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ORIGINAL RESEARCH

# Nowcasting bitcoin's crash risk with order imbalance

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## Abstract

The spectacular nature of bitcoin price crashes baffles market spectators and prompts routine warnings from regulators cautioning that cryptocurrencies behave in contra to the fundamental properties that traditionally define what constitutes money. Arguably most concerning to the public is, first, bitcoin's unprecedented price volatility relative to other asset classes and, second, its seemingly detached price behavior relative to time-honored economic and market fundamentals. In an attempt to create an early warning system of bitcoin price crash risk using generalized extreme value (GEV) and logistic regression modeling, this study integrates order flow imbalance, along with several control factors which reflect blockchain activity and network value, in order to *nowcast* bitcoin's price crashes. From a data analysis perspective, and despite their dissimilar distributional underpinnings, the GEV and logistic models perform comparably. When evaluating the type I and type II errors which these models yield, it is shown that their performance is comparable in terms of accuracy. In addition, it is also shown how the proportion of type I and type II errors can shift dramatically across probability cutoff tolerances. Towards the end of this study, time varying probabilities of a price crash are shown and evaluated. The sample range in this study encompasses the SARS-CoV-2 (Covid-19) time period as well as the recent scandal and collapse of the FTX cryptocurrency exchange.

**Keywords** Bitcoin · Cryptocurrencies · Extreme value modeling

**JEL Classification** C5 · G10 · G17

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## 1 Introduction

This decade has witnessed a rise in blockchain-based technologies and a plethora of offshoot fintech firms that specialize in building cryptographic ledger systems for banks, stock exchanges, and credit card companies.[1] A distinguishing feature of blockchain is the distributed public ledger system which stores transactions between parties without the supervision or regulatory oversight of a central authority. In that way, two or more unacquainted and anonymous parties can create an irrevocable transaction that is forever recorded on the ledger and which is unreservedly viewable by the public (Nakamoto 2008).

Bitcoin is the very first application of blockchain technology. Since the first transaction involving bitcoin in January of 2009 until today, its value has experienced incomparably high volatility relative to what is observable in other traditional assets such as equities, bonds, commodities or currencies.[2] While presently there are over 2000 decentralized cryptocurrencies in circulation, bitcoin constitutes over 60% of the total market capitalization of all cryptocurrencies in existence, with an average daily trading volume in 2022 that far exceeded $30 billion.[3]

There are starkly divergent views on the legitimacy of bitcoin as a viable currency or the potential for blockchain technology to disrupt business and financial services (Abadi and Brunnermeier 2018; Bowden et al. 2021; Chiu and Koeppl 2019; Cong and He 2019; Harvey 2016; US Senate 2013; Yermack 2017). Arguably the dominant argument for why bitcoin cannot become a mainstream currency is because its supply and its seemingly explosive price fluctuations are exogenously determined (Lo and Wang 2014). Because a central bank's monetary policy decisions, based on the state of the economy, have no bearing on the supply of bitcoin, its price is thus independent of shifts in economic fundamentals.[4]

Among all its characteristics, bitcoin's price behavior has arguably garnered the most attention in the financial press and among investors (Wall Street Journal 2019). To illustrate, in early July of 2010, one bitcoin was worth less than $0.05 and, by December of 2017, was worth more than $18,000 before crashing to about $3000 in December of 2018. In April of 2021 it was somewhat over $60,000 before crashing to $31,000 in July 2021. In November 2021 it reached a record high of just over $64,000 while in subsequent months it lost more than half its value and market capitalization. During the height of the FTX cryptocurrency exchange scandal in early November of 2022, bitcoin lost approximately 25% of its value in a matter of days. Presently, it is trading at around $20,000.

[1] See for example *Forbes*' 2019 Fintech 50 list here: https://www.forbes.com/fintech/2019/. In 2010, the multinational consulting firm Accenture launched a "Fintech Innovation Lab" that is designed to bring together Fintech startups with financial institutions: https://www.accenture.com/us-en/service-fintech-innovation-lab.

[2] On January 3, 2009, the bitcoin network was born when Satoshi Nakamoto, the mystery creator, mined the genesis block of bitcoin (the first block in the blockchain—block 0). The coinbase parameter contained the following encoded text message: "The Times 03/Jan/2009 Chancellor on brink of second bailout for banks" (see https://en.bitcoin.it/wiki/Genesis_block). This message is in reference to an article headline in *The Times* for that day (Duncan and Elliott 2009) and serves as a time stamp for proof that the block was created on this date. Apart from serving as a time stamp, it is arguably a manifesto decrying the instability of big banks and the social costs they impose.

[3] See https://coinmarketcap.com.

[4] While economists and policymakers view monetary instruments whose supply cannot be regulated by a central bank as potentially hazardous (Lo and Wang 2014), proponents of bitcoin argue that this very characteristic is what gives bitcoin its value and protects it from inflationary forces (Athey et al. 2016; Bolt and van Oordt 2016; Dwyer 2015; Pagnotta and Buraschi 2018).

While this type of price volatility is alluring for certain speculators seeking upside rewards, it also means a high probability of extreme downside risk. This begs the question that is the motivation for this study: *Can we foresee when bitcoin will experience a price crash and, if so, on the basis of what factors?*

The merits of the first part of this question are immediately self-evident to market participants and regulators alike. While investors and firms that accept bitcoin have a vested interest in monitoring and managing bitcoin's downside risk, regulators understand that bitcoin price shocks can affect not only other cryptocurrencies and their market exchanges, but other segments of our global financial system (Adrian and Mancini-Griffoli 2019; Auer and Claessens 2018; Cheah and Fry 2015; Chimienti et al. 2019; King et al. 2021; Meaning et al. 2018). The merits of the second part of this question as to *what factors* are equally important. There is prevailing disagreement however as to what are the forces which drive bitcoin's price fluctuations and, more fundamentally, whether bitcoin has any intrinsic value at all. Some studies invoke models from network theory, such as Metcalfe's Law, which argues that the value of a network grows as a nonlinear function of the number of users (Alabi 2017; Koutmos and Payne 2021; Metcalfe 2013; Peterson 2018; Van Vliet 2018).[5] Other economists, such as Bradford DeLong, argue that networks inherently face diminishing returns and cannot grow uninhibitedly and exponentially.[6] Specifically, networks tend to build the most valuable connections first, and, while subsequent connections may provide value, they do so at a diminishing rate. In spite of this debate, other studies argue that inferences regarding Bitcoin's value are difficult to make altogether since its price may experience manipulation at various points in time (Gandal et al. 2018).

Against this backdrop, this study illustrates the importance of modelling bitcoin's price crash risk using order imbalance data along with blockchain factors which reflect conditions in its microstructure. The concern of this study is to *nowcast* bitcoin's crash risk and begin laying the foundations of an early warning system that can be used by market participants to estimate probabilities of a bitcoin crash. *Nowcasting* is the focus of our study. This is because it seeks to provide a current description of bitcoin's crash risk using variables that reflect the *nearly*-contemporaneous state of the bitcoin's unique ecosystem. Theoretically, our study makes two main contributions to this emerging strand of literature.

First, it shows the importance of order flow imbalance (buying relative to selling activity) as a variable that can *nowcast* bitcoin price crashes. Order imbalances, their manifestations, and their role in explaining or forecasting returns, have received intense empirical attention following Chordia et al. (2002). Such imbalances are important to study because, first, they may signal trading based on private information and, second, can create market-wide illiquidity and inventory problems for market makers. Kumar and Lee (2006) argue that order imbalance can describe investor sentiment, whereby optimism spurs more buying activity while pessimism spurs more selling activity, ceteris paribus.

Second, it shows that when trying to model bitcoin's price behavior, it is important to incorporate factors that reflect shifts in Bitcoin's blockchain. Much of the emerging

[5] Metcalfe's Law is founded on the observation that in some communication network consisting of $n$ nodes (participants), there are $n(n-1)/2$ possible pairwise connections that can be made in total. Thus, and if we make the blanket assumption that all such pairwise connections are equally valuable, the value of the whole network grows by approximately $n^2$. Derived from the same assumption that all connections are equally valuable, Reed's Law argues that with the advent of the internet, nodes can form groups in addition to connecting as pairs (Reed 2001). Given $n$ nodes, there can exist $2^n$ groups. Thus, the value of the network grows by approximately $2^n$, which is a greater estimate relative to Metcalfe's Law.

[6] See Paul Krugman's web article: http://web.mit.edu/krugman/www/metcalfe.htm.

literature on this subject, like the studies referenced earlier, attempt to link bitcoin with shifts in economic and market variables that have been shown to explain the returns of conventional asset classes, such equities or commodities. As some recent studies show, however, it is important to understand that Bitcoin's microstructure is very different from that of conventional assets, both from a technical and economic standpoint (Auer 2019; Böhme et al. 2015; Ma et al. 2018). In the words of Liu and Tsyvinski (2018, p. 3), "…cryptocurrencies comprise an asset class which is radically different from traditional asset classes…".

Econometrically, our study makes the following two main contributions. First, we show that both the logistic and the generalized extreme value (GEV) regression approaches perform comparably in terms of nowcasting errors. This is an important observation that is relevant to all applications of probabilistic forecasting involving a binary-type outcome. This is because the logistic link function is symmetrical around the value of 0.50. This means that the probability of a binary event can approach zero at the same rate in which it can approach one. Czado and Santner (1992) show that assuming such a logistic link function can lead to biases and inabilities in estimating accurate probabilities. The GEV regression approach attempts to augment this shortcoming in logistic regressions because of its asymmetric link function that is based on the GEV distribution, which in extreme value theory, has shown to better model rare events in statistics (Kotz and Nadarajah 2000; Wang and Dey 2010).

Towards the end of our study, we report on the nowcasting performance of the models and how well they classify days where a price crash occurred relative to days when one did not occur. We show how type I errors (false positive classifications) and type II errors (false negative classifications) are important to consider when trying to deduce what estimated probability is a likely indication that a price crash will actually occur. Finally, we show a time-varying forecasted probability for bitcoin price crashes. To the best of our knowledge, this is one of the first attempts to produce such a forecast, which can serve as a basis for an early warning system for bitcoin investors, those businesses that accept bitcoin as a form of payment, and regulators and market participants at large.

The remainder of this study is organized as follows. Section 2 presents evidence of the linkages between order flow imbalance and bitcoin returns as well as some preliminary statistical description of bitcoin's downside risk. Section 3 presents the blockchain variables that are integrated into our analysis as well as the econometric approaches in estimating probabilities for a bitcoin price crash. Section 4 provides an economic analysis of our findings and reports on the nowcasting performance of each of our models. Section 5 concludes.

## 2 Motivating evidence

This section presents initial evidence on the linkages between bitcoin order flow imbalance and returns, motivating the importance of integrating order flow into models that try to model bitcoin's price behavior. Koutmos (2023) illustrates the challenges with using linear regression modeling to decipher the relation between bitcoin returns and order flow imbalance, especially since some of the most extreme downside price movements occur when there is no significant imbalance.

Instead, we show here that order flow imbalances Granger cause returns more often than returns Granger cause order flow imbalances. This motivates us to incorporate order flow imbalance into our analytical framework (Sect. 3), where we try to determine whether it can serve as an important nowcasting variable. The theory and main evidence from the analysis is presented in Sect. 4.

### 2.1 Order imbalance and returns

The price of bitcoin has experienced sharp perturbations since its inception. Figure 1 shows a time series plot of bitcoin's price (in USD) as well as order flow imbalance for our sample period (April 1, 2013 until January 15, 2023). We describe our sample data and sample range more in Sect. 3.

Chordia et al. (2002) show how order flow imbalance can impact an underlying asset's liquidity and its price. In several recent studies it is shown that it can signal information leakage and that it is an important determinant of (expected) price movements (Bernile et al. 2016; Muravyev 2016). In spirit with these studies, order flow imbalance is estimated as $(\mathrm{B} - \mathrm{S})/(\mathrm{B} + \mathrm{S})$, whereby B (S) is the aggregate buyer-initiated (seller-initiated) trading volume. Estimating order flow imbalance as a ratio rather than as a subtraction between buyer- and seller-initiated volume is advantageous in that it provides a relatively more standardized distribution that is stable across time windows. This is because explosive behaviors, albeit transitory, in either buyer- or seller-initiated volume, will not result, in one direction or another, time series drifts in our order flow imbalance estimation. Removing such drifts ensures stationarity and thus stability in regression errors and across time windows.

As is shown in Fig. 1, bitcoin has experienced sharp bull and bear regimes in its price behavior. In December of 2017, bitcoin reached its first record peak of \$19,270 before declining precipitously to \$6850 in early February of 2018, amid the systematic crackdown and subsequent ban of all cryptocurrencies in mainland China by regulators.[7] Bitcoin reached a subsequent peak in April of 2021, where one bitcoin was worth just over \$60,000 before it crashed to \$31,000 in July of 2021. November of 2021 has been, as of now, its last record high (just over \$64,000), while presently it is trading at around \$20,000. This comes in light of the FTX cryptocurrency exchange scandal which, when became widely public in early November of 2022, resulted in about a 25% decline in bitcoin within only a few days.

Across the overall sample, order flow imbalance in Fig. 1 has a mean of 0.1759 and a standard deviation of 0.2474. In approximately 75% of all trading days it is greater than 0 ($\mathrm{B} > S$) and in 25% of cases it is less than 0 ($\mathrm{B} < S$). Skewness is negative for the overall sample (−0.2312), indicating an asymmetry in the direction of the imbalance. A case can be made that despite bitcoin investors' higher appetites for risk relative to investors who trade in traditional asset classes, they can still behave in accordance with loss aversion theory. Specifically, (expected) negative returns amplify trading behaviors more than do (expected) positive returns of similar magnitude.

A scatter plot of bitcoin log returns against order flow imbalance is shown in Fig. 2. Visual inspection shows that in the relative absence of order flow imbalance ($\mathrm{B} \sim S$), there is a

[7] This report from the Library of Congress details the approach which government officials have taken to eliminate the circulation and mining of cryptocurrencies from mainland China: https://www.loc.gov/law/help/cryptocurrency/china.php.

great degree of dispersion in bitcoin log returns. For instance, the largest negative return in the sample of $-47\%$ is associated with an order flow imbalance of $-0.08$.[8] When inspecting returns that lie on or near zero (the x-axis), we can also see how there is a great degree of dispersion in order flow imbalance in both directions.

At a minimum, the scatter plot shows, first, that a greater proportion of large negative bitcoin returns materialize during points when ($B < S$) and, second, some form of regression analysis involving bitcoin returns and order flow imbalance is not likely to yield a smooth linear relation between the two. This is evident by the near-zero linear fit ($R^2 = 0.013$) and the behavior of the nearest neighbor fit line, which shows sharp nonlinearities throughout the range of the order flow imbalance.

Figure 3 shows rolling window *p* value estimates when bitcoin returns are regressed against order flow imbalance. In regressions when order flow imbalance is contemporaneous and lagged, respectively, we see time varying performance in explanatory power; for example, when order flow imbalance is lagged by one trading day ($t$–1) for the 60-day window length, we see that in several cases it retains significance at the 5% level at least.

A preliminary exploration of whether lags beyond one trading day in order flow imbalance can explain bitcoin returns is shown in the predictive regression analysis in Table 1. While a one trading day lag shows the highest $R^2$ value compared to the other time horizons, it is interesting to see how order imbalance retains its significance (at the 5% level) up to a 7-day lag. Afterwards, it appears that statistical significance diminishes.

## 2.2 Granger causality

To demonstrate the benefits that can be realized by using order flow imbalance, rolling Granger causality tests are also used. As shown in Fig. 4, and for rolling 60-day non-overlapping subsamples, order flow imbalance is evaluated to see whether it can Granger cause bitcoin returns.

Order flow imbalance $\langle OI_t \rangle$ causes bitcoin returns $\langle ret_t \rangle$ in the Granger sense if current *ret* can be better predicted by including lagged values of *OI* while also considering past values of *ret*. Thus, order flow imbalance Granger causes returns if $\sigma^2(ret_t/ret) > \sigma^2(ret_t/ret, OI)$ and whereby $ret = \{ret_{t-1}, \ldots, ret_{t-n}\}$ and $OI = \{OI_{t-1}, \ldots, OI_{t-n}\}$. The minimum predictive variance of $ret_t$ extracted by regressing $ret_t$ on *ret* and *ret* on *OI* is, respectively, $\sigma^2(ret_t/ret)$ and $\sigma^2(ret_t/ret, OI)$.

Implementing a rolling Granger causality test is advantageous in that it allows us to check, first, whether order flow imbalance Granger causes returns or vice versa, and, second, to evaluate which of the two variables relative to one another possesses greater statistical causality power. From Fig. 4, it is observed that there is time series heterogeneity in causality from *OI* to *ret* and vice versa. An important discovery here, however, is that *OI* Granger causes *ret* more frequently than vice versa. This provides some support for the use

[8] This observation point is associated with April 11, 2013. The reasons for why bitcoin crashed during this day and April 12, 2013 are still being debated. During these days, Mt. Gox halted trading and went offline in order to perform network maintenance following distributed denial-of-service (DDoS) attacks. This stirred uncertainty among cryptocurrency traders and whereby public attention on bitcoin peaked; news outlets suggested bitcoin has reached the point where it will crash (historical news articles can be accessed via Google News) while the Bitcoin subreddit became one of the most viewed around the world). While possibly unrelated, Satoshi Nakamoto's final words to the bitcoin community in the volatile month of April 2013 were, "…I've moved on to other things…it's in good hands now…".

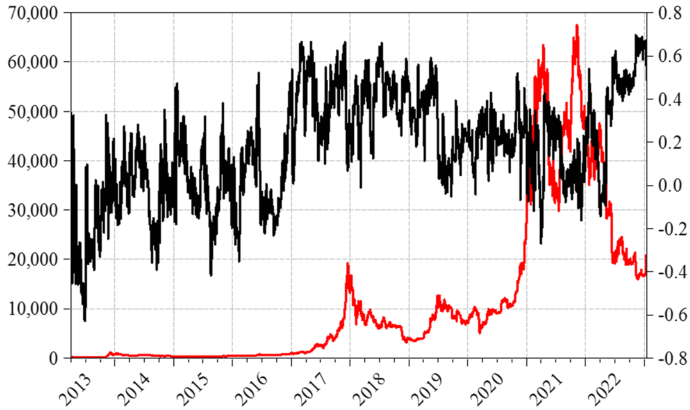


**Fig. 1** Time series plot of BTC price and order imbalance. This figure shows a time series plot of BTC price in USD (in red and left axis) and order imbalance (in black and right axis) for the sample period of April 1, 2013 until January 15, 2023. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail (Color figure online)

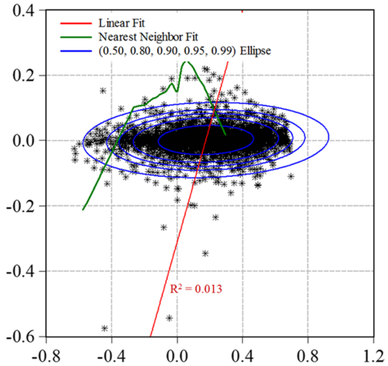


**Fig. 2** Scatter plot of BTC returns and order imbalance. This figure shows a scatter plot of BTC returns (expressed in decimal form and not in percentages on the left axis) against order imbalance (bottom axis) for the sample period of April 1, 2013 until January 15, 2023. A linear regression fit (in red) is estimated with an $R^2$ of 0.013 (expressed in decimal form) while the green line estimates a k-nearest neighbor regression fit. Confidence ellipses are in blue for each of the respective confidence levels. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail (Color figure online)

of order flow imbalance as an important variable for trying to explain or foresee bitcoin price crashes. This finding also contributes to literature which indicates that bitcoin returns are a statistically dominant factor in explaining changes in other blockchain characteristics, such as transaction activity and bitcoin usage volumes, rather than vice versa (Koutmos 2018; Li and Wang 2017).

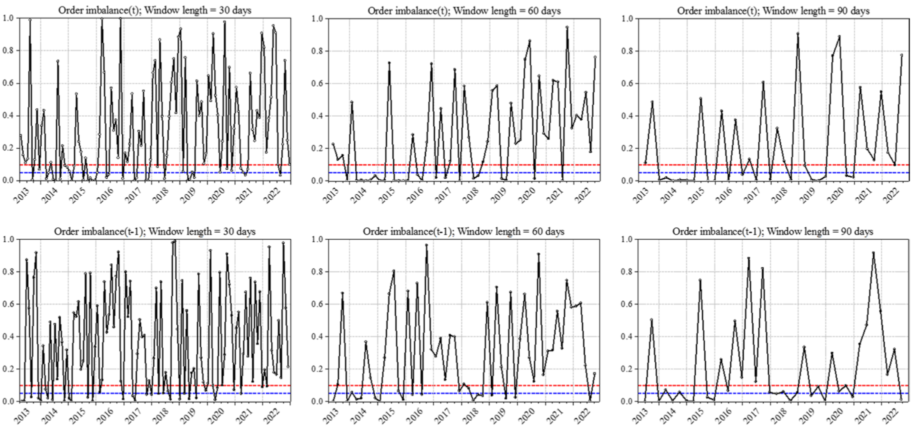


**Fig. 3** Rolling regression *p* value estimates. This figure shows time series plots of *p* value estimates extracted from performing rolling regressions of bitcoin returns against order imbalance. The upper three graphs estimate a contemporaneous rolling regression, $r_t = a + b * OI_t + \varepsilon_t$, across 30-, 60- and 90-day non-overlapping rolling windows, respectively. The lower three graphs estimate a lead-lag rolling regression, $r_t = a + b * OI_{t-1} + \varepsilon_t$, for each of the aforementioned non-overlapping window lengths. *r* and *OI* are bitcoin returns and order imbalance, respectively. The horizontal dashed blue and red lines are drawn across the 0.05 and 0.10 tick marks, respectively, and represent significance at the 5% and 10% levels. The sample period is from April 1, 2013 until January 15, 2023. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail (Color figure online)

## 2.3 Extreme movements in bitcoin prices

The most common criticism against using bitcoin as a medium of exchange or as an investment vehicle is its unprecedented volatility, which far exceeds what is observable in traditional asset classes. In a 2018 speech, Lael Brainard, who is a member of the Board of Governors of the Federal Reserve System, argued that "…[b]itcoin's value has been known to fluctuate by one-quarter in one day alone…such extreme fluctuations limit an asset's ability…[to perform]…the functions of money…"[9] Likewise, Schuhy and Shyz (2016) cite bitcoin's price volatility as a barrier to adoption among risk-averse consumers.

Table 2 provides a basis for comparison between the distributional properties of bitcoin's price changes and those of other traditional assets. It shows the gravity of bitcoin's crash risk and reports statistics for the entire sample that is subsequently used in this study (and which includes weekend price data) as well as for only weekdays (for the sake of comparison). The mean returns are expressed in percentages and thus the mean return for bitcoin over the entire sample is about 0.1503% while a subsample which excludes weekend prices has a mean of about 0.1581%. This positive mean return is driven by the sustained price appreciations bitcoin has experienced throughout the sample period. The mean return for the entire sample (including weekends) is over three times higher than the Nasdaq 100 E-mini, which has the next highest return among all the comparison assets. Overall, the mean returns of the other comparison assets over the sample period pale in comparison

[9] The speech is publicly available online: https://www.federalreserve.gov/newsevents/speech/brainard20180515a.htm.

**Table 1** Predictive regression analysis

| Horizon | a | b | $R^2$ (%) |
|---|---|---|---|
| 0 | −0.0008<br>(−1.08) | **0.0522**<br>(5.184) | 1.320 |
| 1 | −0.0012<br>(−1.645) | **0.0157**<br>(6.165) | 1.052 |
| 2 | −0.0003<br>(−0.438) | **0.0103**<br>(4.026) | 0.452 |
| 3 | −0.0004<br>(−0.052) | **0.0083**<br>(3.251) | 0.295 |
| 4 | −0.0001<br>(−0.076) | **0.0084**<br>(3.283) | 0.301 |
| 5 | −0.0003<br>(−0.372) | **0.0096**<br>(3.769) | 0.396 |
| 6 | −0.0001<br>(−0.098) | **0.0084**<br>(3.279) | 0.300 |
| 7 | 0.0001<br>(0.109) | **0.0074**<br>(2.890) | 0.233 |
| 12 | 0.0006<br>(0.663) | 0.0057<br>(1.936) | 0.158 |
| 24 | 0.0009<br>(1.098) | 0.0024<br>(0.845) | 0.031 |
| 36 | 0.0012<br>(1.295) | 0.0016<br>(0.554) | 0.004 |

This table reports ordinary least squares estimation results for $a$, $b$, and $R^2$ statistics for the predictive regression, $r_{t \to t+h} = \alpha + b * OI_t + \varepsilon_{t \to t+h}$, where $r$ and $OI$ are bitcoin returns and order imbalance, respectively. The sample period is from April 1, 2013 until January 15, 2023. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail. Newey-West heteroskedasticity- and autocorrelation-robust t-statistics are shown in parentheses, while the last column reports $R^2$ across each of the horizons. Coefficients in bold denote significance at the 5% level at least ($p$ value $\leq 0.05$)

to bitcoin's mean returns. Two-sample t-tests for differences of means, assuming unequal variances, are performed (not tabulated) and show that bitcoin historically outperformed the other asset classes and by a significant margin.

This outperformance, however, comes with materially higher volatility and tail risks for investors. The value-at-risk (VaR) and modified VaR (MVaR) calculations for bitcoin far exceed what is calculated for the other assets.[10] Similarly to what is observed in financial

[10] VaR is estimated as: $\text{VaR} = W\left(\mu\Delta t - n\sigma\sqrt{\Delta t}\right)$ whereby $\mu$ is the mean return ($ret$); $W$ is the value of the portfolio; $n$ is the number of standard deviations depending on the confidence level; $\sigma$ is the standard deviation; $\Delta t$ is the time window. MVaR integrates skewness ($S$) and excess kurtosis ($K$) of bitcoin returns and is estimated as: $\text{MVaR} = W\left[\mu - \{z_c + \frac{1}{6}\left(z_c^2 - 1\right)S + \frac{1}{24}\left(z_c^3 - 3z_c\right)K - \frac{1}{36}\left(2z_c^3 - 5z_c\right)S^2\}\sigma\right]$. $z_c$ is the critical value for the probability $(1-\alpha)$ and −1.96 for a 95% probability; $S = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{ret_t - \overline{ret}}{\sigma}\right)^3$; $K = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{ret_t - \overline{ret}}{\sigma}\right)^4 - 3$. More discussion on the estimation of such VaR and Sharpe models can be found in Gregoriou and Gueyie (2003) and Signer and Favre (2002). Iqbal et al. (2020) provide an discussion of some of their alternative distributional extensions.

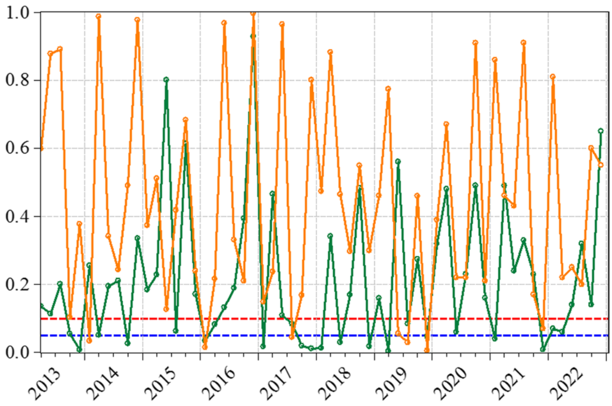


**Fig. 4** Rolling Granger causality *p* value estimates. This figure shows time series plots of *p* value estimates extracted from 60-day non-overlapping Granger causality regressions between returns and order imbalance. The *p* value tests the null hypothesis that there is no Granger causality from returns to order imbalance and vice versa. In green, we see whether order imbalance Granger causes returns ($OI \rightarrow r$). In orange, we see whether returns Granger cause order imbalance ($r \rightarrow OI$). The null hypothesis of no causality is rejected more frequently in the case of $OI \rightarrow r$. On average, the p-value for the case of $OI \rightarrow r$ is 0.2161 while for $r \rightarrow OI$ it is 0.4453. The horizontal dashed blue and red lines are drawn across the 0.05 and 0.10 tick marks, respectively, and represent significance at the 5% and 10% levels. The sample period is from April 1, 2013 until January 15, 2023. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail (Color figure online)

time series data, bitcoin returns exhibit negative skewness and excess kurtosis. This negative skewness is not particularly sizable relative to what is observed with the comparison asset returns and, while bitcoin returns are noticeably leptokurtic, the DJIA E-mini and S&P GSCI crude oil commodity index, respectively, actually exhibit a higher degree of kurtosis risk.

Bitcoin's high volatility risk penalizes its high historical mean returns, thus leading to an estimate for its risk-adjusted returns (Sharpe and modified Sharpe ratios, respectively) that is even lower than what is estimated for, say, the Nasdaq 100 E-mini.[11] Thus, investors seeking to hold bitcoin in order to gain from its price appreciation must also be willing to stomach its high volatility risk.

In this study, a bitcoin price crash (*Crash*) is defined as a return ($ret_t$) that lies one standard deviation or more below the sample mean:

$$Crash = \begin{cases} 1 \text{ if } ret_t \leq \overline{ret} - 1\text{SD}(ret) \\ 0 \text{ if otherwise} \end{cases} \quad (1)$$

This serves as our definition of a bitcoin crash and, as shown in Fig. 5, approximately 8.89% (N = 318) of the return observations lie one standard deviation or more to the left

[11] The Sharpe ratio is estimated as $(ret_t - r_f)/\sigma$ while the modified Sharpe ratio is estimated as $(ret_t - r_f)/\text{MVaR}$. The holding period return for the 1-month treasury bill is used as a proxy for the risk-free rate, $r_f$. For bitcoin returns (the entire sample which includes weekends and thus requires weekend data for $r_f$), a moving average is used to fit in the missing data. Treasury return data from Professor Kenneth French's data library are used in this study; see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

**Table 2** Comparison of bitcoin's risk-return characteristics with conventional assets

| | | Mean | SD | Skew | Kurt | Min | Max | VaR | Modified VaR | Sharpe | Modified sharpe |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Bitstamp sample data | Bitcoin (entire sample) | 0.1503 | 3.7169 | −1.3048 | 19.2187 | −46.6836 | 22.9233 | −7.1348 | −13.4170 | 0.0404 | 0.0112 |
| | Bitcoin (weekdays) | 0.1581 | 3.9572 | −1.4143 | 19.3244 | −46.6836 | 22.9233 | −7.5980 | −14.3481 | 0.0400 | 0.0110 |
| Equity markets | S&P 500 ($50) (E-mini) | 0.0380 | 1.1121 | −0.8215 | 14.0825 | −11.5887 | 8.6731 | −2.1417 | −3.5412 | 0.0342 | 0.0107 |
| | Nasdaq 100 ($20) (E-mini) | 0.0499 | 1.3175 | −0.6914 | 9.1120 | −13.1492 | 8.9347 | −2.5324 | −3.6969 | 0.0379 | 0.0135 |
| | DJIA ($5) (E-mini) | 0.0348 | 1.1079 | −1.0011 | 22.4347 | −13.6521 | 10.4533 | −2.1367 | −4.2081 | 0.0314 | 0.0083 |
| | Euro Stoxx 50 (€10) (futures) | 0.0112 | 1.3625 | −1.1442 | 13.3466 | −13.3124 | 8.9911 | −2.6593 | −4.3869 | 0.0082 | 0.0026 |
| | FTSE 100 (£10) (futures) | 0.0066 | 1.0170 | −0.9871 | 14.0374 | −12.2387 | 8.5895 | −1.9867 | −3.2986 | 0.0065 | 0.0020 |
| | Nikkei 225 (¥1,000) (futures) | 0.0326 | 1.3201 | −0.2435 | 7.2676 | −8.2529 | 7.7313 | −2.5548 | −3.3550 | 0.0247 | 0.0097 |
| | SSE 180 (spot) | 0.0215 | 1.4511 | −0.7175 | 9.0673 | −10.5361 | 9.4528 | −2.8227 | −4.1109 | 0.0148 | 0.0052 |
| Commodity indices | S&P GSCI Crude Oil | 0.0189 | 2.8234 | −0.3574 | 23.1902 | −28.2777 | 22.5134 | −5.5150 | −10.4403 | 0.0067 | 0.0018 |
| | S&P GSCI Commodity Index | 0.0057 | 1.3532 | −0.4824 | 8.8883 | −12.7617 | 7.5958 | −2.6466 | −3.7364 | 0.0042 | 0.0015 |
| | Invesco DB Commodity Fund | −0.0039 | 1.1055 | −0.7414 | 7.8739 | −8.2777 | 4.6874 | −2.1707 | −3.0684 | −0.0035 | −0.0013 |

This table compares summary statistics of percent BTC returns with those of other assets for the sample period of April 1, 2013 until January 15, 2023 (mean, minimum, and maximum returns are expressed as a percentage). The comparison assets consist of equity markets and commodities. Bitcoin returns and the returns of the other comparison assets are estimated from prices in terms of USD. While for the main findings of this paper weekend BTC data is also utilized (entire sample), this table also shows summary statistics for BTC returns observable only on weekdays to allow for comparisons with the equity market and commodity indices. Continuous futures data is used for all the equity markets (with the exception of the Shanghai Stock Exchange 180 index, which uses spot prices). The Bloomberg default roll-over methodology is used to create these continuous contract prices (see the Bloomberg GFUT function). The commodity indices consist of the S&P Goldman Sachs crude oil and commodity index, respectively (more information on these indices can be found here: https://www.goldmansachs.com/what-we-do/FICC-and-equities/business-groups/sts-folder/gsci/index.html). The Invesco Deutsche Bank commodity fund index is also included for the sake of comparison (more information on this index can be found here: https://www.invesco.com/us/financial-products/etfs/product-detail?audienceType=Investor&ticker=DBC). The last four columns report the value-at-risk (VaR), modified VaR (MVaR), Sharpe ratio, and modified Sharpe ratio, respectively. Since the risk-free rate (the daily holding period return of the 1-month treasury bill) is zero for most of the sample period under consideration here, it is omitted from the calculations herein. Calculations for VaR, MVaR, and the respective Sharpe ratios, can be found in footnotes (10) and (11)

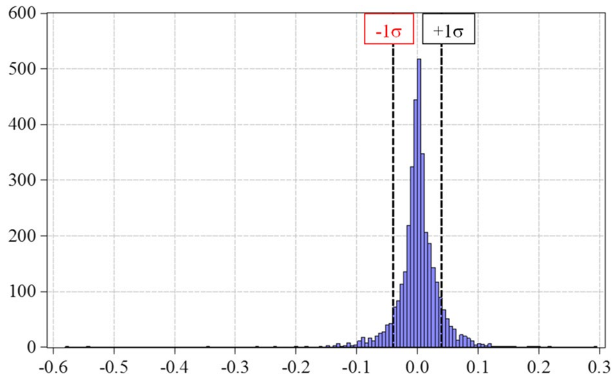


**Fig. 5** Frequency histogram of BTC returns. This figure shows a frequency (left axis) histogram of BTC returns (expressed in decimal form and not in percentages on the bottom axis). The sample period is from April 1, 2013 until January 15, 2023. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail. Approximately 8.89% of observations lie to the left of $-1\sigma$

of the sample mean. Our full sample period, which we describe further in Sect. 3, is from April 1, 2013 until January 15, 2023 (N = 3577). Thus, a return of about − 3.57% or less is classified as a bitcoin crash in our study.

Choosing a one standard deviation length to the left of the sample mean requires striking a balance between two opposite constraints. On the one hand, it is necessary to have a sufficient sample of observations for which we can test the statistical explanatory power of the variables used in this study and their ability to nowcast. On the other hand, and from an investor's point of view, a daily return of − 3.57% may not necessarily be reason for alarm considering the nature of bitcoin's price behavior.

When isolating returns which we classify as bitcoin crashes and checking their distributional characteristics (untabulated for brevity but available upon request) we find the following. First, several of these negative returns are grouped together within at least one week apart. This means that within one trading week, an investor can lose about 10–20% on average if they were holding bitcoin during a crash episode. Second, a significant portion of these total crash observations are daily returns which are at least − 7.00% (this is the mean of all "crash" returns). Thus, in order to maintain a sufficient subsample of crash observations, and in light of the fact that crash observations tend to cluster together and, therefore, early detection of an initial crash observation is useful, this study classifies returns that are one standard deviation or more in length to the left of the sample mean.

Since our study is concerned with modeling extreme values to the left of the return distribution, our methodological approach is motivated by extreme value theory (EVT) as a tool for describing the probabilistic characteristics of extreme events—in our case here, the extreme events are bitcoin crashes. It is worth noting that various other branches of science, such as materials engineering, climatology and hydrology, as well as sociology, to name a few, have been concerned with the empirical modeling of extreme observations in order to answer questions such as[12]: *What is the probability of a flood, or, drought? What is the probability of a heat wave? What is the probability that a part within a machine or a system will fail?*

These types of questions are concerned with estimating the probabilities of sample extrema (maxima or minima), which lie in the tails of the distribution. Unlike the central

[12] See Chavez-Demoulin and Davison (2005) and references therein on using the GEV distribution to model extreme temperatures. From an engineering view, Toshkova et al. (2020) discuss how it can be used as a warning system to detect failures in a mechanical system.

limit theorem which is concerned with describing the sample mean, EVT is concerned with measuring the shape and thickness of a distribution's tails, thus enabling accurate probabilistic measurements of extreme outcomes. According to EVT, the asymptotic distribution of extrema can be described by the Gumbel, Fréchet, or Weibull limit distributions depending on the shape parameter of the tail, which we will denote as $\xi$ for our immediate discussion (Coles et al. 2001). Jenkinson (1955) lays the foundation for the generalized extreme value (GEV) distribution, which combines each of these three limit distributions into a single family of models.

Given we are working with bitcoin returns, and since we are seeking to model the probabilistic distribution of minima (bitcoin crashes), we have the following: $\widetilde{M}_n = \min\{ret_1, \dots, ret_n\}$, where a given *ret* represents a daily return that is independent and identically distributed (iid).[13] Now, if we let $Y_i = -X_i$ for $i = 1, \dots, n$, then the change of sign means small values of $X_i$ correspond to large values for $Y_i$. Thus, if $\widetilde{M}_n = \min\{X_1, \dots, X_n\}$ and $M_n = \max\{Y_1, \dots, Y_n\}$, then $\widetilde{M}_n = -M_n$. For our sample of $n$, and since we are modeling minima, the GEV distribution is expressed as

$$\begin{aligned}
\Pr\left\{\tilde{M}_n \le z\right\} &= \Pr\left\{-M_n \le z\right\} \\
&= \Pr\left\{M_n \ge -z\right\} \\
&= 1 - \Pr\left\{M_n \le z\right\} \\
&\approx 1 - \exp\left\{-\left[1 + \xi(-z - \mu)/\sigma\right]^{-1/\xi}\right\} \\
&= 1 - \exp\left\{-\left[1 - \xi(-z - \tilde{\mu})/\sigma\right]^{-1/\xi}\right\}
\end{aligned} \tag{2}$$

on $\left\{z : 1 - \xi(z - \widetilde{\mu})/\sigma > 0\right\}$, where $\widetilde{\mu} = -\mu$. The shape parameter $\xi$ determines whether we obtain the Gumbel ($\xi = 0$), Fréchet ($\xi > 0$), or Weibull ($\xi < 0$) distribution. Of the three limit distributions, the Weibull distribution has no tail because after a certain point there are no extrema and, while the Fréchet distribution has a fat and slowly decreasing tail, the Gumbel distribution has a thin and rapidly decreasing tail. The independent standard variable $z$ is reduced by the location parameter $\mu$ and scale parameter $\sigma$.

Maximum likelihood estimation (MLE) is utilized in this study to estimate parameters for the GEV distribution. MLE is shown to provide unbiased estimates with minimal variance (Coles et al. 2001). Smith (1985) shows the efficacy of MLE across a range of estimates for $\xi$. Extant studies seeking to describe probabilistic outcomes of extrema, such as those mentioned in Footnote (12), use MLE for a wide range of applications. In this study, and in order to estimate parameters for our GEV distribution, MLE is performed using the optimization algorithm of Berndt et al. (1974).

## 3 Data and regression models

In order to nowcast bitcoin's price crash risk, this study posits a series of regression models that combine order imbalance with control variables that serve as a description of exchange trading activity and shifts in blockchain activity. The purpose is to estimate the probability

[13] Smith (1985) shows that an iid assumption is not a necessary requirement for modeling probabilistic outcomes of extrema with limit distributions. See Coles et al. (2001) as well as the references in Footnote (12) for discussions on modeling maxima.

**Table 3** Predictor data, transformations and descriptions

| Source | Data series | Abbrev | Transf | Expected sign | Description |
|---|---|---|---|---|---|
| Bitstamp | Order imbalance | *OI* | lv | − | Estimated as $(Buys - Sells)/(Buys + Sells)$ whereby *Buys* and *Sells* denote buyer- and seller-initiated trades, respectively |
| | Range volatility | *RV* | lv | + | The difference in the natural logs of the daily highest and lowest BTC prices (quoted in USD); $\ln(High) - \ln(Low)$. Serves as a proxy for intraday volatility risk |
| | Trade volume | *VOL* | ln | −/+ | The total USD value of trading volume. Serves as a proxy for bitcoin news and information flow into the market |
| | Trades per min | *TPM* | ln | −/+ | The average number of trades per minute. Serves as a proxy for the intensity of Bitcoin news and information flow into the market |
| Blockchain | Generated coins | *COIN* | ln | −/+ | The number of new bitcoins generated through mining. This number is halved over time until a total of 21 million bitcoins are in circulation |
| | Active addresses | *ADDR* | Δln | −/+ | The number of active bitcoin addresses (as opposed to all unique addresses). A bitcoin address is a unique identifier that serves as a destination for a bitcoin payment |
| | Transaction fees | *FEE* | Δln | − | The fees that miners earn to process transactions |
| | Mining revenue | *REV* | Δln | − | The sum of block rewards and transaction fees. Block rewards are the fees that miners earn when validating transactions and discovering new blocks |
| | Block count | *BLOC* | ln | + | The number of blocks discovered by miners. It takes approximately 10 min on average for blocks to be discovered. These blocks record data relating to bitcoin transactions and, individually, serve like pieces of a whole ledger. When these blocks are added to the blockchain, they cannot be changed (the transactions data are permanent) |

This table lists the sources and data series that are used as predictors for bitcoin's crash risk (in the first and second columns). Abbreviations for each are shown in the third column while transformations necessary to induce stationarity are shown in the fourth column. *ln* denotes the natural logarithm, $\Delta ln$ denotes the first difference of the natural logarithm, and *lv* denotes the level of the series. The fifth column hypothesizes as to the data series' expected sign in nowcasting crash risk. The last column provides a brief description of each of the series. The sample range for the data is April 1, 2013 until January 15, 2023 of a bitcoin price crash, as defined in Eq. (1), using the GEV model in Eq. (2). In addition, a logistic model is also used to allow for cross-model comparisons.

The data variables, described in Table 3, are either exchange-specific or are blockchain-wide and reflect the health or activity of the entire blockchain. The exchange-specific variables consist of order imbalance (*OI*), along with following control variables: range volatility (*RV*), trading volume (*VOL*), and trades per minute (*TPM*), respectively. The blockchain-wide variables consist of generated coins (*COIN*), active addresses (*ADDR*), transaction fees (*FEE*), mining revenue (*REV*), and block count (*BLOC*), respectively.

The data for *OI* are sourced from Bitstamp, a major cryptocurrency exchange that has gained popularity in recent years for its relatively low transaction fees and the ability to trade fiat currencies with bitcoin as well as other cryptocurrencies.[14] The data contains historical tick-by-tick information on the order book for the Bitstamp exchange. A cryptocurrency exchange's order book is the list of orders (buys and sells) at various prices for a particular cryptocurrency (in our case, bitcoin). Among the many services they provide, cryptocurrency exchanges use a matching engine that, at any given time, determines which orders in the flow are to be fully (or partially) executed depending on supply and demand.[15] At any given time, an imbalance can occur in the order book if there is a mismatch in buy and sell orders at given price points. In this study, we aggregate this tick-by-tick data to create a daily time series of *OI*, as mentioned in Sect. 2.1 and shown in Fig. 1, to estimate price crash probabilities. The Bitstamp exchange is used for empirical testing because it is one of the largest cryptocurrency exchanges and one that has, comparably speaking, a long history within the cryptocurrency community of investors (it presently offers active markets for a wide range of cryptocurrencies).[16]

The remaining exchange-specific data variables are obtained from Bitstamp using APIs and are discussed in turn. (1) *RV* is the range volatility estimated from the highest and lowest bitcoin (BTC) price for the day (quoted in US dollars): $\ln(High) - \ln(Low)$. Because higher volatility risk is linked with thicker tails in the distribution of price changes, it is expected that it should increase the probability of a bitcoin crash. (2) *VOL* is the total USD value of trading volume. The time series movements in volume reflect the stochastic arrival of news and information into the market and the subsequent incorporation of this news in asset prices (Black et al. 2023). For conventional investment assets, such as equities or index funds, a strong contemporaneous link is often shown between trading volume and return volatility. Following Clark (1973) and Tauchen and Pitts (1983), empirical specifications of the volume-volatility relation are motivated by the mixture of distribution hypothesis (MDH), which posits a joint dependence of returns and volume on a common latent information flow variable. Given this joint dependence structure, and in light of the positive volume-volatility relation that is often found with traditional asset classes, it is expected that increases in trade volume will be associated with higher volatility and, therefore, a higher probability of a bitcoin crash. This may or may not be the case however given differences in bitcoin's technological ecosystem or the degrees of risk aversion among its

[14] See https://www.bitstamp.net/api/#order-book. Trading days with suspensions or other such trading frictions are omitted. Consequently, and in all, less than 1% of observations are omitted. One such particular example is on January 6, 2015 (and several trading days thereafter), whereby Bitstamp temporarily suspended service due to a hack.

[15] An example of a live order book can be found here: https://www.bitstamp.net/s/webapp/examples/order_book_v2.html.

[16] The Bitstamp exchange enables investors to trade a range of cryptocurrencies (https://coinmarketcap.com/exchanges/bitstamp/).

investors. (3) *TPM* is the average number of trades per minute. This serves as a proxy for the velocity, or, intensity of news that flows to the market. During periods of accelerating investor attention in bitcoin stemming from news, both positive or negative, it is expected that *TPM* will rise.

Unlike the aforementioned variables which are specific to Bitstamp, the blockchain-wide variables used here reflect the overall health of bitcoin's ecosystem. They are not specific to an exchange but rather describe various characteristics in bitcoin's blockchain.[17] Such variables reflect network activity and health in the overall bitcoin blockchain and, insofar as bitcoin market prices depend on exchange-specific idiosyncrasies such as liquidity and platform security, affect market prices and the demand for bitcoin across all the many exchanges.

These blockchain-wide variables are now discussed in turn. (1) *COIN* is the number of new bitcoins generated through mining. Unlike fiat currencies, whose quantity is determined by central banking policies, bitcoin's supply is limited to 21 million. Specifically, bitcoins are minted whenever a miner discovers a new block. The number of generated and newly minted bitcoins per block began at 50 and has been set to decrease geometrically, with a halving taking place in the generated coins every 210,000 blocks (approximately every 4 years). Thus, the cumulative supply of bitcoin can be expressed as $\left\{ \sum_{i=0}^{32} 210,000\left[50 * 10^8/2^i\right] \right\}/10^8$. The level of difficulty of the mathematical problem for hashing blocks is adjusted in order to maintain a constancy of about 6 blocks per hour. The reason for this limit in bitcoin's supply has never been directly addressed in Nakamoto (2008). Some speculate that this corresponds with a four year reward halving arrangement. Others posit philosophical rather than empirical or technical reasons and note that if one were to form a cube of all the gold ever mined globally, each side would have a length of approximately 21 m.[18] Whatever the reason may be, the supply of bitcoins generated through mining are an important determinant of value. (2) *ADDR* is the number of active bitcoin addresses. An address is a unique identifier that serves as a destination for a bitcoin payment. It is akin to an email address, which is required in order to send or receive an email. From the perspective of telecommunications and computer network value theory, each additional unique address that participates in transactions within bitcoin's ecosystem contributes nonlinearly to the value of the overall bitcoin network (see Footnote 5). (3) *FEE* is the average fees that miners earn to process transactions. In the bitcoin ecosystem, miners compete to verify transactions and will naturally prioritize transactions that have larger transaction fees appended to them (relative to the size of the transaction). An interesting feature of the bitcoin network is that transaction fees are voluntary and, while senders of bitcoin are not required to submit a payment, miners are not required to verify and process transactions. If a bitcoin payment has been submitted with too small of an appended fee for the miners (or no fee at all), the transaction is likely to sit in the pool of unconfirmed transactions, known as the mempool, or, will not be confirmed and thus not be added to the blockchain. There are many publicly available online calculators that help bitcoin users estimate a (optimal) fee which they can append to their payments (depending

[17] See blockchain.com/charts for market data, block details, mining information, and network activity, respectively.

[18] See updated data from the World Gold Council here https://www.gold.org/about-gold/gold-supply/gold-mining/how-much-gold.

on transaction size and how quickly they want their transaction to be confirmed).[19] (4) *REV* is the sum of block rewards and transaction fees earned by all miners from validating transactions as well as discovering new blocks. When the number of bitcoins approaches the supply limit of 21 million, miners will only be able to earn a reward for verifying transactions to append to blocks. (5) *BLOC* is the number of blocks that are discovered by miners. Blocks contain information about transactions that take place among users (such as, for example, when a transaction took place, the amount and the participants). The computational work of miners hashes blocks (to distinguish them from other blocks) and to connect them to the blockchain, which at that point become immutable records. Miners "discover" blocks when they solve the hash function in order to append blocks to the blockchain. It takes approximately 10 min on average for blocks to be discovered.

In addition to order imbalance (*OI*), each of the three exchange-specific variables (*RV*, *VOL* and *TPM*) and blockchain-wide variables (*COIN*, *ADDR*, *FEE*, *REV* and *BLOC*) should serve as important determinants of bitcoin's value. Table 3 also shows the data transformation that is performed to ensure that each of the time series are stationary for the upcoming analysis. For example, while *OI* and *RV* are stationary and are used in their level (lv) form, first-differences in the natural logs ($\Delta$ln) of *ADDR*, *FEE* and *REV*, respectively, are used to induce stationarity. The natural logs (ln) are applied to the remainder of the variable series (*VOL*, *TPM*, *COIN* and *BLOC*, respectively). The frequency of all the data is daily and includes weekend observations (7-days a week).

Their expected sign in explaining the probability of a bitcoin crash is also shown in Table 3. For example it is expected that, everything else equal, rises in buy orders, and thus a rise in *OI*, is expected to decrease the probability of a bitcoin crash (hence the negative sign in Table 3). A rise in volatility, *RV*, for example, should increase the probability of crash risk, especially since bitcoin returns are negatively skewed and leptokurtic (as shown in Table 2). For other variables, such as *VOL*, we may be less confident in surmising what the relation is a priori. If extant literature of the volume-volatility relation of well-known asset classes is any guide, we know that trading volume is positively related to volatility (Lamoureux and Lastrapes 1990). If volume is linked with higher volatility, and, if higher volatility is expected to be linked with higher crash risk, it is then expected that rises in volume should be linked with a higher probability of crash risk. But this may not be the case with bitcoin given how its microstructure is different from that of, say, equities and bonds. It is also possible that rises in volume are associated with enhanced liquidity and thus a lower probability of a price crash. A priori, and similar also to *VOL*, it is not obvious the sign which *TPM* will exhibit in our regression tests.

Similar to *VOL*, and of the blockchain-wide variables, it is not self-evident a priori what sign *ADDR* will have. While rises in the number of addresses means more users (and this should result in a rise in the value of the bitcoin ecosystem) it can also mean more nefarious nodes, or networks of nodes, which are engaged in ransomware, cryptocurrency laundering, or other forms of cybercrime (Turner et al. 2020). Rises in *FEE* and *REV*, respectively, reflect increases in the demand for services provided by miners. This likely serves as an indication of growing transaction activity in the Bitcoin blockchain, and, a priori, a growing ecosystem. *BLOC* also represents blockchain activity, since with the computational problem-solving of miners, new blocks are discovered and transactions are appended

[19] See https://www.buybitcoinworldwide.com/fee-calculator/. More description of the mining process, its importance and the role of fees can be found here: https://support.blockchain.com/hc/en-us/articles/360000939903-Transaction-fees.

**Table 4** Correlation matrix among bitcoin returns and predictor variables

| | *r* | *OI* | *RV* | *VOL* | *TPM* | *COIN* | *ADDR* | *FEE* | *REV* | *BLOC* |
|---|---|---|---|---|---|---|---|---|---|---|
| *r* | – | −0.2354 | −0.2142 | −0.0417 | −0.0348 | 0.0137 | 0.0690 | 0.1499 | 0.2184 | 0.0358 |
| *OI* | −0.2354 | – | 0.0029 | −0.0827 | 0.0475 | −0.0679 | 0.0020 | −0.0546 | −0.0737 | −0.0374 |
| *RV* | −0.2142 | 0.0029 | – | 0.1232 | −0.0084 | 0.0733 | 0.0242 | −0.0141 | −0.0585 | 0.0679 |
| *VOL* | −0.0417 | −0.0827 | 0.1232 | – | 0.3116 | 0.2957 | 0.0884 | 0.1028 | −0.0031 | −0.0203 |
| *TPM* | −0.0348 | 0.0475 | −0.0084 | 0.3116 | – | −0.7230 | 0.0330 | 0.0390 | −0.0058 | −0.3050 |
| *COIN* | 0.0137 | −0.0679 | 0.0733 | 0.2957 | −0.7230 | – | 0.0269 | −0.0324 | 0.0957 | 0.4850 |
| *ADDR* | 0.0690 | 0.0020 | 0.0242 | 0.0884 | 0.0330 | 0.0269 | – | 0.2729 | 0.2608 | 0.1048 |
| *FEE* | 0.1499 | −0.0546 | −0.0141 | 0.1028 | 0.0390 | −0.0324 | 0.2729 | – | −0.0568 | −0.1697 |
| *REV* | 0.2184 | −0.0737 | −0.0585 | −0.0031 | −0.0058 | 0.0957 | 0.2608 | −0.0568 | – | 0.4408 |
| *BLOC* | 0.0358 | −0.0374 | 0.0679 | −0.0203 | −0.3050 | 0.4850 | 0.1048 | −0.1697 | 0.4408 | – |

This table shows a pair-wise correlation matrix among bitcoin returns, $r$, and the predictor variables. Abbreviations for each of the variables are explained in Table 3, which also provides a description of the variables and the transformations performed to ensure their stationarity. The sample period is from April 1, 2013 until January 15, 2023 to the blockchain. The difficulty of the computational problem is adjusted every 2016 blocks to hold steady the rate at which blocks are generated. In the continuous limit of bitcoin's mining activity, and like *COIN*, the computational power, on average, has a tendency to rise, while periods where it does decline signals an exodus of miners in the race to discover blocks and, ultimately, to mine new coins.[20] Whereas rises in *BLOC* can signal network activity, it also signals a rise in computational difficulty and a marginally declining rate at which newly minted bitcoins are minted. In the limit, mining will be cost prohibitive for most miners and the network will rely on transaction fees to operate. As such, a positive coefficient is postulated for *BLOC*.

Pairwise correlations of each of the transformed variables is shown in Table 4. The variables' correlation with bitcoin returns is also shown in the first column (and row) of the table as a preliminary check of comovement between the respective transformed variables and bitcoin returns. Among all the pairwise correlations, the highest negative correlation is between *COIN* and *TPM* (−0.7230). However, rolling window correlations between them (not tabulated) show considerable time variability in the correlation dynamics between these variables. Conversely, among all the pairwise correlations, *REV* and *BLOC* have the highest positive correlation (0.4408).

The regression models we posit aim to, first, provide perspectives of the how order imbalance is linked to crash risk probabilities and, second, to see whether the statistical sign and magnitude of *OI* is robust with the inclusion of exchange-specific and blockchain-wide control variables:

$$Crash_t = \lambda_{1,0} +_{1,1} OI_t + \varepsilon_{1,t} \tag{3.1}$$

$$Crash_t = \lambda_{2,0} + \lambda_{2,1} OI_{t-1} + \varepsilon_{2,t} \tag{3.2}$$

$$Crash_t = \lambda_{3,0} + \lambda_{3,1} OI_t + \lambda_{3,2} OI_{t-1} + \varepsilon_{3,t} \tag{3.3}$$

$$Crash_t = \lambda_{4,0} + \lambda_{4,1} OI_{t-1} + \lambda_{4,2} \sigma^2 \left\{ OI_{t-1} \right\} + \varepsilon_{4,t} \tag{3.4}$$

$$\begin{aligned} Crash_t = {} & \lambda 5,0 + \lambda 5,1 OI_t + \lambda 5,2 OI_{t-1} + \lambda 5,3 \sigma^2 \left\{ OI_{t-1} \right\} + \dots \\ & \dots + \lambda 5,4 RV_t + \lambda 5,5 VOL_t + \lambda 5,6 TPM_t + \lambda 5,7 COIN_t + \dots \\ & \dots + \lambda 5,8 ADDR_t + \lambda 5,9 FEES_t + \lambda 5,10 REV_t + \lambda 5,11 BLOC_t + \varepsilon_{5,t} \end{aligned} \tag{3.5}$$

$$\begin{aligned} Crash_t = {} & \lambda 6,0 + \lambda 6,1 OI_t + \lambda 6,2 \sigma^2 \left\{ OI_{t-1} \right\} + \lambda 6,3 RV_t + \lambda 6,4 VOL_t + \dots \\ & \dots + \lambda 6,5 TPM_t + \lambda 6,6 COIN_t + \lambda 6,7 ADDR_t + \dots \\ & \dots + \lambda 6,8 FEES_t + \lambda 6,9 REV_t + \lambda 6,10 BLOC_t + \varepsilon_{6,t} \end{aligned} \tag{3.6}$$

where *Crash* is defined in Eq. (1) and whereby Eqs. (3.1) through (3.6) are estimated using the GEV probability distribution function for minima, as shown in Eq. (2). The variable $\sigma^2 \left\{ OI_{t-1} \right\}$ denotes the conditional variance of lagged *OI*, whereby the variance is estimated and fitted using a standard GARCH model. This variable reflects uncertainty, or, an imbalance between informed and uninformed traders (Chordia et al. 2019). Inclusion of

[20] See https://www.blockchain.com/charts/difficulty.

this variable in our tests controls for information asymmetry at time $t-1$. For the sake of comparison, Eqs. (3.1) through (3.6) are also estimated using a logistic model

$$\text{logit}\left(\pi_i\right) = \ln\left(\pi_i/\left(1-\pi_i\right)\right) = \lambda_{i,0} + \sum\nolimits_{j=1}^{p} \lambda_j x_{ji} = \omega_i \tag{4}$$

where $\pi$ is the odds of a bitcoin crash, as defined in Eq. (1), for each of the $i$ models in Eqs. (3.1) through (3.6) and for all the respective $x$ covariates. Equation (4) can be written in terms of the odds of a crash, where $\pi_i = 1/\left(1+\exp\left(\omega_i\right)\right)$.

The reason for the comparison between GEV and logistic regression modeling is motivated by differences in the link function structures between the two. Logistic link functions are symmetric in nature. This implies that the slope probability of approaching $Crash = 1$ is equal to the slope probability of approaching $Crash = 0$. As Wang and Dey (2010) argue, first, market observations often depart from such symmetries and, second, the GEV link function effectively detects such departures and can adjust the shape parameter $\xi$, as is discussed in Eq. (2).

## 4 Main results

This section discusses parameter estimates for Eqs. (3.1) through (3.6), when estimated using GEV and logistic regression approaches, respectively (Sect. 4.1.). The purpose here is threefold. First, to evaluate the signs and statistical significance of the coefficients and assess to what extent they depart from our a priori expectations. Second, to compare the performance of the logistic and GEV regressions, respectively, for accuracy as well as for robustness in the parameter estimates. Third, and as discussed in Sect. 4.2., to construct time varying probability estimates for bitcoin price crashes and assess how well they statistically perform.

Apart from helping us understand how activity in bitcoin's market microstructure is linked to extreme price movements, this analysis is also useful from a risk management perspective in that it lays a potential foundation for trying to build an early warning system for a bitcoin price crash. This is useful to not only investors but to the growing number of companies and organizations that accept bitcoin for payment for goods or services rendered.[21] Thus, depending on its fraction of consumers using bitcoin, a company's cash flows inevitably have some degree of exposure to bitcoin crash risk. In addition, this analysis is useful from a regulatory standpoint, especially in light of the increasing number of initial coin offerings (ICOs) and initial exchange offerings (IEOs) that have attracted large sums of money from investors from cryptocurrency exchanges.[22] Finally, because there is much discussion as of late regarding the price level of bitcoin and if it resembles that of a bubble, it is beneficial to see how the aforementioned variables contribute to bitcoin's crash probability and whether the signs of their contributions align, to some extent, with our a priori intuitions.

[21] Major companies such as Microsoft, Overstock, and Newegg are beginning to accept bitcoins for some (or all) of their goods and services. A list of such companies can be found here: https://paybis.com/blog/companies-that-accept-bitcoin/.

[22] See https://www.sec.gov/oiea/investor-alerts-and-bulletins/ia_initialexchangeofferings.

**Table 5** Standardized (beta) regression estimates

| | Logit | | | | | | Generalized extreme value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n =$ | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| $\lambda_{n,1}$ | **−0.1159** | **−0.1326** | 0.2608 | **−0.1331** | 0.0168 | **−0.8464** | **−0.0436** | **−0.0504** | 0.0624 | **−0.0506** | 0.1281 | **−0.2856** |
| | (−1.977) | (−2.291) | (1.485) | (−1.940) | (0.062) | (−8.325) | (−1.960) | (−2.248) | (1.432) | (−2.056) | (1.038) | (−6.939) |
| $\lambda_{n,2}$ | | | **−0.3862** | **−0.1532** | **−0.9413** | **−0.3175** | | | **−0.0914** | **−0.0530** | **−0.4451** | **−0.1402** |
| | | | (−2.168) | (−2.068) | (−3.420) | (−2.655) | | | (−2.062) | (−2.163) | (−3.542) | (−2.712) |
| $\lambda_{n,3}$ | | | | | **−0.3521** | **0.1731** | | | | | **−0.1541** | 0.1617 |
| | | | | | (−2.932) | (2.697) | | | | | (−3.125) | (5.102) |
| $\lambda_{n,4}$ | | | | | **0.1738** | **1.7503** | | | | | 0.1516 | **0.6283** |
| | | | | | (2.763) | (9.777) | | | | | (1.434) | (10.201) |
| $\lambda_{n,5}$ | | | | | **1.8136** | **0.4491** | | | | | **0.6612** | **0.2507** |
| | | | | | (10.031) | (1.956) | | | | | (9.529) | (2.717) |
| $\lambda_{n,6}$ | | | | | **0.4426** | **−0.8136** | | | | | **0.2496** | **−0.2465** |
| | | | | | (1.934) | (−3.120) | | | | | (2.762) | (−2.619) |
| $\lambda_{n,7}$ | | | | | **−0.8499** | **0.2324** | | | | | **−0.2618** | **0.0819** |
| | | | | | (−3.263) | (2.845) | | | | | (−2.657) | (2.357) |
| $\lambda_{n,8}$ | | | | | **0.2345** | **−0.3286** | | | | | **0.0852** | **−0.1241** |
| | | | | | (2.871) | (−4.205) | | | | | (2.467) | (−3.376) |
| $\lambda_{n,9}$ | | | | | **−0.3441** | **−0.6375** | | | | | **−0.1314** | **−0.2763** |
| | | | | | (−4.359) | (−7.608) | | | | | (−3.948) | (−7.489) |
| $\lambda_{n,10}$ | | | | | **−0.6604** | **0.3405** | | | | | **−0.2849** | **0.1608** |
| | | | | | (−7.778) | (3.192) | | | | | (−7.811) | (3.800) |
| $\lambda_{n,11}$ | | | | | **0.3597** | | | | | | **0.1674** | |
| | | | | | (3.345) | | | | | | (3.792) | |
| McFadden $R^2$ (%) | 0.182 | 0.238 | 0.287 | 0.337 | 30.740 | 30.129 | 0.183 | 0.242 | 0.271 | 0.336 | 29.953 | 29.275 |
| RMSE | | | | 0.280 | | 0.246 | | | | 0.280 | | 0.247 |
| MAE | | | | 0.155 | | 0.114 | | | | 0.156 | | 0.119 |
| MAPE | | | | 7.873 | | 6.025 | | | | 7.873 | | 6.245 |

**Table 5** (continued)

| | Logit | | | | | | Generalized extreme value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n =$ | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| Theil inequality | | | | 0.740 | | 0.540 | | | | 0.740 | | 0.558 |

This table shows regression estimates for the six nowcasting regression models shown in (3.1) through (3.6). These estimates are extracted using, respectively, the logit approach shown in Eq. (4) as well as the generalized extreme value (GEV) approach shown in Eq. (2). The parameters on the left-hand-side of the table contain two subscripts. The first subscript corresponds to the respective model being estimated while the second signifies the coefficient in that model. T-statistics are shown in parentheses while coefficients in bold denote significance at the 5% level at least ($p$ value $\leq 0.05$). The last rows of the table show McFadden $R^2$ estimates for each of the models while the forecast error metrics are shown for model 6 of the logit and GEV approaches, respectively. The algorithm of Berndt et al. (1974) is used for likelihood optimization of all the models.

The sample period is from April 1, 2013 until January 15, 2023

### 4.1 Regression estimates

Regression estimates for Eqs. (3.1) through (3.6) are shown in Table 5 using the logistic and GEV frameworks in Eqs. (2) and (4), respectively. From an econometrical stand point, it is of interest to compare the nowcasting performance of the GEV relative to the logistic regression since, as mentioned previously, its link function is asymmetric while the logistic link function is symmetrical around the value of 0.50. This means that the probability of a binary event can approach zero ($Crash = 0$) at the same rate in which it can approach one ($Crash = 1$).

To evaluate goodness-of-fit and performance, the following summary measures are shown: the McFadden Pseudo-$R^2$, root-mean-squared error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and the Theil inequality coefficient (U), respectively, defined as:

$$R^2_{\text{McFadden}} = 1 - l\left(\hat{\lambda}\right)/l\left(\tilde{\lambda}\right) \tag{5}$$

$$\text{RMSE} = \sqrt{\sum_{t=T+1}^{T+h} \left(\widehat{Crash} - Crash\right)^2 / h} \tag{6.1}$$

$$\text{MAE} = \sum_{t=T+1}^{T+h} \left|\widehat{Crash} - Crash\right| / h \tag{6.2}$$

$$\text{MAPE} = 100 \sum_{t=T+1}^{T+h} \left|\left(\widehat{Crash} - Crash\right)/\widehat{Crash}\right| / h \tag{6.3}$$

$$\text{U} = \left(\sqrt{\sum_{t=T+1}^{T+h} \left(\widehat{Crash} - Crash\right)^2 / h}\right) \Bigg/ \left(\sqrt{\sum_{t=T+1}^{T+h} \widehat{\text{Crash}}^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} Crash^2 / h}\right) \tag{6.4}$$

where $Crash$ and $\widehat{Crash}$ are the actual and the predicted binary dependent variable signifying whether a crash has occurred, as defined in Eq. (1).

The coefficient estimates shown in Table 5 are standardized for the very fundamental reasons outlined in Mayer and Younger (1976) and Menard (2004, 2011), among others. Specifically, standardized coefficients transform the variables into a common metric (which, in this case, are standard deviation units). Given that our variables (as shown in Table 3) represent different information content, come from different sources, and exhibit dissimilar scales in their time series behaviors, it is appropriate to estimate standardized coefficients to allow for meaningful comparisons. Furthermore, and as discussed in Menard (2004, 2011), and under certain circumstances, standardized coefficients in binary regression-type models can also allow for some comparisons in the relative strength between two or more variables.

Table 5 thus reports findings for Eqs. (3.1) through (3.6), which respectively shed unique light on the role which the variables play in the probability of a bitcoin price crash. Column 1 (Eq. (3.1.)) for the logistic and GEV regressions, respectively, shows a negative coefficient (for $\lambda_{1,1}$) and that as order imbalance rises in value ($B > S$) the probability of a bitcoin price crash declines. This finding is consistent with both the logistic regression and the GEV estimation, albeit the value of the coefficient is higher in absolute terms for the logistic model. The McFadden $R^2$ estimates (in percent) are 0.182 and 0.183 for the logistic and GEV regressions, respectively. These values are relatively low, despite the significance of the coefficients at the 5% statistical level, and denote the challenges which empiricists face in trying to explain the variation of bitcoin prices.

We can also observe how the coefficient $\lambda_{2,1}$, which denotes lagged order imbalance, maintains its significance at the 5% level. This shows that order imbalance, even in its lagged form, contains important information that has some predictive capabilities. In spite of its significance, the McFadden $R^2$ estimates remain relatively low, as in $\lambda_{1,1}$, for both the logistic and GEV regressions. As we incorporate coefficients for both the contemporaneous and lagged order imbalance variables (in Eq. (3.3)) as represented by $\lambda_{3,1}$ and $\lambda_{3,2}$, respectively, we see that lagged order imbalance maintains its negative sign. The McFadden $R^2$ increases only marginally when estimating Eq. (3.3) for both the logistic and GEV regressions. When we integrate the lagged conditional variance of order imbalance, $\sigma^2\left\{OI_{t-1}\right\}$, as shown in Eq. (3.4), we see a small rise in the McFadden $R^2$ for both the logistic (0.337) and the GEV (0.336) regression models. As mentioned, $\sigma^2\left\{OI_{t-1}\right\}$ is intended to capture the uncertainty component associated with buy and sell orders. Postulating on its sign, a priori, may not be entirely possible, as it is for traditional asset classes (Chordia et al. 2019), given the speculative forces that underlie bitcoin's price movements. In our case here, $\lambda_{4,2}$ bears a negative sign, meaning that rises in the conditional variance of limit orders actually reduces the probability of a bitcoin price crash. This result is an interesting one and merits further investigation; but a possible explanation is that the variance in the order book may be linked to speculative behaviors and price appreciations. The pairwise correlation (not tabulated for brevity) between $\sigma^2\left\{OI_{t-1}\right\}$ and bitcoin returns and the range volatility (*RV*) in bitcoin prices is, respectively, $-0.06$ and $-0.08$. While its correlation with returns merits more investigation, it does appear, at first glance, that there is some negative relation with bitcoin's price volatility.

In Eqs. (3.5) and (3.6), we expand our model with all the variables; whereby the difference between these two equations is that (3.5) also contains lagged order imbalance, $OI_{t-1}$, whereas (3.6) does not. We see that the McFadden $R^2$ estimates are highest for Eq. (3.5) and are 30.740 and 29.953 for the logistic and GEV regressions, respectively. When examining some of the other microstructure variables, we see interesting relations unfold. As postulated in Table 3, we see, for example, that rises in range volatility (*RV*), trading volume (*VOL*), and block count (*BLOC*), are linked to a rise in the probability of a bitcoin price crash. Conversely, and, as mentioned, similarly to order imbalance in its contemporaneous and lagged form, as well as its conditional variance, rises in mining revenue (*REV*) are negatively related to the probability of a bitcoin price crash. The results in Eq. (3.6) for both the logistic and GEV regressions, respectively, yield somewhat similar qualitative conclusions, despite a lower McFadden $R^2$ than what was observed when compared with the estimation of Eq. (3.5).

When observing the goodness-of-fit statistics of all the models (RMSE, MAE, MAPE, and U, respectively), we see that the logistic and GEV regressions performed similarly, with no indication of one significantly outperforming the other.

### 4.2 Nowcasting accuracy

If we turn our attention to Table 6 which shows the trade-off between the proportion of false positives, whereby $Crash = 0$ but the model predicted a price crash, and, false negatives, whereby $Crash = 1$ and the model did not predict a price crash, we see some interesting dynamics emerge. First, and due to the explosive nature of bitcoin prices, the probability cutoffs, $C$, cannot be too high. While other studies which utilize binary-type regression models for conventional assets may report results with probability cut-offs at 0.50 or higher, this does not seem possible with bitcoin prices. In fact, and as shown in the table, when using a cutoff of 0.10 (i.e., $C = 0.10$), the false negative rate is estimated to be 90.26% (for Model 3.4 when using the logistic regression, as shown in Panel A). This is an important finding because it shows the explosive nature of bitcoin price changes makes them difficult to model empirically, or, classify, in early warning systems that use binary-type regressions.

While it is false negatives that can be more devastating than false positives, Table 6 and Panel A (which shows results for Model 3.4) shows we need to be careful in trying to select an optimal cutoff threshold that can fit the data well. For example, and for both the logistic and GEV regressions in Panel A, a probability cutoff of 0.08 (i.e., $C = 0.08$) classifies 75% of crash days correctly and 25% of crash days incorrectly. While this is a promising result, we also see that, for $C = 0.08$, we have a high degree of false positives (approximately 72%).

When looking at Panel B of Table 6 (which shows results for Model 3.6), we see that, relative to Model 3.4, there is a significant improvement in correctly classifying crash days across all the cutoff periods. For example, and for $C = 0.07$, we see that the GEV regression correctly classified 88.96% crash days ($Crash = 1$) and 69.95% non-crash days ($Crash = 0$).

Panel C of Table 6 summarizes all the marginal advantages (or, disadvantages) of using Model 3.6 over Model 3.4. Overall, the results strongly favor the extended model of 3.6.

These findings show that, due to the explosive nature of bitcoin prices, assigning a low probability cutoff maximizes the usefulness of our proposed models and increases the chance of successfully detecting trading days with bitcoin price crashes. The explosive nature of bitcoin prices, as portrayed in Table 2, is well-documented in academic literature (Cheah and Fry 2015) and by regulators (United States Senate 2013).

Finally, and in an attempt to begin discourse on developing a fragility index for bitcoin, Fig. 6 shows a time series plot of the probability of a bitcoin price crash, using Eq. (3.6). For illustrative purposes, and using the logistic regression, the regression coefficients that are found to be significant are utilized for this purpose (as shown in Table 5). These estimates can be used to create a time series model of the probability of a bitcoin crash, as follows (transformation of variables, as mentioned, are found in Table 3):

$$P\left(Crash = \frac{1}{X}\right) = \frac{\begin{bmatrix} exp(-0.8464OI_t - 0.3175\sigma^2\left\{OI_{t-1}\right\} + 0.1731RV_t \\ +1.7503VOL_t + 0.4491TPM_t - 0.8136COIN + 0.2324ADDR \\ -0.3286FEE - 0.6375REV_t + 0.3405BLOC_t) \end{bmatrix}}{\begin{bmatrix} 1 + exp(-0.8464OI_t - 0.3175\sigma^2\left\{OI_{t-1}\right\} + 0.1731RV_t \\ +1.7503VOL_t + 0.4491TPM_t - 0.8136COIN + 0.2324ADDR \\ -0.3286FEE - 0.6375REV_t + 0.3405BLOC_t) \end{bmatrix}} \tag{7}$$

The time series plot, with shaded regions denoting an actual crash, as defined in Eq. (1), show rises during periods when bitcoin's prices experienced crashes. One notable example is the December 2017 period when bitcoin futures were introduced and bitcoin experienced consistent price declines over multiple trading days.

**Table 6** Predictive accuracy across probability cutoffs, $C$

| | | $C = 0.07$ | | $C = 0.08$ | | $C = 0.09$ | | $C = 0.10$ | |
|---|---|---|---|---|---|---|---|---|---|
| | *Crash* = | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| *Panel A: Model 3.4* | | | | | | | | | |
| Logit | $P(Crash) \leq C$ | 443 | 18 | 911 | 77 | 1695 | 150 | 3102 | 278 |
| | $P(Crash) > C$ | 2825 | 290 | 2357 | 231 | 1573 | 158 | 166 | 30 |
| | Total | 3268 | 308 | 3268 | 308 | 3268 | 308 | 3268 | 308 |
| | *%Correct* | *13.56* | *94.16* | *27.88* | *75.00* | *51.87* | *51.30* | *94.92* | *9.74* |
| | *%Incorrect* | *86.44* | *5.84* | *72.12* | *25.00* | *48.13* | *48.70* | *5.08* | *90.26* |
| GEV | $P(Crash) \leq C$ | 468 | 20 | 904 | 77 | 1666 | 149 | 3098 | 277 |
| | $P(Crash) > C$ | 2800 | 288 | 2364 | 231 | 1602 | 159 | 170 | 31 |
| | Total | 3268 | 308 | 3268 | 308 | 3268 | 308 | 3268 | 308 |
| | *%Correct* | *14.32* | *93.51* | *27.66* | *75.00* | *50.98* | *51.62* | *94.80* | *10.06* |
| | *%Incorrect* | *85.68* | *6.49* | *72.34* | *25.00* | *49.02* | *48.38* | *5.20* | *89.94* |
| *Panel B: Model 3.6* | | | | | | | | | |
| Logit | $P(Crash) \leq C$ | 2411 | 44 | 2505 | 53 | 2580 | 61 | 2650 | 69 |
| | $P(Crash) > C$ | 857 | 264 | 763 | 255 | 688 | 247 | 618 | 239 |
| | Total | 3268 | 308 | 3268 | 308 | 3268 | 308 | 3268 | 308 |
| | *%Correct* | *73.78* | *85.71* | *76.65* | *82.79* | *78.95* | *80.19* | *81.09* | *77.60* |
| | *%Incorrect* | *26.22* | *14.29* | *23.35* | *17.21* | *21.05* | *19.81* | *18.91* | *22.40* |
| GEV | $P(Crash) \leq C$ | 2286 | 34 | 2380 | 42 | 2466 | 48 | 2525 | 53 |
| | $P(Crash) > C$ | 982 | 274 | 888 | 266 | 802 | 260 | 743 | 255 |
| | Total | 3268 | 308 | 3268 | 308 | 3268 | 308 | 3268 | 308 |
| | *%Correct* | *69.95* | *88.96* | *72.83* | *86.36* | *75.46* | *84.42* | *77.26* | *82.79* |
| | *%Incorrect* | *30.05* | *11.04* | *27.17* | *13.64* | *24.54* | *15.58* | *22.74* | *17.21* |
| *Panel C: Gain of Model 3.6 over Model 3.4* | | | | | | | | | |
| Logit | $\Delta$*%Correct* | *+60.22* | *−8.45* | *+48.77* | *+7.79* | *+27.08* | *+28.89* | *−13.83* | *+67.86* |
| GEV | $\Delta$*%Correct* | *+55.63* | *−4.55* | *+45.17* | *+11.36* | *+24.48* | *+32.80* | *−17.54* | *+72.73* |

This table shows the percentage of event days that are correctly classified by models 4 (panel A) and 6 (panel B) from Eqs. (3.4) and (3.6), respectively, across various probability cutoffs, $C$. Models 4 and 6 are estimated using the logit and generalized extreme value (GEV) frameworks shown in Eqs. (4) and (2). A sample day with an event (when there is a bitcoin crash) is denoted by $Crash = 1$ while a day without an event (when there is no bitcoin crash) is denoted by $Crash = 0$. Bitcoin crashes are defined in Eq. (1). In panels A and B, the *%Correct* and *%Incorrect* denote the percentage of sample days that are classified correctly and incorrectly, respectively. "False positive" classifications (type 1 errors) are shown for the *%Incorrect* when $Crash = 0$. "False negative" classifications (type 2 errors) are shown for the *%Incorrect* when $Crash = 1$. Panel C shows the gain in estimating model 6 (the full model) over model 4 (which only uses order imbalance as a variable). The sample period is from April 1, 2013 until January 15, 2023

## 5 Concluding remarks

This study takes a step in developing a fragility index for bitcoin on the basis of its price crash risk. It utilizes logistic and GEV regression models in order to begin painting a clearer picture in terms of what variables are relevant in trying to forecast (or, nowcast) bitcoin price crashes. Using transaction level buy and sell orders (in bitcoin's order book), among other key microstructure variables, this study makes the following important findings. First, it shows the importance of order flow imbalance (buying

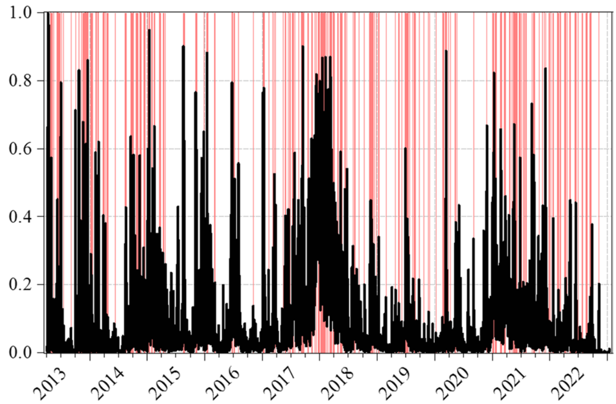


**Fig. 6** Forecasted probability of bitcoin crashes. This figure shows the forecasted probability of bitcoin crashes. This probability is estimated from model 6 (Eq. (3.6)) using the logit approach. The shaded red lines correspond to an actual bitcoin price crash, as defined in Eq. (1). The probability model is expressed in Eq. (7). The sample period is from April 1, 2013 until January 15, 2023. The data are sourced from the Bitstamp cryptocurrency exchange. Footnotes (14) and (15), respectively, discuss the data sample and sources in more detail

relative to selling activity) as a variable that is linked to bitcoin price crashes. While order imbalance has received attention in the context of traditional asset classes (such as in Chordia et al. 2002, for example), more work is needed in determining its importance in the context of digital money such as bitcoin.

Second, this study shows that when trying to model bitcoin's price behavior, it is important to incorporate factors that reflect shifts in Bitcoin's blockchain. Much of the literature on this subject that is growing at a rapid rate, like some of the studies referenced here, attempt to link bitcoin with shifts in economic and market variables that have been shown to explain the returns of conventional asset classes, such equities or commodities. However, and it discussed here, bitcoin's microstructure and its clientele are very different than what is observed in traditional asset classes. In the words of Liu and Tsyvinski (2018, p.3), "…cryptocurrencies comprise an asset class which is radically different from traditional asset classes…".

Third, and from an econometric perspective, our study makes the following two main contributions. First, we show that both the logistic and GEV regression approaches perform comparably in terms of nowcasting errors. This is an important observation that is relevant to all applications of probabilistic forecasting involving a binary-type outcome. This is because the logistic link function is symmetrical around the value of 0.50. This means that the probability of a binary event can approach zero at the same rate in which it can approach one. Czado and Santner (1992), among others, show that assuming such a logistic link function can lead to biases and inabilities in estimating accurate probabilities. The GEV regression approach attempts to augment this shortcoming in logistic regressions because of its asymmetric link function that is based on the GEV distribution, which in extreme value theory, has shown to better model rare events in statistics (Kotz and Nadarajah 2000; Wang and Dey 2010). However, we show that, given the explosive behavior of bitcoin prices, both the logistic and GEV regressions perform similarly and that we need to utilize a low probability cutoff in order for the models herein to be most effective.

Finally, this study aims to begin academic discourse on constructing a fragility index that can be used by academics, policy-makers and the private and public sector at large, to determine the degree of price crash risk present in the Bitcoin ecosystem. As is outlined in United States Senate (2013), bitcoin may pose opportunities but also several threats to investors, consumers and the financial system at large.

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