# Myopic capital market concerns and investment incentives in business alliances 

Hui Chen ${ }^{1}$ (1) . Thomas Pfeiffer ${ }^{2}$

Accepted: 27 January 2023
© The Author(s) 2023


#### Abstract

We study a publicly traded firm that cares about its short-term stock market performance while collaborating with a privately owned firm in a business alliance. The firms each undertake a relation-specific investment and then bargain over the allocation of the joint surplus generated by the alliance. The public firm's myopic market concerns affect both the total size of the surplus and how the firms divide the surplus. While the public firm always becomes more aggressive and obtains more of the surplus, the total size of the surplus may become larger or smaller, due to the effect of myopic market concerns on the firms' investment incentives. We establish conditions under which the investment and the value of each firm increase or decrease with market concerns. The market concerns could mitigate or exacerbate the holdup problem between the two firms and thus could either benefit or harm the whole business alliance. We also study two extensions with (i) the two investments being substitutes instead of complements and (ii) both firms being publicly listed. In both cases, the insights from our main model still hold.


Keywords Business alliance • Capital market concerns • Holdup problem • Investment spillover • Myopic behavior • Specific investment • Supply chain

JEL Classification M40 • M41

[^0]
## 1 Introduction

Strategic alliances are a significant phenomenon in the business world, with firms jointly working on various activities, such as research and development, marketing, production, and distribution. Recent years have seen a significant growth in these alliances (e.g., (Saada and Gomes-Casseres 2019)). ${ }^{1}$ These alliances can constitute an essential part of firm value, which the capital market considers when pricing publicly listed firms. Das et al. (1998, p. 27) argue that "investors ought to recognize the cash flow consequences of strategic alliances when these alliances are publicly announced. If the alliances lead to additional cash flow, the stock price reaction to alliance announcements should be favorable."

While the empirical literature examining the impact of business alliances on firm value consistently reports that the financial market reacts positively to firms' decisions to form alliances (e.g., (Chan et al. 1997; Das et al. 1998; McConnell and Nantell 1985; Koh and Venkatraman 1991)), there is only scant theory literature on the impact of financial markets on business alliances. We thus examine how a public firm that cares about its stock market performance could manage the market expectation by collaborating with a partner firm in a strategic alliance. We are particularly interested in how the public firm's concern for stock price affects the investment incentives in the alliance as well as in the effects of these investments on the value of the individual firms and the efficiency of the whole alliance.

It is well known that publicly traded companies care about their stock performance. In a single firm setting, Stein (1989) uses a signal jamming model and shows that a firm's short-term concern for the capital market could induce it to take actions that temporarily inflate market belief but at the expense of the long-term firm value. We also apply a signal jamming model to explore the potential effects of a firm's market concerns but focus on a business alliance with two firms interacting with each other. Our paper thus complements the literature by providing a partial analysis of the effects of public firms' market concerns, among other important factors that may also influence the formation of alliances. We show that a firm's capital market concerns affect not only its own investment decision and efficiency but also those of its business partner and the whole partnership. We identify conditions under which these effects from market concerns are positive or negative and show that market concerns could benefit the individual firms as well as the whole alliance under plausible circumstances.

In the baseline model, we consider a business alliance consisting of a publicly listed firm and a privately held partner firm. The collaboration between them comprises a significant part of their respective businesses. ${ }^{2}$ At the beginning of the

[^1]alliance, each firm must undertake a relationship-specific investment. The production technology of the alliance follows a Cobb-Douglas function, with the two firms' investments serving as the two production inputs. The output elasticities reflect the importance of the respective investments for the collaboration. Consistent with the hold-up literature, the undertaken investments can be perfectly observed by the firms in the alliance but not by outsiders. The firms then bargain over how much each firm receives from the jointly generated surplus. An interim stock price is formed for the public firm at the end of the first period, based on the capital market's expectation of the public firm's long-term cash flow. Due to its myopia about the stock price, the public firm has an incentive to inflate the market expectation of its future performance. We analyze how the public firm's capital market concerns affect the firms' decisions and the subsequent profitability.

We show that the public firm's market concerns make it a more aggressive bargainer during the negotiation. This result is intuitive, as the public firm now has a higher stake to negotiate for with its total payoff being scaled up by its market concerns. Consequently, the public firm's share of the joint surplus from the alliance increases in its degree of market concerns, while the share of the joint surplus for the nonpublic firm decreases in the public firm' market concerns. However, the public firm's concern for the stock market price does not necessarily prompt it to make a higher investment. Depending on the relative importance of the two firms' investments in their joint business, the public firm's incentive to invest may increase or decrease. When the public firm's investment is sufficiently more important for the alliance, the complementarity implies that the investments of both firms increase in the public firm's market concerns. However, if the nonpublic firm's investment is sufficiently more important than the public firm's investment in the alliance, the public firm with market concerns would make an investment lower than without market concerns. This result is obtained because the public firm's aggressive bargaining leaves the nonpublic firm less incentivized to invest in the alliance. Given that the nonpublic firm's investment matters more for the alliance, its dampened incentive is anticipated by the public firm, which then also reduces its own investment due to the inherent complementarity between the two investments.

The public firm's market concerns affect both the total size of the alliance and the allocation of the trading surplus generated by it. While the market concerns always help the public firm in securing more of the joint surplus, the effect of these concerns on the total size of the surplus can be both positive or negative, depending on the investment decisions of the two firms. The market concerns lead to an improved value for the public firm when its own investment is sufficiently more important than the nonpublic firm's investment. This result complements the conventional wisdom from signal jamming models in a single-firm setting, which typically suggests myopic firms overinvest at the cost of long-term firm value. The public firm's market concerns could also result in a lower firm value when the nonpublic firm's investment is significantly more important than the public firm's investment. Under these circumstances, the public firm still receives a higher share of the trade surplus, but the total surplus is smaller due to the underinvestment. In this case, the public firm can be worse off in the presence of market concerns.

From the perspective of the whole business alliance, a classical hold-up problem arises when the public firm does not face any market concerns. Both firms underinvest as they each have to bear the full cost of their respective investment but cannot receive the full benefits. The effect of the public firm's market concerns on the efficiency of the whole business alliance depends on the relative importance of these investments. When the public firm's investment is sufficiently more important, both firms invest more, which mitigates the hold-up problem and improves joint efficiency. However, the public firm's market concerns cannot lead the firms to make optimal investments at the first-best level and therefore cannot restore joint efficiency of the whole alliance. Furthermore, the public firm's market concerns are not always beneficial. If the nonpublic firm's investment is sufficiently more important than the public firm's investment, the public firm's market concerns exaggerate the inherent underinvestment problem and reduce joint efficiency.

We also consider two extensions to our main model. First, we allow the two investments to be complements as well as substitutes following Noeldeke and Schmidt (1998). Using a more general production function, we show that all of the equilibrium results obtained using the Cobb-Douglas function are still valid. Specifically, when the investments are substitutes, the public firm's investment always increases in its market concerns, while the nonpublic firm's investment always decreases. On balance, both firms and the whole business alliance can still benefit from the public firm's market concerns. However, the public firm is less likely to benefit, while the nonpublic firm is more likely to benefit from the public firm's market concerns.

Second, we examine the case where both firms in the alliance are publicly listed. Although most business alliances seem to be public-private (Lindsey 2008), publicpublic alliances often involve larger firms, which are more visible in the media. We model two symmetric firms that share the same levels of market concerns, bargaining power, and importance of investments in the alliance. As in the case with one public firm and one private firm, the market concerns influence the firms' investments, the amount of joint surplus received, and the payoffs. We confirm again that a certain degree of market concerns can benefit the firms individually as well as the whole alliance.

The results of our analysis have managerial implications. When firms form a business alliance, they should consider a public firm's concerns for the capital market as well as the relative importance of their inputs. When a private firm partners with a public firm, the private firm should be aware that the public firm faces pressure from the capital market. The market concerns will turn the public firm into a more aggressive bargainer in the negotiation, but that is not necessarily bad for the private firm. In fact, if the public firm's input into the business alliance is sufficiently important, its market concerns could encourage investment in the alliance, which improves the efficiency of the joint operation and benefits both firms involved. On the other hand, a public firm with high market concerns and a private firm with a more important input in the partnership would not constitute a good match. The public firm's aggressiveness will only further discourage the private firm from investing in the alliance, thus resulting in a more severe hold-up problem and reductions of efficiency.

Our paper contributes to three streams of literature. First, it contributes to the wellestablished literature on hold-up problems, including Grossman and Hart (1986),

Hart (1995), Hart and Moore (1999), and Segal and Whinston (2012). This literature takes the perspective of the welfare of the whole partnership and studies how a potential hold-up between individual parties can result in the undersupply of relation-specific investments and inefficiencies for all parties involved. The holdup problem has been widely applied to a variety of different inter- and intra-firm coordination problems. Of particular interest is how different agreements between the parties exacerbate or mitigate the hold-up problem (e.g., (Edlin and Reichelstein 1995; 1996; Noeldeke and Schmidt 1995; 1998; Anctil and Dutta 1999; Baldenius and Reichelstein 1999; Arya et al. 2000; Taylor and Plambeck 2007; Pfeiffer et al. 2011; Baldenius and Michaeli 2017; 2019; 2020; Dutta and Reichelstein 2021). Our paper shows how the market concerns of a publicly listed firm affect the hold-up problem through the investments of the parties, the distribution of the joint surplus, and the performance of the individual parties and the whole business alliance.

Second, our paper also closely relates to the signal jamming literature, including Holmstrom (1982), Fudenberg and Tirole (1986), Gibbons (1985), and Stein (1989). Holmstrom (1982) shows that career concerns in the labor market provide incentives for employees to overexert effort in the early stages of their career, to raise the labor market's assessment of their ability. Stein (1989) describes a scenario where managers with a myopic preference for a higher stock price engage in nonproductive actions/investments to manipulate market expectation. Signal jamming models are also applied in accounting research. Reviews provided by Kanodia (2006) and Kanodia and Sapra (2016) highlight how different accounting treatments of performance measures interact with the firm's investment decisions in the presence of capital market concerns. Liang and Wen (2007) show that firms may overor underinvest, depending on the accounting system used to measure their cash flows and investments. Kanodia et al. (2005) show that some imprecision in measuring a firm's investment can be beneficial when the profitability of the investment cannot be directly communicated to the capital market. Dutta and Reichelstein (2005) show that incentive schemes based on the stock price and the residual income measure induce proper managerial investment incentives and prevent myopic investment decisions of purely stock-based incentive schemes. Dutta and Nezlobin (2019) show that risk-averse shareholders can benefit from the nondisclosure of the firm's investment decision. Our study differs by examining signal jamming in a strategic alliance between two firms.

Third, our analysis contributes to the emerging but still limited research on how the capital market affects a firm's behavior in its partnership with other firms, such as in an alliance or a supply chain. Many firms engaged in partnerships are publicly listed and care about their stock market performance. However, the majority of theory research in this area remains silent on the importance of capital market concerns in the firms' operational decisions, with only a few exceptions. Lai et al. (2011) examine how a myopic manager of a public firm may use channel stuffing to mislead the market. The manager reports a sales level that is higher than the actual demand by pushing leftover inventory to downstream firms. Based on the signaling framework, Lai et al. (2012), Schmidt et al. (2015), and Lai and Xiao (2017) examine the effect of inventory stocking decisions in a newsvendor model to signal a firm's private information about its expected consumer demand to the capital market. Our analysis
shows that market concerns affect the hold-up problem between the members of a business alliance and could have a positive or negative effect under certain conditions.

## 2 Setup

We consider two risk-neutral firms that form a business alliance, which could be a joint venture for research and development or a supply chain. One of them is nonpublic and privately owned (denoted with the index " $N$ "), while the other is publicly traded (denoted with the index " $P$ "). The nonpublic firm cares about its value, whereas the public firm cares about the weighted average of the market price and firm value. The key difference between the firms is that the public firm faces pressure from the capital market, which induces myopic behavior, and the nonpublic firm does not.

While our analysis focuses on an alliance, both firms also independently generate profits from their own regular business in each of the two periods. We denote these profits as $\tilde{\pi}_{1 P}, \tilde{\pi}_{2 P}, \tilde{\pi}_{1 N}, \tilde{\pi}_{2 N}$, which are all normally distributed with means $\mu_{P}$ and $\mu_{N}$, and variances $\sigma_{P}^{2}$ and $\sigma_{N}^{2}$. The profits from the two different periods are positively correlated, with $\operatorname{Cov}\left[\widetilde{\pi}_{1 P}, \tilde{\pi}_{2 P}\right] \geq 0$ and $\operatorname{Cov}\left[\tilde{\pi}_{1 N}, \tilde{\pi}_{2 N}\right] \geq 0$. This implies that, when the first-period profit is high, the second-period profit is also likely to be high. All other random variables are uncorrelated.

### 2.1 Timeline

On date I-1, each of the two firms simultaneously undertakes a relation-specific investment, $I_{P}$ and $I_{N}$ respectively, to generate a joint surplus. The firms' specific investments are unobservable and unverifiable to outsiders. This is because many investments involve such activities as market research and technical development, which are inseparable from the firms' regular businesses and difficult to measure and report. To make these investments, the public firm incurs a cost of $w_{P}\left(I_{P}\right)=I_{P}$, and the nonpublic firm incurs a cost of $w_{N}\left(I_{N}\right)=I_{N} .{ }^{3}$

On date I-2, the firms decide whether to cooperate and establish a joint venture and, depending on this decision, then negotiate how to divide the joint surplus generated by that venture. The indicator variable $\tau$ denotes whether the two firms cooperate, with $\tau \in\{0,1\}$. For the sake of exposition, we describe the generated joint surplus when the firms cooperate with a Cobb-Douglas function, that is, $M(I, 1)=I_{P}^{a_{P}} \cdot I_{N}^{a_{N}}$ with $I=\left(I_{P}, I_{N}\right), a_{P}, a_{N} \geq 0$, and $a_{P}+a_{N}<1$ ensures decreasing returns to scale and a well-behaved problem. The parameters, $a_{P}$ and $a_{N}$, reflect the importance of the two firms' investments, respectively. When $a_{P}$ is higher than $a_{N}$, the public firm's investment is more important for the collaboration than the nonpublic firm's investment (and vice versa). The advantage of the

[^2]Period I


Fig. 1 Timeline of the events

Cobb-Douglas function is that it captures the complementary relation between the firms' investments and conceptualizes the surplus function in terms of the importance of the investments. When the firms do not cooperate, then they do not generate a joint surplus; that is, $M(I, 0)=0$ for all $I$. Since $M(I, 1)$ is always positive, it is rational for the firms to cooperate in our model.

When the firms cooperate on date I-2, as a result of the bargaining, which we describe later, the public firm receives a cash flow of $M_{P}(I, 1)$ and the nonpublic firm receives a cash flow of $M_{N}(I, 1)$, with $M_{P}(I, 1)+M_{N}(I, 1)=M(I, 1)$. Given $(I, \tau)$, the public and nonpublic firms' cash inflows are realized on date I-3 and equal

$$
\begin{align*}
& \tilde{x}_{1 P}(I, \tau)=\tilde{\pi}_{1 P}+M_{P}(I, \tau) \\
& \tilde{x}_{1 N}(I, \tau)=\tilde{\pi}_{1 N}+M_{N}(I, \tau) . \tag{1}
\end{align*}
$$

Whether the two firms have established a business alliance is public information, for example, due to some public announcement. The total cash inflow of the public firm is publicly observable but not its individual components, as they are often not reported or difficult to disentangle. ${ }^{4}$ The total cash inflow of the nonpublic firm is not observable.

To separate the cash flows from two periods, we assume that the public firm's cash flows generated at the first period are distributed back to the shareholders. Then the stock market updates its information about the public firm and forms an interim market price $p$. In period II, the firms' final cash flows are realized. Figure 1 depicts the timeline of the events.

The structure of the game is common knowledge. All players know the objective functions of other players, the structure of the cash flows, the characteristics of the

[^3]underlying technology, and the structure of the bargaining game. At date I-1, the firms simultaneously invest. They then observe each others' investments before they bargain at date I-II. However, the investments are unobservable and unverifiable by outsiders, such as the capital market. At date I-3, the capital market observes whether the two firms cooperate as well as the public firm's cash flow from the first period but not its individual components.

### 2.2 Market price and payoffs

The market price $p$ for the public firm on date I-3 reflects the market's expectation of future cash flows in period II, given the market's information and conjectures. The market knows whether the firms have formed an alliance and observes the period-I cash flow of the public firm. The market price of the public firm is $p=E\left[\widetilde{x}_{2 P} \mid\right.$ $\left.\left\{x_{1 P}, \tau, \widehat{M}_{P}(\widehat{I}, \tau)\right\}\right]$, where market conjectures are denoted with " "". Specifically, the market conjectures, $\widehat{M}_{P}(\widehat{I}, \tau)$, are based on the publicly available information, $\tau$, about whether the firms have collaborated.

The Bayes' theorem implies that the market updates its belief about the firm value using the realized period-I cash flow and the familiar covariance-variance ratio, $\beta=\frac{\operatorname{Cov}\left[\tilde{\pi}_{1 P}, \tilde{\pi}_{2 P}\right]}{\operatorname{Var}\left[\tilde{\pi}_{1 P}\right]}>0$. The market price on date I-3 equals

$$
\begin{equation*}
p=E\left[\tilde{x}_{2 P}\right]+\beta \cdot\left(\pi_{1 P}+M_{P}(I, \tau)-E\left[\tilde{\pi}_{1 P}\right]-\widehat{M}_{P}(\widehat{I}, \tau)\right) . \tag{2}
\end{equation*}
$$

The realized period-I cash flow, that is, $x_{1 P}(I, \tau)=\pi_{1 P}+M_{P}(I, \tau)$ from (1), reflects the decisions undertaken by the two firms, whereas $E\left[\widetilde{\pi}_{1 P}\right]+\widehat{M}_{P}(\widehat{I}, \tau)$ reflects the market's expectation regarding those decisions. If the cash flows of the two periods are uncorrelated, the market price does not contain any weight on the period-I cash flow and equals the expected period-II cash flow; that is, $\beta=0$ if $\operatorname{Cov}\left[\widetilde{\pi}_{1 P}, \widetilde{\pi}_{2 P}\right]=0$. If the correlation increases, the period-I cash flow entails more information about the period-II cash flow, and the market assigns more weight on the realized period-I cash flow.

Given (2), the expected market price on dates I-1 and I-2, respectively, equals

$$
\begin{equation*}
E[\widetilde{p}]=E\left[\widetilde{x}_{2 P}\right]+\beta \cdot\left(M_{P}(I, \tau)-\widehat{M}_{P}(\widehat{I}, \tau)\right) \tag{3}
\end{equation*}
$$

When the firms cooperate, the expected market price increases in the public firm's payoff from the business alliance, that is, $M_{P}(I, 1)$. The public firm thus has an incentive to increase the expected market price by undertaking high investments and negotiating more aggressively to obtain a higher payoff. In equilibrium, however, the market anticipates this behavior and adjusts its conjectures correctly.

The public firm is motivated by both the firm's interim market price and long-term cash flows, weighted with $\alpha \in[0,1]$ and $(1-\alpha)$, respectively. The parameter, $\alpha$, reflects the preferences of the shareholders and is exogenously given and not a choice variable. ${ }^{5}$ The public firm's expected total payoff, $E\left[U_{P}\right]$, equals

$$
\begin{equation*}
E\left[U_{P}\right]=(1-\alpha) \cdot E\left[\tilde{x}_{2 P}\right]+\alpha \cdot E[\widetilde{p}]+E\left[\tilde{x}_{1 P}\right]-w_{P}\left(I_{P}\right), \tag{4}
\end{equation*}
$$

[^4]where $(1-\alpha) E\left[\widetilde{x}_{2 P}\right]+\alpha E[\widetilde{p}]$ is the weighted average of the period-II cash flow and the expected interim market price from (3), and $E\left[\tilde{x}_{1 P}\right]-w_{P}$ is the public firm's net payoff for period I. Given the period-I cash flow and the expected market price from (1) and (3), the public firm's expected total payoff (4) can be restated as follows:
\[

$$
\begin{equation*}
E\left[U_{P}\right]=2 \mu_{P}+(1+\alpha \beta) \cdot M_{P}(I, \tau)-\alpha \beta \cdot \widehat{M}_{P}(\widehat{I}, \tau)-w_{P}\left(I_{P}\right) . \tag{5}
\end{equation*}
$$

\]

Absent the public firm's market concerns, the expected total payoff equals the sum of the expected net cash flows. This is equivalent to two nonpublic firms forming a business alliance, which has been discussed in the hold-up literature.

The nonpublic firm does not face pressure from the capital market and cares about its value. Given (1), the value of the nonpublic firm, $E\left[U_{N}\right]$, is the sum of the expected net cash flows of the two periods; that is,

$$
\begin{align*}
E\left[U_{N}\right] & =E\left[\widetilde{x}_{2 N}\right]+E\left[\widetilde{x}_{1 N}\right]-w_{N}\left(I_{N}\right) \\
& =2 \mu_{N}+M_{N}(I, \tau)-w_{N}\left(I_{N}\right) . \tag{6}
\end{align*}
$$

## 3 Analysis

In this section, we examine the impact of the public firm's market concerns on the decisions of both firms as well as the implications for the whole business alliance. We are interested in how the market concerns affect the investment levels and the shares of surplus that firms obtain through negotiation and whether the public firm's market concerns benefit or harm the firms and the whole business alliance.

### 3.1 Bargaining and investments

We examine the firms' decisions using backward induction. As the market price will be determined on date I-3, the firms have to take the market conjectures as "fixed" when they make their decisions on dates I-2 and I-1. On date I-2, both firms decide whether to establish a business alliance and then negotiate over the amount that each receives from the generated joint surplus. By date I-2, the investments have already been made, and the costs are sunk. Depending on whether the two firms cooperate, the public firm's expected total payoff from (5) is:

$$
\begin{align*}
& E\left[U_{P}(1)\right]=2 \mu_{P}+(1+\alpha \beta) \cdot M_{P}(I, 1)-\alpha \beta \cdot \widehat{M}_{P}(\widehat{I}, 1) \\
& E\left[U_{P}(0)\right]=2 \mu_{P}-\alpha \beta \cdot \widehat{M}_{P}(\widehat{I}, 0) \tag{7}
\end{align*}
$$

Similarly, the nonpublic firm's expected total payoff equals $E\left[U_{N}(1)\right]=2 \mu_{N}+$ $M_{N}(I, 1)$ and $E\left[U_{N}(0)\right]=2 \mu_{N}$ from (6).

The two firms negotiate over the amount that each receives from the joint surplus, à la generalized Nash bargaining (e.g., (Myerson 1991)). This well-established approach provides a characterization of bargaining outcomes without requiring an explicit representation of the bargaining process; that is,

$$
\begin{align*}
& \left(E\left[U_{P}(1)\right]-E\left[U_{P}(0)\right]\right)^{b} \cdot\left(E\left[U_{N}(1)\right]-E\left[U_{N}(0)\right]\right)^{1-b} \rightarrow \max _{\left\{M_{P}(I, 1), M_{N}(I, 1)\right\}} \\
& \text { s.t. } M_{P}(I, 1)+M_{N}(I, 1)=M(I, 1), \tag{8}
\end{align*}
$$

where $b \in[0,1]$ reflects the exogenous bargaining power of the public firm and $(1-b)$ reflects the nonpublic firm's bargaining power.

Solving the bargaining problem in (8) shows that the public firm receives a payoff of

$$
\begin{equation*}
M_{P}(I, 1)=\left(b+\alpha \beta \cdot \frac{1-b}{1+\alpha \beta} \frac{\widehat{M}_{P}(\widehat{I}, 1)-\widehat{M}_{P}(\widehat{I}, 0)}{M(I, 1)}\right) \cdot M(I, 1) \tag{9}
\end{equation*}
$$

Absent market concerns, the public firm would receive a share of the joint surplus that equals its bargaining power, that is, $M_{P}(I, 1)=b \cdot M(I, 1)$ if $\alpha=0$, which is a standard result in the hold-up literature. With market concerns, the public firm's net expected total payoff is scaled up to $(1+\alpha \beta) \cdot M_{P}(I, 1)$, as depicted in (7), while the nonpublic firm's interest remains at the same level at $M_{N}(I, 1)$. The increase of the public firm's stake in the alliance beyond its bargaining power results in it bargaining more aggressively.

Since the structure of the game is known by the capital market, the market conjectures correctly that $\widehat{M}_{P}(\widehat{I}, 1)=M_{P}(I, 1)$ and $\widehat{M}_{P}(\widehat{I}, 0)=0$, yielding the following bargaining outcome

$$
\begin{equation*}
M_{P}(I, 1)=\gamma \cdot M(I, 1) \text { with } \gamma=b \cdot \frac{1+\alpha \beta}{1+b \alpha \beta} \tag{10}
\end{equation*}
$$

and $M_{N}(I, 1)=(1-\gamma) \cdot M(I, 1)$ where $\gamma$ denotes the public firm's equilibrium share of the joint surplus. Equation (10) implies that the public firm's share $\gamma$ equals its bargaining power $b$ only if it has (1) zero bargaining power, (2) full bargaining power, or (3) no market concerns; that is, $\gamma=b$ if $b \in\{0,1\}$ or $\alpha=0 .{ }^{6}$ Given $b \in(0,1)$, the public firm's share of surplus increases in its degree of market concerns; that is,

$$
\begin{equation*}
\frac{\partial \gamma}{\partial \alpha}=\frac{b(1-b) \beta}{(1+b \alpha \beta)^{2}}>0 \tag{11}
\end{equation*}
$$

Since each firm receives a positive payoff from the alliance, the firms will cooperate.

Next we examine the firms' investment decisions on date I-1. Each of the firms chooses an investment level that maximizes its total payoff, anticipating they will divide the joint surplus according to (10). They also anticipate that the capital market will determine the market price based on the resulting period-1 cash flow, $\widetilde{\pi}_{1 P}+$ $\gamma M(I, 1)$. Consistent with (2) and (3), the expected market price equals $E[\widetilde{p}]=$ $\mu_{P}+\beta \cdot\left(\gamma M(I, 1)-\widehat{M}_{P}(\widehat{I}, 1)\right)$ and the firms' objective functions from (5) and (6) are as follows:

$$
\begin{align*}
& \max _{I_{P}} 2 \mu_{P}+(1+\alpha \beta) \cdot \gamma M\left(I_{P}, I_{N}, 1\right)-\alpha \beta \cdot \widehat{M}_{P}(\widehat{I}, 1)-w_{P}\left(I_{P}\right) \\
& \max _{I_{N}} 2 \mu_{N}+(1-\gamma) M\left(I_{P}, I_{N}, 1\right)-w_{N}\left(I_{N}\right) \tag{12}
\end{align*}
$$

Each firm accounts for the other firm's investment choice as is standard in a twoplayer simultaneous move game. Since the structure of the game is known by both

[^5]firms, the public and nonpublic firms' first-order conditions constitute the reaction functions, and the equilibrium investments satisfy:
\[

$$
\begin{align*}
& (1+\alpha \beta) \cdot \gamma \frac{\partial M\left(I_{P}, I_{N}, 1\right)}{\partial I_{P}}=w_{P}^{\prime}\left(I_{P}\right) \\
& (1-\gamma) \frac{\partial M\left(I_{P}, I_{N}, 1\right)}{\partial I_{N}}=w_{N}^{\prime}\left(I_{N}\right) . \tag{13}
\end{align*}
$$
\]

For both firms, the marginal investment costs (i.e., RHS) equal the marginal benefits of the investments (i.e., LHS). Without market concerns, the marginal benefit of the public firm's investment equals its share of the marginal joint surplus. Consistent with the literature on signal jamming, the additional term, $\alpha \beta \cdot \gamma \frac{\partial M(I, 1)}{\partial I_{P}}$, reflects the public firm's incentive to inflate its market price by increasing its investment. The nonpublic firm does not have any market concerns and equates marginal benefits with its share of the marginal joint surplus. The investment incentives of both firms are linked through the jointly generated profit margin that exhibits a positive complementarity between the two investments.

Proposition 1 establishes the firms' equilibrium payoffs from the business alliance and the equilibrium investments made by the two firms.

Proposition 1 In equilibrium, the firms' payoffs from the business alliance are $M_{P}(I, 1)=\gamma M(I, 1)$ and $M_{N}(I, 1)=(1-\gamma) M(I, 1)$ with $\gamma=b \cdot \frac{1+\alpha \beta}{1+b \alpha \beta}$. The equilibrium investments made by the public firm and the nonpublic firm are

$$
\begin{aligned}
& I_{P}=\left((1+\alpha \beta) \gamma a_{P}\right)^{\frac{1-a_{N}}{1-a_{P}-a_{N}}} \cdot\left((1-\gamma) a_{N}\right)^{\frac{a_{N}}{1-a_{P}-a_{N}}} \\
& I_{N}=\left((1+\alpha \beta) \gamma a_{P}\right)^{\frac{a_{p}}{1-a_{P}-a_{N}}} \cdot\left((1-\gamma) a_{N}\right)^{\frac{1-a_{p}}{1-a_{P}-a_{N}}} .
\end{aligned}
$$

Proposition 1 shows that the public firm's market concerns make it a more aggressive bargainer and increase its share of the surplus at the expense of the nonpublic firm. One might thus conjecture that the public firm's equilibrium investment would increase in its market concerns, whereas the nonpublic firm's equilibrium investment decreases. This conjecture is only true when the complementarity between the two investments is low. When the complementarity between them is high, a spillover effect is generated. As a result, both investments increase in $\alpha$ when the public firm's investment is important, and both investments decrease in $\alpha$ when the nonpublic firm's investment is important. Intuitively, if the public firm's investment is sufficiently important and the public firm invests more, the increase in the nonpublic firm's marginal benefit from the increased surplus offsets the lower share of surplus the nonpublic firm receives. Similarly, if the nonpublic firm's investment is sufficiently important and the nonpublic firm invests less, the public firm's marginal benefit decreases so severely that even a higher share of surplus cannot offset this effect.

For the sake of convenience, we state the marginal investments in terms of the public firm's share of the joint surplus and the investments from Proposition 1; that is,

$$
\begin{align*}
\frac{\partial I_{P}}{\partial \alpha} & =\frac{2-\gamma-2 a_{N}}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)} \cdot \beta I_{P} \\
\frac{\partial I_{N}}{\partial \alpha} & =\frac{2 a_{P}-\gamma}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)} \cdot \beta I_{N} \tag{14}
\end{align*}
$$

Equation (14) confirms our intuition that we summarize in Corollary 3.1.
Corollary 3.1 The equilibrium investments exhibit the following comparative static results with respect to the public firm's market concerns; that is, (i) $\frac{\partial I_{P}}{\partial \alpha} \geq 0$ and $\frac{\partial I_{N}}{\partial \alpha} \geq 0$ iff $a_{N} \leq 1-\frac{\gamma}{2}$ and $a_{P} \geq \frac{\gamma}{2}$, (ii) $\frac{\partial I_{P}}{\partial \alpha} \geq 0$ and $\frac{\partial I_{N}}{\partial \alpha} \leq 0$ iff $a_{N} \leq 1-\frac{\gamma}{2}$ and $a_{P} \leq \frac{\gamma}{2}$, and (iii) $\frac{\partial I_{P}}{\partial \alpha} \leq 0$ and $\frac{\partial I_{N}}{\partial \alpha} \leq 0$ iff $a_{N} \geq 1-\frac{\gamma}{2}$ and $a_{P} \leq \frac{\gamma}{2}$.

Corollary 3.1 shows that the public firm's market concerns can lead to higher or lower firm investments, depending on the relative importance of the two firms' investments in the alliance. Figure 2 presents the three cases outlined in Corollary 3.1 by numerically illustrating the impact of the public firm's market concerns on the firms' investment decisions.


Fig. 2 A illustrates the impact of the public firm's market concerns on the public firm's investment decision (blue line) and the nonpublic firm's investment decision (orange line). B illustrates the impact of the public firm's market concerns on the value of the public firm (blue line), nonpublic firm (orange line) and the whole alliance (green line)

### 3.2 Impact of market concerns on the individual firm values

We now examine how the public firm's market concerns affect the value of each firm. For the public firm, given the equilibrium values from Proposition 1, its value equals the expected total payoff from (5), $V_{p}(\alpha)=2 \mu_{P}+\gamma(\alpha) M(I(\alpha), 1)-w_{P}\left(I_{P}(\alpha)\right)$, where the expectation is taken before the investments are made. The public firm's market concerns affect $V_{p}$ through its effects on both investments and on the public firm's share of the joint surplus. Given (13), the marginal firm value reflects the impact of these three forces and equals: ${ }^{7}$

$$
\begin{equation*}
\frac{d V_{p}}{d \alpha}=-\alpha \beta \gamma \cdot \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}+\frac{\partial \gamma}{\partial \alpha} \cdot M+\gamma \cdot \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha} . \tag{15}
\end{equation*}
$$

The first term captures the effect of the public firm's market concerns on its value via its investment. As the public firm with market concerns overinvests to manage market expectations, this term has a negative sign when the public firm's investment increases in $\alpha$. This result is consistent with Stein's (1989) insight that myopic behavior induces value destroying investment incentives. While the effect of market concerns in Stein (1989) is always unidirectional, the public firm's investment in our model could increase or decrease in $\alpha$, implying that the first term has either a negative or positive value. The second term shows that higher market concerns result in a higher share of the joint surplus for the public firm, which raises firm value. The third term reflects the effect of the public firm's market concerns on firm value via the nonpublic firm's investments. This term has either a positive or a negative value, depending on whether the nonpublic firm's investment increases or decreases in $\alpha$.

Substituting the marginal investments from (14) and $\gamma$ from Proposition 1 into the marginal firm value (15) yields:

$$
\begin{equation*}
\frac{d V_{p}}{d \alpha}=\left(k_{0}+k_{1} \alpha+k_{2} \alpha^{2}\right) \cdot \frac{b \beta M(I)}{\left(1-a_{P}-a_{N}\right)(1+b \beta \alpha)^{2}} . \tag{16}
\end{equation*}
$$

The sign of (16) is determined by the sign of the quadratic function with coefficients $k_{0}=(1-b)\left(1-a_{P}\right)-a_{N}\left(1-2 a_{P}\right)$, and $k_{1}$ and $k_{2}$ are depicted in the proof for Proposition 2. The proof shows that the sign of the intercept $k_{0}$ matters. When $k_{0}<0$, the public firm's firm value unambiguously decreases in its market concerns. When $k_{0}>0$, the firm value increases in its market concerns for $\alpha \leq \bar{\alpha}_{P}$ and decreases for $\alpha \geq \bar{\alpha}_{P}$, where $\bar{\alpha}_{P}$ is the unique level that maximizes the firm value. The sign of the intercept $k_{0}$ can be expressed in terms of the importance of the public firm's and nonpublic firm's investments, as outlined in Proposition 2.

Proposition 2 (i) When the public firm's investment is sufficiently important, relative to the nonpublic firm's investment, that is, when either (a) $a_{P}<\frac{1}{2}$ and $a_{N}<\frac{(1-b)\left(1-a_{P}\right)}{1-2 a_{P}}$ or $(b) a_{P} \geq \frac{1}{2}$, the value of the public firm increases in its market concerns for $\alpha \leq \bar{\alpha}_{P}$ and decreases for $\alpha \geq \bar{\alpha}_{P}$, with $\bar{\alpha}_{P}=$ (A8).

[^6](ii) When the nonpublic firm's investment is sufficiently important, relative to the public firm's investment, that is, when $a_{P}<\frac{1}{2}$ and $a_{N}>\frac{(1-b)\left(1-a_{P}\right)}{1-2 a_{P}}$, the value of the public firm decreases in its market concerns.

Now we turn our attention to the nonpublic firm. The value of the nonpublic firm equals the expected total payoff from (6); that is, $V_{N}=E\left[U_{N}\right]$. Given the equilibrium values from Proposition 1, the firm value is $V_{N}(\alpha)=2 \mu_{N}+(1-\gamma(\alpha))$. $M(I(\alpha), 1)-w_{N}\left(I_{N}(\alpha)\right)$. On the one hand, the nonpublic firm is disadvantaged because the public firm becomes an aggressive bargainer, which reduces the nonpublic firm's share of surplus in the negotiation. On the other hand, the nonpublic firm may benefit from the public firm's market concerns when they increase the public firm's investment. The nonpublic firm's investment itself does not play a role, as the nonpublic firm invests efficiently from its own perspective. Applying the Envelope Theorem shows that the nonpublic firm's expected marginal payoff reflects these two forces:

$$
\begin{equation*}
\frac{d V_{N}}{d \alpha}=-\frac{\partial \gamma}{\partial \alpha} \cdot M+(1-\gamma) \cdot \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha} . \tag{17}
\end{equation*}
$$

Balancing these two forces in (17), Proposition 3 shows under which circumstances the nonpublic firm benefits or suffers from the public firm's market concerns.

Proposition 3 (i) When the public firm's investment is sufficiently important for the alliance, that is, when $a_{P}>\frac{b}{2}$, the value of the nonpublic firm increases in the public firm's market concerns for $\alpha \leq \bar{\alpha}_{N}$ and decreases for $\alpha \geq \bar{\alpha}_{N}=$ (A10).
(ii) When the public firm's investment is insufficiently important for the alliance, that is, when $a_{P}<\frac{b}{2}$, the value of the nonpublic firm decreases in the public firm's market concerns.

Figure 2 illustrates the impact of the public firm's market concerns on the values of the public firm and the nonpublic firm, respectively.

### 3.3 Impact of market concerns on the value of the whole alliance

In this section, we change our perspective and focus on the whole business alliance. From the perspective of the whole alliance, a hold-up problem arises since each firm must bear the full cost of the investment but does not receive the full benefits. Corollary 3.1 shows that the market concerns of the public firm can increase or reduce the investments of the firms. We will show that the public firm's market concerns mitigate the hold-up problem and increase the joint efficiency when the public firm's investment is more important than the nonpublic firm's investment. Conversely, when the nonpublic firm's investment is more important than the public firm's investment, the public firm's market concerns instead exacerbate the hold-up problem and further reduce joint efficiency.

To verify this intuition, we first consider the first-best scenario under which the business alliance undertakes the investments to maximize its total value, which equals the expected total payoff, $V_{T}=2\left(\mu_{P}+\mu_{N}\right)+M(I, 1)-w_{P}\left(I_{P}\right)-w_{N}\left(I_{N}\right)$ from
(5) and (6). The resulting first-best investments, $I^{*}=\left(I_{P}^{*}, I_{N}^{*}\right)$, satisfy the following standard first-order conditions:

$$
\begin{equation*}
\frac{\partial M\left(I^{*}, 1\right)}{\partial I_{P}}=w_{P}^{\prime}\left(I_{P}^{*}\right) \text { and } \frac{\partial M\left(I^{*}, 1\right)}{\partial I_{N}}=w_{N}^{\prime}\left(I_{N}^{*}\right) \tag{18}
\end{equation*}
$$

The marginal investment costs (i.e., LHS) equal the marginal benefits from the investments (i.e., RHS).

Next we consider the impact of the public firm's market concerns on the value of the whole business alliance when the individual firms undertake the investments as outlined in (13). The marginal value of the whole alliance equals the sum of the marginal values of the two firms from (15) and (17); that is,

$$
\begin{equation*}
\frac{d V_{T}}{d \alpha}=(1-\gamma-\alpha \beta \gamma) \cdot \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}+\gamma \cdot \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha} . \tag{19}
\end{equation*}
$$

The marginal value in (19) depends on how the public firm's market concerns affect the total size of the joint surplus via the investments but not on how the firms divide the surplus. From the perspective of the whole alliance, the nonpublic firm underinvests, and, as the second term in (19) shows, the whole business alliance benefits when the nonpublic firm's investment increases. The logic for the public firm is more nuanced. From the perspective of the whole alliance, the public firm overinvests if its market concerns are high, that is, if $\alpha \geq[1-\gamma] /[\gamma \beta]$, and underinvests otherwise. In the case of underinvestment, the first term in (19) shows that the value of the whole alliance increases when the public firm's investment increases in the public firm's market concerns. This logic reverses in the case of overinvestment and the value of the whole alliance decreases when the public firm's investment increases.

Balancing the two forces in (19), Proposition 4 shows under which conditions the value of the whole business alliance increases or decreases in the public firm's market concerns, with $\bar{\alpha}_{T}$ denoting the level of the public firm's market concerns that maximizes the value of the alliance.

Proposition 4 (i) When the public firm's investment is sufficiently important, relative to the nonpublic firm's investment, that is, when $a_{P}>\frac{b^{2}}{(1-b)(2-b)+a_{N}(4 b-2)} a_{N}$ and $a_{N}<\frac{(1-b)(2-b)}{2-4 b}$ if $b<\frac{1}{2}$, the value of the whole business alliance increases in the public firm's market concerns for $\alpha \leq \bar{\alpha}_{T}$ and decreases for $\alpha \geq \bar{\alpha}_{T}$, with $\bar{\alpha}_{T}=$ (A13).
(ii) When the nonpublic firm's investment is sufficiently important, relative to the public firm's investment, that is, when $a_{N}>\frac{(1-b)(2-b)}{b^{2}+a_{P}(2-4 b)} a_{P}$ and $a_{P}<\frac{b^{2}}{4 b-2}$ if $b>$ $\frac{1}{2}$, the value of the whole business alliance decreases in the public firm's market concerns.

Figure 2 illustrates the impact of the public firm's market concerns on the value of the whole business alliance. Figure 3 illustrates under which conditions in terms of the importance of the investments, $a_{P}$ and $a_{N}$, the public firm, the nonpublic firm, and the whole business alliance benefit from the public firm's market concerns, as established in Propositions 2, 3, and 4.


Fig. 3 The first panel illustrates the case of the public firm, the second panel illustrates the case of the nonpublic firm, and the last panel illustrates the case of the whole alliance. Area I depicts the area under which the public firm's market concerns have a positive impact on the respective value and Area II depicts the area under which the public firm's market concerns have a negative impact. Parameter is $b=1 / 2$ for all panels

## 4 Extensions

In this section of the paper, we consider extensions to the main model. Specifically, we allow (i) the two investments to be complements or substitutes and (ii) both firms to be publicly listed and have market concerns. Our analyses show that the insights of our main model still hold in both extensions.

### 4.1 Investments are complements or substitutes

So far we have used the Cobb-Douglas function for the joint surplus of the business alliances, which implies that the two investments are complements. The literature (e.g., (Noeldeke and Schmidt 1998)) has defined investments as complements at the margin if a larger investment by one firm increases the marginal benefit of the other firm's investment, that is, if $\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}>0$; as substitutes at the margin if $\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}<0$;
and as independent if $\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}=0$. In this section, we show that our key insights using the Cobb-Douglas function remain valid for a general joint surplus function $M(I)$ that accommodates all three relations of investments. The surplus function satisfies standard regularity conditions and has a positive determinant that ensures an unique equilibrium; that is, $\frac{\partial M}{\partial I_{i}}>0, \frac{\partial^{2} M}{\partial I_{i}^{2}}<0$ and $\operatorname{det}[M]=\frac{\partial^{2} M}{\partial I_{P}^{2}} \frac{\partial^{2} M}{\partial I_{N}^{2}}-\left(\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}\right)^{2}>0$ for $i=P, N$.

Our derivation of each firm's share of surplus from bargaining in (10) and the reaction functions of the investment decisions in (13) do not rely on the specific structure of the joint surplus. Thus these results do not change with the surplus function. When the investments are substitutes, we show the public firm's investment always increases in its own market concerns, whereas the nonpublic firm's investment decreases. This amplifies the public firm's overinvestment problem while discouraging the nonpublic firm from investing in the alliance. Consequently, the public firm's value is reduced, but it still benefits from a bigger share of the surplus. The nonpublic firm benefits from the higher investment of the public firm but suffers from a smaller share of the surplus. On balance, both firms as well as the whole business alliance can benefit from the public firm's market concerns.

To examine the effect of the public firm's market concerns $\alpha$ on the firms' investment decisions, we apply the implicit function theorem to the first-order conditions in (13), which yields two equations for the public firm and the nonpublic firm, respectively (see Equation (A15) in the Appendix):

$$
\begin{align*}
& \frac{\partial I_{P}}{\partial \alpha}=\beta \cdot \overbrace{-(2-b(1-\alpha \beta))(1-b) \cdot \frac{\partial^{2} M(I(\alpha), 1)}{\partial I_{N}^{2}}-(1+\alpha \beta)^{3} b^{2} \cdot \overbrace{\frac{\partial^{2} M(I(\alpha), 1)}{\partial I_{P} \partial I_{N}}}^{\text {cross effect }(+/-)}}^{(1+\alpha \beta)^{3}(1-b) b \cdot \operatorname{det}[M]} \begin{array}{c}
\overbrace{(1+\alpha \beta)^{3} b^{2} \cdot \frac{\partial^{2} M(I(\alpha), 1)}{\partial I_{P}^{2}}+(2-b(1-\alpha \beta))(1-b) \cdot \frac{\partial^{2} M(I(\alpha), 1)}{\partial I_{P} \partial I_{N}}}^{\text {cross effect (-/+)}}
\end{array})  \tag{20}\\
& \frac{\partial I_{N}}{\partial \alpha}=\beta \cdot \overbrace{(1+\alpha \beta)^{3}(1-b) b \cdot \operatorname{det}[M]}^{\text {direct effect }(+)}
\end{align*}
$$

The sign of the denominator is always positive. The numerators are composed of two terms, one for the direct effect of $\alpha$ on the firm's own investment and the other for the cross effect of $\alpha$ that reflects the relation between the two investments.

For the public firm, the direct effect is always positive, while the cross effect depends on the relation between the two firms' investments. Obviously, when the investments are independent, the cross effect is zero. When the investments are substitutes, the cross effect is positive, and $\frac{\partial I_{P}}{\partial \alpha}$ is always positive. When the investments are complements, the cross effect is negative, and $\frac{\partial I_{P}}{\partial \alpha}$ can be positive or negative, depending on the degree of complementarity. The result for the nonpublic firm mirrors that of the public firm with reversed signs. The direct effect is always negative, while the cross effect is negative/positive when the two investments are substitutes/complements. In sum, three cases emerge. Both $I_{P}$ and $I_{N}$ can increase
or decrease together, or $I_{N}$ can decrease when $I_{P}$ increases. ${ }^{8}$ These results resonate with Corollary 3.1, which identifies three cases of comparative static results using the Cobb-Douglas surplus function. No additional case emerges as a result of using a general surplus function.

Next we examine the impact of $\alpha$ on the value of each firm. As outlined in (16) and (17), the marginal value of the public firm and the nonpublic firm, respectively, are given by:

$$
\begin{align*}
\frac{d V_{p}}{d \alpha} & =\left(-\alpha \beta \gamma \cdot \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}+\gamma \cdot \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha}\right)+\left(\frac{\partial \gamma}{\partial \alpha} \cdot M\right) \\
\frac{d V_{N}}{d \alpha} & =\left((1-\gamma) \cdot \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}\right)-\left(\frac{\partial \gamma}{\partial \alpha} \cdot M\right) . \tag{21}
\end{align*}
$$

The marginal firm values depend on the effect of $\alpha$ via the investment decisions (first terms in (21)) and the marginal share of the surplus each firm receives (second terms). The interpretations of these terms follow as discussed in (16) and (17). As in our main analyses with the Cobb-Douglas function, the second terms imply that the public firm always benefits from $\alpha$ through a larger share of surplus while the nonpublic firm always suffers due to a smaller share. Conceptually, the signs of the first terms are consistent with that of the Cobb-Douglas function, when the investments are substitutes as well as complements. In more detail, when the two investments are substitutes, the public firm's investment increases in $\alpha$, whereas the nonpublic firm's investment decreases in $\alpha$. This implies that $\alpha$ has a negative impact on the public firm's firm value in (21) via both investments. Conversely, $\alpha$ has a positive impact on the nonpublic firm's firm value in (21) via the public firm's investment. Conceptually, this is consistent with our main analyses using the Cobb-Douglas function and satisfying the conditions of case (ii) in Corollary 3.1.

To determine the sign of the marginal firm value of each firm in (21), we have to consider the marginal share that each firm receives from the joint surplus (i.e., the second terms in equations (21)). For the purpose of tractability, we consider a secondorder Taylor series approximation for the joint surplus function around a point $Z$; that is,

$$
\begin{equation*}
M(I)=\left(M_{P}-\frac{M_{P P}}{2} I_{P}\right) \cdot I_{P}+\left(M_{N}-\frac{M_{N N}}{2} I_{N}\right) \cdot I_{N}+M_{P N} \cdot I_{P} I_{N} \tag{22}
\end{equation*}
$$

where $M_{i}=\frac{\partial M(Z)}{\partial I_{i}} \geq 0, M_{i i}=-\frac{\partial^{2} M(Z)}{\partial I_{i}^{2}} \geq 0$, and $M_{P N}=\frac{\partial^{2} M(Z)}{\partial I_{P} \partial I_{N}}$ denote the value of $M$ and the derivatives of $M$ at point $Z$. Besides tractability, the Taylor series approximation allows for a particularly convenient way to describe the degree of complementarity or substitutability with the cross partial derivative, $M_{P N}$. In particular, $M_{P N}$ measures the degree of complementarity if it takes a positive

[^7]value or substitutability if it takes a negative one. The investments are independent if $M_{P N}=0$.

Balancing the different forces in (21), Proposition 5 identifies sufficient conditions under which the value of one of the two firms or of the whole alliance, respectively, increases in $\alpha$. Ceteris paribus, greater substitutability reduces the set under which these conditions are satisfied for the public firm and for the whole alliance but increases the set for the nonpublic firm. We denote the level of the public firm's market concerns that locally maximizes the value of the public firm, the nonpublic firm, and the whole alliance with $\bar{\alpha}_{p}, \bar{\alpha}_{N}$ and $\bar{\alpha}_{T}$, respectively. Proposition 5 holds true for both cases when investments are substitutes and complements.

Proposition 5 (i) Suppose $b^{2} \cdot\left[(1-b)^{3} M_{N}^{2}-(1+b)\right] \cdot M_{P P}+(1-b)^{3} \cdot\left[b^{2} M_{P}^{2}-\right.$ $1] \cdot M_{N N}>-2(1-b) b \cdot\left[(1-b)^{2} b M_{N} M_{P}+1\right] \cdot M_{P N}$, then the value of the public firm increases in $\alpha$ for $\alpha \leq \bar{\alpha}_{p}$. (ii) Suppose $b^{3} \cdot\left[1-(1-b)^{2} M_{N}^{2}\right] \cdot M_{P P}+(1-b)^{2}$. $\left[4-b-b^{3} M_{P}^{2}\right] \cdot M_{N N}>2(1-b)^{2} b^{3} M_{N} M_{P} \cdot M_{P N}$, then the value of the nonpublic firm increases in $\alpha$ for $\alpha \leq \bar{\alpha}_{N}$. (iii) Suppose $(1-b)^{3}(2-b) \cdot M_{N N}-b^{4} \cdot M_{P P}>$ $-(1-b) b^{2} \cdot M_{P N}$, then the value of the whole business alliance increases in $\alpha$ for $\alpha \leq \bar{\alpha}_{T}$.

### 4.2 Both firms are publicly listed

As discussed earlier, the Thomson Financial Strategic Alliances and Joint Ventures Data show that $47 \%$ of all alliances are formed between a public firm and a private firm, while $25 \%$ are formed between two public firms (Lindsey 2008). In the main analysis, we focused on the public-private alliance. While the public-public alliances are not the majority of business alliances, they are often more prominent and visible. For example, the most well-known recent examples of business alliances, such as Amazon-Berkshire-JPMorgan, Toyota-Microsoft, and Starbucks-Nestle, are all public-public combinations. We examine a strategic alliance with two identical public firms. In this case, the investments of both increase in their market concerns. Therefore the market concerns can increase the value for both firms as well as the value of the whole alliance.

For clarity, we relabel the firms with $A$ and $B$ and $M(I, 1)=I_{A}^{a} \cdot I_{B}^{a}$ with $0 \leq$ $a<\frac{1}{2}$. We slightly extend the definition of the firms' cash flows by introducing a common noise term, that is, $\widetilde{x}_{1 i}\left(\gamma_{i}, I\right)=\widetilde{\pi}_{1 i}+\widetilde{\theta}_{1}+M_{i}(I, \tau)$ and $\widetilde{x}_{2 i}=\widetilde{\pi}_{2 i}+\widetilde{\theta}_{2}$, with $\widetilde{\pi}_{t i} \sim N\left(\mu, \sigma^{2}\right), \widetilde{\theta}_{t} \sim N\left(0, \sigma_{\theta}^{2}\right), \operatorname{Cov}\left[\widetilde{\pi}_{1 i}, \widetilde{\pi}_{2 i}\right]=\rho \geq 0$, and $\operatorname{Cov}\left[\widetilde{\theta}_{1}, \widetilde{\theta}_{2}\right]=\rho_{\theta} \geq 0$, for $i=A, B$ and $t=1,2$. All other random variables are uncorrelated. ${ }^{9}$

The market knows whether the parties cooperate and observes the period-I cash flows of both firms. The market price for firm $i$ reflects the market's expectation of firm $i$ 's future cash flows; that is, $p_{i}=E\left[\widetilde{x}_{2 i} \mid\left\{x_{1 A}, x_{1 B}, \tau, \widehat{M}_{A}(\widehat{I}, \tau), \widehat{M}_{B}(\widehat{I}, \tau)\right\}\right]$. Applying the Bayes' theorem for normally distributed random variables implies that

[^8]the expected market price for firm $i$ on dates I-1 and I-2 equals: $E\left[\widetilde{p}_{i}\right]=\mu+\beta$. $\left(M_{i}(I, \tau)-\widehat{M}_{i}(\widehat{I}, \tau)\right)-\delta \cdot\left(M_{j}(I, \tau)-\widehat{M}_{j}(\widehat{I}, \tau)\right)$, where $\beta=\frac{\sigma_{\theta}^{2} \rho+\sigma^{2}\left(\rho_{\theta}+\rho\right)}{\sigma^{4}+2 \sigma^{2} \sigma_{\theta}^{2}}$ and $\delta=\frac{\sigma_{\theta}^{2} \rho-\sigma^{2} \rho_{\theta}}{\sigma^{4}+2 \sigma^{2} \sigma_{\theta}^{2}}$ are the assigned weights. Firm $i$ 's objective function from (4) is:
\[

$$
\begin{equation*}
E\left[U_{i}(\tau)\right]=2 \mu+M_{i}(I, \tau)+\alpha_{i} \beta\left(M_{i}(I, \tau)-\widehat{M}_{i}(\widehat{I}, \tau)\right)-\alpha_{i} \delta\left(M_{j}(I, \tau)-\widehat{M}_{j}(\widehat{I}, \tau)\right) \tag{23}
\end{equation*}
$$

\]

On date I-2, the two firms bargain over the amount that each firm receives from the jointly generated surplus. Both firms have equal bargaining power, and, in equilibrium, firm $i$ receives a share of the surplus

$$
\begin{equation*}
M_{i}(I, 1)=\gamma_{i} \cdot M(I, 1) \text { with } \gamma_{i}=\frac{1+\alpha_{i}(\beta+\delta)}{2+\left(\alpha_{i}+\alpha_{j}\right)(\beta+\delta)} \tag{24}
\end{equation*}
$$

Equation (24) extends our prior finding in Proposition 1. In particular, firm $i$ 's share of surplus increases in the degree of its own market concerns, $\alpha_{i}$, and decreases in the degree of firm $j$ 's market concerns, $\alpha_{j}$. If the firms have the same degree of market concerns, then each receives half of the surplus.

On date I-1, anticipating the bargaining outcome, each firm determines its investment to maximize its expected payoff in (23), implying the following first-order condition:

$$
\begin{equation*}
\gamma_{i} \cdot \frac{\partial M(I, 1)}{\partial I_{i}}+\alpha_{i}\left(\beta \gamma_{i}-\delta \gamma_{j}\right) \cdot \frac{\partial M(I, 1)}{\partial I_{i}}=w_{i}^{\prime}\left(I_{i}\right) \tag{25}
\end{equation*}
$$

That is, the marginal investment costs equal the marginal benefits accounting for the market effect. For $\alpha_{i}=0$, the marginal benefit of the investment equals firm $i$ 's share of the joint surplus. The additional term, $\alpha_{i}\left[\beta \gamma_{i}-\delta \gamma_{j}\right] \cdot \frac{\partial M(I, 1)}{\partial I_{i}}$, reflects firm $i$ 's incentives to inflate its market price by adjusting its investment. Firm $i$ 's incentives increase in its share of surplus and in the weight that the market assigns to its cash flow, $\beta$, and decreases in firm $j$ 's share of surplus and the weight that the market assigns to firm $j$ 's cash flow, $\delta$. In equilibrium, the firm $i$ 's investments are given by $I_{i}=\left(\left(\gamma_{i}+\alpha_{i}\left[\beta \gamma_{i}-\delta \gamma_{j}\right]\right) a\right)^{\frac{1-a}{1-2 a}}\left(\left(\gamma_{j}+\alpha_{j}\left[\beta \gamma_{j}-\delta \gamma_{i}\right]\right) a\right)^{\frac{a}{1-2 a}}$.

Next we examine the marginal impact of market concerns on the value of firm $i$. To simplify the computation, we assume that the market concerns of both firms are equivalent; that is, $\alpha_{i}=\alpha_{j}=\alpha$. As in (15), the marginal value of firm $i$ consists of three terms that reflect the impact of $\alpha$ on the two investments and on firm $i$ 's share of surplus. With the same degree of market concerns, each firm $i$ receives half of the surplus, that is, $\gamma_{i}=\frac{1}{2}$ from (24). As the firms' shares of surplus no longer depend on $\alpha$, the first-order conditions (25) imply that both investments increase in $\alpha$. Firm $i$ benefits from the higher investment of the other firm but suffers from the increase of its own investment, as this exaggerates the overinvestment problem. For low values of $\alpha$, the first effect dominates and the firm benefits from a higher $\alpha$. Conversely, for high values of $\alpha$, the second effect dominates and the firm suffers from a higher $\alpha$. Technically, the value of firm $i$ has an inverted U -shape in $\alpha$ with a maximum at $\alpha=\frac{1}{\beta-\delta}>1$, and so is the value of the whole business alliance, as it is simply the sum of both firm values. Further, the first-best efficiency is not achievable, as $\alpha$ cannot exceed one. Proposition 6 summarizes our findings.

Proposition 6 For the case of symmetric firms, the value of each firm $i$ and the value of the whole business alliance increases in $\alpha \in[0,1]$.

## 5 Conclusion

Business alliances and joint ventures are generally considered as value-adding by the financial market, which may affect how publicly traded firms interact and engage with alliance partners. We study the effects of market concerns of a publicly traded firm in a business alliance. We derive the equilibrium investments of both firms, the payoff that each receives from the joint surplus through negotiation, and the value of each and the whole alliance. The market concerns always help the public firm obtain a higher share of the joint surplus through bargaining but can have positive or negative effects on the total size of the joint surplus. Ultimately, the effect of the public firm's myopic market concerns on the value of the public firm, the private firm, or the whole business alliance depends on the relative importance of the firms' investment.

While empirical research has provided ample evidence that business alliances are value relevant, the theory literature is surprisingly silent on the market perspective. Specifically, studies of how firms behave when collaborating can change in response to the market expectation are still very limited. Our study helps fill the literature gap by examining how the pressure from the financial market, among other factors, affects investment decisions of publicly traded firms in business alliances. Our results can be easily generalized to other settings, such as supply chains, distribution channels, and an intra-firm environment with divisional managers who are granted with different compensation incentives. As long as the involved parties face both external pressure from the capital market and the hold-up problem in the relationship-specific investments, our insights would apply. Broadly stated, our paper speaks to an emerging branch of literature that empirically studies various spillovers along the supply chain, including investment decisions (e.g., (Bustamante and Fresard 2021; Jia et al. 2022)).

Our study also provides several interesting avenues for future research. First, we do not explicitly model strategic benefits for firms to form a business alliance, such as entering new markets or obtaining a better position in the supply chain. As these are important factors that crucially impact investment decisions in alliances, future research could examine their interplay with capital market concerns. Second, our model can be extended to multiple periods. Myopic market concerns might induce the public firm to prefer the alliance to generate more surplus earlier rather than later, which in turn influences how the firms optimally structure the bargaining protocol intertemporally. Third, the degree of myopia can be endogenized. Conceptually our analysis suggests that it can be optimal to base parts of the managerial compensation on the market price rather than on measures of firm value. Inducing managerial myopia through compensation contract might be helpful in improving efficiency. Fourth, recent literature examines how integration of firms may affect the managerial incentives and the hold-up problem (such as (Baldenius and Michaeli
2019)). It would also be interesting to combine these insights and examine how market concerns in conjunction with divisional incentive problems affect the level of investments.

## Appendix: Proofs

Proof for Proposition 1 For simplicity, we abbreviate the (conjectured) payoff values as follows: $M(\tau)=M(I, \tau), M_{i}(\tau)=M_{i}(I, \tau)$, and $\widehat{M}_{i}(\tau)=M_{i}(\widehat{I}, \tau)$. We also use the following shortcuts: $\Delta_{P}=E\left[U_{P}(1)\right]-E\left[U_{P}(0)\right]=M_{P}(1)+\alpha \beta \cdot\left(M_{P}(1)-\right.$ $\left.\widehat{M}_{P}(1)+\widehat{M}_{P}(0)\right)$ and $\Delta_{N}=E\left[U_{N}(1)\right]-E\left[U_{N}(0)\right]=M(1)-M_{P}(1)$.

First, we consider the bargaining problem in (8); that is, $\max \Delta_{P}^{b} \Delta_{N}^{1-b}$. Noting that $\frac{\partial \Delta_{P}}{\partial M_{P}(1)}=(1+\alpha \beta)$ and $\frac{\partial \Delta_{N}}{\partial M_{P}(1)}=-1$, we get the following first-order conditions with respect to $M_{P}(1)$; that is,

$$
\begin{align*}
0 & =b(1+\alpha \beta) \Delta_{P}^{b-1} \Delta_{N}^{1-b}-(1-b) \Delta_{P}^{b} \Delta_{N}^{-b} \\
& =\left[b(1+\alpha \beta) \Delta_{N}-(1-b) \Delta_{P}\right] \cdot \Delta_{P}^{b-1} \Delta_{N}^{-b} \tag{A1}
\end{align*}
$$

Solving the first-order condition in (A1), that is, $0=\left[b(1+\alpha \beta) \Delta_{N}-(1-\right.$ b) $\left.\Delta_{P}\right]=b(1+\alpha \beta) \cdot\left(M(1)-M_{P}(1)\right)-(1-b) \cdot\left(M_{P}(1)+\alpha \beta \cdot\left(M_{P}(1)-\widehat{M}_{P}(1)+\right.\right.$ $\left.\widehat{M}_{P}(0)\right)$ ) for $M_{P}(1)$, yields (9).

The second-order condition is satisfied; that is,

$$
\begin{equation*}
-(1-b) b \cdot\left[M(1)+\alpha \beta \cdot\left(M(1)-\widehat{M}_{P}(1)+\widehat{M}_{P}(0)\right)\right]^{2} \cdot \Delta_{P}^{-(1+b)} \Delta_{N}^{-(2-b)}<0 . \tag{A2}
\end{equation*}
$$

In equilibrium, the conjectures are correct, that is, $\widehat{M}_{P}(1)=M_{P}(1)$ and $\widehat{M}_{P}(0)=$ 0 , and we obtain the equilibrium value for $M_{P}(1)$ and $M_{N}(1)$.

Given $M(I, 1)=I_{P}^{a_{P}} I_{N}^{a_{N}}$ and $w_{i}^{\prime}\left(I_{i}\right)=1$, the conditions for the equilibrium investments in (13) equal

$$
\begin{align*}
& (1+\alpha \beta) \gamma a_{P} \cdot\left(I_{P}^{a_{P}-1} I_{N}^{a_{N}}\right)=1 \\
& (1-\gamma) a_{N} \cdot\left(I_{P}^{a_{P}} I_{N}^{a_{N-1}}\right)=1 \tag{A3}
\end{align*}
$$

and imply $I_{P}=(1+\alpha \beta) \frac{\gamma}{(1-\gamma)} \frac{a_{P}}{a_{N}} \cdot I_{N}$. Substituting this value in the equilibrium condition (A3) for $I_{N}$ gives $I_{N}^{-\left(1-a_{P}-a_{N}\right)} \cdot\left((1+\alpha \beta) \gamma a_{P}\right)^{a_{p}} \cdot\left((1-\gamma) a_{N}\right)^{1-a_{P}}=1$. Solving the equation gives $I_{N}$, and the equilibrium condition (A3) for $I_{P}$ yields $I_{P}$. The associated second-order conditions are satisfied as $\frac{\partial^{2} M}{\partial^{2} I_{i}}<0$ for $i=P, N$.

Proof for Corollary 3.1 Given the investments from Proposition 1, the marginal investments of the public firm and the nonpublic firm are

$$
\begin{align*}
\frac{\partial I_{P}}{\partial \alpha} & =\frac{1-b+(1+b \alpha \beta)\left(1-2 a_{N}\right)}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)(1+b \alpha \beta)} \beta I_{P} \\
& =\frac{2-\gamma-2 a_{N}}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)} \beta I_{P} \tag{A4}
\end{align*}
$$

and

$$
\begin{align*}
\frac{\partial I_{N}}{\partial \alpha} & =\frac{2 a_{P}(1+b \alpha \beta)-b(1+\alpha \beta)}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)(1+b \alpha \beta)} \beta I_{N} \\
& =\frac{2 a_{P}-\gamma}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)} \beta I_{N}, \tag{A5}
\end{align*}
$$

where we insert the equilibrium value $\gamma$ from Proposition 1. Points (i), (ii), and (iii) follow from $\operatorname{sgn}\left[\frac{\partial I_{P}}{\partial \alpha}\right]=\operatorname{sgn}\left[2-\gamma-2 a_{N}\right]$ and $\operatorname{sgn}\left[\frac{\partial I_{N}}{\partial \alpha}\right]=\operatorname{sgn}\left[2 a_{P}-\gamma\right]$. Finally, we show that $\operatorname{sgn}\left[\frac{\partial I_{N}}{\partial \alpha}\right]=\operatorname{sgn}\left[2 a_{P}-\gamma\right]>0$ and $\operatorname{sgn}\left[\frac{\partial I_{P}}{\partial \alpha}\right]=\operatorname{sgn}\left[2-\gamma-2 a_{N}\right]<$ 0 cannot be fulfilled simultaneously. Suppose $a_{P}>\frac{\gamma}{2}$, then the regularity condition, $1>a_{P}+a_{N}$, implies $a_{N}<\frac{\gamma}{2}$ and $\operatorname{sgn}\left[2-\gamma-2 a_{N}\right]>0$, that is, a contradiction.

Proof for Proposition 2 We first apply the chain rule to obtain the marginal firm value for $V_{p}(\alpha)=2 \mu_{P}+\gamma(\alpha) \cdot M(I(\alpha), 1)-w_{P}\left(I_{P}(\alpha)\right)$. We then restate this value using the first-order condition from (13), that is, $(1+\alpha \beta) \gamma \frac{\partial M}{\partial I_{P}}=w_{P}^{\prime}$, and $\frac{\partial M}{\partial I_{P}}=\frac{a_{P} M}{I_{P}}$ and $\frac{\partial M}{\partial I_{N}}=\frac{a_{N} M}{I_{N}}$, and the marginal investments from (14), that is, $\frac{\partial I_{P}}{\partial \alpha}$ and $\frac{\partial I_{N}}{\partial \alpha}$. That is, we get:

$$
\begin{align*}
\frac{d V_{p}}{d \alpha} & =\left(\gamma \frac{\partial M}{\partial I_{P}}-w_{P}^{\prime}\right) \frac{\partial I_{P}}{\partial \alpha}+\frac{\partial \gamma}{\partial \alpha} M+\gamma \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha} \\
& =-\alpha \beta \gamma \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}+\frac{\partial \gamma}{\partial \alpha} M+\gamma \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha} \\
& =\left(-\alpha \beta \gamma \frac{a_{P} \frac{\partial I_{P}}{\partial \alpha}}{I_{P}}+\frac{\partial \gamma}{\partial \alpha}+\gamma \frac{a_{N} \frac{\partial I_{N}}{\partial \alpha}}{I_{N}}\right) \cdot M \\
& =\left(-\frac{\alpha \beta \gamma a_{P} \cdot\left(2-\gamma-2 a_{N}\right) \beta}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)}+\frac{\partial \gamma}{\partial \alpha}+\frac{\gamma a_{N} \cdot\left(2 a_{P}-\gamma\right) \beta}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)}\right) \cdot M . \tag{A6}
\end{align*}
$$

Next we substitute $\gamma$ from Proposition 1 and $\frac{\partial \gamma}{\partial \alpha}$ from (11) into (A6) and get:

$$
\begin{equation*}
\frac{d V_{p}}{d \alpha}=\left(k_{0}+k_{1} \alpha+k_{2} \alpha^{2}\right) \cdot \frac{b \beta M}{\left(1-a_{P}-a_{N}\right)(1+b \beta \alpha)^{2}}, \tag{A7}
\end{equation*}
$$

with $k_{0}=(1-b)\left(1-a_{P}\right)-a_{N}\left(1-2 a_{P}\right), k_{1}=-\left(2 a_{P}\left(1-b a_{N}-a_{N}\right)-b\left(a_{P}-\right.\right.$ $\left.a_{N}\right)$ ) $\beta<0$, and $k_{2}=a_{P}\left(2 a_{N}-1\right) b \beta^{2}$.

The sign of (A7) is determined by the sign of the quadratic function, $k_{0}+k_{1} \alpha+$ $k_{2} \alpha^{2}$. In the following, we will show that the sign of the intercept $k_{0}$ matters. If $k_{0}<0$, then $\frac{d V_{p}}{d \alpha}<0$. If $k_{0}>0$, then $\frac{d V_{p}}{d \alpha}>0$ for $\alpha \leq \bar{\alpha}_{P}$ and $\frac{d V_{p}}{d \alpha}<0$ for $\alpha \geq \bar{\alpha}_{P}$, where $\bar{\alpha}_{P}$ is the unique maximizer ( $\bar{\alpha}_{P}$ is depicted in (A8) below).
(i) Assume a positive intercept, $k_{0}>0$, and denote the two potential zeros of the quadratic function with $z_{1}=\frac{-k_{1}-\sqrt{k_{1}^{2}-4 k_{0} k_{2}}}{2 k_{2}}$ and $z_{2}=\frac{-k_{1}+\sqrt{k_{1}^{2}-4 k_{0} k_{2}}}{2 k_{2}}$. Three cases emerge. (a) If the quadratic function has a U -shape, $k_{2}>0$, then the function has a negative value of $k_{0}-\frac{k_{1}^{2}}{4 k_{2}}<0$ at the vertex, $-\frac{k_{1}}{2 k_{2}}>0$. This implies that the quadratic function has two different zeros, that is, $\sqrt{k_{1}^{2}-4 k_{0} k_{2}}>0$. Since
the quadratic function decreases at the intercept, $\left.\frac{\partial\left(k_{0}+k_{1} \alpha+k_{2} \alpha\right)}{\partial \alpha}\right|_{\alpha}=k_{1}<0$, we know that $\bar{\alpha}_{P}$ equals the smaller root, $\bar{\alpha}_{P}=z_{1}$. In fact, we can show the larger root is not within the interval $[0,1]$. To do so, we assume $z_{2}=\frac{-k_{1}-\sqrt{k_{1}^{2}-4 k_{0} k_{2}}}{2 k_{2}} \leq 1$, which implies $-\sqrt{k_{1}^{2}-4 k_{0} k_{2}} \leq\left(2 k_{2}+k_{1}\right)$. This constitutes a contradiction, as $z_{1}=\frac{-k_{1}+\sqrt{k_{1}^{2}-4 k_{0} k_{2}}}{2 k_{2}} \leq 1$ implies $\sqrt{k_{1}^{2}-4 k_{0} k_{2}} \leq\left(2 k_{2}+k\right)$. (b) If the quadratic function has an inverted $U$-shape, $k_{2}<0$, then $\bar{\alpha}_{P}$ equals the larger root, $\bar{\alpha}=z_{1} .{ }^{10}$ (c) If the quadratic function collapses to a linear function, $k_{2}=0$, then we get $\bar{\alpha}_{P}=-\frac{k_{0}}{k_{1}} \geq 0$ as $k_{1} \leq 0$. Further, $k_{2}=0$ is true if and only if $a_{N}=\frac{1}{2}$.

For the sake of convenience, we summarize the optimal value in terms of the coefficients of the quadratic function as follows:

$$
\bar{\alpha}_{P}= \begin{cases}\frac{-k_{1}-\sqrt{k_{1}^{2}-4 k_{0} k_{2}}}{2 k_{2}} & \text { if } a_{N} \neq \frac{1}{2}  \tag{A8}\\ -\frac{k_{0}}{k_{1}} & \text { if } a_{N}=\frac{1}{2}\end{cases}
$$

(ii) Assume a negative intercept, $k_{0}<0$, then three cases emerge. (a) If the quadratic function collapses to a linear function, $k_{2}=0$, then $k_{0}+k_{1} \alpha \leq 0$ for all $\alpha$ since $k_{1} \leq 0$. (b) If the quadratic function has an inverted U-shape, $k_{2}<0$, then the quadratic function has a negative value at $\alpha=0$ and decreases as $\frac{\partial\left(k_{0}+k_{1} \alpha+k_{2} \alpha\right)}{\partial \alpha}=k_{1}+2 k_{2} \alpha<0$. (c) If the quadratic function has a U-shape, $k_{2}>0$, then the quadratic function has a negative value at $\alpha=0$ and increases for larger $\alpha$. We show that the function has a negative value at $\alpha=1$ by noting that $k_{2} \geq 0$ implies $a_{N} \leq \frac{1}{2}$ and $k_{0} \leq 0$ implies $a_{P} \leq \frac{1}{2}$ and $a_{N}>\frac{(1-b)\left(1-a_{P}\right)}{1-2 a_{P}}$. W $\log \beta=1$, then $k_{0}+k_{1}+k_{2}=\left[1-b-a_{N}(1+b)\right]-a_{P}\left[3-b-4(1+b) a_{N}\right]$ is linear in $a_{P}$ and is maximized at the boundary values $a_{P}=\left\{0, \frac{1}{2}\right\}$. ${ }^{11}$ For $a_{P}=0$, we get $(1-b)-a_{N}(1+b)<a_{N} \frac{1-2 a_{P}}{1-a_{P}}-a_{N}(1+b)=-a_{N} \frac{b+a_{P}(1-b)}{1-a_{P}} \leq 0$, where we use $(1-b)<a_{N} \frac{1-2 a_{P}}{1-a_{P}}$. For $a_{P}=\frac{1}{2}$, we get $k_{0}+k_{1}-k_{2}=-\frac{(1+b)\left(1-2 a_{N}\right)}{2}<0$, where $a_{P}=\frac{1}{2}$ implies $a_{N}<\frac{1}{2}$. Hence $k_{0}+k_{1}+k_{2} \leq 0$. This proves the proposition.

Proof for Proposition 3 We first apply the chain rule to obtain the marginal firm value for $V_{N}(\alpha)=2 \mu_{N}+(1-\gamma(\alpha)) \cdot M(I(\alpha), 1)-w_{N}\left(I_{N}(\alpha)\right)$. We then restate the marginal value using the first-order condition from (13), that is, $(1-\gamma) \frac{\partial M}{\partial I_{N}}=w_{N}^{\prime}$,

[^9]and $\frac{\partial M}{\partial I_{P}}=\frac{a_{P} M}{I_{P}}$, and the marginal investment from (14), that is, $\frac{\partial I_{P}}{\partial \alpha}$. We then obtain the last line by substituting $\gamma$ from Proposition 1 and $\frac{\partial \gamma}{\partial \alpha}$ from (11). That is, we get:
\[

$$
\begin{align*}
\frac{d V_{N}}{d \alpha} & =\left((1-\gamma) \frac{\partial M}{\partial I_{N}}-w_{N}^{\prime}\right) \frac{\partial I_{N}}{\partial \alpha}-\frac{\partial \gamma}{\partial \alpha} M+(1-\gamma) \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha} \\
& =\left(-\frac{\partial \gamma}{\partial \alpha}+(1-\gamma) \frac{a_{P} \frac{\partial I_{P}}{\partial \alpha}}{I_{P}}\right) \cdot M \\
& =\left(-\frac{\partial \gamma}{\partial \alpha}+\frac{(1-\gamma) a_{P} \cdot\left(2-\gamma-2 a_{N}\right) \cdot \beta}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)}\right) \cdot M \\
& =\left(2 a_{P}-b+\left(2 a_{P}-1\right) \beta b \alpha\right) \cdot \frac{(1-b)\left(1-a_{N}\right)}{\left(1-a_{P}\right)(1+\alpha \beta)(1+b \alpha \beta)^{2}} \cdot \beta M . \tag{A9}
\end{align*}
$$
\]

The sign of the marginal value is determined by the sign of the linear function, $2 a_{P}-b+\left(2 a_{P}-1\right) \beta b \alpha$. Three cases emerge. (i) if $a_{P}<\frac{b}{2}$, then $\frac{d V_{N}}{d \alpha}<0$ for all $\alpha$. (ii) if $\frac{b}{2} \leq a_{P} \leq \frac{1}{2}$, then $\frac{d V_{N}}{d \alpha}>0$ for $\alpha \leq \frac{2 a_{P}-b}{\left(1-2 a_{P}\right) b \beta}$ and $\frac{d V_{N}}{d \alpha}<0$ for $\alpha \geq \frac{2 a_{P}-b}{\left(1-2 a_{P}\right) b \beta}$. (iii) if $a_{P}>\frac{1}{2}$, then $\frac{d V_{N}}{d \alpha}>0$ for all $\alpha$. Collectively, we get:

$$
\bar{\alpha}_{N}= \begin{cases}\frac{2 a_{P}-b}{\left(1-2 a_{P}\right) b \beta} & \text { if } a_{P}<\frac{1}{2}  \tag{A10}\\ 1 & \text { if } a_{P} \geq \frac{1}{2} .\end{cases}
$$

Proof for Proposition 4 We first apply the chain rule to obtain the marginal firm value for $V_{T}(\alpha)=2\left(\mu_{P}+\mu_{N}\right)+M(I(\alpha), 1)-w_{P}\left(I_{P}(\alpha)\right)-w_{N}\left(I_{N}(\alpha)\right)$. Then we apply the first-order conditions from (13), that is, $\gamma(1+\alpha \beta \gamma) \frac{\partial M}{\partial I_{P}}=w_{P}^{\prime}$ and $(1-\gamma) \frac{\partial M}{\partial I_{N}}=$ $w_{N}^{\prime}$, and $\frac{\partial M}{\partial I_{P}}=a_{P} \frac{M}{I_{P}}$ and $\frac{\partial M}{\partial I_{N}}=a_{N} \frac{M}{I_{N}}$, and the marginal investments from (14), that is, $\frac{\partial I_{P}}{\partial \alpha}$ and $\frac{\partial I_{N}}{\partial \alpha}$. We obtain the last line by substituting $\gamma$ from Proposition 1 and $\frac{\partial \gamma}{\partial \alpha}$ from (11). That is, we get:

$$
\begin{align*}
\frac{d V_{T}}{d \alpha} & =\left(\frac{\partial M}{\partial I_{P}}-w_{P}^{\prime}\right) \frac{\partial I_{P}}{\partial \alpha}+\left(\frac{\partial M}{\partial I_{N}}-w_{N}^{\prime}\right) \frac{\partial I_{N}}{\partial \alpha} \\
& =(1-\gamma-\alpha \beta \gamma) \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}+\gamma \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha} \\
& =\left((1-\gamma-\alpha \beta \gamma) \frac{a_{P} \frac{\partial I_{P}}{\partial \alpha}}{I_{P}}+\gamma \frac{a_{N} \frac{\partial I_{N}}{\partial \alpha}}{I_{N}}\right) M \\
& =\left(\frac{(1-\gamma-\alpha \beta \gamma) a_{P} \cdot\left(2-\gamma-2 a_{N}\right) \beta}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)}+\frac{\gamma a_{N} \cdot\left(2 a_{P}-\gamma\right) \beta}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)}\right) M \\
& =P(\alpha) \frac{\beta M}{\left(1-a_{P}-a_{N}\right)(1+\alpha \beta)(1+b \alpha \beta)^{2}}, \tag{A11}
\end{align*}
$$

where $P(\alpha)=c_{0}+c_{1} \alpha+c_{2} \alpha^{2}+c_{3} \alpha^{3}$ denotes a cubic function with coefficients $c_{0}=$ $(2-b)(1-b) a_{P}+a_{P} a_{N}(4 b-2)-a_{N} b^{2}, c_{1}=-\left(2 b a_{N}+a_{P}\left(1-2 a_{N}-4 b a_{N}\right)\right) b \beta$, $c_{2}=-\left(b a_{N}+a_{P}\left(2-2 a_{N}-4 b a_{N}\right)\right) b \beta^{2}$, and $c_{3}=a_{P}\left(2 a_{N}-1\right) b^{2} \beta^{3}$.

The sign of $\frac{d V_{T}}{d \alpha}$ is determined by the sign of the cubic function, $P(\alpha)$. In the following, we will show that $\frac{d V_{T}}{d \alpha}<0$ if $c_{0}<0$. For $c_{0}>0$, we will show that $\frac{d V_{T}}{d \alpha}>0$ for $\alpha \leq \bar{\alpha}_{T}$ and $\frac{d V_{T}}{d \alpha}<0$ for $\alpha \geq \bar{\alpha}_{T}$, with $\bar{\alpha}_{T}$ from (A13) below.

To do so, we first note that, for $a_{N} \neq \frac{1}{2}$, the discriminant, $\Delta=18 c_{0} c_{1} c_{2} c_{3}-$ $4 c_{2}^{3} c_{0}+c_{2}^{2} c_{1}^{2}-4 c_{1}^{3} c_{3}-27 c_{3}^{2} c_{0}^{2}>0$, is positive, and the cubic function has 3 different real zeros, $\widehat{z}_{1 / 2 / 3}$, and two critical values, $\alpha_{1 / 2}^{c r i t}=-\frac{c_{2}+-\sqrt{c_{2}^{2}-3 c_{1} c_{3}}}{3 c_{3}}$, with $P^{\prime}\left(\alpha_{1 / 2}^{c r i t}\right)=$ 0 . Wlog we assume

$$
\begin{equation*}
\widehat{z}_{3}<\alpha_{2}^{\text {crit }}<\widehat{z}_{2}<\alpha_{1}^{c r i t}<\widehat{z}_{1} . \tag{A12}
\end{equation*}
$$

(i) Assume a positive intercept, $c_{0}>0$. This implies $c_{2}<0$, and three cases emerge. (a) For $c_{3}<0$, the cubic function has positive values within $\left(-\infty, \widehat{z}_{3}\right)$ and $\left(\widehat{z}_{2}, \widehat{z}_{1}\right)$. Since $\alpha_{2}^{c r i t}=\frac{-c_{2}+\sqrt{c_{2}^{2}-3 c_{1} c_{3}}}{3 c_{3}}$ is negative (note that $c_{2}<0$ ), the positive intercept must lie in the interval ( $\widehat{z}_{2}, \widehat{z}_{1}$ ), and $\alpha=\widehat{z}_{1}$. (b) For $c_{3}>0$, the cubic function has positive values within $\left(\widehat{z}_{3}, \widehat{z}_{2}\right)$ and $\left(\widehat{z}_{1}, \infty\right)$ and has a negative value at $\alpha_{1}^{\text {crit }}$; that is, $P\left(\alpha_{1}^{c r i t}\right)<0$. Since $\alpha_{1}^{c r i t}=\frac{-c_{2}+\sqrt{c_{2}^{2}-3 c_{1} c_{3}}}{3 c_{3}}$ is positive (note that $c_{2}<0$ ), we get $\alpha=\widehat{z}_{2} .{ }^{12}$ (c) For $c_{3}=0$, the cubic function collapses to an inverse Ushaped quadratic function with a positive intercept and two zeros, $\tilde{z}_{1}>\tilde{z}_{2}$, implying that $\alpha=\widetilde{z}_{1} .{ }^{13}$

Noting that $c_{3}=0$ if $a_{N}=\frac{1}{2}$, our findings for $\bar{\alpha}_{T}$ can be summarized as follows:

$$
\bar{\alpha}_{T}= \begin{cases}\widehat{z}_{2} & \text { if } a_{N}>\frac{1}{2}  \tag{A13}\\ \widehat{z}_{1} & \text { if } a_{N}<\frac{1}{2} \\ \widetilde{z}_{1} & \text { if } a_{N}=\frac{1}{2}\end{cases}
$$

Finally, $c_{0}>0$ if $a_{P}>\frac{b^{2}}{(1-b)(2-b)+a_{N}(4 b-2)} a_{N}$, where $a_{N}<\frac{(1-b)(2-b)}{2-4 b}$ ensures a positive denominator if $b<\frac{1}{2}$.
(ii) Assume a negative intercept, $c_{0}<0$, that is, $a_{P} \leq \bar{a}_{P}=$ $\frac{b^{2} a_{N}}{(1-b)(2-b)+a_{N}(4 b-2)} \geq 0$ and $\bar{a}_{P} \geq 0$. We use these slightly different conditions for technical reasons. Further, $c_{0}<0$ implies $c_{1}<0$. Three cases emerge. (a) For $c_{3}<0$, the cubic function has negative values within $\left(\widehat{z}_{3}, \widehat{z}_{2}\right)$ and $\left(\widehat{z}_{1}, \infty\right)$ and has a positive value at the critical value $\alpha_{1}^{c r i t}=\frac{-c_{2}-\sqrt{c_{2}^{2}-3 c_{1} c_{3}}}{3 c_{3}} \in\left(\widehat{z}_{2}, \widehat{z}_{1}\right)$, that is, $P\left(\alpha_{1}^{c r i t}\right)>0$. Notice $\alpha_{1}^{\text {crit }} \leq 0$ since $\alpha_{1}^{\text {crit }}>0$ implies $\sqrt{c_{2}^{2}-3 c_{1} c_{3}}>-c_{2}$ and $-3 c_{1} c_{3}>0$, contradicting $c_{1} c_{3} \geq 0$. Since $\alpha_{1}^{\text {crit }} \leq 0$ and the cubic function has a negative value at the intercept, we know that $\widehat{z}_{1}<0$. (b) If $c_{3}>0$, then the cubic function has negative values within $\left(-\infty, \widehat{z}_{3}\right)$ and $\left(\widehat{z}_{2}, \widehat{z}_{1}\right)$. Since the critical

[^10]values satisfy $\alpha_{1}^{\text {crit }}>0>\alpha_{2}^{c r i t}, \widehat{z}_{1}$ can lie within [0, 1]. However, we will show that $P(1)<0$, implying that $\widehat{z}_{1}>1 .{ }^{14}$ To do so, note that $P(1)$ is linear in $a_{P}$; that is,
\[

$$
\begin{equation*}
P(1)=a_{P}\left((2-6 b)+a_{N}(2+2 b)(5 b-1)\right)-4 b^{2} a_{N}, \tag{A14}
\end{equation*}
$$

\]

and has its maximum at the boundary value, $a_{P}=\left\{0, \bar{a}_{P}\right\}$. Further, $P(1)=$ $-4 b^{2} a_{N}<0$ for $a_{P}=0$ and $P(1)=\left(a_{N}\left(3-4 b+5 b^{2}\right)-3+b-\right.$ $\left.2 b^{2}\right) \frac{2 b^{2} a_{N}}{(1-b)(2-b)+a_{N}(4 b-2)}$ for $a_{P}=\bar{a}_{P}$. The first term, $\left(a_{N}\left(3-4 b+5 b^{2}\right)-3+b-2 b^{2}\right)$, is maximized by the largest value of $a_{N}$; that is, $a_{N}=1-\bar{a}_{P}=1-\frac{b}{2}$, yielding $-\frac{5(1-b)^{2} b}{2} \leq 0$. Since $\frac{2 b^{2} a_{N}}{(1-b)(2-b)+a_{N}(4 b-2)}>0$, we get $P(1)<0$. (c) For $c_{3}=0$, the cubic function collapses to an inverse U-shaped quadratic function with a negative intercept and exhibits negative values for $\alpha \in[0,1]$.

Finally, we note that $c_{0}<0$ if $a_{N}>\frac{(2-b)(1-b)}{b^{2}+a_{P}(2-4 b)} a_{P}$, where $a_{P}<\frac{b^{2}}{4 b-2}$ ensures a positive denominator if $b>\frac{1}{2}$.

Derivation of the comparative static results in (20) For sake of convenience, we denote $f_{p}(\alpha)=(1+\alpha \beta) \cdot \gamma(\alpha)$ and $f_{N}(\alpha)=1-\gamma(\alpha)$ with $\gamma(\alpha)=b \cdot \frac{1+\alpha \beta}{1+b \alpha \beta}$ from Proposition 1 and restate the conditions for the equilibrium investments from (13) as:

$$
\begin{align*}
& f_{p}(\alpha) \cdot \frac{\partial M\left(I_{P}(\alpha), I_{N}(\alpha), 1\right)}{\partial I_{P}}-1=0 \\
& f_{N}(\alpha) \cdot \frac{\partial M\left(I_{P}(\alpha), I_{N}(\alpha), 1\right)}{\partial I_{N}}-1=0 . \tag{A15}
\end{align*}
$$

Applying the chain rule to (A15) yields

$$
\begin{array}{r}
\frac{\partial f_{p}}{\partial \alpha} \cdot \frac{\partial M}{\partial I_{P}}+f_{p} \cdot \frac{\partial^{2} M}{\partial I_{P}^{2}} \frac{\partial I_{p}}{\partial \alpha}+f_{p} \cdot \frac{\partial^{2} M}{\partial I_{P} \partial I_{N}} \frac{\partial I_{N}}{\partial \alpha}=0 \\
\frac{\partial f_{N}}{\partial \alpha} \cdot \frac{\partial M}{\partial I_{N}}+f_{N} \cdot \frac{\partial^{2} M}{\partial I_{N}^{2}} \frac{\partial I_{N}}{\partial \alpha}+f_{N} \cdot \frac{\partial^{2} M}{\partial I_{P} \partial I_{N}} \frac{\partial I_{P}}{\partial \alpha}=0 . \tag{A16}
\end{array}
$$

Given $\operatorname{det}[M]=\frac{\partial^{2} M}{\partial I_{P}^{2}} \frac{\partial^{2} M}{\partial I_{N}^{2}}-\left(\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}\right)^{2}$, the solution of the system of equations in (A16) is

$$
\begin{align*}
& \frac{\partial I_{P}}{\partial \alpha}=\frac{-\frac{\partial f_{p}}{\partial \alpha} \frac{1}{f_{P}} \frac{\partial M}{\partial I_{P}} \cdot \frac{\partial^{2} M}{\partial I_{N}^{2}}+\frac{\partial f_{N}}{\partial \alpha} \frac{1}{f_{N}} \frac{\partial M}{\partial I_{N}} \cdot \frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}}{\operatorname{det}[M]} \\
& \frac{\partial I_{N}}{\partial \alpha}=\frac{-\frac{\partial f_{N}}{\partial \alpha} \frac{1}{f_{N}} \frac{\partial M}{\partial I_{N}} \cdot \frac{\partial^{2} M}{\partial I_{P}^{2}}+\frac{\partial f_{p}}{\partial \alpha} \frac{1}{f_{P}} \frac{\partial M}{\partial I_{P}} \cdot \frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}}{\operatorname{det}[M]} \tag{A17}
\end{align*}
$$

[^11]To obtain the comparative static result in (20), we restate (A17) by applying the conditions for the equilibrium investments in (A15), $\frac{\partial M}{\partial I_{P}}=\frac{1}{f_{p}}$ and $\frac{\partial M}{\partial I_{N}}=\frac{1}{f_{N}}$, and noting that $\frac{\partial f_{p}}{\partial \alpha} \frac{1}{f_{p}^{2}}=\beta \cdot \frac{2-b(1-\alpha \beta)}{b(1+\alpha \beta)^{3}}$ and $\frac{\partial f_{N}}{\partial \alpha} \frac{1}{f_{N}^{2}}=-\beta \cdot \frac{b}{(1-b)}$.

Finally, following the literature (e.g., Cachon and Netessine 2006, Theorem 7), a positive determinant implies that the product of the slopes of the best response functions do not exceed one; that is, $\frac{\partial I_{i}}{\partial I_{j}}=-\frac{\partial^{2} M}{\partial I_{i} \partial I_{j}} / \frac{\partial^{2} M}{\partial I_{i}^{2}}$ and $\frac{\partial I_{P}}{\partial I_{N}} \frac{\partial I_{N}}{\partial I_{P}}=$ $\left(\frac{\partial^{2} M}{\partial I_{i} \partial I_{j}}\right)^{2} /\left(\frac{\partial^{2} M}{\partial I_{P}^{2}} \frac{\partial^{2} M}{\partial I_{N}^{2}}\right)<1$, which implies a unique equilibrium.

Proof for Proposition 5 For sake of simplicity, we define $f_{p}=(1+\alpha \beta) \cdot \gamma$ and $f_{N}=$ $1-\gamma$ with $\gamma=b \cdot \frac{1+\alpha \beta}{1+b \alpha \beta}$ from Proposition 1 and $\operatorname{det}[M]=M_{P P} M_{N N}-M_{P N}^{2}>0$. Solving (13) yields the investment decisions of both firms

$$
\begin{align*}
& I_{P}=\frac{\left(f_{p} M_{P}-1\right) f_{N} \cdot M_{N N}+\left(f_{N} M_{N}-1\right) f_{P} \cdot M_{P N}}{f_{P} f_{N} \cdot \operatorname{det}[M]} \\
& I_{N}=\frac{\left(f_{N} M_{N}-1\right) f_{P} \cdot M_{P P}+\left(f_{P} M_{P}-1\right) f_{N} \cdot M_{P N}}{f_{P} f_{N} \cdot \operatorname{det}[M]} . \tag{A18}
\end{align*}
$$

Noting from (22) that $M_{P P}=-\frac{\partial^{2} M}{\partial I_{P}^{2}}$ and $M_{N N}=-\frac{\partial^{2} M}{\partial I_{N}^{2}}$, the marginal investments from (20) are:

$$
\begin{align*}
\frac{\partial I_{P}}{\partial \alpha} & =\beta \cdot \frac{(1-b) \cdot(2-b+b \alpha \beta) \cdot M_{N N}-b^{2} \cdot(1+\alpha \beta)^{3} \cdot M_{P N}}{(1-b) b \cdot(1+\alpha \beta)^{3} \cdot \operatorname{det}[M]} \\
\frac{\partial I_{N}}{\partial \alpha} & =\beta \cdot \frac{(1-b) \cdot(2-b+b \alpha \beta) \cdot M_{P N}-b^{2} \cdot(1+\alpha \beta)^{3} \cdot M_{P P}}{(1-b) b \cdot(1+\alpha \beta)^{3} \cdot \operatorname{det}[M]} \tag{A19}
\end{align*}
$$

(i) Given the (marginal) investments from (A18) and (A19) in conjunction with the conditions for the equilibrium investments in (13), $\frac{\partial M}{\partial I_{P}}=\frac{1}{(1+\alpha \beta) \gamma}=\frac{1+b \alpha \beta}{b(1+\alpha \beta)^{2}}$ and $\frac{\partial M}{\partial I_{N}}=\frac{1}{1-\gamma}=\frac{1+b \alpha \beta}{1-b}$, the marginal value of the public firm equals:

$$
\begin{align*}
\frac{d V_{P}(\alpha)}{d \alpha} & =-\alpha \beta \gamma \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}+\gamma \frac{\partial M}{\partial I_{N}} \frac{\partial I_{N}}{\partial \alpha}+\frac{\partial \gamma}{\partial \alpha} M \\
& =-\frac{\alpha \beta}{1+\alpha \beta} \cdot \frac{\partial I_{P}}{\partial \alpha}+\frac{(1+\alpha \beta) b}{1-b} \cdot \frac{\partial I_{N}}{\partial \alpha}+\frac{(1-b) b \beta}{(1+b \alpha \beta)^{2}} \cdot M \\
& =\beta \cdot \frac{C_{P P}(\alpha) \cdot M_{P P}+C_{N N}(\alpha) \cdot M_{N N}+C_{P N}(\alpha) \cdot M_{P N}}{2(1-b)^{2} b \cdot(1+b \alpha \beta)^{2}(1+\alpha \beta)^{4} \cdot \operatorname{det}[M]}, \tag{A20}
\end{align*}
$$

with coefficients

$$
\begin{align*}
C_{P P}= & b^{2}(1+\alpha \beta)^{4} \cdot\left(1-3 b+3 b^{2}-b^{3}\right) M_{N}^{2} \\
& -b^{2}(1+\alpha \beta)^{4} \cdot\left[1+b+4 b \alpha \beta+2 b^{2} \alpha \beta+5 b^{2} \alpha^{2} \beta^{2}+b^{3} \alpha^{2} \beta^{2}+2 b^{3} \alpha^{3} \beta^{3}\right] \\
C_{N N}= & (1-b)^{2} \cdot\left[(1-b) b^{2}(1+\alpha \beta)^{4} M_{P}^{2}-(1+b \alpha \beta)^{2}\left(1-b+4 \alpha \beta-2 b \alpha \beta+2 b \alpha^{2} \beta^{2}\right)\right] \\
C_{P N}= & 2(1-b) b(1+\alpha \beta)^{2} \cdot\left[(1-b)^{2} b(1+\alpha \beta)^{2} M_{N} M_{P}+(1+b \alpha \beta)^{2}\left(1+2 b \alpha \beta+b \alpha^{2} \beta^{2}\right)\right] . \tag{A21}
\end{align*}
$$

The sign of the marginal value in (A20) is determined by the sign of the nominator, which is a polynomial of degree seven. Condition (i) implies that the nominator has a positive value at $\alpha=0$. That is, $C_{P P}(0) \cdot M_{P P}+C_{N N}(0) \cdot M_{N N}+C_{P N}(0) \cdot M_{P N}>0$, implying that $\frac{d V_{P}(\alpha)}{d \alpha}>0$ for $\alpha \in\left[0, \bar{\alpha}_{p}\right]$.
(ii) Given the (marginal) investments from (A18) and (A19) in conjunction with the conditions for the equilibrium investments from (13), $\frac{\partial M}{\partial I_{P}}=\frac{1}{(1+\alpha \beta) \gamma}=\frac{1+b \alpha \beta}{b(1+\alpha \beta)^{2}}$, the marginal value of the public firm equals:

$$
\begin{align*}
\frac{d V_{N}}{d \alpha} & =(1-\gamma) \cdot \frac{\partial M}{\partial I_{P}} \frac{\partial I_{P}}{\partial \alpha}-\frac{\partial \gamma}{\partial \alpha} \cdot M \\
& =\frac{1-b}{b(1+\alpha \beta)^{2}} \cdot \frac{\partial I_{P}}{\partial \alpha}-\frac{(1-b) b \beta}{(1+b \alpha \beta)^{2}} \cdot M \\
& =\beta \cdot \frac{D_{P P}(\alpha) \cdot M_{P P}+D_{N N}(\alpha) \cdot M_{N N}+D_{P N}(\alpha) \cdot M_{P N}}{2(1-b) b^{2}(1+b \alpha \beta)^{2} \cdot(1+\alpha \beta)^{5} \cdot \operatorname{det}[M]} \tag{A22}
\end{align*}
$$

with coefficients

$$
\begin{align*}
& D_{P P}=b^{3}(1+\alpha \beta)^{5} \cdot\left[(1+b \alpha \beta)^{2}-(1-b)^{2} M_{N}^{2}\right] \\
& D_{N N}=(1-b)^{2} \cdot\left[(1+b \alpha \beta)^{2}(4-b+3 b \alpha \beta)-b^{3}(1+\alpha \beta)^{5} M_{P}^{2}\right] \\
& D_{P N}=-2(1-b)^{2} b^{3}(1+\alpha \beta)^{5} M_{N} M_{P} . \tag{A23}
\end{align*}
$$

The sign of the marginal value in (A22) is determined by the sign of the nominator, which is a polynomial of degree seven. Condition (ii) implies that the nominator has a positive value at $\alpha=0$. That is, $D_{P P}(0) \cdot M_{P P}+D_{N N}(0) \cdot M_{N N}+D_{P N}(0) \cdot M_{P N}>0$, implying that $\frac{d V_{N}(\alpha)}{d \alpha}>0$ for $\alpha \in\left[0, \bar{\alpha}_{N}\right]$.
(iii) The marginal value of the public firm equals:

$$
\begin{align*}
\frac{d V_{T}(\alpha)}{d \alpha} & =\frac{d V_{P}(\alpha)}{d \alpha}+\frac{d V_{N}(\alpha)}{d \alpha} \\
& =\beta \cdot \frac{E_{P P}(\alpha) \cdot M_{P P}+E_{N N}(\alpha) \cdot M_{N N}+E_{P N}(\alpha) \cdot M_{P N}}{(1-b)^{2} b^{2}(1+\alpha \beta)^{5} \cdot \operatorname{det}[M]} \tag{A24}
\end{align*}
$$

with coefficients

$$
\begin{align*}
& E_{P P}=-b^{4}(1+\alpha \beta)^{6} \\
& E_{N N}=(1-b)^{2} \cdot(2-b+b \alpha \beta) \cdot\left(1-b-b \alpha \beta-b \alpha^{2} \beta^{2}\right) \\
& E_{P N}=b^{2} \cdot(1-b) \cdot(1+\alpha \beta)^{3} \cdot\left(1+2 b \alpha \beta+b \alpha^{2} \beta^{2}\right) \tag{A25}
\end{align*}
$$

The sign of the marginal value in (A24) is determined by the sign of the nominator, which is a polynomial of degree six. Condition (iii) implies that the nominator has a positive value at $\alpha=0$. That is, $E_{P P}(0) \cdot M_{P P}+E_{N N}(0) \cdot M_{N N}+E_{P N}(0) \cdot M_{P N}>0$, implying that $\frac{d V_{T}(\alpha)}{d \alpha}>0$ for $\alpha \in\left[0, \bar{\alpha}_{T}\right]$.

Proof for Proposition 6 In the following, we use the following abbreviations $\Delta_{i}=$ $E\left[U_{i}(1)\right]-E\left[U_{i}(0)\right], M(\tau)=M(I, \tau)$, and $\widehat{M}_{i}(\tau)=\widehat{M}_{i}(\widehat{I}, \tau)$. Given $\Delta_{i}=$ $M_{i}(1)+\alpha_{i} \beta \cdot\left[M_{i}(1)-\widehat{M}_{i}(1)+\widehat{M}_{i}(0)\right]-\alpha_{i} \delta \cdot\left[M_{j}(1)-\widehat{M}_{j}(1)+\widehat{M}_{j}(0)\right]$, we consider the bargaining problem, that is, $\max \Delta_{A}^{1 / 2} \cdot \Delta_{B}^{1 / 2}$ subject to $M_{B}(I, 1)=$
$M(I, 1)-M_{A}(I, 1)$. Given $\frac{\partial \Delta_{A}}{\partial M_{A}}=1+\alpha_{A}(\beta+\delta)$ and $\frac{\partial \Delta_{B}}{\partial M_{A}}=-\left(1+\alpha_{B}(\beta+\delta)\right)$, the first-order condition with respect to $M_{A}(I)$ is

$$
\begin{equation*}
0=\frac{\frac{\partial \Delta_{A}}{\partial M_{A}} \cdot \Delta_{B}+\frac{\partial \Delta_{B}}{\partial M_{A}} \cdot \Delta_{A}}{2 \cdot \Delta_{A}^{1 / 2} \cdot \Delta_{B}^{1 / 2}} \tag{A26}
\end{equation*}
$$

In equilibrium, the conjectures are correct, $\widehat{M}_{i}(1)=M_{i}(1)$ and $\widehat{M}_{i}(0)=$ 0 , and the first-order condition in (A26) reduces to $0=\left[1+\alpha_{A}(\beta+\delta)\right]$. $\left[M(1)-M_{A}(1)\right]-\left[1+\alpha_{B}(\beta+\delta)\right] \cdot M_{A}(1)$, yielding (24).

Given (A26), the second-order condition equals:

$$
\begin{equation*}
-\frac{\Delta_{A}^{-\frac{1}{2}} \Delta_{B}^{-\frac{1}{2}}}{4}\left(\frac{\Delta_{B}}{\Delta_{A}}\left(\frac{\partial \Delta_{A}}{\partial M_{A}}\right)^{2}+\frac{\Delta_{A}}{\Delta_{B}}\left(\frac{\partial \Delta_{B}}{\partial M_{A}}\right)^{2}-\frac{\partial \Delta_{A}}{\partial M_{A}} \frac{\partial \Delta_{B}}{\partial M_{A}}\right) \leq 0 \tag{A27}
\end{equation*}
$$

and is satisfied as $\Delta_{A} \geq 0, \Delta_{B} \geq 0$, and $-\frac{\partial \Delta_{A}}{\partial M_{A}} \frac{\partial \Delta_{B}}{\partial M_{A}}=\left[1+\alpha_{A}(\beta+\delta)\right]$. $\left[1+\alpha_{B}(\beta+\delta)\right] \geq 0$.

To calculate the marginal value of firm $i$, we note that symmetry of $\alpha$ implies $\gamma_{i}=$ $\gamma_{j}=\frac{1}{2}$ from (24). Given the firm value $V_{i}(\alpha)=2 \mu_{P}+\frac{1}{2} \cdot M(I(\alpha), 1)-w_{i}\left(I_{i}(\alpha)\right)$, we apply the first-order conditions from (25), $\frac{1+\alpha(\beta-\delta)}{2} \frac{\partial M}{\partial I_{i}}=w_{i}^{\prime}\left(I_{i}\right)$, and exploit $\frac{\partial M}{\partial I_{i}}=\frac{a M}{I_{i}}$. That is, we get:

$$
\begin{align*}
\frac{d V_{i}}{d \alpha} & =\left(\frac{1}{2} \frac{\partial M}{\partial I_{i}}-w_{i}^{\prime}\right) \frac{\partial I_{i}}{\partial \alpha}+\frac{1}{2} \frac{\partial M}{\partial I_{j}} \frac{\partial I_{j}}{\partial \alpha} \\
& =-\frac{\alpha(\beta-\delta)}{2} \cdot \frac{\partial M}{\partial I_{i}} \frac{\partial I_{i}}{\partial \alpha}+\frac{1}{2} \cdot \frac{\partial M}{\partial I_{j}} \frac{\partial I_{j}}{\partial \alpha} \\
& =\left(-\frac{\alpha(\beta-\delta)}{2} \cdot \frac{a}{I_{i}} \frac{\partial I_{i}}{\partial \alpha}+\frac{1}{2} \cdot \frac{a}{I_{j}} \frac{\partial I_{j}}{\partial \alpha}\right) \cdot M . \tag{A28}
\end{align*}
$$

Given $\alpha_{i}=\alpha_{j}=\alpha$ and $\gamma_{i}=\gamma_{j}=\frac{1}{2}$, the investment equals $I_{i}=$ $\left(\frac{1+\alpha(\beta-\delta)}{2} a\right)^{\frac{1}{1-2 a}}$, implying that

$$
\begin{equation*}
\frac{a}{I_{i}} \frac{\partial I_{i}}{\partial \alpha}=\frac{a(\beta-\delta)}{(1-2 a)(1+\alpha(\beta-\delta))}=\frac{a}{I_{j}} \frac{\partial I_{j}}{\partial \alpha} . \tag{A29}
\end{equation*}
$$

Given (A29), (A28) can be restated as:

$$
\begin{equation*}
\frac{d V_{i}}{d \alpha}=\frac{1-\alpha(\beta-\delta)}{2} \cdot \frac{a(\beta-\delta) \cdot M}{(1-2 a)(1+\alpha(\beta-\delta))}, \tag{A30}
\end{equation*}
$$

with a maximum at $\alpha=\frac{1}{\beta-\delta}>1$.
Given $V_{T}(\alpha)=4 \mu+M(I(\alpha), 1)-w_{i}\left(I_{i}(\alpha)\right)-w_{j}\left(I_{j}(\alpha)\right)$, we derive the marginal value of the whole alliance by applying the first-order conditions, $\frac{1+\alpha_{j}(\beta-\delta)}{2} \frac{\partial M}{\partial I_{j}}=$ $w_{j}^{\prime}$ from (25), and applying $\frac{\partial M}{\partial I_{j}}=\frac{a M}{I_{j}}$ and the comparative static results from (A29).

That is, we get:

$$
\begin{align*}
\frac{d V_{T}}{d \alpha} & =\sum_{j=A}^{B}\left(\frac{\partial M}{\partial I_{j}}-w_{j}^{\prime}\right) \frac{\partial I_{j}}{\partial \alpha} \\
& =\sum_{j=A}^{B} \frac{1-\alpha_{j}(\beta-\delta)}{2} \cdot \frac{a M}{I_{j}} \cdot \frac{\partial I_{j}}{\partial \alpha} \\
& =\frac{1-\alpha(\beta-\delta)}{2} \cdot \frac{2 a(\beta-\delta) \cdot M}{(1-2 a)(1+\alpha(\beta-\delta))} \tag{A31}
\end{align*}
$$

with a maximum at $\alpha=\frac{1}{\beta-\delta}>1$. For completeness, note that the marginal firm value of the whole alliance equals the sum of the marginal firm values; that is, $\frac{d V_{T}}{d \alpha}=$ $\frac{d V_{i}}{d \alpha}+\frac{d V_{j}}{d \alpha}$ from (A30) and (A31).

Acknowledgements We thank Stefan Reichelstein (editor), two anonymous reviewers, Anil Arya, Sunil Dutta, Qintao Fan (discussant), Klaus Haider, Haijin Lin (discussant), Brian Mittendorf, Alexander Nezlobin, Alfred Wagenhofer (discussant), Xiao-Jun Zhang, and workshop participants at the University of California at Berkeley, the Management Accounting Section Midyear Meeting, the Hawaii Accounting Research Conference, and the German Economic Association of Business Administration Conference for helpful comments.

Funding Open access funding provided by University of Zurich.
Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visithttp://creativecommons.org/licenses/by/4.0/.

## References

Anctil, R., and S. Dutta. 1999. Negotiated transfer pricing and divisional vs. firm-wide performance evaluation. Accounting Review 74(1): 87-104.
Arya, A., J. Fellingham, J. Glover, and K. Sivaramakrishnan. 2000. Capital budgeting, the hold-up problem, and information system design. Management Science 46(2): 205-216.
Baldenius, T., and S. Reichelstein. 1999. Negotiated versus cost-based transfer pricing. Review of Accounting Studies 2(4): 67-91.
Baldenius, T., and B. Michaeli. 2017. Investments and risk transfers. Accounting Review 92(6): 1-23.
Baldenius, T., and B. Michaeli. 2019. Integrated ownership and managerial incentives with endogenous project risk. Review of Accounting Studies 24(4): 1450-485.
Baldenius, T., and B. Michaeli. 2020. Responsibility centers, decision rights, and synergies. Accounting Review 95(2): 1-29.
Bustamante, M., and L. Fresard. 2021. Does firm investment respond to peers' investment? Management Science 67(8): 4703-4724.
Cachon, G., and S. Netessine. 2006. Game theory in supply chain analysis. In: Handbook of Quantitative Supply Chain Analysis, 13-59. Boston: Kluwer.
Chan, S., J. Kensinger, A. Keown, and J. Martin. 1997. Do strategic alliances create value? Journal of Financial Economics 46(2): 199-221.

Das, S., P. Sen, and S. Sengupta. 1998. Impact of strategic alliances on firm valuation. Academy of Management Journal 41(1): 27-41.
Dutta, S., and A. Nezlobin. 2019. Mandatory disclosure, corporate investment, and shareholder welfare. Working Paper.
Dutta, S., and S. Reichelstein. 2005. Stock price, earnings, and book value in managerial performance measures. Accounting Review 80(4): 1069-1100.
Dutta, S., and S. Reichelstein. 2021. Capacity rights and full-cost transfer pricing. Management Science 67(2): 1303-1325.
Edlin, A., and S. Reichelstein. 1995. Specific investment under negotiated transfer pricing: an efficiency result. Accounting Review 70(3): 275-291.
Edlin, A., and S. Reichelstein. 1996. Holdups, standard breach remedies, and optimal investment. American Economic Review 86(3): 478-501.
Fudenberg, D., and J. Tirole. 1986. A "signal-jamming" theory of predation. Rand Journal of Economics 17(3): 366-376.
Gibbons, R. 1985. Incentives in internal labor markets, mimeo.
Grossman, S., and O. Hart. 1986. The costs and benefits of ownership: a theory of vertical and lateral integration. Journal of Political Economy 94(4): 691-719.
Hart, O. 1995. Firms, contracts and financial structure. New York: Oxford University Press.
Hart, O., and J. Moore. 1999. Foundations of incomplete contracts. Review of Economic Studies 66(1): 115-138.
Holmstrom, B. 1982. Managerial incentive problems: a dynamic perspective. In: Essays in Economics and Management in Honor of Lars Wahlbeck, 209-230. Helsinki: Swedish School of Economics.
Jia, Y., Z. Wang, J. Wu, and Z. Zhang. 2022. The spillover effect of customer CEO myopia on supplier firms. Working Paper.
Kanodia, C. 2006. Accounting disclosure and real effects. Foundations and Trends in Accounting 1(3): 167-258.
Kanodia, C., and H. Sapra. 2016. A real effects perspective to accounting measurement and disclosure: implications and insights for future research. Journal of Accounting Research 54(2): 623-676.
Kanodia, C., R. Sing, and A. Spero. 2005. Imprecision in accounting measurement: can it be value enhancing? Journal of Accounting Research 43(3): 487-519.
Koh, J., and N. Venkatraman. 1991. Joint venture formations and stock market reactions: an assessment in the information technology sector. Academy of Management Journal 34(4): 869-892.
Lai, G., L. Debo, and L. Nan. 2011. Channel stuffing with short-term interest in market value. Management Science 57(2): 332-346.
Lai, G., W. Xiao, and J. Yang. 2012. Supply chain performance under market valuation. Management Science 58(10): 1933-1951.
Lai, G., and W. Xiao. 2017. Inventory decisions and signals of demand uncertainty to investors. Manufacturing \& Service Operations Management 20(1): 113-129.
Liang, P., and X. Wen. 2007. Accounting measurement basis, market mispricing, and firm investment efficiency. Journal of Accounting Research 45(1): 155-197.
Lindsey, R. 2008. Blurring firm boundaries: the role of venture capital in strategic alliances. Journal of Finance 63(3): 1137-1168.
McConnell, J., and T. Nantell. 1985. Corporate combinations and common stock returns: the case of joint ventures. Journal of Finance 40(2): 519-536.
Myerson, R. 1991. Game theory: analysis of conflict. Cambridge: Harvard University Press.
Noeldeke, G., and K. Schmidt. 1995. Option contracts and renegotiation: a solution to the hold-up problem. RAND Journal of Economics 26(2): 163-179.
Noeldeke, G., and K. Schmidt. 1998. Sequential investments and options to own. RAND Journal of Economics 29(4): 633-653.
Pfeiffer, T., U. Schiller, and J. Wagner. 2011. Cost-based transfer pricing. Review of Accounting Studies 16(2): 219-246.
Saada, B., and B. Gomes-Casseres. 2019. Why your next deal may be a partnership. Strategy 94(Spring), Retrieved 10 Feb 2020, from https://www.strategy-business.com/article/ Why-Your-Next-Deal-May-Be-a-Partnership.
Schmidt, W., V. Gaur, R. Lai, and A. Raman. 2015. Signaling to partially informed investors in the newsvendor model. Production and Operations Management 25(3): 383-401.

Segal, I., and M. Whinston. 2012. Property rights. In: The Handbook of Organizational Economics. Princeton: Princeton University Press.
Stein, J. 1989. Efficient capital markets, inefficient firms: a model of myopic corporate behavior. Quarterly Journal of Economics 104(4): 655-670.
Taylor, T., and E. Plambeck. 2007. Supply chain relationships and contracts: the impact of repeated interaction on capacity investment and procurement. Management Science 53(10): 1577-1593.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.


[^0]:    Hui Chen
    hui.chen@business.uzh.ch

    Thomas Pfeiffer
    thomas.pfeiffer@univie.ac.at

    1 University of Zurich, Zurich, Switzerland
    2 University of Vienna, Vienna, Austria

[^1]:    ${ }^{1}$ Some high-profile examples of recent alliances include the Amazon-Berkshire-JPMorgan deal to improve healthcare system efficiency, the Toyota-Microsoft joint venture to use cloud computing for car service, and the Starbucks-Nestle partnership for selling packaged coffee.
    ${ }^{2}$ Alliances between one public firm and one private firm are prevalent in practice. Lindsey's (2008) data from the Thomson Financial Strategic Alliances and Joint Ventures Data shows that almost half ( $47 \%$ ) of alliances formed are public-private, while $29 \%$ of alliances are private-private and $25 \%$ are public-public. In Section 4.2, we also discuss public-public alliances. Private-private alliances have been discussed in the literature on hold-up problems (e.g., (Grossman and Hart 1986; Hart 1995)).

[^2]:    ${ }^{3}$ It is standard in both the hold-up and signal jamming literatures to assume unobservable investments and the associated costs. Our insights remain valid when the costs are observable with uncorrelated noise but unverifiable, that is, $w_{P}\left(I_{P}\right)=I_{P}+\eta_{P}$ with $\eta_{P} \sim N\left(0, \sigma_{\eta_{R}}^{2}\right)$ and $\sigma_{\eta_{R}}^{2}>0$ (Kanodia and Sapra 2016).

[^3]:    ${ }^{4}$ Business alliances are often announced in public, with information about the scope and nature of the deal (e.g., technological or marketing). Such information may also be found in the material definitive agreement in the 8-K Form or through various databases, including Thomson Financial Strategic Alliances and Joint Ventures (e.g., (Lindsey 2008)). The insights of our model remain valid even if the capital market receives an additional noisy signal about the public firm's bargaining outcome, $M_{P}(I, 1)+\epsilon_{S}$, where $\epsilon_{S} \sim N\left(0, \sigma_{S}^{2}\right)$ denotes uncorrelated measurement errors. The reason is that the interim market price would not change and thus investment and bargaining incentives of the firms would remain the same.

[^4]:    ${ }^{5} \mathrm{~A}$ common interpretation is that a portion $\alpha$ of the public firm may be sold at the capital market before the firm is liquidated due to life cycle or liquidity reasons. The remaining ( $1-\alpha$ ) portion will be held by the initial shareholders.

[^5]:    ${ }^{6}$ For completeness, the public firm's share increases in its bargaining power; that is, $\frac{\partial \gamma}{\partial b}=\frac{1+\alpha \beta}{(1+b \alpha \beta)^{2}}>0$.

[^6]:    ${ }^{7}$ The first-order condition in (13) is $(1+\alpha \beta) \cdot \gamma \frac{\partial M}{\partial I_{P}}=w_{P}^{\prime}$, implying that $\gamma \frac{\partial M}{\partial I_{P}}-w_{P}^{\prime}=-\alpha \beta \gamma \cdot \frac{\partial M}{\partial I_{P}}$ in (15).

[^7]:    ${ }^{8}$ Note that $\frac{\partial I_{P}}{\partial \alpha}<0$ requires that the cross effect for $I_{P}$ exhibits a negative sign, which requires $\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}<$ 0 . Conversely, $\frac{\partial I_{N}}{\partial \alpha}>0$ requires that the cross effect for $I_{N}$ exhibits a positive sign, which requires $\frac{\partial^{2} M}{\partial I_{P} \partial I_{N}}>0$. Hence $\frac{\partial I_{P}}{\partial \alpha}<0$ and $\frac{\partial I_{N}}{\partial \alpha}>0$ cannot be satisfied simultaneously.

[^8]:    ${ }^{9}$ Introducing a common term does not change any of our prior insights as the market does not observe the nonpublic firm's cash flows.

[^9]:    ${ }^{10}$ Note that the sign of $k_{2}$ alters the ordering of the zeros, $z_{1}$ and $z_{2}$.
    ${ }^{11}$ Formally, we can recast the quadratic function in terms of $z=\beta \alpha$ and determine the zero $\bar{z}$ and $\bar{\alpha}(\beta)=$ $\bar{z} / \beta$. Since $\bar{\alpha}(1) \geq \bar{\alpha}(\beta)$ for all $\beta$, our result is valid for all $\beta$.

[^10]:    ${ }^{12}$ Note that the sign of $c_{3}$ determines the ordering of the critical values, $\alpha_{1 / 2}^{\text {crit }}$.
    ${ }^{13}$ Note that the quadratic function cannot reduce to a linear function as $c_{0}>0$ implies $c_{2}<0$.

[^11]:    ${ }^{14}$ Wolg we can recast the cubic function in terms of $z=\beta \alpha \in[0,1]$. Also note that $\alpha(\beta)=z / \beta$ decreases in $\beta$.

