



A note on functional form specification in random coefficients stochastic frontier models

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Accepted: 21 August 2023 / Published online: 16 September 2023
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Abstract

This study presents a random coefficients stochastic frontier model that can accommodate the flexible translog functional form without being computationally demanding and thus time consuming to estimate. This is achieved by restricting the second-order frontier parameters to be common to all firms. For comparison, random coefficients stochastic frontier models with Cobb–Douglas, semi-translog and translog specifications with all parameters being firm-specific are estimated. The models are applied to an unbalanced panel of German dairy farms, and Bayesian techniques are used for the estimation. The results suggest that the time needed for the sampler to complete in the proposed model reduces dramatically as opposed to a translog model where all parameters are firm-specific. The elasticities exhibit some differences, depending on the choice of functional form, whilst the efficiency scores are less affected. Bayes factors suggest that the proposed model fits the data best.

Keywords Random coefficients · Stochastic frontier · Functional form · Translog

JEL Classification C11 · C23 · D22 · Q12

1 Introduction

Efficiency measurement using the classical stochastic frontier model requires, among others, the choice of a functional form. In this model, this decision does not constitute a dilemma as the translog is a standard choice thanks to its flexibility. However, choosing a functional form is a more cumbersome task when technology heterogeneity is assumed and a random coefficients stochastic frontier model (Kalirajan and Obwona 1994) is used. This is because this model allows for firm-specific parameters in order to form individual frontiers, and the standard translog specification requires the estimation of a large number of parameters, which is computationally demanding and thus very time consuming. This is due to the precision (inverse variance) matrix of the distribution of the random parameters, whose elements are a quadratic function of the number of independent variables. Hence, related studies have relied on

more parsimonious functional forms such as the Cobb–Douglas or the semi-translog (Tsionas (2002); Emvalomatis (2012); Skevas (2023)).

The present study argues that it is not necessary to use parsimonious functional forms when the random coefficients stochastic frontier model is employed. The reasoning is the following: A typical practice when a translog specification is employed is to normalize the right-hand-side variables by their means so that the first-order terms are interpreted as elasticities. This diminishes the importance of the second-order terms. Furthermore, the second-order terms in a translog specification tend to be insignificant. Therefore, this study specifies a translog functional form with firm-specific first-order terms, whilst restricting the second-order terms to be the same across firms. This approach still results in firm-specific frontiers whilst using the typical translog specification and not having to estimate a large number of parameters, as the elements of the precision matrix of the distribution of the random parameters is a quadratic function of (only) the number of first-order terms. This approach serves as a remedy to the arbitrary choices of parsimonious functional forms made by related

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studies when estimating a random coefficients stochastic frontier model.

2 Modeling approach & estimation

Let i denote firms and t denote time. A random coefficients stochastic frontier model that restricts some parameters to be common to all firms is:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_i + \mathbf{z}'_{it}\boldsymbol{\gamma} + v_{it} - u_{it} \tag{1}$$

where y_{it} is the value of the dependent variable, \mathbf{x}_{it} is a $K \times 1$ vector that stores the values of K independent variables associated with firm-specific coefficients, $\boldsymbol{\beta}_i$ is a $K \times 1$ vector of parameters, \mathbf{z}_{it} is an $L \times 1$ vector that stores the values of L independent variables associated with coefficients common to all firms, $\boldsymbol{\gamma}$ is an $L \times 1$ vector of parameters, v_{it} is a two-sided error term, and u_{it} is a one-sided non-negative error term that captures inefficiency.

The one-sided non-negative inefficiency component u_{it} is assumed to follow a Half-Normal distribution, whilst the two-sided error term v_{it} and the random parameters $\boldsymbol{\beta}_i$ are assumed to follow a Normal distribution:

$$v_{it} \sim \mathcal{N}(0, 1/\tau), u_{it} \sim \mathcal{N}^+(0, 1/\phi), \boldsymbol{\beta}_i \sim \mathcal{N}(\bar{\boldsymbol{\beta}}, \boldsymbol{\Omega}^{-1}) \tag{2}$$

where τ and ϕ are precision parameters to be estimated, $\bar{\boldsymbol{\beta}}$ is a $K \times 1$ vector of parameters that represents the mean of the $\boldsymbol{\beta}_i$ s, and $\boldsymbol{\Omega}$ is a $K \times K$ precision matrix for the distribution of the $\boldsymbol{\beta}_i$ s. Regarding the functional form, the proposal of this study is to use the typical translog, with the first-order terms along with a constant term to be included in the \mathbf{x}_{it} vector, and all the remaining second-order terms to be included in the \mathbf{z}_{it} vector.

The model is estimated using Bayesian techniques. The data likelihood, denoted as $p(\{y_{it}\}, \{\boldsymbol{\beta}_i\}, \{u_{it}\} | \{\mathbf{x}_{it}\}, \{\mathbf{z}_{it}\}, \bar{\boldsymbol{\beta}}, \boldsymbol{\gamma}, \boldsymbol{\Omega}, \phi, \tau)$, is the product of two Normal distributions for the error term v_{it} and for the random parameters $\boldsymbol{\beta}_i$, and one Half-Normal distribution for the inefficiency component u_{it} . The priors, denoted as $p(\bar{\boldsymbol{\beta}}), p(\boldsymbol{\gamma}), p(\boldsymbol{\Omega}), p(\phi)$ and $p(\tau)$, consist of Multivariate Normal distributions for $\bar{\boldsymbol{\beta}}$ and $\boldsymbol{\gamma}$ with 0 prior means and identity prior precision matrices multiplied by 0.001, a Wishart distribution for the precision matrix $\boldsymbol{\Omega}$ with the degrees of freedom parameter being equal to 45 and the prior scale matrix being an identity matrix, and Gamma distributions for τ and ϕ with the shape and rate parameters being equal to 0.001 for τ , and 7 and 0.5 for ϕ , respectively. The model's posterior, denoted as $\pi(\bar{\boldsymbol{\beta}}, \boldsymbol{\gamma}, \boldsymbol{\Omega}, \phi, \tau, \{\boldsymbol{\beta}_i\}, \{u_{it}\} | \{y_{it}\}, \{\mathbf{x}_{it}\}, \{\mathbf{z}_{it}\})$, is proportional to the product of the data likelihood and the parameters' prior distributions.

This study also estimates the typical random coefficients stochastic frontier model that restricts $\mathbf{z}'_{it}\boldsymbol{\gamma} = 0$ in Eq. (1).

This model is represented as:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_i + v_{it} - u_{it} \tag{3}$$

In the empirical application, this model is estimated using a Cobb–Douglas, a semi-translog, and a fully translog specification. The parameterization of priors is the same in all models. Regarding $\boldsymbol{\Omega}$, the imposed Wishart distribution integrates to unity only if the degrees of freedom parameter is greater or equal to the number of independent variables associated with firm-specific coefficients. Hence, in the fully translog specification of Eq. (3) this parameter is set equal to the total number of independent variables from the application that follows (i.e., 45). However, setting the degrees of freedom parameter equal to the total number of independent variables in the remaining models would result in a lower value, as they contain fewer firm-specific coefficients, which would in turn result in less diffuse priors since the variance of the elements of $\boldsymbol{\Omega}$ is multiplied by the degrees of freedom parameter. Therefore, in all models the degrees of freedom parameter is set equal to 45 so that prior diffusion is similar across all models. Bayes factors are used to compare the models, with the marginal density of the data being obtained using the Chib and Jeliazkov (2001) approximation.

3 Dataset & specification

The models are applied to an unbalanced panel of German dairy farms that specialize in milk production and are observed between 1999 and 2009. The dataset consists of 1691 farms and 13,384 observations. Two outputs are distinguished: cow's milk and milk products (y_1), and other products (y_2). Six inputs are specified: capital (K), labor (L), land (A), intermediate inputs (I), animals (S), and purchased feed (F). Table 1 presents summary statistics of the specified variables.

An output distance function is used. Using the linear homogeneity property, choosing y_1 as the normalizing output, taking logs, and making rearrangements yields the

Table 1 Summary statistics of specified variables

Variable (Units)	Mean	Standard Deviation
y_1 (1000€)	144.470	213.840
y_2 (1000€)	26.200	30.440
K (1000€)	195.830	249.130
L (1000 hours)	3.970	6.000
A (hectares)	77.410	132.290
I (1000€)	60.250	98.550
S (livestock units)	108.170	130.410
F (1000€)	27.630	55.760

following estimable form of the translog distance function:

$$\begin{aligned}
 -\log y_{1,it} = & \beta_{0,i} + \sum_{m=1}^m \beta_{m,i} \log x_{m,it} + \beta_{p,i} \log \left(\frac{y_{2,it}}{y_{1,it}} \right) + \beta_{t,i} t \\
 & + \frac{1}{2} \sum_{m=1}^M \sum_r^R \gamma_{mr} \log x_{m,it} \log x_{r,it} + \frac{1}{2} \gamma_p \log \left(\frac{y_{2,it}}{y_{1,it}} \right) \\
 & + \frac{1}{2} \gamma_{mp} \sum_{m=1}^M \log x_{m,it} \log \left(\frac{y_{2,it}}{y_{1,it}} \right) + \gamma_t t^2 + \sum_{m=1}^M \gamma_{mt} \log x_{m,it} t \\
 & + \gamma_{pt} \log \left(\frac{y_{2,it}}{y_{1,it}} \right) t + v_{it} + u_{it}
 \end{aligned} \tag{4}$$

Only the first-order terms and the constant term have firm-specific coefficients, whilst the parameters of all second-order terms are constant across firms. This model is called “Translog 2”. Setting all γ parameters equal to zero yields the “Cobb–Douglas” model, whilst keeping all γ parameters and assigning them an “i” subscript gives the “Translog 1” model. Finally, setting γ_{mr} , γ_p , γ_{mp} , and γ_t equal to zero, and assigning γ_{mt} and γ_{pt} an “i” subscript yields the “Semi-Translog” model. Before estimation, the data are normalized by their geometric means, so that the first-order terms are interpreted as distance elasticities.

4 Results

The results are based on 60,000 draws from the posterior. Table 2 presents the time elapsed for the completion of the markov chain monte carlo (MCMC) sampler in each model. The MCMC sampler in the “Translog 1” model took

Table 2 Time elapsed for the completion of the MCMC sampler in each model

Model	Time elapsed (hours)
Cobb–Douglas	3.000
Semi-Translog	5.353
Translog 1	46.051
Translog 2	3.000

46 hours to complete. However, in the “Translog 2” model that this study proposes the MCMC sampler took only 3 hours to complete. This result highlights that, with the same flexible translog functional form, specifying fewer random parameters reduces the time needed for the sampler to complete dramatically, thus freeing the researcher from having to deviate from the typical translog functional form.

Table 3 presents the parameter estimates of only the first-order terms from all models. The full set of results is presented in Table A1 in the Appendix along with monte carlo standard errors. The distance elasticity of output suggests that a 1% increase in other output leads to a 0.084, 0.090, 0.118, and 0.115% increase in the distance function in the “Cobb–Douglas”, “Semi-Translog”, “Translog 1”, and “Translog 2” models, respectively. Obviously, the choice of the functional form matters, as the output distance elasticity is deflated when parsimonious functional forms are used. Small differences in the distance elasticities of inputs are also observed.

Figure 1 presents boxplots of the efficiency scores for all four estimated models. In all models average efficiency is around 91–92%. Focusing on the two translog specifications, the reported similarities in the estimated efficiency

Table 3 Parameter estimates of first-order terms from all models

Variable	Cobb–Douglas	Semi-Translog	Translog 1	Translog 2
constant	-0.112	-0.100	-0.104	-0.115
$\log(y_2/y_1)$	0.084	0.090	0.118	0.115
$\log K$	-0.010	-0.012	-0.014	-0.008
$\log L$	-0.055	-0.058	-0.060	-0.055
$\log A$	-0.119	-0.123	-0.108	-0.117
$\log I$	-0.104	-0.091	-0.092	-0.109
$\log S$	-0.499	-0.524	-0.527	-0.483
$\log F$	-0.178	-0.171	-0.200	-0.196
t	-0.023	-0.028	-0.028	-0.021
τ	1572.500	2575.750	4638.070	1016.060
ϕ	48.398	72.810	100.716	55.091

Bold values are associated with credible intervals that do not contain zero

Fig. 1 Boxplots of efficiency scores across the four models

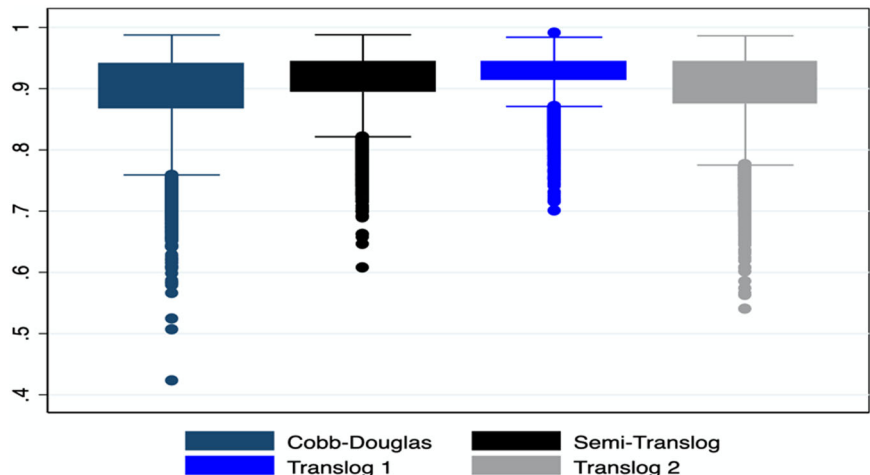


Table 4 Marginal log-likelihoods, prior and posterior model probabilities

Model	Marginal Log-Likelihood	Prior Probability	Posterior Probability
Cobb–Douglas	7434.020	0.250	0
Semi-Translog	5890.400	0.250	0
Translog 1	6674.790	0.250	0
Translog 2	8148.800	0.250	1

scores can be perceived as a good sign, because they highlight that the move to a more parsimonious random coefficients translog specification does not change the efficiency scores. Finally, Table 4 presents the marginal log-likelihoods, the prior and the posterior model probabilities of the four models. The model proposed in this study (e.g., “Translog 2”) is clearly favored by the data against the remaining models.

5 Conclusions

Although the typical practice in random coefficients stochastic frontier models is to specify parsimonious functional forms so that estimation becomes less computationally intensive and thus less time consuming, the present study argues that researchers should not abandon the use of the typical translog functional form. This can be possible by specifying a translog random coefficients stochastic frontier model with fewer firm-specific parameters. Such a model, with firm-specific first-order terms and common to all firms second-order terms is presented and estimated in this study. For comparison, Cobb–Douglas, semi-translog and translog random coefficients stochastic frontier models with all parameters being firm-specific are also estimated.

The results show that the proposed translog model is significantly less time consuming than the translog model in which all parameters are firm-specific. The reported elasticities exhibit some differences, whilst the efficiency scores are similar across the four models. Finally, formal model comparisons suggest that the proposed translog model outperforms all the remaining models. The fact that the efficiency scores do not differ between the translog models is a desirable outcome, as it shows that the choice of a more parsimonious translog model does not affect the efficiency estimates. Furthermore, the proposed method can be more useful in applications where multiple inputs are used to produce multiple outputs, as is the case in dairy farming. However, in applications where a single output is produced

using few inputs, the translog model with all parameters being firm-specific can be preferable. Finally, for future productivity growth studies, specifying the proposed translog distance function implies that the geometric mean of Malmquist output indices results in a Tornqvist index (see Coelli et al. 2005). Exchanging the time dimension with the firm dimension can form the basis for comparing productivity growth between firms.

Data availability

The data used in this study are not publicly available due to a confidentiality agreement signed between the author and the data provider (EU-FADN - DG AGR1).

Supplementary information The online version contains supplementary material available at <https://doi.org/10.1007/s1123-023-00700-4>.

Funding Open access funding provided by HEAL-Link Greece.

Compliance with ethical standards

Conflict of interest The author declares no competing interests.

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