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# Balancing democracy: majoritarianism versus expression of preference intensity

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**Abstract**
This paper evaluates three prominent voting systems: the Majority Rule (MR), Borda Rule (BR), and Plurality Rule (PR). Our analysis centers on the susceptibilities of each system to potential transgressions of two foundational principles: the respect for majority preference (majoritarianism) and the acknowledgment of the intensity of individual preferences. We operationalize the concept of 'cost' as the expected deviation from the aforementioned principles. A comparative assessment of MR, BR, and PR is undertaken in terms of their costs. Our findings underscore the superiority of PR over MR, whilst also highlighting the comparative advantage of MR against BR.

**Keywords** Majority principle · Preference intensity · Scoring rules · Majority rule · Plurality rule · Borda rule · Expected erosion of a principle

## 1 Introduction

Two principles that social-aggregation rules should adhere to are majoritarianism and the recognition of voters' preference intensity. These ideals safeguard the interests of a simple majority as well as protect a minority possessing significantly robust preferences, respectively. The delicate balance between these principles fuels the ongoing discourse between proponents of the majority rule (MR) and supporters of certain scoring rules, particularly the Borda rule (BR) and the prevalent plurality rule (PR).[1]

[1] Under MR, the social preference of an alternative to another one hinges on the existence of a majority of voters who prefer it. According to the Borda rule (m-1), points are assigned to the best out of the *m* alternative ranked at the top, (m-2) points are assigned to the second-best alternative, and so on. (No points are assigned to the worst alternative.) Under PR one point is assigned to the most preferred alternative and no points to the remaining alternatives (from the second-best to the worst).

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Traditionally, the advocacy for MR, BR, and PR has been grounded in axiomatic rationales, demonstrating that these aggregation rules exclusively fulfill the desirable prerequisites concerning the connection between individual and social preferences. There are two distinct justifications for MR: one is the epistemic stance, rooted in Condorcet's Jury Theorem (as referenced in List and Goodin 2001); the other is the consequentialist-utilitarian perspective (as highlighted in Brighouse & Fleurbaey, 2010; Nakada et al., 2023; Rae, 1969), which is particularly pertinent during the constitutional stage, a phase dominated by the veil of ignorance.

Historically, there has been no direct comparison between MR and either BR or PR, considering their inherent drawbacks: the disregarding of preference intensity and the violation of majoritarianism, respectively. This study aims to bridge this gap by first examining MR and BR, and then comparing MR and PR in light of their costs. 'Cost' here represents the degradation of the principle each rule infringes. The two main findings reported below are PR's supremacy over MR and MR superiority over BR. These results cast a fresh perspective on the age-old debate between Condorcet and Borda and their advocates, who fervently criticized PR before championing, in order, MR and BR. Moreover, the first finding also revives the debate about the merits of PR which despite its apparent practical benefits, received limited support from 22 preeminent voting-theory experts who ranked 18 different voting rules, as evidenced by Laslier (2012).

## 2 The novelty of our approach

Let society $N$ consist of $n$ voters, $n>2$, and suppose that the set of social alternatives, $X$, has $m$ elements, $m>2$. $R_i$ denotes the preference relation of individual $i$, which is assumed to be strict ordering, and $\boldsymbol{R}=(R_1, \ldots, R_n)$ is a preference profile. An aggregation rule is a mapping from the set of possible profiles to the set of possible reflexive and complete social-preference relations. Here we do allow for indifference and the typical social-preference relation is $R$. The score of an alternative $x$ under the Borda rule and the Plurality rule on which we focus is denoted by B(x) and P(x). A majority prefers $y$ to $x$ when the number of individuals who prefer $y$ to $x$, $N(y,x)$, is larger than the number of individuals who prefer $x$ to $y$.

Our combined approach of ordinal and ranking-based utilitarianism is very different from the typical unrestricted utilitarian approach that works mostly with the standard principles (Benthamite, Rawlsian, etc.), which is very difficult to apply in comparing alternative voting rules. More explicitly, the typical utilitarian approach compares the expected social welfare (Benthamite, Rawlsian, etc.,) obtained under different voting rules. In contrast, in our approach, the social planner compares MR to a scoring rule based on expected deviation from the two fundamental democratic principles that are assumed to be of equal significance and not on some standard utilitarian principle. Since MR implies ordinal utilities, the proposed measure of the expected deviation of a scoring rule from the majority principle is naturally defined in terms of the number of individuals who prefer one alternative to another one, and not in terms of cardinal interpersonally non-comparable individual utilities. Since scoring rules imply some restricted form of cardinal and personally comparable utilities, the proposed measure of the expected deviation of MR from the principle of allowing expression of preference intensity is naturally defined in terms of the ranking-based utilities of the two scoring rules on which we focus.[2] Both measures of deviation from the fundamental principles depend on the assumed preference distributions in the population and they take into account all possible preference profiles and all possible compared alternatives.

Our focus is on the most widely studied monotonic scoring rule, BR, and the most common weakly monotonic scoring rule, PR. Given the plausibility of a complete veil of ignorance in the constitutional stage, BR is a reasonable representative scoring rule.[3] Thus, we first compare MR with BR and then compare MR with PR.[4]

BR violates the majority principle in those instances, namely, preference profiles and pairs of compared alternatives, where it protects the minority effectively by taking into account its higher preference intensity rather than the majority's lower preference intensity. While the emphasis in Baharad and Nitzan (2002) is on the different degrees of majority-decisiveness amelioration that different scoring rules provide, in the current study the focus is on the comparison between MR and the two most common scoring rules based on the severity of the problems they cause: disregarding expression of preferences, which implies prevention of effective expression of preference intensity by the minority, and violating the majority principle. Let us now define the severity of the two problems in a way that enables a comparison of the "costs" associated with applying the two alternative democratic voting rules and, in turn, the preference of MR or BR.[5]

## 3 The severity of violating the two fundamental principles

One possibility is to measure the severity of a problem by the probability of its occurrence. Baharad and Nitzan (2007, 2011) take such an approach, focusing on comparing alternative scoring rules. Gehrlein and Lepelley (2011, 2017) apply this criterion in assessing different election paradoxes. In our study we focus on the Condorcet–Borda (binary–positional) controversy and the analysis rests on two criteria. First, the comparison between the prior likelihoods of the compared rules to be superior in terms of the deviation from the aforementioned fundamental democratic principles. Second, we take into account not only the severity of a problem in terms of the probability of its occurrence but also in terms of the expected erosion of the two foundational democratic principles. Erosion of the majority principle takes into account all possible preference profiles and any pair of

[2] Interestingly, Bossert and Suzumura (2017) have shown that, with their alternative articulation of the Benthamite greatest-happiness of-the-greatest-number principle and with ordinally measurable and interpersonally non-comparable utilities, the social decision rule chooses those alternatives that maximize the number of individuals who end up with their greatest element. This rule is tantamount to PR.

[3] For further enthusiastic support of BR, see Saari (2006). The two most recent axiomatizations of BR appear in Heckelman and Ragan (2020) and Maskin (2022).

[4] According to Arrow's (1963) Independence of Irrelevant Alternatives (IIA) axiom, social preferences between any two alternatives depend only on the individual preferences between them. IIA implies that social preferences disregard information about individuals' preference intensity. The aggregation rule based on majority comparisons is the clearest example of a rule that satisfies IIA. Note that BR does not satisfy IIA but it does satisfy the weaker Modified IIA recently proposed by Maskin (2022), which allows a particular form of preference-intensity expression. It requires that if two profiles and two alternatives x and y are given, and if every individual ranks the two alternatives the same way in both profiles and ranks the same number of other alternatives between them in both profiles, then the social preference between these two alternatives is the same for both profiles.

[5] Analogous definitions apply for the comparison of MR and PR.

alternatives where the majority's preference is overlooked. Erosion of the second principle, namely, respect of voters' preference intensity, takes into account all possible preference profiles and any pair of alternatives where the disregard of preference intensity implies that the minority's preference intensity is disregarded, even though it is larger than that of the majority.

Our analysis may yield a flexible, "case-dependent" choice between the two aggregation rules. One rule may be superior for certain preference profiles and pairs of alternatives whereas the other rule may prevail for others.[6] The implementation of such flexibility, requiring the practical partitioning of the set of pairs of alternatives, may involve considerable difficulties. Our objective, therefore, is to identify the preferred aggregation rule, just one of the two rules and not a flexible, case-dependent rule, based on its larger likelihood to be superior and its lower expected violation of a fundamental principle. That is, the expected severity of the problem that it raises should be lower than that of the problem associated with using the alternative aggregation rule. The expected severity of the compared rules is referred to as their expected cost. The main contribution of our study is the clarification of the expected costs of the rules and the use of these expected costs to determine the superiority of one of them.

Given a specific situation, namely preference profile **R** and pair of alternatives $x$ and $y$, we first measure the corresponding cost of applying MR in terms of the erosion of the principle that preference intensity must be taken into account, and, in particular, the minority's preference intensity, C(MR, **R**, $x$, $y$); then we measure the cost of applying BR in terms of the erosion of majoritarianism, C(BR, **R**, $x$, $y$); The application of MR is warranted in a specific situation if C(MR, **R**, $x$, $y$) < C(BR, **R**, $x$, $y$); the application of BR is warranted if C(BR, **R**, $x$, $y$) < C(MR, **R**, $x$, $y$). And in case C(MR, **R**, $x$, $y$) = C(BR, **R**, $x$, $y$), the use of either MR or BR is justified.[7] This may ensure an ideal flexible situation-dependent balanced democracy that applies in every particular situation the aggregation rule associated with the lower cost. As already noted, however, such a flexible situation-dependent aggregation rule is difficult to implement because it requires information about the voters' actual preference profile. Therefore, we impose the restriction that the same aggregation rule must be applied to any pair of alternatives and any preference profile. Given this restriction, let us turn to the comparison of MR with BR.

### 3.1 Proposed measures of erosion of the two principles

Consider, first, the cost of applying MR. It involves the possible erosion of the principle of allowing expression of preference intensity, which implies that the minority should win when its preference intensity exceeds that of the majority (Principle 1). This principle is

[6] The justification of deviating from MR in order to protect the minority is typically deemed plausible when the two alternatives result in substantially different long-run irreversible outcomes and is usually implemented by applying a qualified majority rule. Recently, Barberá et al. (2021) studied the hybrid rule proposed by Daunou, (1803) which deviates from MR when a Condorcet winner does not exist by applying PR after eliminating the Condorcet losers. In this case, the reconciliation of conflicting desiderata is based on accommodating them lexicographically.

[7] An analogous criterion is applied in Sect. 4.3, in the comparison between MR and PR where the cost of BR, C(BR,R,x,y), is replaced by the cost of PR, C(PR,R,x,y). The endogenous partition of the set of profiles takes into account the costs of the rules assigning equal weights to these costs. But one can easily enrich the approach by assigning different weights to the costs C(MR, R) and C(BR,R) that are associated with the application of MR and BR

eroded when, given a specific preference profile, MR and BR yield different social preferences between two alternatives *x* and *y*.[8]

Let us present a natural and intuitive measurement of erosion.

Suppose that, given a specific preference profile, alternative *x* is preferred over *y* under MR and that alternative *y* is preferred over *x* under BR because the score of *y*, B(*y*), is higher than the score of *x*, B(*x*). The positive difference between these scores, (B(y)—B(x)), is referred to as the unrealized advantage of the preference intensity of the minority. The first measure is the share of the minority's preference intensity that erodes due to the use of MR that yields the social preference of alternative *x*, despite its inferiority to *y* according to BR. This inferior alternative should not be preferred according to Principle 1. The proposed measure of erosion of Principle 1, for a particular preference profile and two given alternatives *x* and *y*, given that MR and BR result in different social preferences is the unrealized advantage of the preference intensity of the minority relative to its total intensity.

$$\textbf{Measure 1}: \ (\mathrm{B(y)}—\mathrm{B(x)})/\mathrm{B}(y).$$

Given a specific preference profile and pair of alternatives, Measure 1 represents the relative erosion of the minority's ability to effectively express its preference intensity and ensure the superiority that its preferred alternative would have enjoyed had BR been used as the aggregation rule.

To sum up, the proposed measure of the cost of MR in a particular situation is the rate of reduction in the more intense minority's preference of the socially inferior alternative in terms of preference intensity relative to the socially superior alternative under MR.

Consider now the second majority principle: it requires that *x* is socially preferred over *y* by BR when a majority prefers *x* over *y*. The cost of applying BR involves the possible erosion of this principle, Principle 2, and again, this erosion is realized when, given a specific preference profile **R**, MR and BR yield different social preferences between two alternatives *x* and *y*. That is, B(*x*) >B(*y*), however a majority prefers *y* to *x*; the number of individuals who prefer *y* to *x*, N(*y*,*x*), is larger than N(*x*,*y*). Analogously to the measurement of erosion of Principle 1, for a specific preference profile and pair of alternatives *x* and *y*, the measurement of erosion of Principle 2 takes the form:

$$\textbf{Measure 2} = \ (\mathrm{N}(y,x) - \mathrm{N}(x,y))/\mathrm{N}(y,x)$$

The proposed measure of the cost of BR is the disregarded advantage of the majority obtained by alternative *y* (which is superior to alternative *x* under MR), divided by the actual majority of *y*,[9] In other words, Measure 2 represents the erosion of the majority's ability to effectively express its preference and ensure the superiority that its preferred alternative would have enjoyed had MR been used as the aggregation rule.

Note that the lower bound (0) and the upper bound (1) of the two measures represent no violation and maximal violation of the relevant principle.

[8] Notice that if we were to deal with the agreements on the collective ranking between *x* and *y* by MR, BR and PR, while ignoring our proposed measures that focus on the erosion of the two fundamental principles (majoritarianism and respect of preference intensity) by these aggregation rules, we would fall into the problem addressed by Fishburn and Gehrlein (1980, 1981) who focused on the special case of m = 4 and an electorate of infinite size.

[9] Analogous measures are applied for PR.

Given a preference profile and a pair of alternatives, a comparison of the costs of applying MR and BR is the basis for selecting one aggregation rule over the other. The preferred rule is the less costly one.[10]

### 3.2 Illustration

***Example*** Suppose $N = \{1,2,3\}$, $X = \{ w,x,y,z\}$ and the preference profile is $\mathbf{R} = (R_1, R_2, R_3)$,

$$R_1 : yR_1zR_1wR_1x$$

$$R_2 : xR_2yR_2zR_2w$$

$$R_3 : zR_3yR_3wR_{31}x$$

Let us determine simple majority relation and Borda social preference relation corresponding to profile $\boldsymbol{R}$:

It can be verified that

$$y\ R^{maj}\ z\ R^{maj}\ w\ R^{maj}\ x$$

Let B($s$) denote the Borda score of alternative $s$. In the above example, B($w$) = 2, B($x$) = 3, B($z$) = 6 and B($y$) = 7. Hence,

$$yR^BzR^BxR^Bw$$

**Case 1:** Consider the comparison of $y$ and $z$. In such a case,

$$\mathrm{C}(\mathrm{MR}, \mathbf{R}) = \mathrm{C}(\mathrm{BR}, \mathbf{R}) = 0$$

Thus, either rule may be used.

**Case 2:** Consider the comparison of $w$ and $x$. In such a case,

$$\mathrm{C}(\mathrm{MR}, \mathbf{R}) = (\mathrm{B}(x) - \mathrm{B}(w))/\mathrm{B}(x) = (3-2)/3 = 1/3$$

C(BR,**R**)=(N($w$,$x$)) - N($x$,$w$) / N($w$,$x$) =(2-1)/2=1/2. Since C(MR, **R**) = 1/3 < C(BR, **R**) = 1/2, the justified aggregation rule for the comparison of $x$ and $w$ is MR.

### 3.3 Possible alternative measures

Finally note that one could think about alternative measures, such as the absolute deviation between the scores or the support of the alternatives preferred by BR and MR, B($y$)-B($x$) or N($y$)-N($x$) or the relative score or support of these alternatives, B($y$)/B($x$) or N($y$)/N($x$). The former alternatives for Measure 1 and Measure 2 would not allow a meaningful comparison between them. The obvious reason is that such absolute deviations cannot be compared because they apply different notions; one relates to scores and the other to majority and minority support. The latter alternative measures are

[10] Our criterion of comparison between aggregation rules disregards operational simplicity and degree of manipulability. By using pairs of alternatives as our standard of comparison, we avoid the need to take lack of transitivity into account.

not vulnerable to such criticism, that is, they can be compared. However, they do not capture the significant aspect implied by the proposed definitions of the two measures, namely, the relative *disregarded* or *unexploited* advantage of the minority and the majority when the social preference differs from their preference.

Nevertheless, the measures that involve the ratios B($y$)/B($x$), P($y$)/P($x$) and N($y$)/N($x$) are meaningful. In the case of B($y$)/B($x$) and P($y$)/P($x$), this formulation shifts the emphasis from the proportion of the eroded scores of the minority to assessing the severity of relative social injustice against the minority. Its preference intensity is not respected, despite being B($y$)/B($x$) or P(y)/P(x) times stronger than that of the majority. In the case of N($y$)/N($x$), this formulation shifts the emphasis from the proportion of the eroded advantage of the majority to assessing the severity of relative social injustice against the majority. Its advantage is not respected, despite being N($y$)/N($x$) times stronger than that of the minority. We ran simulations verifying that our findings are robust to the application of these alternative metrics to Measure 1 and Measure 2.

The current formulation assigns equal importance to all the compared pairs of alternatives when applying scoring rules. The erosion is calculated as the proportion of the scores eroded out of the scores assigned to the rejected option. This form of "local" normalization doesn't create a common global scale for all the compared pairs, but rather each pair is considered in isolation and the erosion strength for all pairs is given equal weight in the overall expectation calculation. This has its merit as it evaluates each aggregation rule over the choice between any two pairs of alternatives, irrespective of their popularity. We leave to future research the option of giving more importance to the erosion of choices with high scores relative to those with low scores. Several potential avenues for achieving this include:

1. Focusing solely on cases where each rule results in a different winner and calculating erosion exclusively for the pair of winners.
2. Investigating the implementation of a global normalization factor. In the majority rule, the erosion measure naturally interprets what percentage of voters' preferences were eroded when normalized by the total number of voters. We can potentially develop a similar measure for scoring rules by consistently normalizing erosion based on the average score that voters can assign to each option. This would establish a unified scale for measuring erosion across all pairs of alternatives.

## 4 Main findings

In our study, the definite preference of a rule is based on its superiority in terms of the two criteria: the likelihood of being less costly and the difference between the expected costs of the two rules. According to the two possible criteria, the conclusion may hinge on the number of voters $n$ and the number of alternatives $m$. The question is how $n$ and $m$ affect the desirability of MR and BR under the veil of ignorance in the constitutional stage regarding the actual preference profile and the compared alternatives. Before turning to the identification of the preferred rule according to the two possible criteria, we describe the particular statistical model used to generate the preference profiles of the voters.

### 4.1 The simulation

Preference profiles can be generated by several probabilistic models.[11] We base our results on the Cubic and the spatial Euclidean Box models. These approaches have the common feature of generating preferences to reflect or approximate data samples in real elections.[12] The results presented below are based on the Cubic model (the results for the Euclidean Box model are essentially the same and are presented in Appendix A). In the former model, for a particular number of voters, $n$, and $m$ alternatives (the candidates' positions), we generate a matrix of size $n*m$ where each number is sampled from the uniform distribution over the [0, 1] segment. This matrix represents the utility for each voter for each candidate and it yields the preference profile of the voters. In the Euclidean Box model, for a particular number of voters, $n$, and $m$ alternatives, we independently and uniformly sample the alternatives and the positions of the voters from the Box on the assumption that a voter's utility for a candidate is a decreasing function of the distance between the candidate's position and the voter's position.[13] For a particular case of $m$ alternatives and a preference profile **R**, the percentage of pairs of alternatives that result in different preferences by MR and BR in which MR is the superior rule (the erosion of principle 1 by MR, Measure 1, is smaller than the erosion of principle 2 by BR, Measure 2) is equal to:

$$r/t$$

where

$$r = |\{(x,y) : N(x,y) > N(y,x), B(y) > B(x) and \mu(x,y) < \nu(y,x)\}|$$

$$\begin{aligned} B(x) &= \text{The Borda count of } x \\ N(x,y) &= \left|\{i : xR_i y\}\right| \\ \mu(x,y) &= \tfrac{B(y)-B(x)}{B(y)} \\ \nu(x,y) &= \tfrac{N(y,x)-N(x,y)}{N(y,x)} \end{aligned}$$

$$t = |\{(x,y) : N(x,y) > N(y,x)\, and\, B(y) > B(x)\}|$$

Note that $t$ is the number of pairs of alternatives that result in different preferences by MR and BR · Profiles in which BR has equal scores for alternatives $x$ and $y$ are discarded and not considered to create erosion. We decided to only count cases where there is a clear winner and a clear dispute between the results. The same policy is applied later for PR.

For a particular case of $m$ alternatives and a preference profile **R**, the expected costs of MR and BR, the expected erosion of the fundamental principles by these rules applying measures 1 and 2, over all possible pairs of the alternatives that result in different preference by MR and BR are:

[11] The most common models are the Impartial Culture, Impartial and Anonymous Culture and the spatial models.

[12] See https://francois-durand.github.io/svvamp/.

[13] According to Merrill (1984) and Tideman and Plassmann (2013), when generating alternatives (candidates) and voters by means of simulations based on a spatial model, outcomes come very close to describing the distribution of actual outcomes, and ranking data simulated with the spatial model are very similar to observed ranking data. The spatial-model results thus tend to be more realistic.

$$\frac{1}{t}\sum_{\substack{x,y\ :\ N(x,y)>N(y,x)\\ B(y)>B(x)}} \mu(x,y) \text{ and } \frac{1}{t}\sum_{\substack{x,y\ :\ N(x,y)>N(y,x)\\ B(y)>B(x)}} \nu(x,y)$$

In the simulation, we computed the percentage of pairs of alternatives in which MR is superior to BR and the means of the difference of the expected costs of MR and BR in the 100,000 generated cases of the $n$ voters' preference profiles **R**. Analogous computations were made for comparison between MR and PR.[14] For each preference profile, we have compared all possible pairs of the alternatives.[15]

Before delving into the simulated results**,** let us present a theoretical subsection that contains analytical results that prove the superiority of MR to BR and the superiority of PR to MR in terms of the above two criteria for the case of three alternatives and $n$ voters, $n$ = 3,5,7,9,11,13,15,21,31,41,51. These analytical results make it possible to better infer the results of the simulation that are almost identical for m = 3.

## 4.2 Analytical results for the case of three alternatives

When $m = 3$, there are 6 possible strict individual preference relations (rankings) denoted by 1,2, …,6. The possible preference profiles specify the selection of these rankings by the $n$ voters. Since the Cubic model assumes that, for every voter, all preference relations have equal probability, we can disregard the identity of the voters as the voting is anonymous and focus on distinct preference profiles. Each distinct profile is given by (*k1*,*k2*,… .*k6*), where *ki* is the number of voters selecting preference relation i, i = 1, …,6, and *k1* + *k2* + … + *k6* = *n*. The expected occurrence of such a distinct profile can be obtained by using the multinomial theorem. This number, the multinomial coefficient, which is denoted by (*k1*, *k2*, *k3*,…., *k6*)! is equal to *n*!/*k1*!**k2*!*….**k6*!. For each distinct profile, we calculate Measure 1 and Measure 2 for all the pairs of alternatives that result in different preferences by MR and BR, and then compute *r/t*. Assigning to each percentage of such a distinct profile a weight that is equal to its multinomial coefficient and summing up all the weighted percentages corresponding to these distinct profiles, we divide the sum of the weighted percentages by the total number of these distinct profiles and obtain the expected percentage of pairs of alternatives in which MR is superior to BR in all the distinct profiles that result in different preference by MR and BR.

Given the multinomial coefficients of the possible distinct preference profiles, and using the above formulas of the expected costs of MR and BR over all possible pairs of the alternatives for a particular distinct profile, we can compute the expected difference of these costs in all possible distinct profiles (*k1*,*k2*,….*k6*), such that k1 + k2.. + k6 = n.

[14] The random selection of preference orderings may not capture well the presence of proactive, highly motivated minority members who may strategically coordinate their reports of preferences. At the constitutional stage, it is difficult to capture this possibility without adding extra parameters such as an exogenous polarization factor. Therefore, our analysis disregards such possible strategic considerations that are a significant issue with intensity-based scoring rules..

[15] Note that the example in Sect. 3.2 illustrates the costs of the two aggregation rules assuming a particular number of alternatives, a particular preference profile and a particular pair of alternatives. A series of such examples could illustrate the comparison between the expected costs of MR and BR allowing 100,000 profiles over the $m$ alternatives and taking into account all pairs of alternatives where these rules yield different social preferences.

Calculating the expected percentage of pairs of alternatives in which MR is superior to BR and calculating the difference between the expected costs of MR and BR, for $m = 3$ and $n = 3,5,7,9,11,13,15,21,31,41,51$, we obtain the following results:

For three alternatives then, MR is unambiguously superior to BR because, for any n, it is more likely to be superior in pairwise comparisons of the alternatives and its expected cost is smaller (Tables 1, 2).

**Table 1** The expected percentage of pairs of alternatives in all the distinct profiles in which BR is superior to MR for 3 candidates

| VOTERS | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | – | 0 | 0.03 | 0.08 | 0.12 | 0.16 | 0.20 | 0.28 | 0.35 | 0.39 | 0.41 |

**Table 2** Difference between the expected costs of BR and MR over all the pairs that result in erosion, which indicates the advantage of MR for 3 candidates

| Voters | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | – | 0.15 | 0.10 | 0.07 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 |

Applying the multinomial coefficients and the analogous formulas for the comparison between MR and PR, we present in Tables 3 and 4 the expected percentage of pairs of alternatives in which PR is superior to MR and the difference between the expected costs of PR and MR.

For three alternatives then, PR is unambiguously superior to MR because, for any $n$, it is more likely to be superior in pairwise comparisons of the alternatives and its expected cost is smaller.

**Table 3** The expected percentage of pairs of alternatives in all the distinct profiles in which PR is superior to MR for 3 candidates

| Voters | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 1 | 0.94 | 0.90 | 0.88 | 0.86 | 0.82 | 0.77 | 0.73 | 0.70 | 0.67 |

**Table 4** Difference between the expected costs of PR and MR over all the pairs that result in erosion, which indicates the advantage of PR for 3 candidates

| Voters | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | −0.5 | −0.3 | −0.21 | −0.17 | −0.14 | −0.12 | −0.11 | −0.08 | −0.06 | −0.05 | −0.04 |

**Table 5** Percentage of pairwise comparisons in which BR is superior to MR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | – | 0 | 0.04 | 0.08 | 0.12 | 0.15 | 0.2 | 0.28 | 0.35 | 0.39 | 0.41 | 0.42 | 0.41 |
| 4 | 0 | 0.02 | 0.05 | 0.1 | 0.14 | 0.18 | 0.2 | 0.28 | 0.33 | 0.35 | 0.37 | 0.38 | 0.38 |
| 5 | 0 | 0.02 | 0.06 | 0.1 | 0.14 | 0.17 | 0.2 | 0.27 | 0.31 | 0.33 | 0.34 | 0.36 | 0.35 |
| 6 | 0 | 0.03 | 0.06 | 0.1 | 0.14 | 0.17 | 0.19 | 0.25 | 0.3 | 0.31 | 0.32 | 0.35 | 0.35 |
| 7 | 0 | 0.03 | 0.06 | 0.1 | 0.13 | 0.16 | 0.19 | 0.24 | 0.29 | 0.3 | 0.31 | 0.34 | 0.33 |
| 8 | 0 | 0.03 | 0.06 | 0.1 | 0.13 | 0.16 | 0.18 | 0.24 | 0.28 | 0.3 | 0.31 | 0.33 | 0.33 |
| 9 | 0.01 | 0.03 | 0.06 | 0.1 | 0.13 | 0.16 | 0.18 | 0.23 | 0.27 | 0.29 | 0.3 | 0.33 | 0.32 |
| 10 | 0.01 | 0.03 | 0.06 | 0.1 | 0.13 | 0.16 | 0.18 | 0.23 | 0.27 | 0.28 | 0.29 | 0.32 | 0.32 |

### 4.3 General simulation results: the superiority of MR over BR[16]

The simulation results establish the superiority of MR in terms of the above two criteria when the two aggregation rules yield different outcomes. This is true for any combination of the number of alternatives $m$ and the number of voters $n$ when the number of voters is odd, $n=3,5,7,9,11,13,15,21,31,41,51,1001,10001$ and $m=3,4,\ldots,10$.[17]

Table 5 presents the results that illustrate the inferiority of BR in terms of the percentage of cases in which it is superior to MR when the two aggregation rules yield different outcomes. This percentage can be considered as an estimate of the a-priori likelihood that BR is the superior aggregation rule when two alternatives are compared, taking into account all preference profiles that result in the erosion of Principles 1 and 2, that is, MR and BR yield different preferences between the compared alternatives. For any $n$ and $m$, the a-priori likelihood of MR being superior to BR is larger than 50%. Note that our findings are consequential because erosion, i.e., divergent outcomes by MR and BR is a likely possibility. More specifically, for $m>3$, $n>3$, it is obtained in at least 28% of the preference profiles and in 6% of the pairwise comparisons. The likelihood increases with both $m$ and $n$ and for $m=10$, $n=10{,}001$, it is obtained in 100% of the preference profiles and in 17% of the pairwise comparisons. The Tables in Appendix B present these erosion likelihoods for all the combinations of $m$ and $n$.

For a small number of alternatives and a large number of voters, the superiority of MR tends to be less significant. For example, for three alternatives and 51 as well as 100,001 voters, the likelihood of MR being the preferred rule is $1-0.41=0.59$. For 10 alternatives and 10,001 voters, this likelihood increases to 0.68. The results suggest that when the electorate is sufficiently large, the extent of the superiority of MR converges to a limit.

Table 6 confirms the superiority of MR by presenting the mean of the difference in the costs of BR and MR, which is always positive; for any combination of $m$ and $n$, the expected difference is positive. That is, the expected erosion of Principle 1 by MR is always smaller than the expected erosion of Principle 2 by BR, taking into account all possible comparisons of the generated alternatives and preference profiles under any combination of

[16] The code base for running the simulations, analytical calculations, and creating the different statistical reports can be found at https://github.com/adamnitzan/voting-rules-erosion. The repository also includes full result files for 100,000 simulations and the analytical calculations.

[17] We chose to deal with an odd number of voters to avoid dealing with situations of equality in pairwise comparisons of alternatives. The two largest numbers of voters, 1001 and 10,001 illustrate the significance of the findings in the context of elections.

**Table 6** The mean of the differences between the expected costs of BR and MR, which indicates the advantage of MR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
|---|---|---|---|---|---|---|---|
| 3 | – | 0.15 (0.08:0.19) | 0.1 (−0.08:0.15) | 0.07 (−0.13:0.12) | 0.06 (−0.23:0.36) | 0.05 (−0.19:0.31) | 0.04 (−0.21:0.28) |
| 4 | 0.29 (0.17:0.36) | 0.17 (−0.17:0.25) | 0.12 (−0.25:0.53) | 0.09 (−0.27:0.45) | 0.07 (−0.3:0.38) | 0.06 (−0.26:0.51) | 0.05 (−0.29:0.46) |
| 5 | 0.3 (0.0:0.4) | 0.18 (−0.29:0.27) | 0.13 (−0.3:0.55) | 0.1 (−0.3:0.67) | 0.08 (−0.31:0.59) | 0.07 (−0.34:0.53) | 0.06 (−0.31:0.47) |
| 6 | 0.32 (−0.1:0.42) | 0.19 (−0.33:0.7) | 0.14 (−0.32:0.56) | 0.11 (−0.3:0.68) | 0.09 (−0.39:0.6) | 0.08 (−0.28:0.53) | 0.07 (−0.28:0.61) |
| 7 | 0.32 (−0.17:0.44) | 0.2 (−0.39:0.71) | 0.14 (−0.38:0.57) | 0.11 (−0.34:0.69) | 0.1 (−0.31:0.6) | 0.09 (−0.32:0.54) | 0.08 (−0.33:0.62) |
| 8 | 0.33 (−0.21:0.45) | 0.2 (−0.39:0.72) | 0.15 (−0.4:0.57) | 0.12 (−0.3:0.69) | 0.1 (−0.3:0.6) | 0.09 (−0.41:0.68) | 0.08 (−0.39:0.62) |
| 9 | 0.33 (−0.25:0.45) | 0.2 (−0.35:0.72) | 0.15 (−0.39:0.58) | 0.12 (−0.52:0.7) | 0.11 (−0.31:0.74) | 0.09 (−0.32:0.68) | 0.08 (−0.3:0.62) |
| 10 | 0.34 (−0.28:0.46) | 0.21 (−0.42:0.72) | 0.15 (−0.43:0.8) | 0.13 (−0.37:0.7) | 0.11 (−0.32:0.61) | 0.1 (−0.34:0.69) | 0.09 (−0.29:0.62) |

| VOTERS CANDIDATES | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|
| 3 | 0.02 (−0.19:0.34) | 0.02 (−0.2:0.25) | 0.01 (−0.17:0.27) | 0.01 (−0.15:0.24) | 0.003 (−0.04:0.07) | 0.001 (−0.02:0.02) |
| 4 | 0.04 (−0.25:0.47) | 0.03 (−0.23:0.35) | 0.03 (−0.21:0.35) | 0.02 (−0.17:0.34) | 0.0061 (−0.05:0.1) | 0.002 (−0.0:0.03) |
| 5 | 0.05 (−0.24:0.48) | 0.04 (−0.23:0.51) | 0.03 (−0.24:0.41) | 0.03 (−0.23:0.39) | 0.007 (−0.05:0.11) | 0.002 (−0.03:0.03) |
| 6 | 0.06 (−0.31:0.48) | 0.05 (−0.25:0.49) | 0.04 (−0.2:0.41) | 0.04 (−0.21:0.4) | 0.008 (−0.06:0.12) | 0.003 (−0.02:0.04) |
| 7 | 0.06 (−0.32:0.59) | 0.05 (−0.28:0.44) | 0.04 (−0.22:0.45) | 0.04 (−0.22:0.43) | 0.01 (−0.06:0.12) | 0.003 (−0.02:0.05) |
| 8 | 0.07 (−0.28:0.59) | 0.05 (−0.26:0.52) | 0.05 (−0.22:0.47) | 0.04 (−0.24:0.4) | 0.01 (−0.06.14) | 0.003 (−0.02:0.04) |
| 9 | 0.07 (−0.27:0.59) | 0.06 (−0.29:0.52) | 0.05 (−0.24:0.45) | 0.04 (−0.22:0.48) | 0.01 (−0.06:0.12) | 0.003 (−0.02:0.04) |
| 10 | 0.07 (−0.3:0.59) | 0.06 (−0.25:0.55) | 0.05 (−0.23:0.48) | 0.04 (−0.22:0.44) | 0.01 (−0.05:0.12) | 0.003 (−0.02:0.04) |

$m$ and $n$. The Table also presents in brackets two more statistics: the minimum and maximum difference between the costs of BR and MR .

This difference referred to as the advantage of MR, ranges from the lowest advantage of 0.001 when m = 3 and n = 10,001 to the largest advantage of 0.34 when m = 10 and n = 3. These results confirm all the findings adduced based on the first criterion. However, it seems that for sufficiently large numbers of voters, the difference between the mean costs of BR and MR although still positive decreases with n and becomes negligible. In the alternative Euclidean Box model, a salient observation emerges from our simulation results: the discrepancy in the means of the expected costs of BR and MR amplifies notably for expansive electorates, as delineated in the second Table in Appendix A. To elucidate, for m = 5 and n = 10,001, the anticipated advantage of MR escalates to 2% from the initial value of 0.2% in the Cubic distribution simulation. This phenomenon underscores the robustness of our conclusions, particularly when grounded in more realistic assumptions about preference profiles, as expounded upon in footnote 14. A worthwhile task for future research is the analytical study of the limit behavior of the superiority of MR over BR when the number of voters is sufficiently large.

The Table clarifies that the expected cost of MR, the expected violation of Principle 1, is smaller than the expected cost of BR, the expected violation of Principle 2, for any $m$ and $n$. This implies that in the constitutional stage, where the number of alternatives and their identity, the number of voters, and their preferences are all unknown, there is a very good reason to apply MR rather than BR if one focuses on the comparison between their fundamental weaknesses, namely, violation of one of the two fundamental democratic principles: majoritarianism and suitable recognition of preference intensity. Recall that the comparison between MR and BR takes into account only the situations where these rules result in different social preferences.

The underlying reason for MR's superiority can be elucidated by delving into its inherent expressive capability. Under MR, the comparison between two alternatives is based upon the electorate's allocation of n uniform scores to these alternatives. Contrarily, under the Borda Rule (BR), this juxtaposition is predicated upon the n voters' potential allocation of as many as m disparate scores to these alternatives. MR, in comparison to BR, possesses a constrained proficiency in encapsulating and delineating voter preferences. It is plausible to conjecture that this more restricted expressive capability is advantageous because it contributes to the lower erosion of principle 1 (Measure 1) relative to the higher magnitude of degradation of majoritarianism by BR (Measure 2).

**Table 7** Percentage of pairwise comparisons in which PR is superior to MR

| VOTERS<br>CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 1 | 0.94 | 0.9 | 0.88 | 0.86 | 0.81 | 0.77 | 0.73 | 0.69 | 0.67 | 0.57 | 0.56 |
| 4 | 1 | 1 | 0.91 | 0.86 | 0.86 | 0.82 | 0.78 | 0.76 | 0.73 | 0.72 | 0.7 | 0.61 | 0.59 |
| 5 | 1 | 1 | 0.92 | 0.85 | 0.88 | 0.83 | 0.79 | 0.77 | 0.75 | 0.73 | 0.71 | 0.64 | 0.63 |
| 6 | 1 | 1 | 0.93 | 0.86 | 0.91 | 0.85 | 0.81 | 0.79 | 0.78 | 0.77 | 0.74 | 0.67 | 0.65 |
| 7 | 1 | 1 | 0.95 | 0.88 | 0.93 | 0.88 | 0.83 | 0.82 | 0.82 | 0.8 | 0.78 | 0.69 | 0.67 |
| 8 | 1 | 1 | 0.96 | 0.9 | 0.94 | 0.9 | 0.85 | 0.85 | 0.84 | 0.83 | 0.8 | 0.71 | 0.69 |
| 9 | 1 | 1 | 0.97 | 0.91 | 0.95 | 0.91 | 0.87 | 0.87 | 0.86 | 0.84 | 0.83 | 0.72 | 0.71 |
| 10 | 1 | 1 | 0.97 | 0.93 | 0.96 | 0.93 | 0.89 | 0.89 | 0.88 | 0.86 | 0.85 | 0.74 | 0.72 |

Two unequivocal findings come to light regarding the effect of $m$ and $n$ on the advantage of MR. For a given number of voters $n$, the advantage of MR increases with the number of alternatives $m$. For a given number of alternatives $m$, the advantage of MR decreases with the number of voters $n$.

Note that the maximum difference between the costs of BR and MR is always positive. But the minimum difference can also be positive, which indicates that in all pairwise comparisons between the alternatives MR is the superior rule. This is the case when $m=3$, $n=5$, and when $m=4$, $n=3$. In the remaining combinations of $m$ and $n$, the minimum difference is negative, indicating specific cases where BR is the preferred rule and the maximum difference is positive, indicating that MR is the preferred rule.

Finally, for $m=3$, we confirmed that the simulation results are very close to the analytical ones. We have increased the reliability of the simulation results by generating 1,000,000 preference profiles instead of 100,000 and obtained that, for the combinations of 3 alternatives and $n$ voters, the average deviation of the simulation results from the analytical results, in terms of the first criterion, was 0.27%. The average deviation of the simulation results from the analytical results, in terms of the second criterion, was only 0.16%.

## 4.4 The superiority of PR over MR

The most common and best-known scoring rule is the plurality rule, PR, which is an extreme weakly monotonic scoring rule, that is, only the score assigned to the best alternative exceeds that of the second-best and all other alternatives, so significance is assigned only to every voter's most preferred alternative. Applying the same methodology for comparing MR and PR based on their costs, the extent of erosion of the principle they violate, PR emerges unambiguously as a superior aggregation rule when $n \leq 10{,}001$.

Our findings establish the superiority of PR in terms of its higher likelihood to be superior (Table 7) and its lower mean cost (Table 8), all simulation outcomes taken into account. That is, for any combination of a number of alternatives $m$ and a number of voters $n$, PR outperforms MR in terms of the two criteria we have applied in the previous section. The negative difference between the means of the expected costs of PR and MR represents the advantage of PR. This advantage ranges from $-0.57$ which is the largest advantage, when $m=10$ and $n=7$, to $-0.002$ which is the smallest advantage, when $m=3$ and $n=10{,}001$.

Table 7 illustrates the superiority of PR in terms of the expected a priori likelihood of being superior in pairwise comparisons of alternatives when the two aggregation rules yield different outcomes. This likelihood always exceeds 0.56. This is of significance because the likelihood of erosion now is considerably higher than that in the preceding section. More specifically, for $m>3$ and $n>3$, it is obtained in at least 55% of the preference profiles and in 13% of the pairwise comparisons. The likelihood increases with both $m$ and $n$ and for $m=10$, $n=10{,}001$, it is obtained in 100% of the preference profiles and in 35% of the pairwise comparisons. The last two tables in Appendix B present these erosion likelihoods for all the combinations of $m$ and $n$.

Table 8 yields two unequivocal findings regarding the effect of $m$ and $n$ on the advantage of PR. For a given number of voters $n$, $n>3$, the advantage of PR increases with the number of alternatives $m$. For a given number of alternatives $m$, the advantage of PR decreases with the number of voters $n$.

As in the comparison between MR and BR, we conjecture that the superiority of PR is partly due to its limited expressive capability. Under MR, the comparison between two

**Table 8** The mean of the differences between the expected costs of PR and MR, which indicates the advantage of PR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
|---|---|---|---|---|---|---|---|
| 3 | −0.5 (−0.5:−0.5) | −0.29 (−0.67:−0.17) | −0.21 (−0.75:0.1) | −0.17 (−0.8:0.21) | −0.14 (−0.83:0.29) | −0.13 (−0.86:0.37) | −0.11 (−0.88:0.42) |
| 4 | −0.5 (−0.5:−0.5) | −0.41 (−0.67:−0.17) | −0.32 (−0.75:0.1) | −0.26 (−0.8:0.21) | −0.22 (−0.83:0.29) | −0.2 (−0.86:0.37) | −0.18 (−0.88:0.42) |
| 5 | −0.5 (−0.5:−0.5) | −0.47 (−0.67:−0.17) | −0.4 (−0.75:0.1) | −0.34 (−0.8:0.21) | −0.3 (−0.83:0.29) | −0.26 (−0.86:0.37) | −0.24 (−0.88:0.42) |
| 6 | −0.5 (−0.5:−0.5) | −0.51 (−0.67:−0.17) | −0.46 (−0.75:0.1) | −0.41 (−0.8:0.21) | −0.36 (−0.83:0.29) | −0.32 (−0.86:0.37) | −0.29 (−0.88:0.42) |
| 7 | −0.5 (−0.5:−0.5) | −0.53 (−0.67:−0.17) | −0.5 (−0.75:0.1) | −0.45 (−0.8:0.21) | −0.41 (−0.83:0.29) | −0.37 (−0.86:0.37) | −0.34 (−0.88:0.42) |
| 8 | −0.5 (−0.5:−0.5) | −0.55 (−0.67:−0.17) | −0.53 (−0.75:0.1) | −0.5 (−0.8:0.21) | −0.45 (−0.83:0.29) | −0.42 (−0.86:0.37) | −0.38 (−0.88:0.42) |
| 9 | −0.5 (−0.5:−0.5) | −0.56 (−0.67:−0.17) | −0.55 (−0.75:0.1) | −0.53 (−0.8:0.21) | −0.49 (−0.83:0.29) | −0.46 (−0.86:0.37) | −0.43 (−0.88:0.42) |
| 10 | −0.5 (−0.5:−0.5) | −0.56 (−0.67:−0.17) | −0.57 (−0.75:0.1) | −0.55 (−0.8:0.21) | −0.52 (−0.83:0.29) | −0.49 (−0.86:0.37) | −0.46 (−0.88:0.42) |

| | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|
| 3 | −0.09 (−0.77:0.43) | −0.06 (−0.76:0.42) | −0.05 (−0.67:0.41) | −0.04 (−0.55:0.4) | −0.006 (−0.14:0.124) | −0.002 (−0.05:0.04) |
| 4 | −0.14 (−0.91:0.49) | −0.11 (−0.83:0.47) | −0.09 (−0.8:0.44) | −0.08 (−0.76:0.46) | −0.01 (−0.23:0.15) | −0.004 (−0.07:0.05) |
| 5 | −0.19 (−0.91:0.49) | −0.14 (−0.94:0.54) | −0.12 (−0.85:0.51) | −0.11 (−0.83:0.51) | −0.02 (−0.30:0.15) | −0.006 (−0.10:0.06) |
| 6 | −0.23 (−0.91:0.51) | −0.18 (−0.94:0.54) | −0.15 (−0.95:0.51) | −0.13 (−0.88:0.51) | −0.03 (−0.32:0.17) | −0.008 (−0.11:0.07) |
| 7 | −0.27 (−0.91:0.49) | −0.21 (−0.94:0.56) | −0.18 (−0.95:0.51) | −0.16 (−0.96:0.48) | −0.03 (−0.35:0.19) | −0.01 (−0.12:0.06) |
| 8 | −0.31 (−0.91:0.51) | −0.24 (−0.94:0.54) | −0.21 (−0.95:0.55) | −0.18 (−0.96:0.53) | −0.04 (−0.42:0.18) | −0.01 (−0.15:0.07) |
| 9 | −0.35 (−0.91:0.51) | −0.27 (−0.94:0.51) | −0.23 (−0.95:0.55) | −0.2 (−0.96:0.56) | −0.04 (−0.41:0.2) | −0.014 (−0.14:0.06 |
| 10 | −0.38 (−0.91:0.51) | −0.3 (−0.94:0.56) | −0.26 (−0.95:0.51) | −0.23 (−0.96:0.53) | −0.05 (−0.41:0.21) | −0.015 (−0.17:0.08) |

alternatives is contingent upon the electorate's distribution of $n$ uniform scores to these alternatives. Contrarily, under PR, the comparison typically hinges on voters allocating fewer than $n$ such uniform scores. That is, PR allows electorates to only indicate their top-preferred choice, thereby limiting the expressiveness of their preferences. This characteristic endows PR with a distinct advantage when one is evaluating the potential severity of degrading majoritarianism, as outlined in Measure 2, relative to the erosion of principle 1 by MR as articulated in Measure 1.

Although our results establish that PR is superior to MR which is in turn superior to BR, we cannot naively conclude that PR is superior to BR, not in general or when MR is used as the yardstick. The reason is the following. In the comparisons between MR and BR or PR, the majoritarian principle is fully respected by MR, and the consideration of preference intensity is respected, albeit in a particular form by the prominent scoring rules on which we focus. Comparison between BR and PR cannot be carried out by applying the two measures we have proposed because both of these scoring rules respect a particular form of preference intensities yet violate majoritarianism. We could compare, however, PR and BR based on an alternative single criterion: the extent of their consistency with majoritarianism. As is well known, in terms of this criterion BR is superior to PR,[18] but of course, this result does not imply transitivity as the framework used in our study is different from that applied in such a comparison between the two scoring rules.

The results provide a novel justification for the widely used PR beyond its practical advantages. There are certainly other scoring rules that are superior to MR. The identification of this set of scoring rules is a task worth pursuing in future research.

## 5 Conclusion

The root cause of the fervent discussion regarding the use of the simple-majority rule versus a scoring rule stems from the interplay between two critical principles under our focus: the ability for voters to express preference intensity, thus offering some defense for the minority (Principle 1), and safeguarding the majority by honoring majoritarianism (Principle 2). The Borda method of counts is a monotonic scoring rule. The plurality rule is weakly monotonic. These rules have drawn the most interest as two scoring rules. Since these principles cannot be concurrently upheld, adherence to Principle 1 (or Principle 2) inherently results in the infringement of Principle 2 (or Principle 1). In practical terms, if MR is applied, Principle 1 is compromised, while the use of BR disrupts Principle 2. These violations can be construed as the costs incurred by employing these widely examined aggregation rules. In applying a sensible measure of these costs, the predicted cost of the two rules in instances of outcome divergence, and leveraging an intuitive concept of relative erosion of Principles 1 and 2, our research offers a key contribution by asserting that PR outperforms MR which, in turn, is superior to BR based on the proposed metrics of expected erosion of two fundamental democratic principles. The primary findings indicate that when a rule has a more restricted capacity to express preferences, it benefits from a reduced undermining of the principle it breaches. Specifically, PR's more constrained nature compared to MR makes it superior. Similarly, MR's relative limitation in expressing preferences compared to BR gives it an advantage. Our study not only rekindles

[18] See Gehrlein and Lepelley (2011, 2017).

the historic Borda-Condorcet debate by adding novel support to MR but also revives the debate about the merits of PR challenging the minimal endorsement PR garnered from 22 esteemed voting rule experts who evaluated 18 well-established voting rules, as articulated by Laslier (2012).

Our conclusions rest on the presumption that both principles are assigned equal significance.[19] They are derived from simulations that utilize the Cubic model, along with a more feasible, realistic probabilistic model to generate alternatives and voter preferences, the Box model. These outcomes augment the comprehensive discourse on the advantages and drawbacks of the three most rigorously scrutinized aggregation rules, offering an innovative justification for favoring PR over MR and MR over BR during the constitutional phase, where the veil of ignorance reigns. They cast fresh insights on the attractiveness of PR and BR, considering MR as the alternative to these scoring rules. The first finding rationalizes the actual revealed superiority of PR over MR. With the second finding, it becomes possible to decide between the approaches of Condorcet and Borda and to explain the low prevalence of BR relative to MR

## Appendix A

The simulation results based on the Box model are presented in e Tables 9, 10, 11, 12

**Table 9** Percentage of pairwise comparisons in which BR is superior to MR under the Box model

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | – | 0 | 0.08 | 0.19 | 0.26 | 0.31 | 0.37 | 0.42 | 0.46 | 0.48 | 0.46 | 0.45 | 0.47 |
| 4 | 0 | 0.04 | 0.10 | 0.17 | 0.22 | 0.27 | 0.32 | 0.38 | 0.42 | 0.44 | 0.43 | 0.45 | 0.44 |
| 5 | 0 | 0.04 | 0.10 | 0.16 | 0.22 | 0.25 | 0.30 | 0.36 | 0.40 | 0.40 | 0.42 | 0.43 | 0.43 |
| 6 | 0.01 | 0.045 | 0.10 | 0.15 | 0.20 | 0.25 | 0.27 | 0.34 | 0.37 | 0.39 | 0.40 | 0.43 | 0.43 |
| 7 | 0.01 | 0.04 | 0.09 | 0.15 | 0.19 | 0.24 | 0.27 | 0.32 | 0.37 | 0.38 | 0.39 | 0.42 | 0.42 |
| 8 | 0.01 | 0.04 | 0.09 | 0.14 | 0.19 | 0.23 | 0.26 | 0.32 | 0.36 | 0.37 | 0.38 | 0.41 | 0.41 |
| 9 | 0.01 | 0.04 | 0.09 | 0.14 | 0.18 | 0.22 | 0.25 | 0.31 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 |
| 10 | 0.01 | 0.04 | 0.08 | 0.13 | 0.18 | 0.22 | 0.24 | 0.30 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 |

[19] The results extend to the case of asymmetric weight assignment, of course. Here, MR remains the superior aggregation rule as long as the ratio of costs associated with the use of BR and MR exceeds the ratio of the weights assigned to Principle 2 and Principle 1.

**Table 10** The mean of the differences between the expected costs of BR and MR under the Box simulations, which indicates the advantage of MR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
|---|---|---|---|---|---|---|---|
| 3 | – | 0.15 (0.08:0.19) | 0.09 (−0.08:0.15) | 0.05 (−0.18:0.12) | 0.04 (−0.23:0.36) | 0.03 (−0.27:0.32) | 0.02 (−0.3:0.29) |
| 4 | 0.28 (0.17:0.36) | 0.16 (−0.17:0.25) | 0.11 (−0.31:0.54) | 0.08 (−0.38:0.45) | 0.06 (−0.43:0.58) | 0.05 (−0.45:0.52) | 0.04 (−0.45:0.61) |
| 5 | 0.3 (0.0:0.4) | 0.18 (−0.29:0.27) | 0.12 (−0.42:0.56) | 0.09 (−0.44:0.59) | 0.07 (−0.48:0.6) | 0.06 (−0.49:0.67) | 0.05 (−0.46:0.61) |
| 6 | 0.32 (−0.1:0.42) | 0.19 (−0.37:0.7) | 0.13 (−0.48:0.57) | 0.1 (−0.54:0.67) | 0.08 (−0.55:0.6) | 0.07 (−0.54:0.58) | 0.06 (−0.5:0.62) |
| 7 | 0.33 (−0.17:0.44) | 0.19 (−0.42:0.71) | 0.13 (−0.51:0.57) | 0.1 (−0.57:0.69) | 0.09 (−0.62:0.61) | 0.07 (−0.5:0.68) | 0.06 (−0.48:0.62) |
| 8 | 0.33 (−0.21:0.45) | 0.2 (−0.45:0.72) | 0.14 (−0.49:0.58) | 0.11 (−0.55:0.69) | 0.09 (−0.58:0.61) | 0.08 (−0.56:0.69) | 0.07 (−0.57:0.62) |
| 9 | 0.34 (−0.25:0.45) | 0.2 (−0.48:0.72) | 0.14 (−0.54:0.58) | 0.11 (−0.55:0.7) | 0.09 (−0.58:0.76) | 0.08 (−0.57:0.69) | 0.07 (−0.5:0.62) |
| 10 | 0.35 (−0.28:0.46) | 0.21 (−0.5:0.72) | 0.15 (−0.54:0.82) | 0.11 (−0.57:0.7) | 0.09 (−0.54:0.74) | 0.08 (−0.54:0.67) | 0.07 (−0.55:0.63) |

| VOTERS CANDIDATES | 21 | 31 | 41 | 51 | **1001** | 10001 |
|---|---|---|---|---|---|---|
| 3 | 0.02 (−0.4:0.4) | 0.01 (−0.4:0.43) | 0.01 (−0.4:0.4) | 0.01 (−0.4:0.4) | 0.01 (−0.4:0. 4) | 0.01 (−0.4:0.4) |
| 4 | 0.03 (−0.5:0.6) | 0.03 (−0.5:0.5) | 0.02 (−0.5:0.6) | 0.02 (−0.4:0.5) | 0.01 (−0. 4:0.4) | 0.01 (−0.4:0.5) |
| 5 | 0.04 (−0.6:0.6) | 0.03 (−0.5:0.6) | 0.03 (−0.6:0.6) | 0.03 (−0.5:0.5) | 0.02 (−04:0.4) | 0.02 (−0.5:0.5) |
| 6 | 0.05 (−0.6:0.5) | 0.04 (−0.5:0.6) | 0.03 (−0.5:0.6) | 0.03 (−0.5:0.5) | 0.02 (−0.4:0.5) | 0.02 (−0.4:0.5) |
| 7 | 0.05 (−0.5:0.7) | 0.04 (−0.6:0.6) | 0.04 (−0.6:0.5) | 0.03 (−0.5:0.6) | 0.4 (−0.4:05) | 0.02 (−0.4:0.5) |
| 8 | 0.05 (−0.5:0.7) | 0.04 (−0.6:0.5) | 0.04 (−0.6:0.6 | 0.03 (−0.5:0.5) | 0.02 (−04:0.5) | 0.02 (−0.5:0.4) |
| 9 | 0.06 (−0.7:0.6) | 0.04 (−0.6:0.6) | 0.04 (−0.6:0.6) | 0.03 (−0.6:.0.6) | 0.02 (−0.4:052) | 0.02 (−0.4:0.4) |
| 10 | 0.06 (−06:0.6) | 0.05 (−0.5:0.6) | 0.04 (−0.6:0.6) | 0.03 (−0.5:0.5) | 0.02 (−0.4:0.5) | 0.02 (−0.5:0.5) |

**Table 11** Percentage of pairwise comparisons in which PR is superior to MR under the Box model

| VOTERS<br>CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 1 | 0.86 | 0.81 | 0.80 | 0.75 | 0.69 | 0.63 | 0.638 | 0.61 | 0.59 | 0.56 | 0.54 |
| 4 | 1 | 1 | 0.86 | 0.79 | 0.80 | 0.73 | 0.69 | 0.65 | 0.63 | 0.61 | 0.59 | 0.55 | 0.55 |
| 5 | 1 | 1 | 0.89 | 0.80 | 0.82 | 0.75 | 0.70 | 0.67 | 0.64 | 0.61 | 0.60 | 0.54 | 0.53 |
| 6 | 1 | 1 | 0.90 | 0.83 | 0.85 | 0.77 | 0.73 | 0.691 | 0.65 | 0.63 | 0.61 | 0.54 | 0.53 |
| 7 | 1 | 1 | 0.92 | 0.85 | 0.87 | 0.80 | 0.76 | 0.70 | 0.68 | 0.64 | 0.62 | 0.54 | 0.53 |
| 8 | 1 | 1 | 0.93 | 0.87 | 0.89 | 0.82 | 0.78 | 0.73 | 0.69 | 0.66 | 0.63 | 0.53 | 0.53 |
| 9 | 1 | 1 | 0.94 | 0.88 | 0.90 | 0.84 | 0.80 | 0.75 | 0.71 | 0.67 | 0.64 | 0.53 | 0.52 |
| 10 | 1 | 1 | 0.95 | 0.90 | 0.91 | 0.86 | 0.82 | 0.77 | 0.73 | 0.69 | 0.66 | 0.53 | 0.52 |

**Table 12** The mean of the differences between the expected costs of PR and MR under the Box simulations, which indicates the advantage of PR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
|---|---|---|---|---|---|---|---|
| 3 | −0.5 (−0.5:0.5) | −0.35 (−0.67:0.17) | −0.26 (−0.75:0.1) | −0.22 (−0.8:0.21) | −0.18 (−0.83:0.29) | −0.16 (−0.86:0.37) | −0.15 (−0.88:0.42) |
| 4 | −0.5 (−0.5:0.5) | −0.41 (−0.67:0.17) | −0.32 (−0.75:0.1) | −0.28 (−0.8:0.21) | −0.24 (−0.83:0.29) | −0.21 (−0.86:0.37) | −0.19 (−0.88:0.42) |
| 5 | −0.5 (−0.5:0.5) | −0.45 (−0.67:0.17) | −0.38 (−0.75:0.1) | −0.32 (−0.8:0.21) | −0.28 (−0.83:0.29) | −0.25 (−0.86:0.37) | −0.22 (−0.88:0.42) |
| 6 | −0.5 (−0.5:0.5) | −0.47 (−0.67:0.17) | −0.41 (−0.75:0.1) | −0.36 (−0.8:0.21) | −0.33 (−0.83:0.29) | −0.29 (−0.86:0.37) | −0.26 (−0.88:0.42) |
| 7 | −0.5 (−0.5:0.5) | −0.48 (−0.67:0.17) | −0.44 (−0.75:0.1) | −0.4 (−0.8:0.21) | −0.36 (−0.83:0.29) | −0.32 (−0.86:0.37) | −0.29 (−0.88:0.42) |
| 8 | −0.5 (−0.5:0.5) | −0.5 (−0.67:0.17) | −0.46 (−0.75:0.1) | −0.42 (−0.8:0.21) | −0.38 (−0.83:0.29) | −0.35 (−0.86:0.37) | −0.32 (−0.88:0.42) |
| 9 | −0.5 (−0.5:0.5) | −0.51 (−0.67:0.17) | −0.47 (−0.75:0.1) | −0.44 (−0.8:0.21) | −0.4 (−0.83:0.29) | −0.37 (−0.86:0.37) | −0.35 (−0.88:0.42) |
| 10 | −0.5 (−0.5:0.5) | −0.51 (−0.67:0.17) | −0.49 (−0.75:0.1) | −0.46 (−0.8:0.21) | −0.42 (−0.83:0.29) | −0.39 (−0.86:0.37) | −0.37 (−0.88:0.42) |

| VOTERS CANDIDATES | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|
| 3 | −0.11 (−0.91:0.51) | −0.1 (−0.94:0.61) | −0.08 (−0.95:0.66) | −0.07 (−0.96:0.7) | −0.05 (−0.945:0.85) | −0.04 (−0.90:0.90) |
| 4 | −0.15 (−0.91:0.51) | −0.11 (−0.94:0.61) | −0.09 (−0.95:0.66) | −0.08 (−0.96:0.7) | −0.04 (−0.94556:0.93) | −0.04 (−0.93:0.91) |
| 5 | −0.17 (−0.91:0.51) | −0.13 (−0.94:0.61) | −0.1 (−0.95:0.66) | −0.09 (−0.96:0.7) | −0.03 (−0.97:0.93) | −0.03 (−0.97:0.95) |
| 6 | −0.2 (−0.91:0.51) | −0.15 (−0.94:0.61) | −0.12 (−0.95:0.66) | −0.1 (−0.96:0.7) | −0.03 (−0.96:0.93) | −0.03(−0.95:0.96) |
| 7 | −0.23 (−0.91:0.51) | −0.17 (−0.94:0.61) | −0.13 (−0.95:0.66) | −0.11 (−0.96:0.7) | −0.03 (−1.00:0.93) | −0.02 (−0.98:0.96) |
| 8 | −0.25 (−0.91–0.51) | −0.19 (−0.94–0.61) | −0.15 (−0.95–0.66) | −0.13 (−0.96–0.7) | −0.02 (−0.96785:0.936) | −0.02 (−0.98:0.96) |
| 9 | −0.28 (−0.91:0.51) | −0.21 (−0.94:0.61) | −0.17 (−0.95:0.66) | −0.14 (−0.96:0.7) | −0.02 (−0.99:0.93) | −0.02 (−0.97:0.96) |
| 10 | −0.3 (−0.91:0.51) | −0.22 (−0.94:0.61) | −0.18 (−0.95:0.66) | −0.15 (−0.96:0.7) | −0.02 (−0.97:0.93) | −0.02 (−0.98:0.95) |

## Appendix B

The percentage of preference profiles and pairwise comparisons resulting in erosion are presented in Tables 13, 14, 15, 16

**Table 13** Percentage of preference profiles resulting in erosion when comparing BR and MR

| VOTER-SCANDI-DATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | – | 0.04 | 0.07 | 0.09 | 0.12 | 0.13 | 0.15 | 0.17 | 0.19 | 0.21 | 0.21 | 0.25 | 0.26 |
| 4 | 0.16 | 0.28 | 0.34 | 0.38 | 0.40 | 0.42 | 0.43 | 0.45 | 0.47 | 0.49 | 0.50 | 0.54 | 0.55 |
| 5 | 0.41 | 0.55 | 0.60 | 0.64 | 0.66 | 0.67 | 0.68 | 0.71 | 0.72 | 0.73 | 0.74 | 0.76 | 0.77 |
| 6 | 0.64 | 0.76 | 0.80 | 0.82 | 0.83 | 0.85 | 0.85 | 0.86 | 0.87 | 0.89 | 0.88 | 0.90 | 0.90 |
| 7 | 0.82 | 0.89 | 0.91 | 0.92 | 0.93 | 0.94 | 0.94 | 0.95 | 0.95 | 0.96 | 0.96 | 0.97 | 0.97 |
| 8 | 0.91 | 0.96 | 0.97 | 0.97 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 |
| 9 | 0.96 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.99 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table 14** Percentage of erosion in pairwise comparisons by BR and MR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | – | 0.02 | 0.03 | 0.04 | 0.052 | 0.06 | 0.06 | 0.07 | 0.08 | 0.08 | 0.08 | 0.10 | 0.10 |
| 4 | 0.03 | 0.06 | 0.07 | 0.08 | 0.09 | 0.09 | 0.10 | 0.10 | 0.11 | 0.11 | 0.12 | 0.13 | 0.13 |
| 5 | 0.05 | 0.08 | 0.09 | 0.10 | 0.11 | 0.11 | 0.12 | 0.12 | 0.13 | 0.13 | 0.13 | 0.14 | 0.15 |
| 6 | 0.08 | 0.10 | 0.11 | 0.12 | 0.12 | 0.13 | 0.13 | 0.13 | 0.14 | 0.14 | 0.14 | 0.15 | 0.16 |
| 7 | 0.08 | 0.11 | 0.12 | 0.13 | 0.13 | 0.13 | 0.14 | 0.14 | 0.15 | 0.15 | 0.15 | 0.16 | 0.16 |
| 8 | 0.09 | 0.12 | 0.13 | 0.13 | 0.14 | 0.142 | 0.14 | 0.15 | 0.15 | 0.15 | 0.16 | 0.16 | 0.17 |
| 9 | 0.10 | 0.12 | 0.13 | 0.14 | 0.14 | 0.15 | 0.15 | 0.15 | 0.16 | 0.16 | 0.16 | 0.17 | 0.17 |
| 10 | 0.10 | 0.13 | 0.14 | 0.14 | 0.15 | 0.15 | 0.15 | 0.16 | 0.16 | 0.16 | 0.16 | 0.17 | 0.17 |

**Table 15** Percentage of preference profiles resulting in erosion when comparing PR and MR

| VOTERS CAN-DIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0.22 | 0.27 | 0.33 | 0.34 | 0.38 | 0.39 | 0.39 | 0.42 | 0.43 | 0.46 | 0.45 | 0.50 | 0.50 |
| 4 | 0.44 | 0.55 | 0.62 | 0.65 | 0.68 | 0.69 | 0.70 | 0.73 | 0.76 | 0.76 | 0.76 | 0.79 | 0.79 |
| 5 | 0.59 | 0.73 | 0.79 | 0.83 | 0.85 | 0.86 | 0.87 | 0.89 | 0.91 | 0.91 | 0.92 | 0.93 | 0.94 |
| 6 | 0.69 | 0.83 | 0.89 | 0.92 | 0.93 | 0.94 | 0.95 | 0.96 | 0.97 | 0.97 | 0.97 | 0.98 | 0.99 |
| 7 | 0.76 | 0.90 | 0.93 | 0.95 | 0.97 | 0.98 | 0.98 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 0.805 | 0.93 | 0.96 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 9 | 0.84 | 0.95 | 0.98 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.87 | 0.97 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table 16** Percentage of erosion in pairwise comparisons by PR and MR

| VOTERS CANDIDATES | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 21 | 31 | 41 | 51 | 1001 | 10001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0.05 | 0.08 | 0.10 | 0.11 | 0.12 | 0.13 | 0.13 | 0.14 | 0.15 | 0.16 | 0.16 | 0.19 | 0.19 |
| 4 | 0.09 | 0.13 | 0.15 | 0.16 | 0.16 | 0.17 | 0.18 | 0.19 | 0.20 | 0.205 | 0.21 | 0.24 | 0.25 |
| 5 | 0.11 | 0.15 | 0.17 | 0.18 | 0.19 | 0.20 | 0.202 | 0.22 | 0.23 | 0.23 | 0.24 | 0.27 | 0.28 |
| 6 | 0.11 | 0.16 | 0.18 | 0.19 | 0.20 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.29 | 0.30 |
| 7 | 0.11 | 0.16 | 0.18 | 0.20 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.31 | 0.32 |
| 8 | 0.10 | 0.16 | 0.18 | 0.20 | 0.22 | 0.22 | 0.23 | 0.25 | 0.26 | 0.27 | 0.27 | 0.32 | 0.33 |
| 9 | 0.10 | 0.15 | 0.18 | 0.20 | 0.22 | 0.23 | 0.24 | 0.25 | 0.27 | 0.28 | 0.29 | 0.33 | 0.34 |
| 10 | 0.10 | 0.15 | 0.18 | 0.20 | 0.22 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.34 | 0.35 |

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