Decoherence dynamics estimation for superconducting gate-model quantum computers

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Abstract

Superconducting gate-model quantum computer architectures provide an implementable model for practical quantum computations in the NISQ (noisy intermediate scale quantum) technology era. Due to hardware restrictions and decoherence, generating the physical layout of the quantum circuits of a gate-model quantum computer is a challenge. Here, we define a method for layout generation with a decoherence dynamics estimation in superconducting gate-model quantum computers. We propose an algorithm for the optimal placement of the quantum computational blocks of gate-model quantum circuits. We study the effects of capacitance interference on the distribution of the Gaussian noise in the Josephson energy.

Keywords Gate-model quantum computers · Quantum computers · Quantum computations

1 Introduction

Quantum computers exploit the fundamentals of quantum mechanics [1–14], making it possible to solve problems more efficiently than with any possible traditional computer [8–25]. The information processing network of a gate-model quantum computer uses traditionally uninterpretable phenomena such as quantum superposition or quantum entanglement [26–36]. The quantum computations are performed on some initial quantum states via a sequence of unitary operators [30,37–49]. The unitary operators

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can formulate a larger unit called unitary block. A quantum circuit integrates these unitary blocks with a measurement array to measure the resulting quantum systems [2–4,50–62]. In a quantum computer architecture, an important issue is the development of entangled connections between the quantum systems of the quantum circuit [15,17–25]. A practical way to establish entangled connection is to use entangling gates in a gate-model quantum computer [38], along with some constraints on the circuit depths of the quantum circuits [15,17–25]. For near-term quantum computers, superconducting architectures [1,63] and gate-model quantum computing [15,17–26,38,64,65] provide a well-implementable model. The development of experimental implementations of gate-model quantum computers has already been started [15,17–20,38], and the results look promising to arrive in the commercial quantum computers in the near future. The superconducting qubit architectures [1,63] allow a straightforward experimental implementation in the NISQ (noisy intermediate scale quantum) technology era [2], with non-error-corrected qubits to realize quantum simulations and quantum algorithms. In the physical layout of a gate-model quantum computer, the quantum states and quantum gates (unitary operators) are positioned in space such that the quantum gates are constrained by some hardware restrictions. Specifically, in the quantum circuit computation model of gate-model quantum computers, several hardware restrictions have to be satisfied simultaneously. A particular constraint is that some quantum gates have to be applied in multiple rounds [38]. Another constraint could be the preservation of some entangled connections between the selected gates, or the connection topology in the quantum circuit. In our approach, we assume that these constraints are present in the development phase; thus, the quantum circuits are evolved with respect to the hardware restrictions of gate-model quantum computers. Another important aspect is the application of gate-model quantum computations in near-term quantum devices of the quantum Internet [30–32,66–120].

Here, we define a method for quantum circuit layout generation for gate-model quantum computers. The layout generation is achieved through several algorithms, and we define a grid structure¹ for the development of gate-model quantum circuits. The quantum circuits integrate a number of unitary computational blocks, and each unitary block consists of unitary quantum gates. In the physical layout, each quantum circuit has a particular depth (circuit depth) and a number of output wires. The grid structure encodes the physical layer placement of the unitary blocks, and it provides a tool to find the optimal placement of the unitary blocks. The grid structure can be used to iterate on the possible placements until the optimal placement is determined: The optimal placement minimizes the area of the quantum circuit such that the connections and the entanglement between the blocks are preserved. The optimal grid and the optimal placement are in a one-to-one correspondence relation. We define an algorithm for the determination of the optimal placement from an arbitrary input placement. First, the

¹ The grid structure model is motivated by the space-time volume of a quantum circuit computation in a gate-model quantum computer setting [38]. In this particular model, an array of qubits is arranged on a grid with a particular height and depth. The qubit array is initialized by preparing the quantum state of each qubit. In a qubit gate-model setting, a layer of 1-qubit gates and a layer of 2-qubit gates are applied on the qubits. In the gate structure, the 2-qubit gates are applied in several rounds due to hardware restrictions, because as a constraint, one qubit cannot participate in two gates simultaneously. Note that in the grid structure model, as the circuit depth increases above a critical limit, the qubits become entangled with each other at some particular setting, see also [38].

input placement is converted into a set of grids that encodes different placements. Then, the optimal placement is found in the form of a grid, using this set of grid instances. We prove the optimality of the method and extend it with a decoherence analysis. We define a decoherence estimation method for charge qubit [121] structures² (a charge qubit circuit integrates Josephson junctions and capacitor elements). In the system model, the charge qubit circuits are integrated into a multilayer circuit structure, with a different number of circuits in each layer. We study the capacitor interference in the multilayer quantum circuit structure and prove its effects on the decoherence dynamics. The decoherence of a particular charge qubit circuit is modeled by a Gaussian noise in the Josephson energy. Gaussian noise is a standard model of noise in the control parameters in a superconducting charge qubits. Particularly, an imperfect control impacts the qubit dynamics, since it induces a Gaussian noise in the Josephson energy. A detailed study on the Gaussian noise and its impacts in the Josephson energy can be found in [128].

The novel contributions of our manuscript are as follows:

1. We define a decoherence estimation method for charge qubit structures. The charge qubit circuits are integrated into a multilayer circuit structure in a superconducting quantum computer, with a different number of circuits in each layer.
2. We study the capacitor interference in the multilayer quantum circuit structure and prove its effects on the decoherence dynamics. The decoherence of a particular charge qubit circuit is modeled by a Gaussian noise in the Josephson energy.
3. We define a method for quantum circuit layout generation for gate-model quantum computers. We define a grid data structure for the development of gate-model quantum circuits. The grid structure encodes the physical layer placement of the unitary blocks.

This paper is organized as follows. Section 3 gives the problem statement. In Sect. 3, the layout generation framework is proposed. In Sect. 4, the decoherence estimation model is defined. Finally, Sect. 5 concludes the paper. Supplemental information is included in Appendix.

2 Problem statement

The problems to be solved are as follows.

Problem 1 *Define a data structure for an optimal physical layer placement of the unitary blocks.*

Problem 2 *Evaluate the capacitor interference in multilayer superconducting quantum circuits.*

Problem 3 *Prove the effects of capacitor interference on decoherence in superconducting charge qubit circuits.*

² Decoherence is typically considered through the environmental interactions in circuit QED and cavity QED settings. A charge qubit is a particular type of superconducting qubit. Compared to other solid-state implementations, the superconducting gap protects the condensate from the environment to some extent [121]. In the proposed model, the decoherence is associated with the interference of capacitors. Note that decoherence in a charge qubit setting can be connected to other sources, such as imperfect voltage pulses applied to the gates [121]. These cases are well studied in the literature, and we suggest [122–127].

The resolutions to Problems 1–3 are proposed in the theorems and lemmas of the manuscript. The steps are detailed in the Algorithms.

3 Methods

In this section, we define a grid data structure for the optimal physical layer placement of entangled quantum circuits. The optimality requires that the quantum circuit area and the length of quantum wires are minimized such that the entangled connections defined by the developer are preserved. The grid structure is used to determine the optimal placement from a non-optimal input placement. The non-optimal placement is converted into a set of grids, and after some manipulations, the optimal grid is determined that encodes the optimal physical layer placement.

3.1 Grid structure

Proposition 1 *There exists a d -dimensional, N size grid G^d to encode the $\mathcal{P}$ physical placement of the m unitary computational blocks $U = \{U_1, U_2, \dots, U_m\}$ of a quantum circuit QG , with k entangled connections between the computational blocks.*

Let $U = \{U_1, U_2, \dots, U_m\}$ refer to the set of m unitary blocks, where the i th unitary block, U_i , integrates unitary operators such that the circuit depth of U_i is D_i , where the number of quantum wires N_i is evaluated as

$$N_i = \max(n_i^{in}, n_i^{out}), 1 \leq i \leq m, \quad (1)$$

where n_i^{in} and n_i^{out} are the number of input and output quantum wires of U_i , and area A_i as

$$A_i = D_i N_i, 1 \leq i \leq m. \quad (2)$$

To achieve a physical layer placement $\mathcal{P}$ of the unitary blocks, let (x_i, y_i) refer to the bottom-left corner of U_i [129–133]. For an optimal placement $\mathcal{P}^*$, the following conditions have to be satisfied: First, the total area A of the quantum circuit is minimized, with no overlap between the modules, such that all unitary blocks are compacted in both x and y directions; thus,

$$A = \sum_{i=1}^m D_i N_i = \min \kappa(\mathcal{P}^*), \quad (3)$$

where $\kappa(\mathcal{P}^*)$ is the minimum bounding rectangle of $\mathcal{P}^*$.

Second, the length ℓ of quantum wires of $\mathcal{P}^*$ is minimized:

$$\ell = \sum_{i=1}^m w_i = \min \beta(\mathcal{P}^*), \quad (4)$$

where w_i is the total quantum wire length associated with U_i and $\beta(\mathcal{P}^*)$ is the sum of the half-bounding box of interconnections between the unitary blocks.

Third, all of these conditions have to be satisfied such that the set of k entangled connections

$$E = \{E_1, E_2, \dots, E_k\} \quad (5)$$

between the blocks is preserved in the optimal placement $\mathcal{P}^*$.

The construction method of the G^d , d -dimensional, N size grid for the optimal placement $\mathcal{P}^*$ with constraints (3), (4), and (5) is summarized in Procedure 1.

Procedure 1 Grid Construction

- Step 1.** Let there be m unitary blocks $U = \{U_1, U_2, \dots, U_m\}$ with k entangled connections $E = \{E_1, E_2, \dots, E_k\}$ between the blocks. Determine dimension d and size N of the grid G^d . Select the center of a grid for U_0 , which represents $(x_0, y_0) = (0, 0)$ in $\mathcal{P}^*$.
- Step 2.** Place the remaining $m - 1$ blocks into the intersections of the grid G^d such that the blocks are connected by only left and right auxiliary diagonal lines.
- Step 3.** If a j th unitary block, U_j , $j > i$, is on the right-hand side and adjacent to U_i , then connect U_i and U_j by a left diagonal auxiliary line.
- Step 4.** If U_j , $j > i$, is above U_i , then connect U_i and U_j by a right diagonal auxiliary line.
- Step 5.** Repeat steps 3–4 until all unitary blocks are placed.
- Step 6.** Delete the auxiliary diagonal lines.
- Step 7.** Connect the entangled U_i unitary blocks according to set E .
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3.1.1 Discussion

A detailed description of Procedure 1 is as follows.

In Step 1, the development constraints are defined. The aim of the procedure is to determine an optimal physical layer placement of the unitary blocks such that the entangled connections between the particular blocks are preserved.

Step 2 assures that the first unitary block U_0 is placed at the center of a particular grid, while the other blocks can be placed only at the intersections of G^d . The unitary blocks are connected by auxiliary diagonal lines that identify the placement of the blocks in the physical layer minimization process. A given U_i can be connected by at most two auxiliary diagonal lines: One left, U_{i+1} , and one right unitary block, U_{i+2} , are allowed [129–133].

Step 3 achieves the $\mathcal{P}^*$ physical layer placement of the unitary block U_j with respect to U_i , such that [132,133]

$$x_j = x_i + D_i \quad (6)$$

in $\mathcal{P}^*$, where x_i and x_j are the x -coordinates of the bottom-left corner of U_i and U_j , while D_i is the circuit depth of U_i .

Step 4 achieves the physical layer placement of the unitary block U_j with respect to U_i , such that

$$x_j = x_i \quad (7)$$

in $\mathcal{P}^*$. Note that (6) and (7) ensures that $y_j \in [x_j, x_j + D_j]$.

Step 5 achieves the placement of all unitary blocks of the quantum circuit.

Step 6 is verified by the fact that the grid structure of G^d uniquely identifies the placement $\mathcal{P}^*$ (see Lemma 1).

Step 7 ensures the entangled circuit topology between the particular blocks.

The proof is concluded here.

Additional information is included in Sect. 1.

3.1.2 Optimality

Lemma 1 *The G^d grid structure provides an optimal structure for the placement of the m computational blocks with the preservation of the k entangled connections of a QG quantum circuit.*

Proof The optimality of the G^d grid structure means that there is a one-to-one correspondence between an optimal placement $\mathcal{P}^*$ and the optimal grid G^{d*} . As a corollary, the optimal placement of the unitary blocks can be determined by a packing procedure in the grid structure.

The optimality in the physical layer is interpretable in terms of packing area. In G^d , U_0 identifies a unique base bottom-left coordinate $(x_0, y_0) = (0, 0)$ in $\mathcal{P}^*$, and all remaining computational blocks are adjacent to at least one other module, if it exists.

For the determination of a U_j left-diagonal connection of U_0 , let us assume as the first case that there is a set $\mathcal{S}_r$ of unitary blocks placed right adjacent to U_0 [132,133]. In this case, the block U' with the smallest y -coordinate from $\mathcal{S}_r$ is also unique; therefore, U_0 and U' can be connected by a left-diagonal line in the grid; thus, $U_j = U'$. Then, as a second case, let us assume that $\mathcal{S}_r = \emptyset$. In this case, there is no left-diagonal line from U_0 and any other computational block; thus, $U_j = \emptyset$.

For the determination of a U_j right-diagonal connection of U_0 , let $\mathcal{S}_a$ be the set of unitary blocks above U_0 [132,133]. Then, a right-diagonal connection U_j from U_0 identifies the block U' with the smallest y -coordinate from $\mathcal{S}_a$ such that $x' = x_0$, where x' is the x -coordinate of U' . Assuming that $\mathcal{S}_a \neq \emptyset$, there always exists such a unique U' , since $\mathcal{P}^*$ is optimal; thus, $U_j = U'$. If $\mathcal{S}_a = \emptyset$, then such a unique U' does not exist; thus, $U_j = \emptyset$.

These assumptions can be extended to all computational blocks; therefore, there is a one-to-one correspondence between an optimal placement $\mathcal{P}^*$ and its unique grid structure G^{d*} [129–133].

As a corollary, in the physical layer, the coordinates of all computational blocks can be uniquely determined from G^{d*} using the construction rules (6) and (7).

Note that by some fundamental theory on tree data structures [129–133], the possible combinations for a $d = 2$ -dimensional grid G^2 at m unitary blocks is as

$$\mathcal{O}\left(\frac{m!2^{2m}}{m^{1.5}}\right). \quad (8)$$

The proof is concluded here. $\square$

3.2 Grid set generation

Let us assume that the developer inputs a placement $\mathcal{P}$. The problem then is to find a set of different G^d grid encodings that encode the input placement. Then, the

optimal grid G^{d*} has to be determined for the optimal placement $\mathcal{P}^*$. For the resolution of the former problem, we define algorithm $\mathcal{A}_1$, while for the latter problem, we define algorithm $\mathcal{A}_2$.

Lemma 2 *For an input placement $\mathcal{P}$, there exists a set $\mathcal{S}_{\mathcal{P}}(G^d) : \{G_1^d, \dots, G_c^d\}$ of c different grid instances that encode $\mathcal{P}$ such that the k entangled connections are satisfied.*

Proof In an input placement $\mathcal{P}$, a U_i unitary block is represented by a box with particular x_i and y_i coordinates in the bottom-left corner, along with the depth D_i and N_i .

The aim of algorithm $\mathcal{A}_1$ is to convert the non-optimal input placement $\mathcal{P}$ with the particular constraints into a set $\mathcal{S}_{\mathcal{P}}(G^d) : \{G_1^d, \dots, G_c^d\}$ of c different grids that encode $\mathcal{P}$. During the grid construction, the fact that a left-diagonal connection U_j of a block U_i in G^d represents that U_j is above U_i in $\mathcal{P}$ can be exploited.

The steps of $\mathcal{A}_1$ are summarized in Algorithm 1.

Algorithm 1 Grid Set Extraction from an Input Placement ($\mathcal{A}_1$)

Step 1. Let $\mathcal{P}$ be the input placement of the m unitary blocks $U = \{U_1, U_2, \dots, U_m\}$ with k entangled connections $E = \{E_1, E_2, \dots, E_k\}$ between the blocks. Select the bottom-left-most unitary block to U_0 in G^d .

Step 2. Create a list L_1 of the $m - 1$ unitary blocks ordered by their y -coordinates. Select a next unitary block U_j with a minimal y and minimal x .

Step 3. Connect U_j and U_0 via a corresponding diagonal line. Delete U_j from L_1 .

Step 4. If some unitary blocks are overlapping in L_1 of $\mathcal{P}$ with respect to their y -coordinates, and

- if U_j is to the right of U_i in $\mathcal{P}$, then
 U_j is added to the grid G^d as a right-diagonal connection of U_i .
- if U_j is to the left of U_i in $\mathcal{P}$, then
 U_j replaces the U_i , and U_j is added as a right-diagonal connection of U_i in G^d .

Step 5. If U_j is above the U_i in $\mathcal{P}$ and

- if U_j is above with some x -coordinate overlapping in $\mathcal{P}$, then
 U_j is a left-diagonal connection of U_i in G^d .
- if U_j is to the right of U_i in $\mathcal{P}$, then a grid-instance G_1^d is generated from the actual grid structure G^d , and U_j is a left-diagonal connection of U_i in G^d , while U_j is a right-diagonal connection of U_i in G_1^d .

Step 6. Repeat steps 2–5 until all $m - 1$ unitary blocks are added to G^d .

Step 7. Output the set $\mathcal{S}_{\mathcal{P}}(G^d) : \{G_1^d, \dots, G_c^d\}$ of c different grid instances that encode $\mathcal{P}$ such that the k entangled connections are satisfied.

The proof is concluded here. □

3.2.1 Computational complexity

By some fundamental assumptions, at m unitary blocks, the computational complexity of $\mathcal{A}_1$ is

$$\mathcal{O}(\mathcal{A}_1) = \mathcal{O}(m). \quad (9)$$

3.3 Optimal solution

Theorem 1 *The G^{d*} optimal grid can be determined from a set $\mathcal{S}_{\mathcal{P}}(G^d)$ of grid instances output by algorithm $\mathcal{A}_1$ such that all k entangled connections are preserved in $\mathcal{P}^*$.*

Proof The aim of this step is to produce the optimal placement via the set of grid instances. Motivated by the fundamentals of simulated annealing [134,135], we define algorithm $\mathcal{A}_2$ to determine the optimal G^{d*} grid from the set $\mathcal{S}_{\mathcal{P}}(G^d)$ of grid instances output by $\mathcal{A}_1$ such that all input constraints (gate connections, entangled connections, or decoherence estimation; see Sect. 4) are satisfied.

The steps of $\mathcal{A}_2$ are summarized in Algorithm 2.

Algorithm 2 Optimal Grid from the Grid Set ($\mathcal{A}_2$)

Step 1. Let set $\mathcal{S}_{\mathcal{P}}(G^d) : \{G_1^d, \dots, G_c^d\}$ be the input set of $\mathcal{A}_2$ with the c different grid instances that encode $\mathcal{P}$ such that the k entangled connections are satisfied for all grid instances.

Step 2. Select randomly a given grid instance G_i^d from $\mathcal{S}_{\mathcal{P}}(G^d)$.

Step 3. Apply the set $\mathcal{S}_O : \{O_R, O_M, O_S\}$ of operations to G_i^d , where O_R is the rotation operator, O_M is the movement operator, and O_S is the swapping operator.

Step 4. Use the operations of $\mathcal{S}_O$ on G_i^d until a frozen grid state, $\tilde{G}_i^d$, is reached.

Step 5. Stop the iteration on G_i^d and output $\tilde{G}_i^d$.

Step 6. Repeat steps 3–5 for all remaining $c - 1$ grid instances of $\mathcal{S}_{\mathcal{P}}(G^d)$. Put the resulting grids into set $\mathcal{S}'_{\mathcal{P}}(G^d) : \{\tilde{G}_1^d, \dots, \tilde{G}_c^d\}$.

Step 7. Select the $G^{d*} = \tilde{G}_z^d$ optimal grid from $\mathcal{S}'_{\mathcal{P}}(G^d)$ for which the $A_{\mathcal{E}}$ empty area between the modules is minimal, $A_{\mathcal{E}} = \min_{\forall i} \{A_{\mathcal{E}}(\tilde{G}_1^d), \dots, A_{\mathcal{E}}(\tilde{G}_c^d)\}$, where $A_{\mathcal{E}}(\tilde{G}_i^d)$ is the empty area associated with the placement of $\tilde{G}_i^d$.

The discussion of algorithm $\mathcal{A}_2$ is as follows. After the preliminaries of Steps 1–2, in Step 3, a set of operations $\mathcal{S}_O$ [129–133] are used for the construction of a better grid from the current input grid. The application of O_R in a module makes it possible to determine a novel solution when a node is inserted into G_i^d . Operation O_R can be executed twice at each position. The complexity of O_R is $\mathcal{O}(O_R) = \mathcal{O}(1)$. The O_M movement operator is applied for the deletion and re-insertion [132,133] of a particular block in the grid; therefore, it can be decomposed as $O_M = o_D + o_I$, where o_D is the delete sub-operator and o_I is the insert sub-operator. The o_D sub-operator of O_M can be applied to any unitary block of the grid; thus, the given U_s of G_i^d selected for deletion can have zero, one, or two diagonal connections in the grid. If U_s has only one diagonal connection, then U_s is removed, and the diagonal connection is placed at the position of the removed node. If U_s has two diagonal connections, then the target unitary block is replaced by the left- or right-diagonal connection, and a diagonal connection is moved to the original position of U_s . The o_I sub-operator of O_M is used for the addition of new modules into the grid structure [132,133]. A new node can be inserted into a place between two modules (internal position) or into a

position pointed by a null pointer (external position). The updating process of O_M therefore has a complexity $\mathcal{O}(O_M) = \mathcal{O}(1 + h)$, where h is the height of the current grid.

The O_S swapping operator [132,133] is applied for the swapping of some unitary blocks in the grid structure. Since O_S can also be decomposed as $O_S = o_D + o_I$, the complexity of O_S is $\mathcal{O}(O_S) = \mathcal{O}(1 + h)$.

In Step 4, the desired grid $\tilde{G}_i^d$ is achieved from G_i^d only by application of the O_R , O_M , O_S operations from $\mathcal{S}_O$.

In Step 7, as the G^{d*} optimal grid is determined, the optimal placement $\mathcal{P}^*$ is trivially yielded by the one-to-one correspondence of G^{d*} and $\mathcal{P}^*$. The grid G^{d*} therefore identifies the optimal placement $\mathcal{P}^*$ with the k entangled connections between the unitary blocks of the quantum circuit, that concludes the proof. $\square$

3.3.1 Computational complexity

From the definition of $\mathcal{A}_2$, for the selection of the optimal placement from the set of c solutions of $\mathcal{S}_P(G^d)$ at m unitary blocks, the computational complexity of $\mathcal{A}_2$ is

$$\begin{aligned}\mathcal{O}(\mathcal{A}_2) &= \mathcal{O}(c \cdot m (O_R + O_M + O_S)) \\ &= \mathcal{O}(c \cdot m (O_R + 2(o_D + o_I)))\end{aligned}\quad (10)$$

4 Decoherence dynamics estimation

In this section, we define a method for the decoherence dynamics estimation of superconducting quantum circuits. In the description, we assume that a particular unitary block U_j integrates a number of Josephson junction units J and capacitor units (C is the capacitance of a capacitor); for simplicity, we assume charge qubit circuits [121] in the system model. The unitary blocks formulate a larger unit called a quantum circuit. The quantum circuits are integrated into a multilayer structure with L layers and K columns. A quantum circuit in the i th layer l_i and k th column of $G_{QG}^{L,K}$ is referred to as $C^{l_i,k}$. We study a scenario in which the capacitor units of the quantum circuits are interfering with each other [129–131] and reveal the effects of the interference on the distribution of the Gaussian noise in the Josephson energy [121].

4.1 Capacitor interference in the charge qubit model

Let $C^{l_i,k}$ be a reference charge qubit $\mathcal{C}_q$ quantum circuit of the i th layer l_i and k th column of $G_{QG}^{L,K}$. Due to the presence of interference between the capacitor units of the charge qubit circuits, we define the interference coefficients [129–131] α , φ , and Φ as follows.

Let α be the area capacitance interference defined between an area of the reference quantum circuit $C^{l_i,k}$ and areas of $C^{l_{i-1},k}$ and $C^{l_{i+1},k}$. Let coefficient φ be the fringe capacitance interference coefficient defined between a side of the reference circuit $C^{l_i,k}$ and any parts of $C^{l_{i-1},k}$ and $C^{l_{i+1},k}$. Finally, let Φ be the coupling capacitance

interference coefficient defined on the same layer l_i from a side of the reference circuit $C^{l_i,k}$ to a given side $C^{l_i,k+1}$ [129–131].

Let us assume further that $C^{l_i,k}$ contains $u^{l_i,k}$ unitary blocks, U_i , $1 \leq i \leq u^{l_i,k}$. Let A_i be the area (see (2)) of a particular U_i . Let us assume that a given $C^{l_i,k}$ quantum circuit of the i th layer l_i , where k identifies the column number in the $G_{QG}^{L,K}$ multilayer grid structure, contains $u^{l_i,k}$ unitary blocks; therefore, the area of $C^{l_i,k}$ is

$$A(C^{l_i,k}) = \sum_{i=1}^{u^{l_i,k}} A_i = \sum_{i=1}^{u^{l_i,k}} D_i N_i, \quad (11)$$

where N_i (i.e., height of $C^{l_i,k}$) is given by (1), while D_i is the depth (i.e., width of $C^{l_i,k}$).

As the capacitor interference is analyzed for a particular $C^{l_i,k}$ with nonzero $A(C^{l_i,k})$, some geometrical considerations also come up because the capacitor interference does not just depend on the amount of overlap between the circuits, but it also differs for the interlayer (quantum circuits from different layers of $G_{QG}^{L,K}$) and intralayer (quantum circuits in the same layer of $G_{QG}^{L,K}$) quantum circuits.

The coefficients α , φ , and Φ are illustrated in Fig. 1 for a $G_{QG}^{L,K}$ multilayer charge qubit $\mathcal{C}_q$ quantum circuit structure. These quantities are used for the decoherence estimation in the multilayer charge qubit quantum circuit model.

4.1.1 Interlayer interference at full overlap

Let $\bar{A}(C^{l_i,k}, C^{l_j,k})$ be the overlap between the $A(C^{l_i,k})$ and $A(C^{l_j,k})$ areas of circuits $C^{l_i,k}$ and $C^{l_j,k}$ in the layers of l_i and l_j , given as

$$\bar{A}(C^{l_i,k}, C^{l_j,k}) = \sum_{i=1}^{w^{l_i,j,k}} D_i N_i, \quad (12)$$

where $w^{l_i,j,k}$ identifies the number of overlapped unitary blocks. If $C^{l_i,k}$ is in a full overlap relation with $C^{l_j,k}$, then the quantity $\bar{A}(C^{l_i,k}, C^{l_j,k})$ is as

$$\bar{A}(C^{l_i,k}, C^{l_j,k}) = A(C^{l_i,k}). \quad (13)$$

Let $\alpha(\bar{A}(C^{l_i,k}, C^{l_j,k}))$ be the area capacitance interference at overlapped area $\bar{A}(C^{l_i,k}, C^{l_j,k})$ between the $A(C^{l_i,k})$ area of the reference quantum circuit $C^{l_i,k}$, and the $A(C^{l_j,k})$ area of $C^{l_j,k}$. Using (13),

$$\alpha(\bar{A}(C^{l_i,k}, C^{l_j,k})) = \alpha(A(C^{l_i,k})) = \alpha(D^{l_i,k}) N^{l_i,k}, \quad (14)$$

where $D^{l_i,k} = \sum_{i=1}^{u^{l_i,k}} D_i$, and $N^{l_i,k} = \sum_{i=1}^{u^{l_i,k}} N_i$.

Fig. 1 Representation of the interference coefficients α , φ , and Φ with respect to a reference charge qubit quantum circuit $C^{l_i,k}$ (depicted in green) for the decoherence estimation in a $G_{QG}^{L,K}$ multilayer structure C_q . Each layer contains several unitary blocks U_j with charge qubit circuits. Each U_j integrates Josephson junction elements, capacitors, wires, and other elements. The quantum circuit of the i th layer, l_i , is referred to as $C^{l_i,k}$, where k is the column number in the $G_{QG}^{L,K}$ multilayer grid structure. The capacitors of the charge qubit quantum circuits are interfering with each other, which induces different capacitance coefficients between the quantum circuits. With respect to the $C^{l_i,k}$ reference quantum circuit of the i th layer l_i , the quantity α refers to the area capacitance (between an area of the reference quantum circuit $C^{l_i,k}$ and areas of $C^{l_{i-1},k}$ and $C^{l_{i+1},k}$), the quantity φ is the fringe capacitance coefficient (from a side of the reference circuit $C^{l_i,k}$ to any parts of $C^{l_{i-1},k}$ and $C^{l_{i+1},k}$), and Φ is the coupling capacitance coefficient defined on the same layer (from a side of the reference circuit $C^{l_i,k}$ to a side $C^{l_i,k+1}$)

Then, let be the $\varphi(d(C^{l_i,k}, C^{l_j,k}))$ capacitance interference coefficient defined between the sides of the reference circuit $C^{l_i,k}$ and the area of $C^{l_j,k}$, where $d(C^{l_i,k}, C^{l_j,k})$ is the vertical distance between circuits $C^{l_i,k}$ and $C^{l_j,k}$. Since

$$d(C^{l_i,k}, C^{l_j,k}) = L2(l_i, l_j), \quad (15)$$

where $L2(l_i, l_j) = |x_i - x_j| + |y_i - y_j| + |z_i - z_j| f_c$ is the distance function between the layers l_i, l_j in $G_{QG}^{L,K}$, where $f_c(\cdot)$ is a cost function between the layers, thus the quantity $\varphi(d(C^{l_i,k}, C^{l_j,k}))$ can be rewritten as

$$\varphi(d(C^{l_i,k}, C^{l_j,k})) = \varphi(L2(l_i, l_j)). \quad (16)$$

Using (14), (15) and (16), the $\mathcal{D}_{l_i, l_j}^{A(C^{l_i,k})}(C^{l_i,k}, C^{l_j,k})$ interlayer interference at full overlapping circuits $C^{l_i,k}$ and $C^{l_j,k}$ between layers l_i, l_j is defined as

$$\mathcal{D}_{l_i, l_j}^{A(C^{l_i,k})}(C^{l_i,k}, C^{l_j,k}) = \alpha(D^{l_i,k}) N^{l_i,k} + 2\varphi(L2(l_i, l_j)) N^{l_i,k}. \quad (17)$$

4.1.2 Interlayer interference at partial or zero overlap

If the circuits $C^{l_i,k}$ and $C^{l_j,k}$ in the layers of l_i and l_j have a partial or zero overlap, then

$$\bar{A}(C^{l_i,k}, C^{l_j,k}) < A(C^{l_i,k}), \quad (18)$$

and

$$\alpha(\bar{A}(C^{l_i,k}, C^{l_j,k})) = \alpha(\bar{D}^{l_i,k}) \bar{N}^{l_i,k} < \alpha(D^{l_i,k}) N^{l_i,k}, \quad (19)$$

where $\bar{D}^{l_i,k} = \sum_{i=1}^{o^{l_i,k}} D_i$ (overlap width), and $\bar{N}^{l_i,k} = \sum_{i=1}^{o^{l_i,k}} N_i$ (overlap length), where $o^{l_i,k}$ identifies the amount of overlap of $C^{l_i,k}$, $o^{l_i,k} < u^{l_i,k}$.

By similar assumptions, the $\mathcal{D}_{l_i,l_j}^{\bar{A}(C^{l_i,k}, C^{l_j,k})}(C^{l_i,k}, C^{l_j,k})$ interlayer interference at partial overlapping circuits $C^{l_i,k}$ and $C^{l_j,k}$ between two layers l_i, l_j is defined as

$$\begin{aligned} \mathcal{D}_{l_i,l_j}^{\bar{A}(C^{l_i,k}, C^{l_j,k})}(C^{l_i,k}, C^{l_j,k}) \\ = \alpha(\bar{D}^{l_i,k}) \bar{N}^{l_i,k} + 2\varphi(L_2(l_i, l_j)) \bar{N}^{l_i,k}. \end{aligned} \quad (20)$$

As the overlap between $C^{l_i,k}$ and $C^{l_j,k}$ is zero, $\bar{A}(C^{l_i,k}, C^{l_j,k}) = 0$, then

$$\mathcal{D}_{l_i,l_j}^0(C^{l_i,k}, C^{l_j,k}) = 2\varphi(h(C^{l_i,k}, C^{l_j,k})) N^{l_i,k} \varsigma, \quad (21)$$

where $h(C^{l_i,k}, C^{l_j,k})$ is the horizontal distance between $C^{l_i,k}$ and $C^{l_j,k}$, while ς is a constant (decay factor [129–131]).

4.1.3 Intralayer interference

Let $h(C^{l_i,k}, C^{l_i,f})$ be the horizontal distance between two parallel circuits $C^{l_i,k}$ and $C^{l_i,f}$ in the same layer l_i . Then, $\bar{A}(C^{l_i,k}, C^{l_i,f}) = 0$, and the intralayer interference is between $C^{l_i,k}$ and $C^{l_i,f}$

$$\mathcal{D}_{l_i,l_j}^0(C^{l_i,k}, C^{l_i,f}) = \Phi(h(C^{l_i,k}, C^{l_i,f})) N^{l_i,k}, \quad (22)$$

where $\Phi(h(C^{l_i,k}, C^{l_i,f}))$ is coupling capacitance coefficient.

The same result holds for two perpendicular circuits $C^{l_i,k}$ and $C^{l_i,f}$ in the same layer l_i with $v(C^{l_i,k}, C^{l_i,f})$ vertical distance, as

$$\mathcal{D}_{l_i,l_j}^0(C^{l_i,k}, C^{l_i,f}) = \Phi(v(C^{l_i,k}, C^{l_i,f})) N^{l_i,k}. \quad (23)$$

4.1.4 Capacitor interference estimation

For the determination of quantities (17), (20) and (22), the $\mathcal{A}_3$ interference estimation algorithm is defined. The algorithm also finds the coefficients $\alpha(U_j)$, $\varphi(U_j)$ and $\Phi(U_j)$ of a particular unit U_j . The steps of $\mathcal{A}_3$ are detailed in Algorithm 3.

Algorithm 3 Capacitor Interference Estimation for Charge Qubit Circuits ($\mathcal{A}_3$)

- Step 1.** Let assume that $C^{l_i,k}$ is the quantum circuit of the k th column of the i th layer l_i , in the $G_{QG}^{L,K}$ multilayer QG quantum gate structure $G_{QG}^{L,K}$, with L layers and K columns, $1 \leq i \leq L$, $1 \leq k \leq K$. Let a given $C^{l_i,k}$ contain $N^{l_i,k}$ unitary blocks U_j , $1 \leq j \leq N^{l_i,k}$.
- Step 2.** Select a given $C^{l_i,k}$.
- Step 3.** For all $C^{l_g,k}$, $g \neq i$, $1 \leq g \leq L$ quantum circuits in the remaining $L - 1$ layers of the k th column:
- For all U_j , $1 \leq j \leq N^{l_g,k}$, of $C^{l_g,k}$, determine $\alpha(U_j)$ between U_j and the χ_k remaining unitary blocks, $\chi_k = \sum_{i=1}^L N^{l_i,k} - 1$, of column k .
 - For all U_j , $1 \leq j \leq N^{l_g,k}$, of $C^{l_g,k}$, determine $\varphi(U_j)$ between U_j and the χ_k remaining unitary blocks, $\chi_k = \sum_{i=1}^L N^{l_i,k} - 1$, of column k .
- Step 4.** Compute $\alpha = \sum_{j=1}^{N^{l_g,k}} \alpha(U_j)$ and $\varphi = \sum_{j=1}^{N^{l_g,k}} \varphi(U_j)$.
- Step 5.** Output $\mathcal{D}_{l_i,l_j}^A(C^{l_i,k}, C^{l_g,k})$ via (17) for all g , and $\mathcal{D}_{l_i,l_j}^0(C^{l_i,k}, C^{l_g,k})$ via (21) for all g .
- Step 6.** For all $C^{l_i,m}$, $m \neq k$, $1 \leq m \leq K$ remaining $K - 1$ quantum circuits in the i th layer l_i :
- For all U_j , $1 \leq j \leq N^{l_i,k}$, of $C^{l_i,k}$, determine $\Phi(U_j)$ between U_j and the λ_{l_i} remaining unitary blocks, $\lambda_{l_i} = \sum_{k=1}^K N^{l_i,k} - 1$, of layer l_i .
- Step 7.** Compute $\Phi = \sum_{j=1}^{N^{l_i,k}} \Phi(U_j)$.
- Step 8.** Output $\mathcal{D}_{l_i,l_j}^0(C^{l_i,k}, C^{l_i,m})$ via (22) for all m .
- Step 9.** Repeat steps 2-8 for all i and all k .
-

The result on the effect of interference coefficients $\alpha(U_j)$, $\varphi(U_j)$, $\Phi(U_j)$ of U_j on the distribution of the decoherence of U_j is concluded in Theorem 2.

4.1.5 Decoherence distribution

Theorem 2 For a $\mathcal{C}_q : U_j$ charge qubit structure with a Gaussian noise $\mathcal{D}(U_j)$, any nonzero $\Omega(U_j)$, where $\Omega(U_j) = \alpha(U_j) + \varphi(U_j) + \Phi(U_j)$, modifies the noise distribution $\mathcal{D}(U_j) \in \mathcal{N}(\bar{E}_J, \sigma_{E_J}^2)$ of the Josephson energy E_J of U_j , where $\bar{E}_J$ is the mean, and $\sigma_{E_J}^2$ is the variance.

Proof Let assume that the quantum circuits in $G_{QG}^{L,K}$ multilayer grid are implemented by $\mathcal{C}_q$ charge qubit superconducting qubit structures. Each $\mathcal{C}_q$ is equipped with a capacitor with capacitance C , and Josephson junction J with energy E_J that is biased by a voltage U [121]. Let U_j be a reference $\mathcal{C}_q$ in the $G_{QG}^{L,K}$ multilayer quantum circuit structure.

To describe the interactions and the evolution of a superconducting charge qubit circuit, we utilize the circuit Hamiltonian.

Let n be the number of excess Cooper-pairs on the island, then the $H(C_q)$ circuit Hamiltonian of C_q is as [63]

$$H(C_q) = 4E_C(n - n_g)^2 - E_J \cos \phi, \quad (24)$$

where E_C is the charging energy, E_J is the Josephson energy, and n_g is a control parameter (effective offset charge), as

$$n_g = -\frac{CV_0}{2e}, \quad (25)$$

where V_0 is a gate voltage, while ϕ is the superconducting wave function phase difference across J (a 2π -periodic operator of the phase difference). Note, the operators satisfy the commutation relation [63],

$$[\phi, n] = i. \quad (26)$$

Then, applying a voltage that yields $n_g = 1/2$ for the period $t_0 < t < t_0 + \Delta t$, the $|\psi\rangle$ system state of U_j will oscillate between states $|0\rangle$ and $|1\rangle$. Therefore, $|\psi(t)\rangle$ is as [121]

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\hat{T}\int_{t_0}^t\hat{H}(t')dt'\right)|\psi(t_0)\rangle, \quad (27)$$

where $\hat{H}$ is a Hamiltonian, $\hat{T}$ is the time-ordering operator, and $\hbar$ is the reduced Planck constant. If the Hamiltonian is constant, then (27) can be rewritten as

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}(t-t_0)}|\psi(t_0)\rangle. \quad (28)$$

Using the eigenstates $1/\sqrt{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $1/\sqrt{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ at $n_g = 1/2$, $|\psi(t)\rangle$ can be rewritten as [121]

$$\begin{aligned} |\psi(t)\rangle &= e^{-\frac{i}{\hbar}\hat{H}(t-t_0)}\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) \\ &= e^{+\frac{i}{\hbar}\frac{E_J}{2}(t-t_0)}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - e^{-\frac{i}{\hbar}\frac{E_J}{2}(t-t_0)}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \end{aligned} \quad (29)$$

therefore $|\psi(t)\rangle$ is yielded from (29) as

$$|\psi(t)\rangle = \left(e^{+\frac{i}{\hbar}\frac{E_J}{2}(t-t_0)} - e^{-\frac{i}{\hbar}\frac{E_J}{2}(t-t_0)}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \left(e^{+\frac{i}{\hbar}\frac{E_J}{2}(t-t_0)} + e^{-\frac{i}{\hbar}\frac{E_J}{2}(t-t_0)}\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (30)$$

The $\Pr(|\psi(t)\rangle = |0\rangle)$ probability at $t = t_0 + \Delta t$ is therefore as

$$\Pr(|\psi(t_0 + \Delta t)\rangle = |0\rangle) = |\langle 0|\psi(t_0 + \Delta t)\rangle|^2 = \cos^2(E_J/2\hbar\Delta t) \quad (31)$$

and $\Pr(|\psi(t)\rangle = |1\rangle)$ at $t = t_0 + \Delta t$ is as

$$\Pr(|\psi(t_0 + \Delta t)\rangle = |1\rangle) = |\langle 1|\psi(t_0 + \Delta t)\rangle|^2 = \sin^2(E_J/2\hbar\Delta t). \quad (32)$$

The $\Pr(|\psi(t_0 + \Delta t)\rangle = |0\rangle)$ and $\Pr(|\psi(t_0 + \Delta t)\rangle = |1\rangle)$ probabilities are therefore oscillated with an angular frequency f_0 as

$$f_0 = \frac{E_J}{\hbar}. \quad (33)$$

As one can conclude, setting Δt to $\Delta t = \pi\hbar/2E_J$ yields the equal superposition of the charge states.

Let assume that the $\mathcal{D}(U_j)$ decoherence of the Josephson energy E_J of U_j is a Gaussian noise [121]

$$\mathcal{D}(U_j) \in \mathcal{N}(\bar{E}_J, \sigma_{E_J}^2), \quad (34)$$

with mean $\bar{E}_J$ and variance $\sigma_{E_J}^2$, with standard deviation σ_{E_J} as

$$\sigma_{E_J} = \Delta E_J = |E_J^{(0)} - E_J^{(1)}|, \quad (35)$$

where ΔE_J is the width of the Gaussian distribution of E_J centered on $\bar{E}_J$, and $E_J^{(0)}$ and $E_J^{(1)}$ are parameterize the width ΔE_J .

Thus, in the presence of (34), the $\Pr(|\psi(t)\rangle = |1\rangle)$ probability in (32) is yielded as [121]

$$\begin{aligned} & \Pr_{\mathcal{D}(U_j)}(|\psi(t_0 + \Delta t)\rangle = |1\rangle) \\ &= 1/\sigma_{E_J}\sqrt{2\pi} \int_{-\infty}^{\infty} e^{-\frac{(E_J - \bar{E}_J)^2}{2\sigma_{E_J}^2}} \sin^2(E_J/2\hbar\Delta t) dE_J, \end{aligned} \quad (36)$$

where E_J is a Gaussian random variable,

$$E_J \in \mathcal{N}(\bar{E}_J, \sigma_{E_J}^2). \quad (37)$$

The probability in (36) can be rewritten as

$$\begin{aligned} & \Pr_{\mathcal{D}(U_j)}(|\psi(t_0 + \Delta t)\rangle = |1\rangle) \\ &= 1/2 \left(1 - e^{-\left(\frac{\Delta t(\sigma_{E_J})}{\hbar}\right)^2} \cos(\bar{E}_J/\hbar\Delta t) \right) \\ &= 1/2 \left(1 - e^{-(\Delta t\zeta)^2} \cos(\bar{E}_J/\hbar\Delta t) \right), \end{aligned} \quad (38)$$

thus in the presence of a Gaussian noise $\mathcal{D}(U_j)$ the system oscillates with frequency $f = \bar{E}_J/\hbar$, and it decreases exponentially with a time constant [121]

$$\zeta = \frac{\sigma_{E_J}}{\hbar}, \quad (39)$$

where σ_{E_J} is the standard deviation of $\mathcal{D}(U_j)$ in the Josephson energy [121].

The decoherence associated with the capacitance interference is derived as follows. Let us evaluate the effect of a nonzero $\Omega(U_j)$, where $\Omega(U_j)$ is the sum of all capacitance interference coefficients associated with U_j as

$$\Omega(U_j) = \alpha(U_j) + \varphi(U_j) + \Phi(U_j), \quad (40)$$

where the coefficients $\alpha(U_j) \geq 0$, $\varphi(U_j) \geq 0$ and $\Phi(U_j) \geq 0$ are determined for a particular U_j via Algorithm 3.

At any nonzero $\Omega(U_j)$, the capacitance C of U_j can be rewritten as C' as

$$C' = C + \Omega(U_j), \quad (41)$$

and the resulting n'_g is as

$$n'_g = -\frac{C'V_0}{2e} = -\frac{(C + \Omega(U_j))V_0}{2e}. \quad (42)$$

Using (42), the Josephson energy E'_J at a nonzero $\Omega(U_j)$ is as

$$E'_J = \frac{4E'_C(n - n'_g)^2 - H'(\mathcal{C}_q)}{\cos \phi}, \quad (43)$$

which is a Gaussian random variable

$$E'_J \in \mathcal{N}(\bar{E}'_J, \sigma_{E'_J}^2) \quad (44)$$

with mean $\bar{E}'_J$ and standard deviation $\sigma_{E'_J}$

$$\sigma_{E'_J} = \Delta E'_J = |E_J^{(0)} - E_J^{(1)}| = \sigma_{E'_J} + \sigma_\Omega, \quad (45)$$

where $\Delta E'_J$ is the width of the Gaussian distribution of E'_J centered on $\bar{E}'_J$, while $E_J^{(0)}$ and $E_J^{(1)}$ parameterize the width of $\Delta E'_J$.

Thus, σ_Ω is as

$$\sigma_\Omega = \left| |E_J^{(0)} - E_J^{(1)}| - |E_J^{(0)} - E_J^{(1)}| \right|, \quad (46)$$

while the Hamiltonian $H'(\mathcal{C}_q)$ in (43) is as,

$$H'(\mathcal{C}_q) = 4E'_C \left(n - n'_g \right)^2 - E'_J \cos \phi. \quad (47)$$

Therefore, the noise evolution of U_j at the degeneracy point $n'_g = 1/2$ in the presence of $\Omega(U_j) > 0$ is as

$$\begin{aligned} & \Pr_{\mathcal{D}(U_j)}^{\Omega(U_j) > 0} (|\psi(t_0 + \Delta t)\rangle = |1\rangle) \\ &= 1/\sigma_{E'_J} \sqrt{2\pi} \int_{-\infty}^{\infty} e^{-\frac{(E'_J - \bar{E}'_J)^2}{2\sigma_{E'_J}^2}} \sin^2(E'_J/2\hbar\Delta t) dE'_J, \end{aligned} \quad (48)$$

that can be rewritten as

$$\begin{aligned} & \Pr_{\mathcal{D}(U_j)}^{\Omega(U_j) > 0} (|\psi(t_0 + \Delta t)\rangle = |1\rangle) \\ &= 1/2 \left(1 - e^{-\left(\frac{\Delta t \sigma_{E'_J}}{\hbar}\right)^2} \cos(\bar{E}'_J/\hbar\Delta t) \right) \\ &= 1/2 \left(1 - e^{-(\Delta t \zeta')^2} \cos(\bar{E}'_J/\hbar\Delta t) \right), \end{aligned} \quad (49)$$

where

$$\zeta' = \frac{\sigma_{E'_J}}{\hbar} = \frac{\sigma_{E_J} + \sigma_{\Omega}}{\hbar} = \frac{\Delta E'_J}{\hbar}. \quad (50)$$

Thus, in the presence of a $\Omega(U_j) > 0$, the noisy system oscillates with frequency

$$f' = \frac{\bar{E}'_J}{\hbar}, \quad (51)$$

and it decreases exponentially with a time constant as given in (50).

Therefore, at a nonzero $\Omega(U_j)$, the distribution of the $\mathcal{D}(U_j)_{\Omega(U_j)}$ Gaussian noise of U_j is yielded as

$$\mathcal{D}(U_j)_{\Omega(U_j)} \in \mathcal{N}(\bar{E}'_J, \Delta E'_J) = \mathcal{N}(\bar{E}'_J, \sigma_{E'_J}^2 + \sigma_{\Omega}^2), \quad (52)$$

that concludes the proof. $\square$

By assuming charge qubit structure in all computational units of circuits of the QG quantum circuit, the results of Theorem 2 can be extended, and the effects of the interlayer interference between full (see (17)) and partial overlapping (see (21)) circuits $C^{l_i, k}$ and $C^{l_j, k}$ between layers l_i, l_j , and the intralayer interference (see (22)) between two circuits $C^{l_i, k}$ and $C^{l_i, f}$ in the same layer can also be yielded.

4.2 Decoherence estimation method

The decoherence estimation method $\mathcal{A}_4$ is defined in Algorithm 4. From the output of $\mathcal{A}_4$, the $\mathcal{P}^*$ optimal placement can be re-determined by algorithms $\mathcal{A}_1$ and $\mathcal{A}_2$, by adding the constraint of the minimization of the corresponding quantities $\Delta = |\tilde{E}'_J - \tilde{E}_J|$ and $\gamma = |\Delta E'_J - \Delta E_J|$.

Algorithm 4 Decoherence Estimation for $C^{l_i,k}$ with $u^{l_i,k}$ unitary blocks $U_i : C_q$ ($\mathcal{A}_4$)

Step 1. Determine (17), (21) and (22) via $\mathcal{A}_3$.

Step 2. Using Step 1, determine an $\tilde{\mathcal{Q}}(U_j)$ average of (40) for all $u^{l_i,k}$ unitary blocks of $C^{l_i,k}$ via an averaging from (17), (21) and (22).

Step 3. Using Step 1, evaluate an $\tilde{\mathcal{D}}(U_j)$ average of (52) for all $u^{l_i,k}$ unitary blocks of $C^{l_i,k}$ via an averaging from (17), (21) and (22).

Step 4. Determine $\mathcal{D}(C^{l_i,k})$ as $\mathcal{D}(C^{l_i,k}) = \sum_{j=1}^{u^{l_i,k}} \tilde{\mathcal{D}}(U_j)$.

Step 5. Output the resulting distribution of $\mathcal{D}(C^{l_i,k}) \in \mathcal{N}\left(\sum_{j=1}^{u^{l_i,k}} \tilde{E}'_{J,j}, \sum_{j=1}^{u^{l_i,k}} \Delta E'_{J,j}\right)$ of $C^{l_i,k}$, where $E'_{J,j}$ is the Josephson energy of the j th unit U_j .

4.3 NISQ applications

The proposed results can be applied in experimental NISQ [2] settings. The system model particularly focuses on near-term superconducting charge qubit architectures; however, the placement algorithm can be extended to other experimental scenarios such as trapped ion circuits. The decoherence estimation method is directly applicable in superconducting qubit architectures, NISQ algorithm implementations, and in the realization of quantum error correction using superconducting qubits [63]. Another near-term application is in NISQ qubit gate-model quantum computer architectures with a large number of qubits and quantum gates in the physical layer [5,10,11,136]. As a future work, our aim is to analyze the performance of the system model in these extended NISQ scenarios.

5 Conclusions

Here, we defined a method for layout generation and decoherence estimation in superconducting gate-model quantum computers. The optimal physical layout was determined by an iterative procedure that utilizes our grid structure to encode the various placements of the quantum circuit. We set a multilayer grid structure for the analysis of decoherence distribution in the presence of different capacitor interferences between the quantum circuits. The results can be used in superconducting gate-model quantum computers and in the near-term quantum devices of the quantum Internet.

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Competing non-financial interests statement There are no competing non-financial interests.

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Ethical standard This work did not involve any active collection of human data.

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A Appendix

A.1 Placement of unitary blocks

The G^d grid structure, $d = 2$, $N = 4$, and the yielding placement $\mathcal{P}$ of the unitary blocks with the defined entangled connections are depicted in Fig. 2.

Fig. 2 G^2 grid structure of a placement $\mathcal{P}$ of the unitary blocks with a set E of entangled connections (depicted by solid lines) between the U_i blocks. In the grid structure, the first unitary block U_0 is in the center of a square to identify the bottom-left block in the $\mathcal{P}$ physical layer topology. From a given U_i , one left and one right unitary block can be evaluated by auxiliary diagonal lines, such that the $U_i, i > 0$, evaluated blocks are placed in the intersections of the grid. A left-diagonal line represents a right-adjacent block in the physical layer structure, while a right-diagonal line represents an up-adjacent block. The entanglement between the blocks is depicted by the solid lines. **a:** The first block U_0 is evaluated into U_1 and U_2 . From U_1 , a new block, U_3 is evaluated via the diagonal line (topology relations depicted by gray dashed lines). **b:** The physical layer topology of the unitary computational blocks extracted from the grid structure of (a) (entanglement between the blocks is depicted by the thick solid lines). **c:** A grid structure of nine unitary blocks (gray dashed lines for the topology relations are not depicted). **d:** The physical layer topology of the unitary computational blocks is extracted from the grid structure of (c) (entanglement between the blocks is depicted by the thick solid lines)

A.2 Notations

The notations of the manuscript are summarized in Table 1.

Table 1 Summary of notations

Notation	Description
G^d	A d -dimensional, N size grid G^d to encode the physical placement of the m unitary computational blocks.
d	Dimension of the grid.
H	Hamiltonian operator.
QG	Quantum circuit (structure of quantum gates).
U_i	An i th unitary computational block, it contains a block of unitary operators.
m	The number of unitary blocks in the physical layout.
D_i	Circuit depth of U_i .
N_i	Number of quantum wires, $N_i = \max(n_i^{in}, n_i^{out})$, where n_i^{in} and n_i^{out} are the number of input and output quantum wires of U_i .
A_i	Area of U_i , $A_i = D_i N_i$.
$\mathcal{P}^*$	Physical placement of the m unitary computational blocks.
$\kappa(\mathcal{P}^*)$	Optimal placement in the physical layer.
ℓ	Minimum bounding rectangle of $\mathcal{P}^*$.
ℓ	Total length of quantum wires of $\mathcal{P}^*$, $\ell = \sum_{i=1}^m w_i = \min \beta(\mathcal{P}^*)$, where w_i is the quantum wire length associated with U_i , $\beta(\mathcal{P}^*)$ is the sum of the half-bounding box of interconnections between the unitary blocks.
U_0	A unitary block that identify a unique base bottom-left coordinate in $\mathcal{P}^*$.
E	A set of k entangled connections, $E = \{E_1, E_2, \dots, E_k\}$.
S_r	Set of unitary blocks placed adjacent and right to U_0 .
S_a	Set of unitary blocks above U_0 .
G^{d*}	Optimal grid that encodes $\mathcal{P}^*$.
$\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4$	Algorithms of the framework.
$\mathcal{S}(G^d)$	A set of c different grid instances that encode a placement such that the k entangled connections are satisfied in the physical layout $\mathcal{S}(G^d) : \{G_1^d, \dots, G_c^d\}$.
c	Number of solutions in $\mathcal{S}(G^d)$.
L_1	List of the $m - 1$ unitary blocks ordered by the y -coordinates.
S_O	Set of operations, $S_O : \{O_R, O_M, O_S\}$, where O_R is the rotation operator, O_M is the movement operator, and O_S is the swapping operator. $O_M = o_D + o_I$, $O_S = o_D + o_I$, where o_D is the delete sub-operation, and o_I is the insert sub-operator.
$\tilde{G}_i^d$	Frozen grid state.

Table 1 continued

Notation	Description
$\mathcal{S}' \left(G^d \right)$	Auxiliary set of grids.
$A_{\mathcal{E}}$	Empty area between the modules, $A_{\mathcal{E}} = \min_{\forall i} \left\{ A_{\mathcal{E}} \left(\tilde{G}_1^d \right), \dots, A_{\mathcal{E}} \left(\tilde{G}_c^d \right) \right\},$ where $A_{\mathcal{E}} \left(\tilde{G}_i^d \right)$ is the empty area associated with the placement of $\tilde{G}_i^d$.
C_q	Charge qubit model.
J	Josephson junction in C_q .
E_J	Energy of Josephson junction J .
C	Capacitance of a capacitor in C_q .
$G_{QG}^{L,K}$	Multilayer structure of C_q quantum circuits, with L layers and K columns.
$C^{l_i,k}$	Quantum circuit in the i th layer l_i , and k th column of $G_{QG}^{L,K}$.
α, φ, Φ	Interference coefficients, α is the area interference capacitance, φ is the fringe interference capacitance coefficient, Φ is the coupling interference capacitance coefficient.
$u^{l_i,k}$	Number of unitary blocks of $C^{l_i,k}$, U_i , $1 \leq i \leq u^{l_i,k}$.
$A \left(C^{l_i,k} \right)$	Area of $C^{l_i,k}$.
N_i	Circuit height of $C^{l_i,k}$.
D_i	Circuit depth of $C^{l_i,k}$.
$\bar{A} \left(C^{l_i,k}, C^{l_j,k} \right)$	Overlap between the $A \left(C^{l_i,k} \right)$ and $A \left(C^{l_j,k} \right)$ areas of circuits $C^{l_i,k}$ and $C^{l_j,k}$, in the layers of l_i and l_j .
$\alpha \left(\bar{A} \left(C^{l_i,k}, C^{l_j,k} \right) \right)$	Area interference capacitance at overlapped area $\bar{A} \left(C^{l_i,k}, C^{l_j,k} \right)$ between the $A \left(C^{l_i,k} \right)$ area of the reference quantum circuit $C^{l_i,k}$, and the $A \left(C^{l_j,k} \right)$ area of $C^{l_j,k}$.
$\varphi \left(d \left(C^{l_i,k}, C^{l_j,k} \right) \right)$	Capacitance interference coefficient defined between the sides of the reference circuit $C^{l_i,k}$ and the area of $C^{l_j,k}$, where $d \left(C^{l_i,k}, C^{l_j,k} \right)$ is the vertical distance between circuits $C^{l_i,k}$ and $C^{l_j,k}$.
$L2 \left(l_i, l_j \right)$	Distance function between the layers l_i, l_j , $L2 \left(l_i, l_j \right) = x_i - x_j + y_i - y_j + z_i - z_j f_c,$ where $f_c(\cdot)$ is a cost function between the layers.
$\mathcal{D}_{l_i,l_j}^{A \left(C^{l_i,k} \right)} \left(C^{l_i,k}, C^{l_j,k} \right)$	Interlayer interference at full overlapping circuits $C^{l_i,k}$ and $C^{l_j,k}$ between layers l_i, l_j .

Table 1 continued

Notation	Description
$\bar{\mathcal{D}}_{l_i, l_j}^{(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k})}(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k})$	Interlayer interference at partial overlapping circuits $\mathcal{C}^{l_i, k}$ and $\mathcal{C}^{l_j, k}$ between two neighboring layers l_i, l_j .
$\mathcal{D}_{l_i, l_j}^0(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k})$	Interlayer interference at zero overlapping circuits $\mathcal{C}^{l_i, k}$ and $\mathcal{C}^{l_j, k}$ between two neighboring layers l_i, l_j , $\bar{\mathcal{A}}(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k}) = 0$.
$h(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k})$	Horizontal distance between $\mathcal{C}^{l_i, k}$ and $\mathcal{C}^{l_j, k}$.
ς	Constant (decay factor) in $\mathcal{D}_{l_i, l_j}^0(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k})$.
$h(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_i, f})$	Horizontal distance between two parallel running circuits $\mathcal{C}^{l_i, k}$ and $\mathcal{C}^{l_i, f}$ in layer l_i .
$\Phi(h(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k}))$	Coupling capacitance coefficient.
$v(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_j, k})$	Vertical distance of two perpendicular running circuits $\mathcal{C}^{l_i, k}$ and $\mathcal{C}^{l_i, f}$ in the same layer l_i .
$\mathcal{D}_{l_i, l_j}^0(\mathcal{C}^{l_i, k}, \mathcal{C}^{l_i, f})$	Intralayer interference between $\mathcal{C}^{l_i, k}$ and $\mathcal{C}^{l_i, f}$ in layer l_i .
χ_k, λ_{l_i}	Parameters of algorithm $\mathcal{A}_3$.
$\alpha(U_j), \varphi(U_j), \Phi(U_j)$	Interference coefficients of U_j .
$\Omega(U_j)$	Sum of interference coefficients of U_j , as $\Omega(U_j) = \alpha(U_j) + \varphi(U_j) + \Phi(U_j)$.
$\mathcal{D}(U_j)$	Gaussian noise of U_j (decoherence of the Josephson energy E_J of U_j).
$H(\mathcal{C}_q)$	Hamiltonian of $\mathcal{C}_q$.
E_C	Charging energy.
C_J	Capacitance of the Josephson junction.
n	Number of Cooper-pairs to tunnel the junction J .
n_g	Charge on the capacitor in units of Cooper pairs number.
ϕ	Superconducting wave function phase difference across J .
$ \psi\rangle$	System state of U_j .
$\hat{T}$	Time-ordering operator.
$\hbar$	Reduced Planck constant.
f_0	Angular frequency.
$\bar{E}_J$	Mean of E_J of U_j .
σ_{E_J}	Standard deviation of E_J , $\sigma_{E_J} = \Delta E_J = E_J^{(0)} - E_J^{(1)} $, where ΔE_J is the width of the Gaussian distribution of E_J centered on $\bar{E}_J$, and $E_J^{(0)}$ and $E_J^{(1)}$ are parameterize the width ΔE_J .
C'	Capacitance C of U_j at any nonzero $\Omega(U_j)$, $C' = C + \Omega(U_j)$.
E'_C	Charging energy at any nonzero $\Omega(U_j)$.

Table 1 continued

Notation	Description
n'_g	Charge on the capacitor in units of Cooper pairs number at any nonzero $\Omega(U_j)$.
$\sigma_{E'_j}$	Standard deviation of Josephson energy at any nonzero $\Omega(U_j)$.
$\bar{E}'_J$	Mean of Josephson energy at any nonzero $\Omega(U_j)$.
ζ	Time constant of exponentially decreasing probability function, $\zeta = \frac{\sigma_{E'_J}}{\hbar}$, where $\sigma_{E'_J}$ is the standard deviation of $\mathcal{D}(U_j)$ in the Josephson energy.
ζ'	Time constant of exponentially decreasing probability function at any nonzero $\Omega(U_j)$, $\zeta' = \frac{\sigma_{E'_J}}{\hbar} = \frac{\sigma_{E'_J} + \sigma_{\Omega}}{\hbar} = \frac{\Delta E'_J}{\hbar}$.
f'	Oscillation frequency at any nonzero $\Omega(U_j)$, $f' = \frac{\bar{E}'_J}{\hbar}$.
$\mathcal{D}(U_j)_{\Omega(U_j)}$	Gaussian noise of U_j at a nonzero $\Omega(U_j)$.
$\tilde{\Omega}(U_j), \tilde{\mathcal{D}}(U_j)$	Parameters of algorithm $\mathcal{A}_4$.
Δ	Abs. value of the difference of means $\bar{E}'_J$ and $\bar{E}_J$, $\Delta = \bar{E}'_J - \bar{E}_J $.
γ	Abs. value of the difference of standard deviations $\Delta E'_J$ and ΔE_J , $\gamma = \Delta E'_J - \Delta E_J $.

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