

Erratum to: Complement of forms

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The original publication of this paper contained an error in Lemma 2(g). The statement (which is used later in Theorem 6, Lemma 12, and Theorem 11) is false in general.

We will show that $\mathbf{D}_{\mathfrak{w}_k} \mathfrak{t} \uparrow \mathbf{D}_{\mathfrak{w}} \mathfrak{t}$ whenever $\mathfrak{w}_k \uparrow \mathfrak{w}$ and \mathfrak{w} is dominated by \mathfrak{t} . Since this condition is satisfied in Theorem 6, Lemma 12, and Theorem 11, the results remain true (with the same proof).

The first part of the following theorem is the reformulation of (Proposition 5.1 in [1]) for nonnegative sesquilinear forms, the second part is an easy consequence. This is the correct version of Lemma 2(g).

Theorem 1 *Let \mathfrak{t} , \mathfrak{w} , and \mathfrak{w}_k ($k \in \mathbb{N}$) be nonnegative sesquilinear forms on the complex linear space \mathfrak{D} . Assume further that $\mathfrak{w}_k \uparrow \mathfrak{w}$, and $\mathfrak{w} \leq \alpha \mathfrak{t}$ for some $\alpha > 0$. Then*

$$\sup_{k \in \mathbb{N}} (\mathfrak{w}_k : \mathfrak{t}) = \left(\sup_{k \in \mathbb{N}} \mathfrak{w}_k \right) : \mathfrak{t} = \mathfrak{w} : \mathfrak{t} \quad \text{and} \quad \sup_{k \in \mathbb{N}} \mathbf{D}_{\mathfrak{w}_k} \mathfrak{t} = \mathbf{D}_{\sup_{k \in \mathbb{N}} \mathfrak{w}_k} \mathfrak{t} = \mathbf{D}_{\mathfrak{w}} \mathfrak{t}.$$

Proof Since $\mathfrak{w}_k : \mathfrak{t} \leq \mathfrak{w} : \mathfrak{t}$ for all $k \in \mathbb{N}$, the supremum (denoted by \mathfrak{s})

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$$s := \sup_{k \in \mathbb{N}} (\mathfrak{w}_k : t)$$

exists. We will show that $s = \mathfrak{w} : t$. Since

$$s = \sup_{k \in \mathbb{N}} (\mathfrak{w}_k : t) \leq \mathfrak{w} : t \leq (\alpha t) : t = \frac{\alpha}{\alpha + 1} t$$

one can apply (Theorem 3.4 in [3]). I.e., the parallel difference $s \div t$ exists and

$$t : (s \div t) = s = \sup_{k \in \mathbb{N}} (\mathfrak{w}_k : t) \geq \mathfrak{w}_k : t \quad (\forall k \in \mathbb{N}).$$

Hence, according to (Proposition 2.7 in [2]) one obtain that

$$D_t(s \div t) \geq D_t \mathfrak{w}_k \quad (\forall k \in \mathbb{N}).$$

Since \mathfrak{w}_k is almost dominated by t , it follows also that

$$s \div t \geq D_t(s \div t) \geq D_t \mathfrak{w}_k = \mathfrak{w}_k \quad (\forall k \in \mathbb{N}).$$

Taking the supremum in k , $s \div t \geq \mathfrak{w}$ and

$$s = (s \div t) : t \geq \mathfrak{w} : t \geq \sup_{k \in \mathbb{N}} (\mathfrak{w}_k : t) = s.$$

Consequently,

$$\sup_{k \in \mathbb{N}} (\mathfrak{w}_k : t) = \left(\sup_{n \in \mathbb{N}} \mathfrak{w}_k \right) : t = \mathfrak{w} : t.$$

It remains only to show that $\sup_{k \in \mathbb{N}} D_{\mathfrak{w}_k} t = D_{\mathfrak{w}} t$. Observe first that the sequence $(n\mathfrak{w}_k)_{k \in \mathbb{N}}$ is also monotonically nondecreasing, $\sup_{k \in \mathbb{N}} n\mathfrak{w}_k = n\mathfrak{w}$, and $n\mathfrak{w} \leq n\alpha t$. Consequently, we can take the supremum in k :

$$\sup_{k \in \mathbb{N}} D_{\mathfrak{w}_k} t = \sup_{k \in \mathbb{N}} \left(\sup_{n \in \mathbb{N}} t : n\mathfrak{w}_k \right) \geq t : n\mathfrak{w}_k \Rightarrow \sup_{k \in \mathbb{N}} D_{\mathfrak{w}_k} t \geq t : n\mathfrak{w}.$$

This gives immediately that $\sup_{k \in \mathbb{N}} D_{\mathfrak{w}_k} t \geq D_{\mathfrak{w}} t$. Since the inequality $\mathfrak{w}_k \leq \mathfrak{w}$ implies

$$D_{\mathfrak{w}_k} t \leq D_{\mathfrak{w}} t$$

for all $k \in \mathbb{N}$, the proof is complete. □

Corollary 1 *Let t and \mathfrak{w} be nonnegative sesquilinear forms on the complex linear space \mathfrak{D} . Then*

$$D_{\mathfrak{w}} t = D_{D_{\mathfrak{w}} t} t,$$

i.e., $D_{\mathfrak{w}} t$ is a t -quasi-unit for all \mathfrak{w} .

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