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Two-worker Households, Decentralized Employment, and Residential Segregation

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Abstract

The last century was marked by a remarkable improvement in the economic position of women, as reflected in higher labor force participation and wages. This paper extends the Hybrid Tiebout models of residential choice to allow for two-worker households. Our model incorporates both residential choice and labor market choices of households simultaneously and, thus, gives us a unique opportunity to study the impact of changes in the labor market conditions for workers on residential segregation. We develop a general equilibrium model of residential choice with decentralized workplaces in which households face a trade-off among accessibility, space and a public good (education). Education is financed through property taxes, which are determined by majority voting. The quality of education is determined by the spending and the peer group effects. The model is interesting in the sense that (i) households consider the work locations of both male and female working members of the household while making residential choice decisions; (ii) the presence of decentralized workplaces offers an alternative job location to workers; and (iii) the endogenous labor supply decisions for workers. We find that the increase in educational attainment for women and the changes in wages for men and women have had a substantial impact on the spatial distribution of households across metropolitan areas and hence, segregation by income.

Keywords Residential segregation · Household labor supply · Urban location model · Tiebout model

JEL Primary Field Name Urban · Rural · Regional Economics

JEL Classification H7 · I2 · J2 · R2

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Introduction

The twentieth century marked the most dramatic change in female labor supply and urban spatial structure. Between 1950 and 1999, the labor force participation of married females increased approximately by 130 percent. As a result, the proportion of two-worker households increased from 33 percent to 73 percent (Bar and Leukhina, 2011). Among others, the most frequently cited reasons for this demographic change include increases in both the educational attainment and wages of women and factors that reduced the wage gender gap.[1]

To understand the increase in married female labor participation rate, macroeconomists needed to understand how the decisions were made within a family in which couples pooled resources, shared risks, and made decisions that affected both partners and children. The pioneering work of Becker (1973, 1974) on family economics was a breakthrough that macroeconomists needed, and a new literature in macroeconomics has emerged based on Becker's work. Macroeconomic models that incorporated family economics elements were used to answer questions about male and female labor supply, female educational attainment, marriage rates, fertility rates, divorce rates, assortativeness of matings and how they evolved over time. Realizing the decentralization of workplaces, some scholars (e.g. Guler et al., 2012) also studied the joint job-search and workplace location problem of a household formed by a couple.[2] These models proved useful for public policy discussions; for instance, Nishiyama (2019) analyzes the effect of spousal and survivor benefits of the U.S. Social Security program on the labor supply of married households and the overall economy. Even though this literature experienced exponential growth and produced fascinating results, the literature overlooked an important urban economics aspect of family decision-making: e.g. the location of housing and workplaces. In line with Gong and Leung (2020), our paper intends to fill this gap.[3] Table 1 highlights the principal contribution of our paper relative to macroeconomics literature.

As women have entered the labor force, a greater proportion of the women value commuting time, which provides an incentive for more women to live closer to work. This demographic change has important implications for the spatial distribution of households across metropolitan areas: a greater fraction of the population consists of two-worker households now, and their residential location choice decisions also include the workplace for the female worker of a household. On the urban spatial structure side, employment has been suburbanizing[4] while there is considerable heterogeneity across cities and industries (Glaeser and Kahn 2001, Baum-Snow 2020). Moreover, U.S. metropolitan areas have experienced rising residential segregation by

[1] See Goldin (1990) and Costa (2000) for historical trends in female labor supply and possible explanations.

[2] See Doepke and Tertilt (2016) and Greenwood et al. (2017) for a survey of macroeconomics literature that intersects family economics.

[3] Gong and Leung (2020) present a model of household decision-makers in a monocentric city. In our model, households include husband and wife who interact with each other, and the interests of spouses might conflict with each other: e.g. spouses might have different preferences for the location of a house as they might have different workplaces, or each spouse prefers the other one to work more as they pool the resources as a family.

[4] In the past century, employment densities declined. Mills (1972), Macauley (1985).

Table 1 Comparison with selected macroeconomics literature

	Greenwood et al. (2016)	Guler et al. (2012)	Gong and Leung (2020)	This Manuscript
Model	Infinite-Horizon Model	Continuous-Time Search Model	Spatial Equilibrium Model	Urban Location/Tiebout Model
Education	✓		✓	✓
Endogenous Marriage	✓			
Endogenous Fertility			✓	
Endogenous Labor Force Participation	✓	✓		✓
Endogenous Male/Female Labor Supply			Household Supply only	✓
Decentralized Employment		✓		✓
Location of Housing/Workplaces			✓	✓
Endogenous Education Production			✓	✓
Calibration Target	USA, 1960	USA, period not specified	USA, 2010	USA, 2016

income since 1970:[5] in 2000, 19.4 percent of the central city population within MSAs lived in poverty while 7.5 percent of the suburban population was poor.

To understand the spatial distribution of households, various papers [e.g. Hanushek and Yilmaz (2007, 2013, 2015), Hanushek et al. (2011), Leung et al. (2012), de Bartolome and Ross (2003), and Yilmaz (2019)] developed Hybrid Tiebout models that provided a unified treatment of both urban location models and Tiebout models. In those models, a household's residential location is determined by the trade-off among three factors: accessibility, space, and local public good (education).[6] The benchmark model for those papers is assumed to be a monocentric city in general, and, more importantly, with each household having one working member. In the urban location models literature, some researchers (White (1977) and Hotchkiss and White (1993)) have considered the residential location choice of two worker households. In Hotchkiss and White (1993), a simulation model of an urban area is used to explore the residential location patterns where there are multiple household types – two-worker households or single (female) worker households. Among other things, this paper does not consider neighborhood amenities and assumes that the labor supply decisions for each working member of households are exogenously given.

The purpose of this paper is to develop a hybrid Tiebout model of a metropolitan area with two school districts and decentralized employment locations to explore the spatial implications of social trends (such as the increasing rate of educational attainment by women and changing wages for male and female workers).[7] In the model, each household consists of a husband, a wife, and a child. Husbands and wives differ by their skill levels, and households are assumed to be formed with assortative matching. Depending on their education levels, they place a different value on the education that their child receives. Households maximize their utility by choosing a residential location (i.e. a school district and a location in the school district), the amount of composite commodity to be consumed, the size of a house, hours of leisure (hence, labor supply) and a workplace for each working member of the household if the member chooses to work. Commuting has fixed, variable, and time cost components. Except for single-worker households commuting to their workplaces in the Central Business District (CBD) by bus, the means of transportation across the metropolitan area is assumed to be an automobile. Housing is provided by a competitive housing industry that uses capital and land to produce housing. Absentee landlords offer the land to the highest bidder to maximize their revenue from the land. Each school provides education financed through property taxes. The quality of education in each school district is determined by the peers and spending per pupil. As for the property tax rates, they are determined by a majority voting in each school district.

The rest of the paper is organized as follows: "Model" through "Baseline" describe the baseline model and its outcomes. Next, "Demographic Changes" explores the spatial implications of the increasing educational attainment by women and the decreasing

5 See Margo (1992), Mieszkowski and Mills (1993), Mills and Lubuele (1997), and Glaeser et al. (2008).

6 Some other factors affect residential choice: the presence of topographical and historical amenities in the city center Brueckner et al. (1999) or the age of housing stock (Brueckner and Rosenthal (2009)).

7 For the macroeconomic aspects of the housing market, see Leung (2004) and Leung and Ng (2019).

wage gender gap along with increasing wages for female workers. Finally, "Concluding Remarks" concludes the paper.

Model

We now describe the model in detail. Our major contribution is that in addition to the residential location choice of single-worker households, we also consider the residential location choice of two-worker households in a hybrid Tiebout model, which provides a joint treatment of Urban Location Models and Tiebout models of community choice.[8] Suppose that some portion of employment is decentralized in a metropolitan area with a dense radial transportation network, and jobs are offered at two locations: in a central business district (CBD) and a suburban ring (Sub) located at $\bar{r}$ miles from the city center. The metropolitan area also contains two school districts separated by a straight line passing through the CBD, which forms the school district boundary.[9] Without loss of generality, we name those two school districts as West School District and East School District. The metropolitan area is located on a featureless plain land: each location differs by the distance to workplaces and the education and tax package it provides. The model has radial symmetry in the sense that locations with the same distance to the CBD (and hence, with the same distance to the Suburban Ring) in any school district are identical in terms of location and neighborhood amenities they provide.

Households. Our model has households with three members: a male worker, a female worker, and a child.[10] Labor supply decisions along with workplace choices are endogenous, and some workers may choose not to work. In other words, not only are there two-worker households in equilibrium but also there might be single-worker households. A worker can be either a skilled worker, s, or an unskilled worker, u. For the sake of tractability, we assume that a family is formed with assortative matching as in Gong and Leung (2020): hence, both the male and female workers of a household have the same skill level.[11] Thus, we have two household types, depending on their workers' skill levels (i.e. household $i \in \{s, u\}$): households with skilled male and female workers and households with unskilled male and female workers.

A household utility function[12] is defined quality of education, Q, the size of house, H, a composite good, X, the male worker's leisure time, L_M, and the female worker's

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8 See Gong and Leung (2023) for the importance of modeling space to study the housing market.

9 This metropolitan area is a stylized representation of the many cities on state boundaries such as Philadelphia or water boundaries such as Chicago, Cleveland, or New Orleans.

10 We ignore the trade-off between child quantity and quality. See Yilmaz (2018) for a model of the child quantity-quality trade-off.

11 Fernández et al. (2005) find that the sample correlation between spouses' years of schooling in the U.S. is about 0.61. Greenwood et al. (2016) also report that there is a trend for increasing marital sorting in the U.S.

12 Following the footsteps of Becker (1973, 1974), we have a unitary model of the family, which distinguishes between male and female labor supply in the context of a single household utility function.

leisure time, L_F. For household $i \in \{s, u\}$, it is given by

$$U^i = Q^{\alpha_i} H^{\beta_i} X^{\gamma_i} L_M^{\delta_i} L_F^{\epsilon_i} \tag{1}$$

where $\alpha_i > 0$, $\beta_i + \gamma_i + \delta_i + \epsilon_i = 1$, $L_M \in [0, 24 - \bar{h}_{M_i}]$, and $L_F \in [0, 24 - \bar{h}_{F_i}]$. We assume that the skill level of the household determines how much household i values education, and skilled households place a higher valuation on education than unskilled households (i.e. $\alpha_s > \alpha_u$). In this formulation, a household's decision variables are measured daily. Both male workers and female workers of the household spend some time each day on home production. Hence, the maximum amounts of leisure that can be chosen by the male and female workers of household i are $24 - \bar{h}_{M_i}$ and $24 - \bar{h}_{F_i}$, respectively where $\bar{h}_{M_i} \in \{\bar{h}_{M_u}, \bar{h}_{M_s}\}$ and $\bar{h}_{F_i} \in \{\bar{h}_{F_u}, \bar{h}_{F_s}\}$. The variables Q, H, X, L_M, and L_F in the utility function depend on the household type, i, and we suppress those for the sake of the simplicity of the notation. Also, we will drop the subscript i from the worker gender variables, F and M, if the household type does not make any difference in the variables.

In case both male and female members of the household choose to work at workplaces $j_M \in$ {CBD, Sub} and $j_F \in$ {CBD, Sub}, respectively, and commute to their workplaces by automobile, the daily budget constraint for a household $i \in \{s, u\}$ in school district $d \in \{East, West\}$ at a residential location r miles from the CBD is given by

$$(1 + \tau_d) R_d^*(r) H + X + w_{j_M}^{M_i} L_M + w_{j_F}^{F_i} L_F = Y^i(r_{j_M}, r_{j_F}) \tag{2}$$

where τ_d is the property tax rate in school district d, $R_d^*(r)$ is the daily unit housing rent, $w_{j_M}^{M_i} \in \{w_{j_M}^{M_s}, w_{j_M}^{M_u}\}$ and $w_{j_F}^{F_i} \in \{w_{j_F}^{F_s}, w_{j_F}^{F_u}\}$ are the wages, r_{j_M} and r_{j_F} are the distances to the workplaces of male and female workers,[13] and $Y^i(r_{j_M}, r_{j_F}) = (24 - \bar{h}_{M_i}) w_{j_M}^{M_i} - (\bar{a}_A + (\bar{b}_A + \bar{c}_A w_{j_M}^{M_i}) r_{j_M}) + (24 - \bar{h}_{F_i}) w_{j_F}^{F_i} - (\bar{a}_A + (\bar{b}_A + \bar{c}_A w_{j_F}^{F_i}) r_{j_F})$ is the income of household i. We assume that wages at workplaces are given exogenously, and both male workers and female workers of this household commute to their workplaces once a day. There are two modes of commute in the metropolitan area: bus (B) or automobile (A). Commuting has a fixed cost, $\bar{a}_m$, variable cost, $\bar{b}_m$, and time cost, $\bar{c}_m$, component where $m \in \{A, B\}$ is the mode of commute.

Wages are the only source of income in the model, and a household's income comes from the earnings of both male and female workers of the household. Moreover, leisure for both male and female workers is an endogenous variable. Some workers might choose leisure as the maximum time endowment they have and hence, do not work at all (single-worker households). In that case, these workers do not need to commute to any workplace, and their commute cost is to be dropped from the expression for $Y^i(r_{j_M}, r_{j_F})$ in the budget constraint above. As for the mode of commuting choice, we assume that all single-worker household workers with a job in the CBD commute to their workplaces by bus, and all other workers commute to their workplaces by automobile.

[13] Once we know the distance of a house from the CBD, r, we also know its distance to the Suburban Ring. Due to the presence of radial symmetry in the model, the distance of the house to the Suburban Ring is given by $|r - \bar{r}|$: i.e. $r_{CBD}(r) = r$ and $r_{Sub}(r) = |r - \bar{r}|$.

The household i's problem could be formulated as a utility maximization problem subject to its budget constraint in which it chooses the location of the house, the school district, the workplaces for male and female workers, the consumption of all other goods, the size of the house, and the leisure for male and female workers:

$$V^i = max_{r,\, d,\, j_M,\, j_F,\, ,H,\, X,\, L_M \in [0, 24 - \bar{h}_{M_i}],\, L_F \in [0, 24 - \bar{h}^i_F]}\ Q_d^{\alpha_i} H^{\beta_i} X^{\gamma_i} L_M^{\delta_i} L_F^{\epsilon_i}$$
$$\text{subject to } (1+\tau_d) R_d^*(r) H + X + w_{j_M}^{M_i} L_M + w_{j_F}^{F_i} L_F = Y^i(r_{j_M}, r_{j_F})$$

V^i is the indirect utility function for household $i \in \{s, u\}$. It is important to emphasize the fact that this is a constrained optimization problem in which we also impose time constraints on leisure (and hence, labor supply) choices for both male and female workers. Moreover, the same type of households might end up with different house locations, school districts, male and female worker workplaces, house sizes, consumptions of all other goods, and male and female worker leisures, which yield the same utility level in equilibrium.

Housing. As in the class of models originated by Muth (1969), households consume housing in our model, and the housing industry produces the housing through a constant returns to scale production function from capital K and land (lot size) S:

$$H = AK^b S^{1-b} \tag{4}$$

where $A > 0$ and $b \in (0, 1)$. Each profit-maximizing firm in the housing industry behaves as

$$max_{K,S}\ R_d^*(r) AK^b S^{1-b} - K - \Psi_d^*(r) S \qquad \text{at each } r \text{ and } d \in \{East, West\} \tag{5}$$

where $\Psi_d^*(r)$ is the equilibrium land rent per square foot at location r in school district d.

Land Market and Bid-rent Function. In our model, we need to specify the form of land ownership. We assume absentee landownership in which land is owned by absentee landlords. Both skilled and unskilled households bid for land. Moreover, the landlord has the option to spare the land for agriculture, from which it gets $\bar{\Psi}_a$.

To ascertain the household's residential decision-making more simply, we can introduce a new approach by exploiting the fact that there are two types of households (skilled and unskilled) and identical households obtain identical utilities in equilibrium. This method requires the introduction of a bid-rent function, which describes a household's willingness to pay for land under a fixed utility level. For household $i \in \{s, u\}$ with job locations $j = (j_M, j_F)$ in school district d, the house bid-rent function at location r can be mathematically expressed as

$$R^i_{j,d}(r, \bar{u}_i) = \max_{H,\, X,\, L_M,\, L_F} \left\{ \frac{Y^i(r_{j_M}, r_{j_F}) - X - w_{j_M}^{M_i} L_M - w_{j_F}^{F_i} L_F}{(1+\tau_d) H} \mid U^i(.) = \bar{u}_i \right\} \tag{6}$$

where $L_M \in [0, 24-\bar{h}_{M_i}]$ and $L_F \in [0, 24-\bar{h}_{F_i}]$. This is a constrained optimization problem in which labor supply decisions are endogenous and in equilibrium, some workers might be constrained and choose not to work. Thus, the solution to this maximization problem is one of the following three cases: a two-worker household i with workplaces (j_M, j_F), a one-worker household i with a male worker at workplace j_M, and a one-worker household i with a female worker at workplace j_F.[14]

For a household with a constrained female worker and unconstrained male worker (i.e. one-worker household with a male worker), we have:

$$R^i_{j,d}(r) = R^i_{j_M,d}(r) = \frac{k_{2i}^{1/\beta_i}}{(1+\tau_d) {w^{M_i}_{j_M}}^{\delta_i/\beta_i}} Q_d^{\alpha_i/\beta_i} Y^i(r_{j_M})^{\frac{\beta_i+\gamma_i+\delta_i}{\beta_i}} \bar{L}_F^{\epsilon_i/\beta_i} \bar{u}_i^{-1/\beta_i} \tag{7}$$

$$H^i_{j,d}(r) = H^i_{j_M,d}(r) = \frac{\beta_i}{(\beta_i+\gamma_i+\delta_i)} \frac{Y^i(r_{j_M})}{(1+\tau_d)R^i_{j_M,d}(r)} \tag{8}$$

$$L^i_{M,j,d}(r) = L^i_{M,j_M,d}(r) = \frac{\delta_i}{(\beta_i+\gamma_i+\delta_i)} \frac{Y^i(r_{j_M})}{w^{M_i}_{j_M}}; \quad L^i_{F,j,d}(r) = \bar{L}_F = 24 - \bar{h}_{F_i} \tag{9}$$

where $Y^i(r_{j_M}) = (24-\bar{h}_{M_i})w^{M_i}_{j_M} - (\bar{a}_m + (\bar{b}_m + \bar{c}_m w^{M_i}_{j_M})r_{j_M})$ is the male worker's income endowment net of commuting costs by the mode of commute $m \in \{A, B\}$. If the male worker works and commutes to his workplace at the CBD, his mode of commute is a bus. Otherwise, he commutes to his workplace by automobile. Similarly, we can also find the house bid rent function for a one-worker household i with a female worker.

Solving the maximization problem for a household with unconstrained workers (i.e. a two-worker household), we have:[15]

$$R^i_{j,d}(r) = \frac{k_i^{1/\beta_i}}{(1+\tau_d) {w^{M_i}_{j_M}}^{\delta_i/\beta_i} {w^{F_i}_{j_F}}^{\epsilon_i/\beta_i}} Q_d^{\alpha_i/\beta_i} Y^i(r_{j_M}, r_{j_F})^{\frac{\beta_i+\gamma_i+\delta_i+\epsilon_i}{\beta_i}} \bar{u}_i^{-1/\beta_i} \tag{10}$$

$$H^i_{j,d}(r) = \frac{\beta_i}{(\beta_i+\gamma_i+\delta_i+\epsilon_i)} \frac{Y^i(r_{j_M}, r_{j_F})}{(1+\tau_d)R^i_{j,d}(r)} \tag{11}$$

$$L^i_{M,j,d}(r) = \frac{\delta_i}{(\beta_i+\gamma_i+\delta_i+\epsilon_i)} \frac{Y^i(r_{j_M}, r_{j_F})}{w^{M_i}_{j_M}}; \quad L^i_{F,j,d}(r) = \frac{\epsilon_i}{(\beta_i+\gamma_i+\delta_i+\epsilon_i)} \frac{Y^i(r_{j_M}, r_{j_F})}{w^{F_i}_{j_F}} \tag{12}$$

where $Y^i(r_{j_M}, r_{j_F}) = (24-\bar{h}_{M_i})w^{M_i}_{j_M} - (\bar{a}_A + (\bar{b}_A + \bar{c}_A w^{M_i}_{j_M})r_{j_M}) + (24-\bar{h}_{F_i})w^{F_i}_{j_F} - (\bar{a}_A + (\bar{b}_A + \bar{c}_A w^{F_i}_{j_F})r_{j_F})$ is the household's income endowment net of commuting

[14] No worker household i cannot be a solution because the income for household i becomes zero in that case. For a model with only single-worker households, See Hanushek and Yilmaz (2007, 2013) and Yilmaz (2019).

[15] where $k_i = \frac{\beta_i^{\beta_i} \gamma_i^{\gamma_i} \delta_i^{\delta_i} \epsilon_i^{\epsilon_i}}{(\beta_i+\gamma_i+\delta_i+\epsilon_i)^{(\beta_i+\gamma_i+\delta_i+\epsilon_i)}}$ and $k_{2i} = \frac{\beta_i^{\beta_i} \gamma_i^{\gamma_i} \delta_i^{\delta_i}}{(\beta_i+\gamma_i+\delta_i)^{(\beta_i+\gamma_i+\delta_i)}}$ are constants.

costs by automobile. In the case of the optimal solution being a one-worker household solution, it means that in equilibrium, we cannot have a two-worker household at location r in school district d with workplaces j. What solution we end up with for a household i depends on the parameter values.

To see how the joint labor supply decisions of two-worker households may be distorted compared to the single-worker households, we hypothetically compare the male labor supply choices of two identical households with different labor supply choices at the same location in the same school district: i.e. the labor supply of a one-worker household with a male worker to that of a two-worker household. There are two major differences. First, the members of a two-worker household combine their incomes and hence, they have more income. As shown in Eqs. 9 and 12, the income effect induces the male worker of a two-worker household to consume more leisure. Since his daily time endowment is constant (i.e. $24-\bar{h}_{M_i}-\bar{c}_m r_{j_M}$), the income effect causes a decrease in the male worker's labor supply. The second effect can be seen in the budget share of the leisure for male workers: $\frac{\delta_i}{\beta_i+\gamma_i+\delta_i}$ for single-worker households versus $\frac{\delta_i}{\beta_i+\gamma_i+\delta_i+\epsilon_i}$ for two-worker households. A two-worker household spends a smaller proportion of its budget on leisure because the members of a two-worker household pool resources, and some portion of the pooled budget is allocated to the leisure of the female worker (i.e. $\epsilon_i > 0$). For a two-worker household, the second effect decreases the leisure for the male worker of the household and increases his labor supply. Moreover, the more his spouse likes leisure, the less leisure the male worker consumes and the more he works. Which of those two effects dominates depends on the calibration parameters once again and is verified numerically.

Our model also allows us to calculate the house rent gradient for each household type. For instance, taking the logarithm of both sides in the house bid-rent function for a one-worker household in Eq. 7 and differentiating with respect to the distance from the employment center yield the house rent gradient as

$$\frac{\partial ln(R^i_{j,d}(r))}{\partial r} = -\frac{(\beta_i+\gamma_i+\delta_i)}{\beta_i}\frac{(\bar{b}_m+\bar{c}_m w^{M_i}_{j_M})}{Y^i(r_{j_M})} \approx -\frac{(\beta_i+\gamma_i+\delta_i)}{\beta_i}\frac{(\bar{b}_m+\bar{c}_m w^{M_i}_{j_M})}{(24-\bar{h}_{M_i})w^{M_i}_{j_M}} \tag{13}$$

where we approximate the male worker's income endowment net of commuting costs, $Y^i(r_{j_M})$, under the assumption that the commuting costs, $\bar{a}_m + (\bar{b}_m + \bar{c}_m w^{M_i}_{j_M})r_{j_M}$, are negligible. We can also derive similar formulas for two-worker households. These formulas reveal that the house rent gradient depends on the type of household along with the workplaces and modes of commute for its workers.

As in Muth (1969), the land goes to the highest bidder in our model. To be able to find the outcome of bidding for land at any location, we need to derive and work with land bid-rent functions. Since the housing production function has constant returns to scale, the firms in the housing industry get zero profit at each location in equilibrium. If a house is constructed for a household i at location r in school district d with workplaces j, we have a housing firm being able to offer the following land bid-rent to the landlord:

$$\Psi^{i}_{j,d}(r, \bar{u}_i) = A^{\frac{1}{1-b}} b^{\frac{b}{1-b}} (1-b) R^{i}_{j,d}(r)^{\frac{1}{1-b}} \tag{14}$$

In a competitive environment, the competition among prospective households bids up the price of land. Household i's land bid-rent differs by its workers' workplaces $j \in \{c-, -c, sub-, -sub, cc, csub, subc, subsub\}$, and depending on its workers' workplaces, there are eight different household i bids at location (r, d). In this notation, $j = csub$ represents a male worker with a workplace at the CBD, $j_M = CBD$, and a female worker with a workplace at the Suburban Ring, $j_F = Sub$. Recall that at some workplaces, the optimal solution could be a single-worker household, and the first four elements in this notation correspond to single-worker households. For instance, we represent one-worker households with a male worker at the CBD by $c-$. Thus, the landlord receives 16 bids at any location from households (2 bids from each household $i \in \{s, u\}$ and 8 possible workplace combinations), in addition to the bid from farmers, $\bar{\Psi}_a$. And the land goes to the highest bid. Formally,

$$\Psi^{*}_{d}(r) = \max\left\{ \{\Psi^{i}_{j,d}(r) \,|\, i \in \{s, u\},\ j \in \{c-, -c, sub-, -sub, cc, csub, subc, subsub\}\}, \bar{\Psi}_a \right\} \tag{15}$$

By using the land bid-rent functions and bidding for land, we also handle the endogeneity of workplace choices. Household i is forced by competition to pick up workplaces for its workers that maximize its utility and hence, makes its best offer at location (r, d).

Moreover, it proves quite useful to introduce an equilibrium-type function that keeps track of land use at location (r, d):

$$\theta^{*}_{d}(r) = \operatorname{argmax}\left\{ \{\Psi^{i}_{j,d}(r) \,|\, i \in \{s, u\},\ j \in \{c-, -c, sub-, -sub, cc, csub, subc, subsub\}\}, \bar{\Psi}_a \right\} \tag{16}$$

where $\theta^{*}_{d}(r)$ is the winner of the bidding at location (r, d). Recalling that household i's also differentiated by workplaces of their workers, $\theta^{*}_{d}(r) = (i, j)$ shows the type of winner $i \in \{s, u\}$ along with its worker's workplaces $j \in \{c-, -c, sub-, -sub, cc, csub, subc, subsub\}$. In case a farmer gets the land, it just shows the type of winner as farmers (i.e. $\theta^{*}_{d}(r) = a$).

There are two workplaces in our model, and the land bid-rent for household i is oriented towards one of those two workplaces: i.e. the land bid-rent goes down as we move away from a workplace. As in any other urban location model, the spatial ordering of households around a workplace is determined by the relative steepness of land bid-rent curves oriented towards the workplace. It is worth understanding the parameters that determine the relative steepness of land bid-rent functions between two households with land bid-rent functions oriented towards the same workplace. For example, let us consider a single-worker household i_1 with a male worker at the CBD (i.e. $j_1 = j_{M_{i_1}} = CBD$) and a two-worker household i_2 with a male worker at the CBD and female worker at the Suburban Ring (i.e. $j_2 = (j_{M_{i_2}}, j_{F_{i_2}}) = (CBD, Sub)$). Suppose the land bid-rent for household i_2 is also oriented towards the CBD, and household i_1 has a steeper land bid-rent curve than household i_2: i.e. whenever $\Psi^{i_1}_{j_1,d}(r^*) = \Psi^{i_2}_{j_2,d}(r^*)$ at some r^*:[16]

$$\frac{\partial \Psi^{i_1}_{j_1,d}(r^*)/\partial r}{\partial \Psi^{i_2}_{j_2,d}(r^*)/\partial r} = \frac{\beta_{i_2}}{\beta_{i_1}} \frac{Y^{i_2}(r_{CBD}, r_{Sub})}{Y^{i_1}(r_{CBD})} \frac{(\bar{b}_B + \bar{c}_B\, w^{M_i}_{CBD})}{(\bar{b}_A + \bar{c}_A\, w^{M_i}_{CBD}) - (\bar{b}_A + \bar{c}_A\, w^{F_{i_2}}_{Sub})} > 1 \tag{17}$$

where it must be the case that $(\bar{b}_A + \bar{c}_A\, w^{M_i}_{CBD}) > (\bar{b}_A + \bar{c}_A\, w^{F_{i_2}}_{Sub})$ to have the land bid-rent for household i_2 is also oriented towards the CBD. Then, household i_1 outbids household i_2 at locations closer to the CBD and underbids household i_2 at locations further away from the CBD. The key parameters that determine the relative steepness of land bid-rent curves are the budget share of housing, income endowment, and the marginal cost of commuting. The higher the budget share of housing or endowment income is, the flatter the land bid-rent is. Thus, the household is more likely to be located at locations farther away from the CBD. On the other hand, the higher the marginal cost of commuting is, the steeper the land bid-rent is. The household is more likely to be located at locations closer to the CBD in this case. Based on this analysis, we can expect that the residential locations around a workplace are more likely to be occupied by households with workers at this workplace, and the spatial ordering of households around a workplace is determined by a tug-of-war between income (hence, labor supply decisions) and commuting costs (hence, mode of commute) in equilibrium.

Population. We assume that the population of household $i \in \{s, u\}$ is exogenously given as $\bar{N}^i$. This is a standard assumption in closed city models that is a good conceptual device for analyzing urban land use in large cities or "average cities" of developed countries. Land markets clear in equilibrium. In each school district, the semi-rings around the CBD are identical in terms of accessibility, education and property taxes that are used to finance schools. Thus, in equilibrium, it must be the case that they are occupied by identical households. For a semi-ring of size dr at location (r, d), the amount of land available for housing is $\pi r dr$. Suppose household i with worker's workplaces j wins the bidding at this semi-ring. Each household's land consumption is given by $S^i_{j,d}(r)$, and, hence, the number of type i households at the semi-ring is $\frac{\pi r dr}{S^i_{j,d}(r)}$. To find the population of household i's in school district d, we add the population of all household i's who offer the highest land bid-rent and win the bidding in the school district. Recalling that household i's differ by workplaces $j \in \{c-, -c, sub-, -sub, cc, csub, subc, subsub\}$, the population of household $i \in \{s, u\}$ in school district $d \in \{East, West\}$ is given by:

$$N^i_d = \sum_j \left(\int_{\{\theta^*_d(r)=(i,j)\}} \frac{\pi r}{S^i_{j,d}(r)} dr \right) \tag{18}$$

[16] Uniqueness of r^* is verified numerically. See Hanushek and Yilmaz (2007, 2013) for details.

Some household i's reside in the West School District while some others reside in the East School District. Hence, the clearance of land markets implies the following population constraint:

$$N^i_{East} + N^i_{West} = \bar{N}^i,\ \forall i \in \{s, u\} \tag{19}$$

Schools. From the perspective of each household, the school quality and property tax rate in each school district is exogenously given. We now turn to the determination of those aggregate variables in the model. Following the Hanushek and Yilmaz series (HY 2007, 2013, 2015), we assume education in school district d is provided through the following production function:

$$q_d = [c_0 + c_1\, exp(-\frac{N^u_d}{N^s_d})]\, e^{c_2}_d \tag{20}$$

$c_0 > 0, c_1 > 0$, and $0 < c_2 < 1$ are constants, e_d is the expenditure per pupil in school district d, and *exp* is the base for natural exponential function. In this formulation, higher spending on education is associated with a better quality of education. The production function is concave in spending and thus, the return to spending increases at a decreasing rate. In line with the literature that finds that the peers' academic outcomes have a significant effect on own academic performance,[17] our production function also incorporates peer effects. The peer effect in each school district depends on the population composition where skilled households impose a positive impact and unskilled households inflict a negative impact on others.[18]

Schools in each school district are open to the residents only and do not charge any additional tuition.[19] Hence, the spending per pupil in school district d is given by:

$$e_d = \tau_d \frac{\int_{\Psi^*_d(r) > \bar{\Psi}_a} R^*_d(r)\pi r dr}{N_d},\ \forall d \in \{East, West\} \tag{21}$$

where $N_d = N^s_d + N^u_d$ is the population of school district d, $R^*_d(r)$ is the equilibrium house rent at location r in school district d, and only residential properties are taxed to finance schools.

As for the property taxes, the decision is based on majority voting. For a two-worker household i with workplaces j at location (r,d), his preferred property tax rate is the solution to the following maximization problem:

$$\max_{\tau_d} V^i(.) = \kappa_i \frac{\tilde{q}^{\alpha_i}_d}{(1+\tau_d)^{\beta_i}} \text{ subject to } \tilde{q}_d = \tau_d \frac{\int_{\Psi^*_d(r) > \bar{\Psi}_a} R^*_d(r)\pi r dr}{N_d} \tag{22}$$

17 See Blume and Durlauf (2005), Nechyba (2006), Soetevent (2006), and Sacerdote (2011).

18 See Benabou (1993) for a discussion of incorporating human capital spillovers into education production in this way and the empirical literature on this. Also see Hanushek and Yilmaz (2007, 2013), Leung et al. (2012), and Yilmaz (2019) for alternative formulations of the peer effect function.

19 We see a growing involvement by the states to reduce the spending disparities across school districts over the years (Hanushek & Lindseth, 2009). See Hanushek and Yilmaz (2007, 2013) and Yilmaz (2019) for a study of some prominent state interventions in the U.S. education system.

Where $\beta_i > \alpha_i > 0$ and $\kappa_i = \frac{k_i}{R_d^*(r)^{\beta_i} w_{J_M}^{M_i\ \delta_i} w_{J_F}^{F_i\ \epsilon_i}} Y^i(r_{j_M}, r_{j_F})^{\beta_i+\gamma_i+\delta_i+\epsilon_i}$. In our formulation, voters are myopic[20] as in Epple et al. (1984, 1993) and do not consider the implication of their votes on aggregate variables such as population composition and land prices (i.e. κ_i is a constant in the problem above). More importantly, households proxy the expenditure per pupil on education as a measure of school quality in their school districts.[21] The solution depends on the taste parameters α_i and β_i and takes the simple form $\tilde{\tau}_i = \frac{\alpha_i}{\beta_i - \alpha_i}$.

We are now ready to formally define the general equilibrium of this economy. Formally, it is defined as follows:

Definition: An equilibrium is a set of utility levels $\{\bar{u}_u^*, \bar{u}_s^*\}$, market house-rent functions $\{R_{East}^*(r), R_{West}^*(r)\}$, market land-rent functions $\{\Psi_{East}^*(r), \Psi_{West}^*(r)\}$, quality of education and property tax pairs $\{(q_d, \tau_d) \mid d \in \{East, West\}\}$, and type function $\{\theta_d^*(r) \mid d \in \{East, West\}\}$ that shows the equilibrium occupant of the location (r, d) such that

- Given prices and qualities of education in school districts, households choose a composite good, leisure for male and female workers, location of workplaces, location of a house, size of a house, and school district that maximizes their utilities.
- Firms in a competitive housing industry choose land and capital to maximize profits.
- The city is *closed* city in which the same type of households attain the same utility level.
- There is a bidding for land owned by absentee landlords at each location. Firms in the housing industry and farmers bid for land, which goes to the highest bidder. Moreover, land markets clear.
- Each school district provides a free public education through a production function. In each school district, education is financed through property taxes determined by majority voting.

The timing of events in the model is as follows: based on the quality of education and property tax rate package offered by each school district last period, households make their school district, residential, and workplace decisions along with their consumption and labor supply decisions at the beginning of a period. Households are stuck with the choices they make: i.e. they can neither relocate nor alter their choices till the next period. A majority voting takes place in each school district, and the voting outcome determines the quality of education they receive and property taxes they pay in the current period. The quality of education and tax rate package they receive can be different from what they expected when they made their choices at the beginning of the period. As the economy moves to the next period, households are free to relocate and alter their other choices. The sequence of events repeats once again. This process continues until the current period quality of education and property tax rates packages offered by school districts converge to those of the last period.

[20] See Yinger (1982, 1985) for an alternative formulation of voter behavior.

[21] Alternatively, for a voting model of households with full information about the school production function, see Hanushek and Yilmaz (2007, 2013).

Calibration

We calibrate the model to match several stylized facts for an average U.S. city around 2016 and solve it numerically. Table 2 summarizes the set of parameter values used for calibration. We set the location of the Suburban Ring at $\bar{r} = 6$ miles.[22] In 2016, the median weekly earnings for female workers with a high school and college degree were \$599 and \$1,101, respectively. For males, the median weekly earnings for high school graduates and college graduates were \$769 and \$1,464, respectively. Moreover, the average weekly work hours for female and male workers are 36.4 and 41.1, respectively.[23] Thus, the hourly wage rates for male and female workers with a high school degree are about \$18.7 and \$16.5, respectively, while the hourly wage rate for male and female workers with a college degree are about \$35.6 and \$30.2, respectively. Wages for male and female workers at the Suburban Ring are calibrated based on Ihlanfeldt's (1992) wage gradient estimate of one percent. Also, the daily home production time for male workers, unskilled female workers, and skilled female workers are set to be $\bar{h}_{M_u} = \bar{h}_{M_s} = 1.8$, $\bar{h}_{F_u} = 6.2$, and $\bar{h}_{F_s} = 3.8$.[24]

We choose the parameters of a household's utility function to mimic the behavior of average households in the U.S. For skilled households, our calibration is based on the parameters of a two-worker skilled household with a residence and workplaces at the CBD. One useful property of the Cobb-Douglas production function is the fact that the exponents in the utility function are budget shares for the corresponding commodity. The budget share of leisure for skilled male workers, δ_s, is set to be 0.43 so that his workings are about 41 hours per week. To match working hours for an average female worker in the data, ϵ_s is calibrated to be 0.33. One key parameter in the model, β_s, is assumed to be 0.046 so the budget share of housing for the rich is about 19 percent.[25] Since the sum of utility function parameters is normalized to be one, γ_s becomes 0.196. For unskilled households, the calibration was a little bit more complicated. The utility function parameter calibrations for unskilled households are based on a two-worker unskilled household with a residence and workplaces at the CBD. We set δ_u and ϵ_u to be 0.46 and 0.31, respectively so that the male and female workers of two-worker unskilled households work about 41 and 36 hours per week, respectively. As in the calibration of the utility function parameters for skilled households, β_u and γ_u are calibrated to be 0.056 and 0.182, respectively so that the budget share of housing for a single-worker unskilled household is about 23 percent. To have an equilibrium property tax rate (out of property value) of about two percent in each school district,[26]

[22] We also experimented with different suburban ring locations. Our main findings are robust to the change in the suburban ring location.

[23] The Current Population Survey, U.S. Bureau of Labor Statistics

[24] Time allocation trends: 1965-2003, Bar and Leukhina (2011).

[25] The Consumer Expenditure Survey, U.S. Bureau of Labor Statistics, 2012.

[26] The median property tax rates in 2016 ranged from 2.14 percent in New Jersey to 0.29 percent in Louisiana. Source: American Community Survey, 2016.

Table 2 Calibration parameters and targets

Parameter	Interpretation	Value	Target (in the U.S.)
Macroeconomic Environment			
$\bar{\Psi}_a$	Monthly Agricultural Rent (dollars per acre)	1097	Avg. Population Density of Cities
$\bar{N}_i$	Population of Type i	$\bar{N}_s = 900,000$, $\bar{N}_u = 600,000$	Avg. Population of Cities
Budget Constraints			
w^i_j	Hourly Wage Rate at Workplace j (dollars)	$w_c^{M_s} = 35.6$, $w_c^{F_s} = 30.2$ $w_c^{M_u} = 18.7$, $w_c^{F_u} = 16.5$	Avg. Wages Wage Gradient (Ihlanfeldt (1992))
$\bar{a}_m$	Fixed Cost of Commute by Mode m (dollars per day)	$\bar{a}_B = 6$, $\bar{a}_A = 18.1$	Avg. Fixed Commute Cost
$\bar{b}_m$	Variable Cost of Commute (dollars per round trip mile)	$\bar{b}_B = 0$, $\bar{b}_A = 0.35$	Avg. Variable Commute Cost
$\bar{c}_m$	Time Cost of Commute (hours per round trip mile)	$\bar{c}_B = 0.182$, $\bar{c}_A = 0.057$	Avg. Time Commute Cost
Preferences			
α_i	Valuation of Education	$\alpha_s = 0.022$, $\alpha_u = 0.018$	Avg. Property Tax Rate
β_i	Budget Share of Housing	$\beta_s = 0.046$, $\beta_u = 0.056$	Avg. Budget Share of Housing
γ_i	Budget Share of Composite Good	$\gamma_s = 0.196$, $\gamma_u = 0.182$	$\beta_i + \gamma_i + \delta_i + \epsilon_i = 1$
δ_i	Budget Share of Leisure (Males)	$\delta_s = 0.426$, $\delta_u = 0.455$	Avg. Hours of Work
ϵ_i	Budget Share of Leisure (Females)	$\epsilon_s = 0.332$, $\epsilon_u = 0.307$	Avg. Hours of Work
Production Functions			
c_0, c_1, c_2	Education Production Function Parameters	$c_0 = 7.58$, $c_1 = 3.05$ $c_2 = 0.1$	Quality of Education Targets
A, b	Housing Production Function Parameters	$A = 0.01$, $b = 0.35$	Avg. House-Lot Size Ratio

we also set α_u and α_s to be 0.018 and 0.022, respectively. In line with the *closed city* assumption, the population of our city is assumed to be 1,500,000 households, of which 40 percent are unskilled households.[27]

[27] In 2016, 28.9 percent of women in the labor force held high school degrees or less. Source: Bureau of Labor Statistics.

For commuting by bus, the fixed and time costs of commuting are assumed to be \$6 per round trip[28] and 0.182 hours per round trip mile,[29] respectively, and there are no variable variable costs of commuting. For automobiles, we set the fixed, variable material, and time costs of commuting to be \$18.1 per day, \$0.35 per round trip mile, and \$0.057 hours per round trip mile, respectively.[30]

For the remaining parameters, we take an indirect approach: we set calibration targets for some endogenous variables and calibrate the remaining parameters to match those targets. We set targets for the quality of education in each school district, which determines the spatial distribution of population across the city. Those endogenous calibration targets are reported in Table 4 and Fig. 1 of baseline equilibrium. The agricultural rent determines the fringe distance and hence, the population density in the city. The agricultural rent is set to be $\bar{\Psi}_a =\$1,097$ per acre per month to produce a population density of 4,290.[31] We choose the peer effect parameters as $c_0 = 7.58$, $c_1 = 3.05$, and $c_2 = 0.1$ in school production function to match the endogenous quality of education targets for both school districts. Lastly, the house production parameters A and b are chosen to be 0.01 and 0.35, respectively to produce a house-lot size ratio that ranges from 0.25 at the fringe to 0.5 at the CBD.

Baseline

We display the stylized facts regarding American households and cities along with the corresponding statistics for the baseline equilibrium in Table 3 and summarize the baseline equilibrium outcomes in Figs. 1 and 2 along with Table 4.[32] Due to radial symmetry, we simply graph a cross-section of the city. Figure 1 provides gross market house rents, basic education statistics, and the spatial ordering of households across the city. In equilibrium, West School District chooses a higher property tax rate and expenditure per pupil on education to provide a higher quality of education than East School District. Figure 1 shows market house rents first decline as we move away from the CBD. As we get closer to the suburban workplace, it bounces back and increases.[33] After passing the suburban workplaces, it declines again. It also shows the capitalization of a higher quality of education and property tax around the school

28 Source: The Public Transportation Fare Database, American Public Transportation Association, 2014.

29 The average commuting speeds for automobile commuting and bus commuting around 2010 are 28.87 and 11.42 miles per hour, respectively. Source: Summary of Travel Trends: National Household Travel Survey.

30 Source: Your Driving Costs; American Automobile Association, 2015.

31 The median population per square mile of cities with a population of 1,000,000 or more is 3,871 per square mile. Source: County and City Data Book, 2007.

32 There is also a symmetric equilibrium to this model, which happens to be unstable and have lower utility levels for both skilled and unskilled households. This issue is common in this literature. For instance, see Benabou (1993) or Fernández and Rogerson (1996).

33 A monocentric city model would produce a negative income gradient. The presence of decentralized employment disrupts this and makes the income gradient non-monotonic, consistent with the income-distance relationship presented in Glaeser et al. (2008).

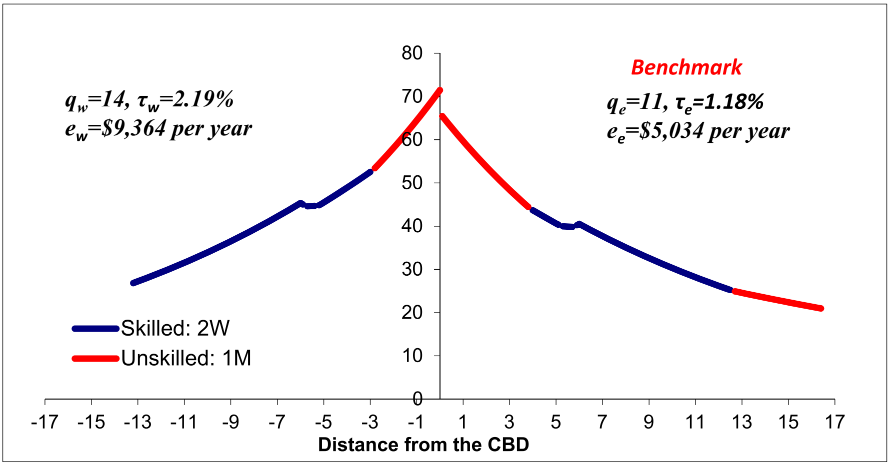

Fig. 1 Monthly market house gross rent (cents per square foot)

district boundary at the CBD: there is a discontinuity in the form of a big jump of the gross market house rent at the school district boundary.[34]

The spatial allocation of population is an important result of urban location models (Alonso, 1964; Muth, 1969).[35] A household's spatial location in a school district is determined by the steepness of its land bid-rent curve, which is previously introduced in Eq. 16. Most of the time, the orientation of gross market house rent shows the workplace of the workers in a household. From the CBD towards the fringe, the spatial ordering of households in equilibrium is as follows: unskilled one-worker households with a male worker at the CBD, skilled two-worker households with both workers at the CBD, skilled two-worker households with a male worker at the CBD and a female worker at the suburban ring, skilled two-worker households with both workers at the suburban ring, and unskilled one-worker households with a male worker at the suburban ring. In Table 4, we report the spatial distribution of household population by the spatial ordering of households from the CBD. The table categorizes households by their female worker's skill level (skilled or unskilled) and their working member(s) workplace. We do not see households with every possible workplace combination in the table. For instance, we do not see a two-worker household with a male worker at the suburban ring and a female worker at the CBD in any school district. We also see heterogeneous school districts in terms of income.[36] The introduction of bus as an alternative mode of commute alters the relative steepness of land bid-rent curves and has important implications for the model, as seen in Fig. 1: unskilled one-worker households with a male worker at the suburban ring have a flatter land bid-rent than

[34] Even though the size of the capitalization of school quality is subject to debate, the literature finds that better schools raise housing prices (e.g. Black (1999) and Bayer et al. (2007). See Black and Machin (2011) for a comprehensive review of this literature.

[35] See Kain (1975) and McDonald and McMillen (2011) for a detailed discussion of the Alonso-Muth model and the related literature.

[36] For a thorough discussion and empirical evidence on imperfect income sorting, see Davidoff (2005) and Hardman and Ioannides (2004) among others.

Table 3 Data and baseline model

	Model		Data	
Labor Supply	Fem	Male	Fem	Male
-Skilled	36.4	41	36.4	41
-Unskilled	36.8	41	36.4	41
Budget Share	Skill	Unskill	Skill	Unskill
-Housing	19	23.5	19.7	21.8
Property Tax Rate	West	East		
	2.19%	1.18%	0.29% - 2.14%	
Population Density				
	4,290		3,871	
Dissimilarity Index				
	0.395		0.37	
Percentage of Jobs				
-CBD	22.7%		25%	
Labor Force Particip				
-Females	60%		56.8%	
Bus Ridership				
	10.1%		10.9%	
Avg Commute Time	Fem	Male	Fem	Male
	29	33.2	39	47
House Rent Gradient				
	-0.10		-0.10	

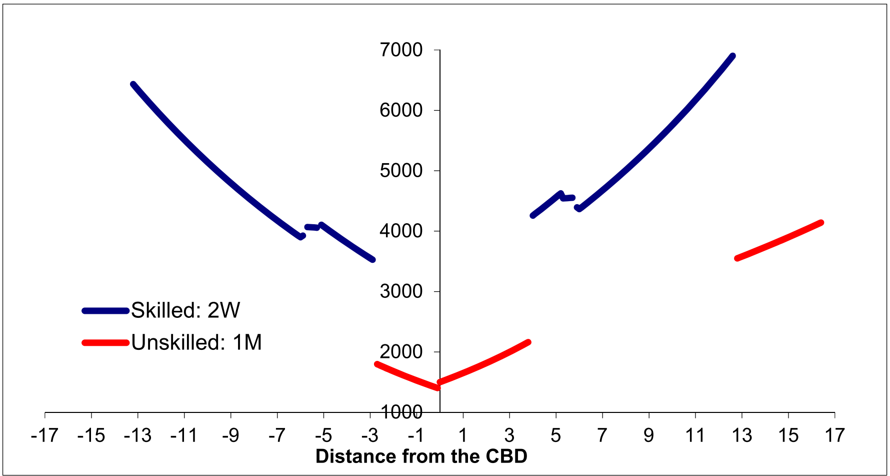

Fig. 2 House size (square feet)

Table 4 Spatial distribution of household population

	U: 1M-c	S: 2W-cc	S: 2W-cs	S: 2W-ss	U: 1M-s
West	6	5.5	1.5	25.7	0
East	10.2	3.1	1.5	22.7	23.8
Total	16.2	8.6	3	48.4	23.8

skilled two-worker households with both workers at the suburban ring while unskilled one-worker households with a male worker at the CBD have a steeper land bid-rent than skilled two-worker households with both workers at the CBD because both two-worker households and one-worker households with a worker at the suburban ring commutes to their workplaces by automobile while one-worker households with a worker at the CBD commute to their workplaces by bus.

In the West, most residential locations are occupied by skilled households: 84.5 percent of the West School District population consists of skilled households. As a result, a high property tax rate prevails as the outcome of majority voting. We see unskilled one-worker households with a male worker around the CBD, and the male workers of those households commute to their workplaces by bus. More importantly, the female workers of these households choose not to work. Those households are neighbored by skilled two-worker households with both workers at the CBD. About 5.3 miles off the CBD, those households are followed by skilled two-worker households with a male worker at the CBD and a female worker at the Suburban Ring. The gross market house rent curve across the locations occupied by these households becomes almost flat because the workers of these households have different workplaces. Moreover, the locations around the suburban ring are occupied by skilled two-worker households with both workers at the suburban ring. In the East, we see a similar pattern even though we observe a much heavier presence of unskilled households, and hence, a lower property tax rate in equilibrium: 55.5 percent of the school district population is unskilled workers. The only major difference with the West is that the locations around the fringe are occupied by unskilled one-worker households with a male worker who commutes to his workplace at the Suburban Ring by automobile. The presence of unskilled households around the fringe adds another interesting prediction to our findings, which is consistent with empirical literature: the income gradient becomes non-monotonic as in Glaeser et al. (2008).

Table 4 also provides us with information about workplaces, commuting patterns, and labor supply choices of households in equilibrium. 22.7 percent of jobs are offered at the CBD,[37] of which 44.5 are held by workers from unskilled households. For unskilled households, we only observe one-worker households with a male worker in equilibrium. In other words, the male workers of these households choose to work while the female workers choose not to work. 59.5 percent of unskilled households have a worker in suburban areas. As for skilled households, we find that all skilled worker households are two-worker households. 14.4 and 80.7 percent of skilled households have both workers at the CBD and Suburban Ring, respectively. 4.9 percent of

[37] Glaeser and Kahn (2004) find that 25 percent of the employees work within three miles of their metropolitan area center.

skilled households have a male worker at the CBD and a female worker at the Suburban Ring. Only one-worker households with a job at the CBD commute by bus, and the ratio of workers commuting by bus is 10.1 percent in our model.[38]

Figure 2 shows the house sizes in equilibrium. House sizes tend to increase as we move away from the CBD, even though the increase is not monotone. As in the spatial distribution of income across the city, the presence of the Suburban Ring and unskilled one-worker households at the fringe generates this pattern. With a higher property tax rate set and a heavier presence of skilled households, West School District spends a much higher amount on education for each child and has better peers. Therefore, it provides a higher quality of education than expected.

By substituting our calibration parameters in Eq. 13, we calculate the house rent gradients for each household type and report them in Table 5. Those values indicate that the house rents are, on average, about 7.3 percent cheaper at locations 1 mile away from the employment centers. The empirical literature on rent gradients varies by the functional forms used to estimate bid-rent functions. Gupta et al. (2022) estimate the relationship between the logarithm of housing prices and the logarithm of 1+r, where r is the distance in kilometers from the CBD, in a pooled regression with CBSA fixed effects and ZIP-level control variables and find a house price gradient of -0.10 for the 30 largest MSAs in the United States. In this model specification, the percentage change in housing rent is the coefficient of the log (1+r) divided by $1 + r$. For locations close to the CBD (i.e. $r \approx 0$), the estimate of this coefficient corresponds to the house price gradient[39] predicted by our model in Table 5. The households around the CBD in the baseline equilibrium of our model are unskilled one-worker households with a male worker at the CBD in both school districts. The house rent gradient for households around the CBD is predicted to be 0.103, which is in line with their estimate. We also see the house rent gradients predicted by our model decrease with distance from the CBD, which is consistent with their model specification. As in Gong and Leung (2020), our model seems to be reliable as a first-order approximation of reality.

We also test two more predictions of our benchmark model in the data. In 2016, the share of women who participated in the labor force was 56.8%.[40] The benchmark model predicts the labor force participation rate for female workers to be 60%, which is consistent with the data. Not only do males and females differ in the various aspects of the labor market, but there is also a substantial gender difference in the length of commute. In the United States, female workers and male workers with school-aged children, on average, spend 39 and 47 minutes on their daily commute, respectively.[41] The benchmark model also predicts a daily commute time of 29 minutes and 33.2 minutes for female and male workers, respectively. Our model, however, has a radial transportation system and does not include the time spent commuting other than the

[38] Workers who lived in a principal city and worked in the metropolitan area of residence had the public transportation usage rate, at 10.9 percent in 2009. Source: McKenzie and Rapino (2011).

[39] In our model, the house price gradient is the same as the house rent gradient because the value of a house, V, is the present value of an infinite stream of rents, R: i.e. $V = \frac{R}{i}$ where i is the interest rate.

[40] Source: U.S. Bureau of Labor Statistics, 2017.

[41] Source: Organization for Economic Cooperation and Development, 2016.

Table 5 House rent gradients predicted by the model

	Avg	U: 1M-c	S: 2W-cc	S: 2W-cs	S: 2W-ss	U: 1M-s
|Gradient|	0.073	0.103	0.071	0.072	0.072	0.046

ride time on a straight line between work and residences. Assuming indirect routes and stop times increase the commute times by about 40 percent, our model provides a good fit for the data. Moreover, it can shed light on the gender commute gap literature: male workers, on average, commute more than female workers because (i) male workers in one-worker households either commute to their workplaces at the CBD by bus, which is slower than automobile; or (ii) they live in the fringe, which is farther away from their workplaces at the Suburban Ring, and commute longer distances.

The joint labor supply decisions of two-worker households may be distorted compared to those of one-worker households. We conducted a simulation in which we assume each household has one male worker, and the female worker of the household does not work. This simulation not only allowed us to illustrate the distortions in joint labor supply decisions but also the distortions in residential choice decisions, and hence, in residential segregation. The findings are summarized in Table 6. In the Male Worker-only model, the average labor supply for male workers increases by about 20%. The income effect is the stronger force in our simulations, and the removal of female workers from the labor force results in less leisure and more working hours for male workers. Two school districts offer the same education quality and property tax package in the equilibrium and, hence, become identical in terms of household distribution; The sorting of households by the valuation of education into school districts disappears. In terms of the spatial distribution of households, we see unskilled one-worker households at locations around CBD and skilled one-worker households at locations around the Suburban Ring in any school district. The average commute time for male workers also increases by about 11%. More importantly, the market house rents go down monotonically as we move away from the CBD even though we have decentralized workplaces. As discussed above, these predictions are inconsistent with the empirical evidence.

Table 6 Single-worker versus two-worker households

Variable	Benchmark	Male Worker-only
Avg. Male Labor Supply (hrs per day)	6.5	7.8
Avg. Male Commute Time (min. per day)	33.2	37
Population Skilled:West	32.7%	30%
Population Skilled:East	27.3%	30%
Population Unskilled:West	6%	20%
Population Unskilled:East	34%	20%

Demographic Changes

Since the beginning of the twentieth century, the labor market in the U.S. has experienced significant changes in the composition of its labor force and in the differential in wages by gender. More women attended college, and the female labor force grew at a spectacular rate during the last century.[42] The wage gap between a college and high school graduate also has been widening substantially since 1980 (Goldin & Katz, 2007). Besides, there is a substantial gap between the earnings of men and women, which has barely closed recently (Blau & Kahn, 2017). The population, on the other hand, has been both urbanizing and suburbanizing. which, has led to a rapid growth in residential segregation by income (Bischoff & Reardon, 2013). We now turn to some experiments to get more insights into the impact of changes in the labor market conditions for female workers on residential segregation. Figure 3 provides a snapshot of the equilibrium for each experiment in comparison to the baseline equilibrium.

A Change in Female Educational Attainment

While the population of the city is being held constant in our model, the first experiment changes the composition of the labor force by increasing the proportion of unskilled households to 65 percent.[43] Since there are no changes in wages, home production times, commuting costs, and utility function parameters, the spatial ordering of households from either the CBD or the suburban ring essentially remains the same in equilibrium. The labor force participation rate for female workers decreases to 35 percent while all the male and skilled female workers choose to work. The decrease in female labor force participation results from the increase in the number of uneducated female workers who choose not to work and rely on their husband's wages. We also have more unskilled one-worker households with male workers who work more than other male workers. As a result, the average weekly working hours for men is 8.1 percent higher than that of the baseline equilibrium while the average weekly working hours for female workers is 0.7 percent lower.

Figure 3 summarizes the spatial distribution of households in equilibrium. Due to lower expenditures per pupil resulting from lower rents, both school districts provide a lower quality of education even though the West remains the best school district. In the figure, skilled households are identified by blue while unskilled households are identified by red. A solid filling shows two-worker households while a pattern fill shows one-worker households. All of the two-worker households happen to be skilled households. In the legend, the households are ranked by their spatial order from the CBD, which we get closer to as we move down the list. The school districts differ by the size of each type, and, sometimes, some household types are not present in some school districts. More skilled households reside in the West. Compared to the benchmark, we observe a more unbalanced distribution of households across school districts. The biggest group of households are unskilled one-worker households with

[42] Source : U.S. Bureau of Labor Statistics, 2017.

[43] In 1979, 65.1 percent of women in the labor force held high school degrees or less. Source: Bureau of Labor Statistics, 2017.

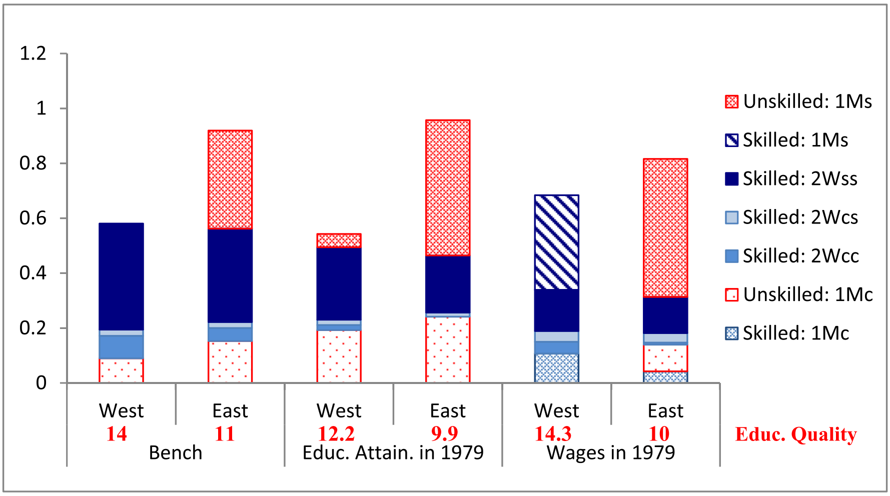

Fig. 3 Distribution of households across school districts after experiments (in millions)

a male worker at the Suburban Ring, who live around the fringe of the city. The locations around the Suburban Ring are occupied by the second largest group, skilled two-worker households with both workers at the Suburban Ring. Around the CBD, we also see a large number of unskilled one-worker households with a male worker at the CBD.

To measure the degree of segregation, we define two income groups based on whether each household has a single worker or two workers. We calculate a dissimilarity index, which measures the percentage of a group's population who would have to change residence for each school district to have the same percentage of that group as the city overall. The dissimilarity index is calculated as 0.33, which is lower than the benchmark value of 0.395.[44] This is a significant change and shows that the increase in educational attainment of women that leads to an increase in income inequality has a major impact on the increase in residential segregation.[45] To see which group benefits more from the change in female educational attainment, we also calculate the change in welfare for skilled and unskilled households, which is the percentage change in a household's benchmark consumption level needed to make the household as well off (i.e. achieving the same utility level) as he or she is at the new equilibrium, as in Hanushek and Yilmaz (2013).[46] Compared to the baseline equilibrium, skilled households are better off while unskilled households are worse off: the consumption of skilled and unskilled households should be decreased by 0.7% and increased by 0.4%, respectively from their benchmark values to make them as better off as they are at the new equilibrium.

44 Massey et al. (2009) report the dissimilarity index for the U.S. in 1990 as 0.43 in 1990 and 0.37 in 2000.

45 Prior research has established a strong link between the rise in income inequality and the rise in income segregation from 1970-2000 (Reardon & Bischoff, 2011), Watson (2009).

46 Our analysis ignores the welfare of absentee landlord. For a more detailed and careful analysis of school finance policies on welfare and equality in educational opportunity, see Hanushek and Yilmaz (2007, 2013).

A Change in Wages

The second experiment sets the wage rates for female and male workers at about their values in 1979 and still keeps the wage gradient at one percent to calculate the wages at the Suburban Ring. In 1979, the inflation-adjusted median weekly earnings (in constant 2016 dollars) for female workers with a high school and college degree were \$571 and \$815, respectively, while those for male workers were \$951 and %1,222, respectively. Moreover, women worked more hours while men worked fewer hours: the average weekly work hours for female and male workers were 34.5 and 42, respectively.[47] Thus, the hourly wage rates for male workers with a high school degree and a college degree are set to be at \$22.6 and \$29.1, respectively while those for female workers are set to be \$16.8 and \$23.6, respectively. With the new wages, the female worker wage rate for skilled households is not only lower but also the college wage premium for female workers is lower than that of the baseline equilibrium. Moreover, the male worker wage rate for unskilled households is higher while the male wage rate for skilled households is lower.

Relative to the baseline equilibrium, the labor force participation rate of female workers decreases to 27% while that of male workers remains at 100%: All female workers for unskilled households still choose not to work, and the female labor force participation rate of skilled households decreases to 45%. This is not surprising because, in 1979, the wages for female workers of skilled households were lower than those in 2016. As a result, more female workers of skilled households choose not to work, and the ones who choose to work reduce their average weekly working hours by 6.5%. Moreover, male workers increase their average weekly working hours by 14% because more wives choose not to work. As shown in Fig. 3, we observe more skilled one-worker households with a male worker at the CBD, fewer skilled two-worker households with both workers at the CBD, and more skilled one-worker households with a male worker at the Suburban Ring in the new equilibrium. All of the two-worker households, once again, happen to be skilled households even though some skilled households are one-worker households. The West provides a better education, and the segregation by household type increases: unskilled households completely disappear from the West. Moreover, the number of skilled two-worker households with both workers at the CBD decreases substantially in any school district: these households are replaced by skilled one-worker households with a male worker at the CBD in the West and, unskilled one-worker households with a male worker at the CBD in the East. Skilled one-worker households with a male worker in the CBD account for 10 percent of the city's household population. In the East, the proportion of unskilled one-worker households with a male worker at the Suburban Ring increases approximately by 50%.

The dissimilarity index is calculated as 0.16, which is much lower than the baseline equilibrium value of 0.395. We can conclude that the change in the wages for male and female workers also has caused an increase in income inequality and, thus, in residential segregation. As for the welfare change relative to the baseline equilibrium, skilled households are worse off as a result of lower wages, and unskilled households are better off as a result of higher wages: the consumption of skilled households should

[47] The Current Population Survey, U.S. Bureau of Labor Statistics

be decreased by 24.6% and that of unskilled households should be increased by 33.6% from their baseline equilibrium values, to make them as better off as they are at the new equilibrium.

Concluding Remarks

Our paper builds a model of residential, workplace, and labor supply choices of households in a hybrid Tiebout model with decentralized workplaces. In addition to a unifying treatment of location and community choices, the model also considers two-worker households, for the working members of which both labor supply and workplace decisions are endogenous. We calibrate our model to match some key empirical observations and test the predictions of the model in the data. Our calibrated model confirms that the female worker of some households chooses to work while the female worker of some others does not, in equilibrium. As illustrated by our Male Worker-only model, this decision has a substantial impact on the equilibrium labor supply decisions and spatial distribution of households across the city through the accessibility of location to workplaces and income channels for households' labor supply and residential choice decisions.

Over the last century, there were important demographic changes in the population: among other things, there has been an increase in the educational attainment of females; and the wages, especially the college wage premium, for both male and female workers have increased substantially. Due to the endogeneity of labor supply decisions along with the explicit modeling of residential choice, our model allows us to see the impact of these changes on the spatial distribution of households across the city. We conducted two experiments and found that both the increase in educational attainment and the changes in wages for male and female workers since 1979 have increased income inequality and, thus, residential segregation.

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