Zheng T (2022) Hedge fund return predictability in the presence of model risk. Eur J Finance 28(18):1892–1916

Ben-David I, Franzoni F, Moussawi R (2012) Hedge fund stock trading in the financial crisis of 2007–2009. Rev Financ Stud 25(1):1–54

Bian Y, Xiong X, Yang J (2022) Investor protection, hedge fund leverage and valuation. N Am J Econ Finance 62:101710

Branger N, Larsen LS (2013) Robust portfolio choice with uncertainty about jump and diffusion risk. J Bank Finance 37(12):5036–5047

Branger N, Larsen LS, Munk C (2013) Robust portfolio choice with ambiguity and learning about return predictability. J Bank Finance 37(5):1397–1411

Buraschi A, Kosowski R, Sritrakul W (2014) Incentives and endogenous risk taking: a structural view on hedge fund alphas. J Finance 69(6):2819–2870

Carpenter JN (2000) Does option compensation increase managerial risk appetite? J Finance 55(5):2311–2331

Cvitanic J, Lazrak A, Martellini L, Zapatero F (2003) Optimal allocation to hedge funds: an empirical analysis. Quant Finance 3(1):28

DeVault L, Sias R (2017) Hedge fund politics and portfolios. J Bank Finance 75:80–97

Drechsler I (2014) Risk choice under high-water marks. Rev Financ Stud 27(7):2052–2096

Escobar M, Ferrando S, Rubtsov A (2015) Robust portfolio choice with derivative trading under stochastic volatility. J Bank Finance 61:142–157

Goetzmann WN, Ingersoll JE Jr, Ross SA (2003) High-water marks and hedge fund management contracts. J Finance 58(4):1685–1718

Guasoni P, Obłój J (2016) The incentives of hedge fund fees and high-water marks. Math Finance 26(2):269–295

Hansen LP, Sargent TJ (2012) Three types of ambiguity. J Monet Econ 59(5):422–445

Hansen LP, Sargent TJ, Turmuhambetova G, Williams N (2006) Robust control and model misspecification. J Econ Theory 128(1):45–90

He XD, Kou S (2018) Profit sharing in hedge funds. Math Finance 28(1):50–81

Heaton J (2019) The unfulfilled promise of hedge fund activism. Va Law Bus Rev 13:317

Hodder JE, Jackwerth JC (2007) Incentive contracts and hedge fund management. J Financ Quant Anal 42(4):811–826

Jiang GJ, Liang B, Zhang H (2022) Hedge fund manager skill and style-shifting. Manag Sci 68(3):2284–2307

Kouwenberg R, Ziemba WT (2007) Incentives and risk taking in hedge funds. J Bank Finance 31(11):3291–3310

Kruttli MS, Monin P, Petrasek L, Watugala SW (2021) Hedge fund treasury trading and funding fragility: evidence from the covid-19 crisis. FEDS Working Paper No. 2021-38

Lan Y, Wang N, Yang J (2013) The economics of hedge funds. J Financ Econ 110(2):300–323

Luo D, Wu X, Xu J, Yan J (2022) Robust leverage decision under locked wealth and high-water mark con-
    tract. Finance Res Lett 46:102428
Maccheroni F, Marinacci M, Ruffino D (2013) Alpha as ambiguity: robust mean-variance portfolio analysis.
    Econometrica 81(3):1075–1113
Maenhout PJ (2004) Robust portfolio rules and asset pricing. Rev Financ Stud 17(4):951–983
Maenhout PJ (2006) Robust portfolio rules and detection-error probabilities for a mean-reverting risk pre-
    mium. J Econ Theory 128(1):136–163
Mu C, Yan J, Liang Z (2021) Optimal risk taking under high-water mark contract with jump risk. Finance
    Res Lett 38:101460
Nishimura KG, Ozaki H (2017) Economics of pessimism and optimism, vol 10. Springer, Japan, pp
    978–984
Owjimehr S, Hasanzadeh Dastfroosh H (2022) Uncertainty governance in the stock market during the
    COVID-19: evidence of the strictest economies in the world. China Finance Review International
    (ahead-of-print)
Ross S (1976) The arbitrage theory of capital asset pricing. J Econ Theory 13(3):341–360
Sias R, Turtle HJ, Zykaj B (2016) Hedge fund crowds and mispricing. Manag Sci 62(3):764–784
Sun Y, Liu S, Chen S (2022) Fund style drift and stock price crash risk–analysis of the mediating effect
    based on corporate financial risk. China Finance Review International (ahead-of-print)
Vrontos SD, Vrontos ID, Giamouridis D (2008) Hedge fund pricing and model uncertainty. J Bank Finance
    32(5):741–753
Yang Y-C, Liu W-P, Shih K-H (2023) The COVID-19 pandemic and firm value: the mediating effect of fin-
    tech applications. Rev Quant Finance Account 60(1):329–344
Zhao L, Huang W, Yang C, Li S (2018) Hedge fund leverage with stochastic market conditions. Int Rev
    Econ Finance 57:258–273