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Comment: Performance measurement and joint production of intended and unintended outputs

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1 Introduction

The paper "Performance Measurement and Joint Production of Intended and Unintended outputs" by Finn Førsund crystallises and extends core ideas of several papers by the same author (Førsund 2009, 2017) on how to model production technologies in the presence of undesirable outputs governed by the materials balance principle. In the current and earlier work, he argues that the popular single-equation framework is an unsuitable way of modelling pollution. The single-equation framework approach has been dominated by the weak disposability and null-jointness assumptions presented in Shephard (1970) and Shephard and Färe (1974), respectively. This framework has long been the standard way of modelling pollution-generating technologies, especially under the influence of the popularisation of Färe, Grosskopf, and co-authors (see for instance Färe et al. 1986, 1989; Chung et al. 1997). Drawing on Frisch (1965), most notably the degree of assortment, Førsund argues that the single-equation framework is inadequate for modelling joint production of intended and unintended outputs. Førsund proposes a factorially determined production technology that separately considers the production of good, intended outputs, on the one hand, and the generation of bad, unintended outputs, on the other. Such a separate consideration has also been proposed by Murty et al. (2012), Murty and Russell (2016, 2020).

The main argument of Førsund made here and in earlier work resonates with us: the multiple-equation framework is adequate for modelling joint production of intended and unintended outputs. Our comment focuses on making a connection with the past and providing possible avenues for future research beyond flow pollutants. Regarding the connection with the past, we link the multiple-equation framework to the material-balance (MB) framework proposed by Coelli et al. (2007). Coelli et al. (2007) formally prove that the single-equation framework can violate the MB condition and propose a framework that complies with the MB condition. Although the multi-equation framework and MB framework have developed separately in the literature, we demonstrate that they can be connected by the duality principle. This link has so far been overlooked.

We discuss two possible avenues for future research. First, we propose to consider the dynamic properties of stock externalities. While discussions have focused on instantaneous reduction of pollution flows, there are a variety of stock externalities that have thus far been largely untouched. Second, we suggest to also incorporate non-material by-products in a production framework.

The remainder of our comment is structured as follows. Section 2 discusses the MB approach of Coelli et al. (2007) and its link with the multiple-equation framework. Section 3 discusses the dynamic aspect related to stocks of undesirable outputs and focuses on by-products other than pollution.

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2 The materials balance principle as an "accounting identity"

Coelli et al. (2007) have provided a novel way of treating bad outputs in a production framework. Their model relies on the so-called "accounting identity" to estimate environmental efficiency using similar concept as iso-cost lines. Let us consider the following mass balance equation

$$z = a'x_M - b'y \tag{1}$$

Førsund disregards Eq. (1) as a simple accounting identity that does not show how bad output b is generated. From his perspective, this equation should not be considered for modelling pollution-generating technologies. While Eq. (1) *seemingly* does not occur as production technology, it is instructive to consider the dual relationship between a value function (such as profit, cost and revenue functions) and the production technology (Chambers 1988). Let us consider the cost, which is defined as:

$$c = w'x \tag{2}$$

Just like Eqs. (1), (2) is an accounting identity, yet its minimisation is dual to the input requirement set. There are two ways to model a production technology: (1) a primal approach that relies on a description of the physical process in terms of quantities and (2) a dual approach that relies on economic behaviour in terms of prices and quantities. The two approaches have been evolving concomitantly to each other. From this perspective, Coelli et al. (2007) have actually designed a *dual* mechanism to modelling pollution-generating technologies. Following the notation of Førsund, they propose the pollution minimisation problem subject to the production technology:

$$\begin{aligned} &\min_{x_M} z \equiv a'x_M \\ &s.t. f(x_S, x_M, y) = 0 \end{aligned} \tag{3}$$

Solving Eq. (3) results in iso-environmental lines analogous to iso-cost lines. Therefore, there is in our view nothing fundamentally wrong with model (3). We furthermore have:

$$x_M \equiv x(a, x_S, y) \tag{4}$$

Given the usual monotonicity assumptions, we have $\frac{dx_M}{dx_S} \leq 0$ and $\frac{dx_M}{dy} \geq 0$. We also have

$$z \equiv a'x(a, x_S, y), \tag{5}$$

which implies $\frac{dz}{dx_S} \leq 0$ and $\frac{dz}{dy} \geq 0$. These trade-offs are the same as the ones presented in the multi-equation model of Førsund (see Eq. (9) herein). The pollution minimisation problem (3) is thus an equivalent representation of the polluting technology.

Further, let us consider the multi-equation model of Førsund:

$$\begin{aligned} y &= f(x_S, x_M) \\ z &= g(x_M) \end{aligned} \tag{6}$$

Using the implicit function theorem, we can write:

$$x_M = h(x_S, y) \tag{7}$$

and $z = g(h(x_S, y)) = j(x_S, y)$

The dual representation of the bad output technology (last equation in (7)) is the same as the mass balance equation in (5), where pollution-generating inputs are valued by their respective emission coefficients. We thus argue that the materials balance equation reflects the duality between the MB-based approach of Coelli et al. (2007) and the bad output technology of Førsund and Murty et al. (2012). This is analogous to the well-known dual relationship between the cost function and the input requirement set.

Should we choose an MB framework or a multiple-equation framework? Information on quantities of inputs and outputs and their respective emission coefficients is needed in the MB framework. The multiple-equation framework only requires quantities of inputs and outputs, but relies on the estimation of distance functions, which can become complicated. Finally, the MB framework allows computing minimum pollution levels for each decision maker, which contrasts with the multiple-equation framework, where pollution levels should only be reduced. The suitability of each approach depends on the specific application. Pollution minimisation is not in line with economic behaviour, but might be necessary.

3 Beyond modelling flow pollutants

The MB- and multiple-equation-based frameworks discussed above focus on flow pollutants. While pertinent in many applications, flow pollutants are only a specific case of a negative externality. In what follows, we extend our outlook to stock externalities and non-material by-products.

3.1 Stock externalities

The long-run economic objective of the firm is arguably to maximise wealth. Wealth is a stock, while profit, cost and revenue are flows. Profit maximisation, cost minimisation and revenue maximisation are often used as behavioural economic assumptions in production models, for they increase the stock of wealth in the short run. The common application by empirical analysts reveals its usefulness. Yet, it is important to emphasise that such behavioural assumptions are static simplifications abstracted from wealth maximisation. We may think of instances where maximising addition to wealth in the short run conflicts with the long-run objective of wealth maximisation. This holds when not all inputs and outputs can be changed to long-run optimal levels. Most notably, investments in quasi-fixed capital inputs are necessary for long-run wealth maximisation, but deter the firm to economically optimise in the short run. Just like wealth, quasi-fixed capital inputs are a stock rather than a flow. Considering the long run instead of the short run requires a dynamic framework instead of a static one. Notable approaches rely on adjustment-cost theory (Silva et al. 2020) and network data envelopment analysis (Färe et al. 2018). We refer to Fallah-Fini et al. (2014) for a comprehensive overview of dynamic approaches.

In Section 2, we have shown that the minimisation of pollution under the MB condition can be seen as the dual of the bad output technology, where pollution-generating inputs are valued by their respective emission coefficients. This is analogous to the dual relationship between the cost function and the input requirement set. Pollution and costs are in both cases treated as flows. Yet, the accumulation of flows often plays an important role in the production process. In this case, we should consider the stock of materials, analogous to modelling stocks of quasi-fixed capital inputs. Stocks can have positive or negative impacts for the decision maker. We give an example of each from the agricultural sector. A positive stock is for example the soil organic matter, which contributes to high yields in the long run. A negative stock is for instance pesticide accumulation, which depresses yields in the long run. The multiple-equation framework is a good starting point to model such stocks.

3.2 Non-material by-products

Thus far, we have solely concentrated on material by-products. However, non-material by-products often play an essential role in the production process, in which impacts can be positive or negative (Dakpo and Ang 2019). Recently, there have been several attempts to model non-material by-products having a positive impact in a production framework. Crop diversity is an important ecosystem service in that it contributes to biodiversity in agricultural production. Sipiläinen and Huhtala (2013) model crop diversity as a conventional output in a single-equation framework. Arguing that such an approach imposes untenable assumptions on the production technology, Ang et al. (2018) compute the dynamic profit function (see Ang and Oude Lansink 2018) (1) when land use cannot be reallocated and (2) when land use can be reallocated. Comparison of the two scenarios allows assessing the opportunity cost of crop diversification without explicitly modelling crop diversity in the production technology. This resembles the MB framework that does not explicitly model pollution in the production technology. Schulte et al. (2018) model animal welfare as a conventional output in a single-equation framework. We note that such an approach implicitly assumes that there always is a non-positive relationship between animal welfare and conventional production. Finally, the multiple-equation framework has recently been applied to various settings to account for social outputs. Applications include the agricultural sector (Chambers and Serra 2018; Sidhoum 2018) and the food and beverages manufacturing sector (Puggioni and Stefanou 2019; Engida et al. 2020).

Non-material by-products having a negative impact have received somewhat less attention. One example is non-performing loans in the banking sector. Earlier attempts have employed a single-equation framework (Park and Weber 2006), which is later extended to a multiple-equation framework (Salim et al. 2017).

In conclusion, although non-material by-products are not governed by physical laws, we can still draw lessons from the above discussion on the MB. Indeed, we observe a shift from single-equation frameworks to multi-equation frameworks and frameworks that exclude the considered by-product from the production technology along the lines of the MB-based framework. Such ongoing developments permit more realistic modelling of by-products, benefiting managerial decision-making.

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References

Ang F, Mortimer SR, Areal F, Tiffin R (2018) On the opportunity cost of crop diversification. J Agric Econ 69:794–814

Ang F, Oude Lansink A (2018) Decomposing dynamic profit inefficiency of Belgian dairy farms. Eur Rev Agric Econ 45:81–99

Chambers RG (1988) Applied production analysis: a dual approach. Cambridge University Press, New-York and Melbourne (Australia)

Chambers RG, Serra T (2018) The social dimension of firm performance: a data envelopment approach. Empir Econ 54:189–206

Chung YH, Fare R, Grosskopf S (1997) Productivity and undesirable outputs: a directional distance function approach. J Environ Manag 51:229–240

Coelli T, Lauwers L, Van Huylenbroeck G (2007) Environmental efficiency measurement and the materials balance condition. J Prod Anal 28:3–12

Dakpo KH, Ang F (2019) Modelling environmental adjustments of production technologies: A Literature Review In: ten Raa T., Greene W. (eds) The Palgrave Handbook of Economic Performance Analysis, Palgrave Macmillan, Cham. https://doi.org/10.1007/978-3-030-23727-1_16

Engida TG, Rao X, Lansink AGO (2020) A dynamic by-production framework for analyzing inefficiency associated with corporate social responsibility. Eur J Oper Res 287:1170–1179

Fallah-Fini S, Triantis K, Johnson AL (2014) Reviewing the literature on non-parametric dynamic efficiency measurement: state-of-the-art. J Prod Anal 41:51–67

Färe R, Grosskopf S, Lovell CAK, Pasurka C (1989) Multilateral productivity comparisons when some outputs are undesirable - a nonparametric approach. Rev Econ Stat 71:90–98

Färe R, Grosskopf S, Margaritis D, Weber WL (2018) Dynamic efficiency and productivity, The Oxford handbook of productivity analysis. Oxford University Press, New-York, United States

Färe R, Grosskopf S, Pasurka C (1986) Effects on relative efficiency in electric-power generation due to environmental controls. Resour Energy 8:167–184

Førsund FR (2009) Good modelling of bad outputs: pollution and multiple-output production. Int Rev Environ Resour Econ 3:1–38

Førsund FR (2017) Multi-equation modelling of desirable and undesirable outputs satisfying the materials balance. Empir Econ 54:67–99

Frisch R (1965) Theory of production. Dordrecht Reidel, Publishing Company. https://link.springer.com/article/10.1007%2Fs11123-021-00599-9#Bib1

Murty S, Russell RR (2016) Modeling emission-generating technologies: reconciliation of axiomatic and by-production approaches. Empir Econ 54:7–30

Murty S, Russell RR (2020) Bad Outputs. In: Ray SC, Chambers R, Kumbhakar S eds. Handbook of production economics. Springer Singapore, Singapore

Murty S, Russell RR, Levkoff SB (2012) On modeling pollution-generating technologies. J Environ Econ Manag 64:117–135

Park KH, Weber WL (2006) A note on efficiency and productivity growth in the Korean Banking Industry, 1992–2002. J Bank Finance 30:2371–2386

Puggioni D, Stefanou SE (2019) The value of being socially responsible: a primal-dual approach. Eur J Oper Res 276:1090–1103

Salim R, Arjomandi A, Dakpo KH (2017) Banks' efficiency and credit risk analysis using by-production approach: the case of Iranian banks. Appl Econ 49:2974–2988

Schulte HD, Armbrecht L, Bürger R, Gauly M, Musshoff O, Hüttel S (2018) Let the cows graze: an empirical investigation on the trade-off between efficiency and farm animal welfare in milk production. Land Use Policy 79:375–385

Shephard RW (1970) Theory of cost and production functions. Princeton University Press, Princeton

Shephard RW, Färe R (1974) The law of diminishing returns. Zeitschrift für Nationalökonomie 34:69–90

Sidhoum AA (2018) Valuing social sustainability in agriculture: an approach based on social outputs' shadow prices. J Cleaner Prod 203:273–286

Silva E, Stefanou SE, Lansink AO (2020) Dynamic efficiency and productivity measurement. Oxford University Press, New-York, United States

Sipiläinen T, Huhtala A (2013) Opportunity costs of providing crop diversity in organic and conventional farming: would targeted environmental policies make economic sense? Eur Rev Agric Econ 40:441–462