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CORRECTION

Correction to: Multi-server queueing systems with multiple priority classes

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We correct the expressions of the matrix $\mathbf{B}^{(\ell)}$ on page 340 and the matrix $\mathbf{L}^{(\ell)}$ on page 341 in [1]. Specifically, the following are the corrected expressions of these matrices:

$$\mathbf{B}^{(\ell)} = \mu_L \begin{pmatrix} \min(2,\ell) & & & \\ & 1 & & \\ & & 1 & \\ & & & \mathbf{0} \end{pmatrix} \tag{1}$$

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$$\mathbf{L}^{(\ell)} = \left(\begin{array}{c|c|c|c|c|c|c|c} \begin{matrix} -\sigma_1 & \lambda_M & \lambda_H \\ \mu_M & -\sigma_2 & \\ \mu_H & & -\sigma_3 \end{matrix} & \lambda_M \mathbf{p}^{(2M,M)} & \lambda_M \mathbf{p}^{(2M,H)} & \begin{matrix} \lambda_H \mathbf{p}^{(MH,M)} \\ \lambda_M \mathbf{p}^{(MH,M)} \end{matrix} & \begin{matrix} \lambda_H \mathbf{p}^{(MH,H)} \\ \lambda_M \mathbf{p}^{(MH,H)} \end{matrix} & \lambda_H \mathbf{p}^{(2H,M)} & \lambda_H \mathbf{p}^{(2H,H)} \\ \hline \mathbf{t}^{(1)} & \mathbf{T}^{(1)} & & & & & & \\ \hline \mathbf{t}^{(2)} & & \mathbf{T}^{(2)} & & & & & \\ \hline \mathbf{t}^{(3)} & & & \mathbf{T}^{(3)} & & & & \\ \hline \mathbf{t}^{(4)} & & & & \mathbf{T}^{(4)} & & & \\ \hline \mathbf{t}^{(5)} & & & & & \mathbf{T}^{(5)} & & \\ \hline \mathbf{t}^{(6)} & & & & & & \mathbf{T}^{(6)} & \end{array}\right) \tag{2}$$

for all $\ell \geq 0$, where the definitions of the notation in the matrices are unchanged from [1] except the zero matrix $\mathbf{0}$ in (1), whose size needs to be corrected to 12×12.

These matrices represent the transitions shown in the left panel of Figure 3 in [1]. The transition from state (1H, 0M, uL) to (1H, 0M, $(u-1)$L), namely the third diagonal element of $\mathbf{B}^{(\ell)}$, was missing in the original expression. The transition rates from (0H, 1M, uL) to two states labeled with (1H, 1M, uL), namely the $(2, k)$ element of $\mathbf{L}^{(\ell)}$ for $8 \leq k \leq 11$, are $\lambda_H \mathbf{p}^{(MH,M)}$ and $\lambda_H \mathbf{p}^{(MH,H)}$, but erroneously were $\lambda_M \mathbf{p}^{(MH,M)}$ and $\lambda_M \mathbf{p}^{(MH,H)}$ in the original expression. Likewise, the transition rates from (1H, 0M, uL) to (1H, 1M, uL), namely the $(3, k)$ element of $\mathbf{L}^{(\ell)}$ for $8 \leq k \leq 11$, are $\lambda_M \mathbf{p}^{(MH,M)}$ and $\lambda_M \mathbf{p}^{(MH,H)}$, but were $\lambda_H \mathbf{p}^{(MH,M)}$ and $\lambda_H \mathbf{p}^{(MH,H)}$ in the original expression.

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Reference

1. Harchol-Balter, M., Osogami, T., Scheller-Wolf, A., Wierman, A.: Multi-server queueing systems with multiple priority classes. Queueing Syst. **51**, 331–360 (2005). https://doi.org/10.1007/s11134-005-2898-7