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A new challenge for contingentists

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Abstract
Contingentism is the view that it is contingent which things exist. Despite its plausibility, advocates of contingentism face a well-known 'challenge' to demonstrate that they can draw what appear to be intelligible modal distinctions (Williamson *Modal Logic as Metaphysics*. Oxford University Press, Oxford, 2013). In this article, I argue that if certain controversial modal principles fail, the challenge contingentists face becomes much more difficult. Whereas extant challenges concern contingentists' inability to draw quite theoretical second-order modal distinctions, I present a challenge which concerns contingentists' inability to draw simpler first-order distinctions. This indicates that in certain modal settings there may well be significant first-order barriers to maintaining contingentism.

Keywords Modality · Necessitism · Contingentism · Second-order logic

Necessitism is the thesis that necessarily everything is necessarily something, where the necessity is understood as metaphysical and the quantifiers as unrestricted. Contingentism is the negation of this thesis, which is equivalent to the claim that possibly something possibly fails to be. Over the last two decades, Timothy Williamson has offered a sustained defence of necessitism. As one of the central pillars of his defence, Williamson (2010, 2013) presents a 'challenge' to contingentists. The challenge is centred on the charge that contingentists can extract no 'kernel of truth' or 'cash-value' from various necessitist claims which appear to draw perfectly intelligible modal distinctions. In contrast, necessitists face no such problem.

In this article, I argue that if certain controversial modal principles fail, the challenge contingentists face becomes much more difficult. In particular, there are simple first-order necessitist claims from which the contingentist is unable to extract

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any cash-value whatsoever. In this respect the challenge becomes much more pressing for contingentists, for Williamson's own challenge concerns only their inability to extract a kernel of truth from quite theoretical second-order claims. Indeed, if contingentists are obliged to meet such challenges, there may well be significant first-order obstacles to maintaining contingentism.

1 Williamson's challenge

To state Williamson's challenge various items of setup are required, including various pieces of terminology. Let *first-order necessitism* be the thesis that necessarily every individual is necessarily something. Using a plural reading of the second-order quantifiers and variables, Williamson (2010) identifies *second-order necessitism* as the thesis that necessarily any things are such that necessarily there are some things which are them. A more natural way to formulate second-order necessitism would be in terms of irreducible second-order quantification (Williamson 2013, Chaps. 5 & 6). However Williamson uses this plural reading because it affords a particularly simple statement of the challenge; it is also motivated by a tradition of interpreting second-order quantification in plural terms (Boolos 1985).[1] Finally we take *necessitism* to be the conjunction of first-order and second-order necessitism.

There are opposing contingentist positions at both the first and second orders. Let *first-order contingentism* be the thesis that possibly some individual possibly fails to be. Similarly, using the plural reading once again, let *second-order contingentism* be the thesis that it is possible that some things are possibly not any things. As before, we take *contingentism* to be the conjunction of first-order and second-order contingentism. Characterised as such, necessitists and contingentists speak the same language. They both employ unrestricted first-order and plural quantification along with the idiom of metaphysical necessity and possibility. Nonetheless they disagree over which sentences of this shared language express true claims.

This second-order modal language can be formalised in the usual way, with lower case letters x, y, z serving as first-order variables, upper case letters X, Y, Z serving as n-adic (for $n \geq 1$) second-order variables, and the propositional operator $\Box$ expressing metaphysical necessity on its intended interpretation (consult Sect. 2 and also Williamson (2010, Appendix) for further details). Williamson's challenge also assumes that the necessitist and contingentist are willing to use certain devices of modal anaphora, $\uparrow$ and $\downarrow$, which were originally formalised in the study of tense logic (Vlach 1973). Effectively, these devices function as generalised actuality operators. In terms of a possible worlds model theory, they allow the

[1] In terms of this plural reading, Williamson understands the second-order variables to range over the 'empty' plurality, those things of which nothing is one. For technical convenience, in the model theory introduce in Sect. 2 second-order variables are required to range over only 'non-empty' pluralities. Williamson is aware that the plural reading does not make immediate sense for n-adic second-order quantification when $n > 1$. He suggests interpreting such quantification either as plural quantification over ordered n-tuples (Rayo and Yablo 2001) or via a familiar combination of plural and mereological resources (Burgess et al. 1991). Williamson also assumes that plural membership is 'constant' in a sense which guarantees that something is (not) one of some things just in case it is necessary that, provided that thing exists and those things exist, it is (not) one of those things; see Sect. 2 below for further discussion.

language to simulate reference to worlds which one has previously ‘considered’. In particular, the $\uparrow$ operator allows one to ‘store’ the current world of evaluation, and at a later point the $\downarrow$ operator allows one to ‘retrieve’ that world and place it as the world of evaluation. To understand how the operators work, consider the following formula.

$$\Diamond\uparrow\Box\forall x(Fx \rightarrow \downarrow Gx)$$

Put in terms of the metaphor of possible worlds, this may be understood as the claim that there is some possible world at which every F in any possible world is a G. Without recourse to talk of possible worlds, we might read the sentence as saying: possibly it was once necessary that every F was then G. In other words, one may think of $\downarrow$ as an actuality operator for the world which has been ‘stored’ by $\uparrow$, and so often I shall speak interchangeably of ‘actuality operators’ and ‘$\downarrow$ operators’ when the context allows. Importantly, these devices of anaphora can be eliminated with the use of plural quantification or other higher-order resources, so worries about their legitimacy can arguably be put aside.[2]

Williamson also assumes that the necessitist and contingentist will agree on the principles of a natural background logic, which he characterises via a class of variable domain models for the shared formal language (see Williamson 2010, Appendix). The choice of variable domains has the effect of making necessitism true in some models and contingentism true in others, which constitutes a mild form of neutrality between necessitism and contingentism. In the models, variable assignments assign first-order variables individuals from a non-empty ‘outer domain’ of which all world domains are subsets. As is typical in variable domain quantified modal logics, this permits variables to be assigned individuals which do not belong to the current world of evaluation. Unlike certain variable domain model theories, however, in this model theory world domains can be empty. Moreover individual variables can be assigned items which do not belong to the domain of any world of evaluation. Hence even extremely weak claims such as $\exists x(x = x)$ or $\Diamond\exists y(x = y)$ will not be valid, which displays just how weak the background logic is. The model theory also requires that the extension of an n-adic non-logical predicate at a world includes only n-tuples taken from that world’s domain, which validates the so-called ‘being constraint’. This is the principle that instantiating a relation requires existence:

Being Constraint $\Box\forall x_1 \ldots \Box\forall x_n\Box(Fx_1 \ldots x_n \rightarrow \exists y\, x_1 = y \wedge \ldots \wedge \exists y\, x_n = y)$

The being constraint is endorsed by some but not all contingentists, but Williamson’s challenge concerns those who accept it.[3] Moreover since all instances of the

[2] A version of this result was first proven by Forbes (1989) in the setting of S5. However Roberts (forthcoming) develops an analogue of Forbes’s technique in the context of a higher-order modal logic where the underlying modal system is merely S4. See Sects. 2 and 4 for closely connected discussion.

[3] Williamson (2013, Chap. 4) argues that contingentists ought to accept the being constraint, although see Fritz and Goodman (2017) and Dorr (2016). Interestingly, Fritz (2013, Sect. 3.3) shows that results analogous to those of Williamson (2010) with respect to generalized quantifiers do not depend on the being constraint. Fritz (2018) also shows that the similar results of Fritz and Goodman (2017) do not depend on the being constraint either. Nevertheless, here I do not investigate whether the results in Sect. 2 require the being constraint.

being constraint are theorems of standard necessitist logics, the constraint will be accepted by both parties. Finally, Williamson assumes that the logic for metaphysical necessity is S5, and the model theory guarantees this by not using accessibility relations.

In addition to these logical resources, Williamson assumes that necessitists and contingentists will also theorise in terms of the predicate 'is chunky', which may be taken to be synonymous with the predicate 'is grounded in the concrete'.[4] According to standard contingentist positions, ontology is exhausted by what is chunky: everything is grounded in the concrete. In contrast, according to standard necessitist positions there are things such as merely possible people which are possibly chunky but not in fact so. Yet, as one would expect, the necessitist and contingentist agree on all matters chunky. The realm of the chunky thus constitutes a neutral domain between the two theories; any difference between either position can be traced back to a dispute over the non-chunky. To make this precise, one can articulate a syntax-based criterion for being a *neutral* formula of the language. To be exact, the set of neutral formulas is defined recursively as the closure of the image of the following function under the relation of logical equivalence from the background logic (in what follows, $X \leq C$ is an abbreviation of $\forall x_1 \ldots \forall x_n(Xx_1 \ldots x_n \rightarrow Cx_1 \wedge \ldots \wedge Cx_n)$, for n-adic X where $n \geq 1$):[5]

$$[Fv_1 \ldots v_n] = Fv_1 \ldots v_n \wedge Cv_1 \wedge \ldots \wedge Cv_n, \text{ for } n\text{-adic non-logical atomic } F$$
$$[Vv_1 \ldots v_n] = Vv_1 \ldots v_n \wedge V \leq C, \text{ for } n\text{-adic variable } V$$
$$[v_1 = v_2] = v_1 = v_2 \wedge Cv_1 \wedge Cv_2$$
$$[\neg A] = \neg[A]$$
$$[A \wedge B] = [A] \wedge [B]$$
$$[\Diamond A] = \Diamond[A]$$
$$[\uparrow A] = \uparrow[A]$$
$$[\downarrow A] = \downarrow[A]$$
$$[\exists x A] = \exists x(Cx \wedge [A])$$
$$[\exists X A] = \exists X(X \leq C \wedge [A])$$

[4] Williamson does not appear to be using 'grounding' here in the sense of either Fine (2001) or Schaffer (2009). According to Williamson (2010, p. 673), in the operative sense of 'grounding', sets are grounded in the concrete whenever their members are all grounded in the concrete, thus ensuring that any pure set is chunky. Williamson (2010, p. 673) also claims that 'numbers may count as grounded in the concrete, perhaps through one or more stages of logicist abstraction'.

[5] Williamson (2010, p. 699, n. 34) chooses a plural reading of the second-order quantifiers partly because he claims that, on the irreducible reading of them, it is not clear what the criterion for a neutral formula ought to be. However Goodman (2016, 2.5) considers whether Williamson's results can be extended to a language with higher-order quantifiers which are read irreducibly. Goodman also offers several suggestions for how one might characterise neutrality in this setting.

According to this definition, the formula $\exists x Fx \wedge Gy$ is not neutral. But if its quantifier and 'F' are forced to occur within chunky contexts, the result is the neutral formula $\exists x(Cx \wedge Fx) \wedge (Gy \wedge Cy)$, on whose truth value both parties will agree. Generalising, the claims expressed by these neutral formulas are not in dispute between the necessitist and contingentist, at least in virtue of their respective commitments to necessitism or contingentism.

Williamson exploits this neutral domain of agreement to develop his challenge. The idea is that the necessitist is to provide a recursive intra-linguistic mapping of formulas to neutral formulas which are equivalent by the contingentist's lights. The contingentist is also to provide a recursive intra-linguistic mapping of formulas to neutral formulas which are equivalent by the necessitist's lights. If both mappings succeed, each theorist can draw the same distinctions and hypotheses as one another, for each theory can be seen as comprising a body of claims which, by its advocates' own lights, is equivalent to a body of neutral claims. Yet if one side is able to map the other side's claims to a neutral body but not vice-versa, the former enjoys an advantage over the latter: the latter may not be able to draw genuine modal distinctions which the other can. Indeed Williamson argues that the contingentist has this shortcoming whereas the necessitist does not: there are genuine modal distinctions which the necessitist but not the contingentist can draw. This should be a source of embarrassment for the contingentist, and is indeed a sign that their theory is impoverished, for the distinctions which they cannot draw are intimately related to distinctions which they do regard as genuine (Williamson 2013, p. 364).[6]

Importantly, Williamson (2013, pp. 309–312; 2016c, pp. 645–646) emphasises that, as he uses the phrase, which 'distinctions one can draw' does not depend only on the literal meaning of one's words. For since the necessitist and contingentist speak the same language, it would then be trivial that they can draw exactly the same distinctions. Rather, which 'distinctions one can draw' depends on one's *theory*, in addition to the language one is using. So Williamson's argument purports to show that every distinction that can be drawn in certain contingentist theories has a working neutral equivalent. In contrast, he argues that there are modal distinctions—indeed intelligible, genuine modal distinctions—that can be drawn in certain necessitist theories which have no working neutral equivalent, and thus cannot be drawn by contingentists.

[6] Goodman (2016, Sect. 2.3) questions Williamson's way of setting up the challenge on several grounds, such as the fact that the ability to identify neutral equivalents of an opponent's position is not in general an ambition which advocates of tenable, and indeed plausible, theories ought to have. (Relatedly, Fine (2016) suggests that the demand to find a neutral *equivalent* of every sentence is needlessly exacting.) However Goodman (2016, pp. 18–19) grants that there is a more forceful challenge to contingentists in the vicinity. This challenge is developed in Fritz and Goodman (2017), where it is argued that, under certain conditions, contingentists are unable to offer a systematic paraphrase of superficially necessitist ways of speaking that appear to communicate seemingly intelligible claims about modal reality. This argument is substantiated by a collection of technical results about the undefinability in various formal languages of certain classes of variable domain models. Williamson (2016c, Sect. 2) responds to Goodman's criticisms, including the aforementioned charge of non-generality; Williamson also suggests that the differences between the two modes of argument are not to be overstated, despite being non-trivial. Whether this latter point of Williamson's is correct or not, a key point is that Fritz & Goodman's (2017) way of setting up the challenge involves the use of model-theoretic techniques similar to those used in what follows. This observation strongly suggests that my extension of Williamson's challenge can be adapted to their setting.

As Williamson (2010, pp. 711–712) puts it, "necessitists can draw distinctions whose genuineness contingentists can neither plausibly deny nor explain on their own terms".

Strictly speaking, Williamson's challenge does not demonstrate that this disparity arises between *any* contingentist and necessitist theory, but only that it arises between natural versions of each view. Williamson focuses on a natural version of contingentism which is axiomatised in the background logic by the assertion that necessarily everything is chunky.

$$\textbf{AuxCon} \quad \Box\forall xCx$$

As Williamson (2013, p. 678) recognises, a peculiarity of this auxiliary contingentist theory is that it does not even include the assertion of contingentism, which has the effect of making AuxCon consistent with necessitism. Nevertheless this auxiliary claim ought to be appealing to contingentists: for them, to be is to be chunky, and of course necessarily everything exists. Moreover it is difficult to imagine why any necessitist would endorse this idea: even if we necessarily exist, we are not necessarily chunky. Alongside AuxCon, Williamson focuses on a natural version of necessitism which is axiomatised in the background logic by the assertions of first-order necessitism, the claim that everything is possibly chunky, and the claim that standing in a relation requires being chunky. In addition, Williamson (2010, p. 688) stipulates that there are only finitely many non-logical predicates in the language in order to axiomatise the theory as a single, finite conjunction. I follow this stipulation of his (where F is an n-adic non-logical atomic predicate for $n \geq 1$, and Rel is the set of non-logical atomic predicates):

$$\textbf{AuxNec} \quad \Box\forall x\Box\exists yx = y \wedge \forall x\Diamond Cx \wedge \bigwedge_{F\in Rel} \Box\forall z_1 \ldots \forall z_n(Fz_1 \ldots z_n \rightarrow (Cz_1 \wedge \ldots \wedge Cz_n))$$

Certain necessitist visions will license the claim that everything is possibly chunky. As Williamson (2013, p. 236) puts it, one can regard such visions as corresponding to a picture of necessitist ontology as the 'minimal rounding out' of the AuxCon-friendly contingentist ontology to make first-order necessitism true.[7] As Williamson is aware, however, it is much less plausible that the necessitist will assert that it is necessary that if some non-logical atomic predicate applies to certain individuals, those individuals are chunky, since that precludes mere possibilia from belonging to the extension of non-logical atomic predicates. Nevertheless Williamson (2013, p. 687) is explicit that this assumption is used only to simplify matters. Indeed, as an idealisation, one might imagine that the necessitist and contingentist are using a restricted language with a choice of non-logical atomic predicates that only apply to that which is chunky. That the challenge for contingentists arises even in that context

[7] Importantly, Williamson (2010, Sect. 7) recognises that, if a necessitist posits impure sets of individually possibly chunky individuals which are not possibly chunky together (compare the examples towards the end of this section), they will reject that everything is possibly chunky. However Williamson (2013, p. 340) stresses the importance of the fact that his challenge arises even if one grants that the necessitist does not posit such sets.

is damning enough. It is also worth noting that AuxNec implies the failure of contingentism, whereas AuxCon is consistent with the assertion of necessitism.

The first step of Williamson's argument is to provide a recursive mapping $[.]^{\mathrm{CON}}$ of formulas to neutral formulas, where AuxCon entails $A \leftrightarrow [A]^{\mathrm{CON}}$ for all formulas A. This mapping provides a neutral analogue of each claim which by AuxCon's lights is equivalent to the initial contingentist claim. Given that $[A]^{\mathrm{CON}}$ must be neutral, each claim must be mapped to a formula which is logically equivalent with a formula that conforms to the syntax-based criterion specified above. More plainly, each claim must be mapped to a claim which is restricted to the chunky. But this suggests a natural way for the necessitist to map the target body of discourse to a neutral body of discourse in a manner which is acceptable to the auxiliary contingentist. For from the necessitist's perspective, the contingentist's ontology is properly included in what there is, and contingentists can be viewed as restricting their attention solely to the realm of the chunky. Indeed, given the plural reading of second-order quantification, this applies to contingentist second-order quantification too. Such observations suggest that the necessitist may just use the previous mapping [.] for their desired mapping of contingentist discourse. To make this explicit, I shall use $[.]^{\mathrm{CON}}$ and [.] to denote one and the same function:

$$[A]^{\mathrm{CON}} = [A], \quad \text{for all formulas } A$$

The choice of mapping generates some welcome results. Consider the claim that some mountain could have failed to exist:

(1) $\exists x(Mx \wedge \Diamond \forall y \neg y = x)$

The necessitist can use $[.]^{\mathrm{CON}}$ to map the this claim to a neutral equivalent:[8]

$[(1)]^{\mathrm{CON}}$ $\exists x(Cx \wedge Mx \wedge \Diamond \forall y(Cy \rightarrow \neg y = x))$

$[(1)]^{\mathrm{CON}}$ states that some chunky mountain is possibly not chunky, a claim which the contingentist will recognise as equivalent to (1). Similarly, suppose the contingentist claims that possibly there are some one or more things no one of which actually exists.

(2) $\uparrow \Diamond \exists X \exists x(Xx \wedge \downarrow \forall y \neg Xy)$

The necessitist can again use $[.]^{\mathrm{CON}}$ to map this claim to the neutral claim that possibly there are some one or more chunky things which actually are not chunky:

$[(2)]^{\mathrm{CON}}$ $\uparrow \Diamond \exists X(X \leq C \wedge \exists x(Cx \wedge Xx \wedge \downarrow \forall x(Cx \rightarrow \neg Xx)))$

These examples indicate that the mapping is on the right track, but the interest is in the more general question of whether every formula A is mapped to a neutral formula $[A]^{\mathrm{CON}}$ which by the auxiliary contingentist's lights is equivalent to A. This is indeed established by Williamson (2010):

[8] In what follows, the values of the mapping will often be represented by formulas which are equivalent to them in the background logic from the perspective of the relevant auxiliary theory.

Theorem 1 *In the background logic, AuxCon entails* $A \leftrightarrow [A]^{\mathrm{CON}}$ *for all formulas* A.

This theorem vindicates the claim that necessitists can draw all the modal distinctions which auxiliary contingentists can.

The salient question now becomes whether the contingentist is in a similar position. In other words, is there a recursive mapping $[.]^{\mathrm{NEC}}$ of formulas to neutral formulas, where AuxNec entails $A \leftrightarrow [A]^{\mathrm{NEC}}$ for all formulas A? One natural candidate is a mapping inspired by efforts to 'translate' what is known as 'possibilist' discourse to 'actualist' discourse (Fine 1977). The basic idea is to simulate necessitist quantification over possible *F*s by modalised neutral quantification over *F*s. The thought is that when necessitists claim that there is some possible *F* which is thus and so, contingentists can map this claim to the neutral claim that possibly there is something chunky which is *F* and thus and so. One crude implementation of this idea can be demonstrated with the following example:

> [there is some possible son of Wittgenstein]$^{\mathrm{NEC}}$ = possibly there is something chunky which is a son of Wittgenstein

This crude strategy does not generalise, however, since it maps necessitist claims to neutral claims which they will not view as equivalent. For example, consider the following result based on the same strategy:

> [there is some possible son of Wittgenstein which is not a son of Wittgenstein]$^{\mathrm{NEC}}$ =
> possibly there is something chunky which is a son of Wittgenstein and is not a son of Wittgenstein

Clearly the initial claim and its neutral putative analogue will not be equivalent according to AuxNec, since the latter implies a contradiction whereas the former does not. Nevertheless a more sophisticated mapping based on essentially the same idea fares better. Put in terms of the possible worlds semantics, the problem with the crude strategy is that the 'possibly' operator shifts the perspective to a world at which one then must evaluate the part of the original claim within the scope of the quantifier. But often this will deliver the wrong result, as in the example above. The required tweak is that one first needs to shift attention to a different possible world at which the quantifier 'collects' a chunky individual, after which attention is shifted *back* to the initial world where the remainder of the necessitist's claim is assessed by assigning the 'collected' individual to the variable. The devices of modal anaphora are key to this strategy and are used in both clauses for the quantifiers in the mapping:

$$[Fv_1 \ldots v_n]^{\mathrm{NEC}} = Fv_1 \ldots v_n \wedge Cv_1 \wedge \ldots \wedge Cv_n, n\text{-adic non-logical atomic } F$$
$$[v_1 \ldots v_n]^{\mathrm{NEC}} = \Diamond(Vv_1 \ldots v_n \wedge V{\leq}C), \text{ for } n\text{-adic variable } V$$
$$[v_1 = v_2]^{\mathrm{NEC}} = \Diamond(v_1 = v_2 \wedge Cv_1 \wedge Cv_2)$$
$$[\neg A]^{\mathrm{NEC}} = \neg[A]^{\mathrm{NEC}}$$
$$[A \wedge B]^{\mathrm{NEC}} = [A]^{\mathrm{NEC}} \wedge [B]^{\mathrm{NEC}}$$
$$[\Diamond A]^{\mathrm{NEC}} = \Diamond[A]^{\mathrm{NEC}}$$
$$[\uparrow A]^{\mathrm{NEC}} = \uparrow[A]^{\mathrm{NEC}}$$
$$[\downarrow A]^{\mathrm{NEC}} = \downarrow[A]^{\mathrm{NEC}}$$
$$[\exists xA]^{\mathrm{NEC}} = \uparrow\Diamond\exists x(Cx \wedge \downarrow[A]^{\mathrm{NEC}})$$
$$[\exists XA]^{\mathrm{NEC}} = \uparrow\Diamond\exists X(X{\leq}C \wedge \downarrow[A]^{\mathrm{NEC}})$$

The crucial point is that quantified claims are mapped to indexed modalised quantified statements. These statements return the perspective to the initial world in order to evaluate the part of the original claim within the scope of the quantifier. To take an example, by this more sophisticated mapping we have the following correct result:

> [there is some possible son of Wittgenstein which is not son of Wittgenstein]$^{\mathrm{NEC}}$=
> $\uparrow$ possibly there is something chunky $\downarrow$ which is a possible son of Wittgenstein but not a son of Wittgenstein

Put in terms of the possible worlds semantics, this indexed modalised quantifier 'stores' a given world w with the initial $\uparrow$, then moves one to another world u via $\Diamond$, at which one picks up some chunky a in the domain of u and assigns it to a given variable, after which one returns to the stored world w via $\downarrow$ to evaluate $[A]^{\mathrm{NEC}}$, with a assigned to the given variable. The result is a neutral analogue of the initial claim which the auxiliary necessitist will view as equivalent to it.

Given that the proposed recursive mapping enjoys some initial success, one might anticipate a more general result. Indeed Williamson (2010) proves the following proposition (hereafter, an FO formula is one which contains no occurrence of any second-order quantifier or second-order predicate variable):

Theorem 2 *In the background logic, AuxNec entails* $A \leftrightarrow [A]^{\mathrm{NEC}}$ *for all FO formulas* A.

Whilst this may seem like a cause for optimism, unfortunately for contingentists the result does not fully generalise; the problem concerns the contingentist's proposed mapping of formulas involving second-order quantifiers and second-order predicate variables. To appreciate the issue in question, it helps first to consider the mapping's treatment of second-order quantified claims of the form $\exists XA$, which are mapped by $[.]^{\mathrm{NEC}}$ to claims of the form $\uparrow\Diamond\exists X(X{\leq}C \wedge \downarrow[A]^{\mathrm{NEC}})$. Less formally, $[.]^{\mathrm{NEC}}$ maps second-order quantified claims of the form 'there are some things such that A' to neutral claims of the form '$\uparrow$ possibly there are some chunky things such that $\downarrow[A]^{\text{NEC}}$'. However if such claims are to be deemed equivalent by the auxiliary necessitist, they must maintain that arbitrary pluralities of things are such that it is possible for all of their members to be chunky together. Yet there would appear to be a plethora of counterexamples to that claim. One standard counterexample is used by Williamson:[9]

> "A human *h* could grow from a sperm *s* and an egg *e*. A human *h** could grow from the sperm *s* and an egg *e** distinct from *e*. But there could not be both a human *h* and a human *h**. For given the essentiality to humans of their origins (conditional on their being chunky), there could be a human *h* only by growing from *s* and *e*, and there could be a human *h** only by growing from *s* and *e**. Given the nature of the entities, *h* cannot grow from from *s* and *e* while *h** grows from *s* and *e**. Thus there could not be both a human *h* and a human *h**. For *h* and *h**, to be chunky is to be human. Therefore, it is impossible for *h* and *h** to be chunky together. Although the chunkiness of *h* and the chunkiness of *h** are separately possible, they are not compossible."
>
> Williamson (2013, p. 337)

In light of such counterexamples, it is natural to doubt that $[.]^{\text{NEC}}$ maps each claim to a neutral claim which the auxiliary necessitist will regard as equivalent to it. One can confirm this doubt by working through some examples of the mapping. For example, consider the claim that for any two individually possible humans there are some things which are exactly them:

(3) $\forall x \forall y(\Diamond Hx \wedge \Diamond Hy \rightarrow \exists X \forall z(Xz \leftrightarrow z = x \vee z = y))$

Roughly speaking, (3) is mapped by $[.]^{\text{NEC}}$ to claim that for any two possible individuals which are individually possible humans, possibly there are some chunky things which comprise exactly those possible individuals. The Vlach operators are redundant in $[(3)]^{\text{NEC}}$ so it may represented by the following equivalent formula below:

$$[(3)]^{\text{NEC}} \quad \Box\forall x(Cx \rightarrow \Box\forall y(Cy \rightarrow (\Diamond(Hx \wedge Cx) \wedge \Diamond(Hy \wedge Cy) \rightarrow \Diamond\exists X(X \leq C \wedge \Box\forall z(Cz \rightarrow (\Diamond(Xz \wedge X \leq C) \leftrightarrow \Diamond(z = x \wedge Cz \wedge Cx) \vee \Diamond(z = y \wedge Cz \wedge Cy)))))))$$

However the auxiliary necessitist will not regard (3) as equivalent to $[(3)]^{\text{NEC}}$, for it is not the case that, for example, possible human *h* and possible human *h** (from the example above) are possibly chunky together. Thus it will not be possible that there are some chunky things which comprise exactly *h* and *h**. Indeed the specifics of the driving examples do not matter a great deal: it is merely required that there be some things which are individually possibly chunky but not possibly chunky together. In other words, unless the auxiliary necessitist theory implies that arbitrary individuals are possibly chunky together, the contingentist's attempted mapping will fail.

Once one has appreciated the manner of the counterexample, one may notice that $[.]^{\text{NEC}}$ also maps atomic predications involving second-order variables to claims which the auxiliary necessitist will not regard as equivalent to them. To take an

[9] Williamson cites (Salmon 1987, pp. 47–48) for this example; but he suggests that there are other, equally plausible examples of the same phenomenon.

example, suppose the necessitist endorses the truth of the formula $Xx_1 \ldots x_n$ (for $n > 1$) under an assignment that assigns individuals which are not possibly chunky together to at least two of the variables $x_1 \ldots x_n$ respectively. By $[.]^{NEC}$ the formula $Xx_1 \ldots x_n$ is then mapped to the neutral formula $\Diamond(Xx_1 \ldots x_n \wedge X{\leq}C)$. However the formula $\Diamond(Xx_1 \ldots x_n \wedge X{\leq}C)$ will be false under the same assignment, since at least two of the variables $x_1 \ldots x_n$ are assigned individuals which cannot be chunky together. Thus the mapping fails even in its basic treatment of certain atomic formulas.[10]

Moreover the problem for the contingentist gets even worse. As Williamson (2010, pp. 705–708; pp. 738–774) shows, there are formulas of the object-language which are not equivalent in the background logic to *any* neutral formula even in the presence of AuxNec. To appreciate this, first note that given the assumption that there cannot be more than two chunky individuals, AuxNec implies that any neutral formula is equivalent to a first-order one. Intuitively this is because one can then simply replace second-order quantifiers which are restricted to the chunky with pairs of first-order quantifiers that are restricted to the chunky. However there are second-order formulas which are not equivalent to any first-order formula even given AuxNec and the assumption that there cannot be more than two chunky individuals. One neat example is a second-order formula which states that there are some things to which some but not all Fs belong that are closed under the possibly-R relation.[11]

(4) $\exists X(\exists x(Fx \wedge Xx) \wedge \exists x(Fx \wedge \neg Xx) \wedge \forall x \forall y(\Diamond Rxy \rightarrow (Xx \rightarrow Xy)))$

This formula can be shown not to be equivalent to any first-order formula, even given AuxNec and the assumption in question, by adapting a technique first introduced by David Kaplan for establishing 'nonfirstorderizability'.[12] Thus (4) lacks a property which every neutral formula possesses, so it cannot be equivalent to a neutral formula (since the neutral formulas are closed under logical equivalence in the background logic). More generally, the problem is not local to the proposed mapping $[.]^{NEC}$.

Theorem 3 *In the background logic, there is a formula A for which there is no neutral formula B such that AuxNec entails A ↔ B.*

[10] Williamson (2010, p. 705) highlights this fact but acknowledges that it is not as philosophically significant as the previously mentioned failure due to open formulas not being the vehicles of speech acts.

[11] A example with this form is given by Williamson (2013, p. 348), who imagines the necessitist dividing pairs of chunky individuals into two lists, one of which includes exactly those chunky individuals which bear the ancestral of possible interbreeding to one another.

[12] See Boolos (1984, pp. 432–433) for an explanation of Kaplan's technique. The technique is used to prove that the Geach-Kaplan sentence 'some critics admire only one another' has no first-order equivalent by showing that it has an arithmetical 'interpretation' which is true in all non-standard models of (first-order) arithmetic but false in all standard models. (All these models are assumed to be 'standard' in the sense that the second-order quantifiers are interpreted as ranging over all subsets of the domain of individuals however.) Williamson adapts this technique by 'interpreting' (4) as a sentence of second-order Peano arithmetic which is true in all non-standard models of (first-order) arithmetic but false in all standard models (all such models validate Williamson's arithmetical 'interpretations' of AuxNec and the assumption that there cannot be more than two chunky individuals).

In this regard, there are deep obstacles facing any contingentist attempt to draw all of the seemingly intelligble modal distinctions which necessitists can.

2 Weakening the background logic

Necessitism and contingentism are theses about whether things must or might not exist. Although both theses are typically discussed within the context of other modal assumptions, it is important to separate the theses from various assumptions. One salient example is the assumption that the logic of metaphysical necessity is S5, which captures the idea that necessity is a non-contingent status of propositions. Yet whereas it is highly plausible that S4 is a lower bound on the logic of metaphysical necessity, to motivate S5 one requires an argument for the so-called B axiom: the principle that whatever is the case is necessarily possible.[13] Nevertheless the status of the B axiom is unclear. Whilst it is widely assumed in practice, the arguments for it are far from decisive. It is naturally motivated on concrete modal realist reductions of modality, on which metaphysical necessity in some sense reduces to patterns of non-modal facts across a pluriverse of spacetimes. Outside such settings, however, the principle is much more contentious.[14] Indeed Dorr et al. (2021, pp. 111) go as far to remark that 'while the B axiom is orthodox, its status as orthodoxy does not seem to have been earned by anyone's having given any good arguments for it'. Such sentiment, along with an emerging interest in B-free theories of metaphysical necessity, makes it salient that the dispute between necessitism and contingentism should be studied in settings weaker than S5.[15]

In the context of merely S4, however, other modal principles become much more contentious. In particular, one loses Prior (1956) famous argument for the necessity of distinctness (which I abbreviate as 'ND') which makes use of the B axiom.[16]

Necessity of Distinctness $\forall x \forall y(x \neq y \rightarrow \Box(\exists z\, z = x \wedge \exists z\, z = y \rightarrow x \neq y))$

[13] The claim that S4 is a lower bound on the logic of metaphysical necessity can be derived from a regimentation of the idea that metaphysical necessity is either a broadest species of necessity, or a broadest species of 'objective' necessity. See Williamson (2016a), Dorr (2016), and Bacon (2018).

[14] It is worth noting that making sense of 'world'-talk and recovering part of its connection with metaphysical necessity requires only S4 and certain auxiliary assumptions, but not the B axiom itself; see Dorr et al. (2021, Chap. 1 ,Sect. 6).

[15] On the metaphysics of modality without the B axiom, see Dorr (2016, pp. 68–70), Bacon (2018), Ditter (2020), Roberts (forthcoming), amongst others. Some of these authors focus on B-free theories of the broadest species of necessity without identifying it with metaphysical necessity, but the necessitism-contingentism debate with respect to the broadest necessity is at least as interesting as it is with respect to metaphysical necessity.

[16] The necessity of identity, i.e. the formula $\forall x \forall y(x = y \rightarrow \Box(\exists z\, z = x \wedge \exists z\, z = y \rightarrow x = y))$, is a theorem of natural contingentist and necessitist modal logics in which necessity obeys only S4; it is also validated in the neutral model theory used below. Moreover, using some elementary modal logic which is also validated by the model theory used below, one can derive from this formula the following formula: $\forall x \forall y(\Box\Diamond(\exists z\, z = x \wedge \exists z\, z = y \wedge x \neq y) \rightarrow \Box x \neq y)$. Yet the B axiom guarantees that whatever is the case is necessarily possible, so with it one can also derive the formula $\forall x \forall y((\exists z\, z = x \wedge \exists z\, z = y \wedge x \neq y) \rightarrow \Box x \neq y)$. From this, more elementary modal logic validated by the model theory below allows one to derive the necessity of distinctness.

Indeed the emerging interest in B-free theories of metaphysical necessity covers pictures on which the necessity of distinctness not only fails, but also does so in quite radical ways.[17] Here I am less concerned with debating the B axiom and the necessity of distinctness than studying the dispute between necessitists and contingentists without either assumption in the background. I shall argue that on such pictures the problem for contingentists becomes particularly acute. For in the context of S5 contingentists are unable to map quite theoretical second-order claims to neutral claims which necessitists will view as equivalent to them. But in an ND-free setting contingentists are also unable to map quite simple first-order claims to neutral claims which necessitists will view as equivalent to them—claims which capture distinctions that contingentists should want to draw. More generally, if the necessity of distinctness fails, contingentists face a particularly difficult challenge even with respect to first-order discourse.

To develop this challenge, care is required when choosing an appropriate background logic. One point that requires particular care concerns the fact that Williamson (2010) uses a semantics for the second-order variables and Vlach operators which validates certain principles about the quantifiers and Vlach operators from which the necessity of distinctness is derivable, even without the B axiom. In the case of the former, the principle states that second-order quantification is *constant* in the sense that: (i) if something is (not) one of some things, then it is necessary that, provided that thing and those things exist, it is (not) one of those things, and (ii) plural membership is modally non-increasing. In the case of 1-adic second-order variables, this may be formalised as follows:[18]

Constancy $\Box\forall X(\forall x(Xx \rightarrow \Box(Ex \wedge EX \rightarrow Xx)) \wedge \forall x(\neg Xx \rightarrow \Box(Ex \wedge EX \rightarrow \neg Xx)) \wedge \forall Y(\forall x(Xx \rightarrow \Box(Ex \wedge EY \rightarrow Yx)) \rightarrow \Box\forall x(Xx \rightarrow (EY \rightarrow Yx))))$

This constancy principle is motivated by the plural reading of the second-order quantifiers and second-order variables (in the context of S5). In the case of the Vlach operators, Williamson's model theory validates the principle that they do not *distort*

[17] See Bacon (2020) and Goodman (MS). Dorr et al. (2021, Chap. 5) formulate (but do not endorse) a thesis of 'extreme anti-essentialism' which predicts a dramatic failure of the necessity of distinctness according to which arbitrary pairs of individuals are possibly identical with one another. On one version of that view, arbitrary pairs of individuals are possibly identical with one another whilst being chunky. However we saw previously that if the auxiliary necessitist endorsed that claim, there would be no cases of incompossibly chunky individuals to prevent the mapping $[.]^{NEC}$ from succeeding. This suggests that such dramatic failures of the necessity of distinctness may end up undercutting some of the examples Williamson uses to motivate his challenge, although they help to generate the first-order challenge I present below. Nevertheless, in connection with the recombinatorial puzzle developed in Fritz (2017), Fritz and Goodman (2017, p. 1082) suggest that there are cases of incompossibility that are not essentialism-induced which would still motivate Williamson's challenge.

[18] Two notes bear some emphasis here. First, hereafter Ex abbreviates the formula $\exists yy = x$, and, where X is an n-adic second-order variable, EX is a formula that results from concatenating a new primitive predicate E with X, *whose intended interpretation is second-order existence*. A natural semantic clause is given for E below; it must be introduced as a primitive predicate in the B-free setting to guarantee that contingentists have an adequate second-order notion of existence. Second, it helps to note that the generalisation of Constancy to the case of an arbitrary n-adic second-order variable is formalised in the obvious way.

actuality in the sense that whatever is actually the case is necessarily actually the case, and whatever is not actually the case is necessarily not actually the case. This intuitive non-distortion principle is formalised schematically as follows:

$$\textbf{Non-Distortion}\quad \Box((\phi \rightarrow \uparrow\Box\downarrow\phi) \wedge (\neg\phi \rightarrow \uparrow\Box\neg\downarrow\phi))$$

Constancy and Non-Distortion supply two respective arguments for the necessity of distinctness.[19] Taking an informal version of the former first, if x and y are distinct then by standard plural logic there are some things X which comprise exactly x. By Constancy, x is necessarily one of those things (provided it exists and those things exist) and y is necessarily not one of those things (including when it exists and those things exist). Moreover standard plural logic guarantees that X exists whenever x exists. Thus, as a matter of necessity, provided x and y exist, x and y are distinguished by virtue of their membership of the things in question, and so by the necessitation of Leibniz's Law they are necessarily distinct when they exist. Taking the argument from Non-Distortion next, if x and y are distinct, then x is x and y is not x. So by Non-Distortion it is necessarily actually the case that x is x and it is necessarily not actually the case that y is not x. Thus, as a matter of necessity, x and y are distinguished by virtue of their actual identity with x, and so by the necessitation of Leibniz's Law they are necessarily distinct.

Since the aim is to develop a challenge without the necessity of distinctness in the background, the new background logic must not validate either Constancy or Non-Distortion. Thus I shall specify the new background logic via a class of variable domain models which involve a treatment of the second-order quantifiers and Vlach operators which is more amenable to the particular ND-free setting of interest. More generally, to avoid validating the necessity of distinctness the models will interpret the identity predicate with a function taking 'worlds' to an equivalence relation on the 'domain' of that 'world' which need not be the identity relation between elements of the 'domain' of that 'world'. This equivalence relation will also be a congruence relation in the sense that the 'extensions' of predicate constants and predicate variables at the relevant 'world' are closed under it. (Hereafter I shall drop the practice of enclosing expressions like 'worlds' in scare-quotes, and leave implicit that 'world' denotes the element of some relevant set-theoretic construct.)

This logic must be specified formally, so we first define the object language explicitly and then specify an appropriate class of models to study it.

Definition The *language* consists of the following items of vocabulary: countably many individual variables x, y, z, ...; countably many n-adic second-order variables, $X^n, Y^n, Z^n, ...$, for each natural number $n \geq 1$; finitely many atomic non-logical predicate constants of finite arity, including the 1-adic constant C; the atomic binary logical predicate $=$; the atomic monadic logical predicate E; the truth-functional

[19] See Linnebo (2013, n. 16), Dorr et al. (2021), and Williamson (1996) for versions of these arguments.

connectives ¬, and ∧; the unary modal operators $\Box$, ↑, and ↓, and the quantifier ∀. The other truth-functional connectives, modal operators and quantifiers are introduced as abbreviations in the usual way. The *well-formed formulas* of the language are defined recursively as follows. Where Φ is an n-adic predicate constant or variable, ψ and χ are well-formed formulas, V is an n-adic predicate variable, and $v, v_1, ..., v_n$ are individual variables, the following are well-formed formulas: $\Phi v_1...v_n$, $\neg\psi, (\psi \wedge \chi), \forall v\psi, \forall V\psi, \Box\psi, \uparrow\psi, \downarrow\psi$.

Definition A *model* is a sextuple $\mathfrak{M} = \langle W, R, D, d, \sim, I\rangle$ in which W and D are non-empty sets, R is a reflexive, transitive binary relation over W, d is a function taking each $w \in W$ to a subset $d(w)$ of D, and $\sim$ is a function taking each $w \in W$ to an equivalence relation $\sim_w$ on $d(w)$ such that if $a \sim_w b$ and $a \in d(u)$ then $a \sim_u b$ whenever Rwu; note that this implies $b \in d(u)$, since $\sim_u$ is an equivalence relation on $d(u)$. Finally, I is a function taking each non-logical n-adic predicate constant F to a function $I(F)$ taking each $w \in W$ to a subset of $d(w)^n$ subject to the condition that if $\langle a_1, \dots, a_n\rangle \in I(F)(w)$ then $\langle b_1, \dots, b_n\rangle \in I(F)(w)$ whenever $a_i \sim_w b_i$ (for $1 \leq i \leq n$).

Definition For each natural number $n \geq 1$, an *n-plural intension* is a function f taking each $w \in W$ to a non-empty subset of D^n such that whenever Rwu:

$$f(u) =$$
$$f(w) \cup \{\langle b_1, \dots, b_n\rangle : \text{ there is some } \langle a_1, \dots, a_n\rangle \in f(w) \text{ and } a_i \sim_u b_i \text{ for } 1 \leq i \leq n\}$$

Notice that when f is an n-plural intension $\langle a_1, \dots, a_n\rangle \in f(w)$ implies $\langle b_1, \dots, b_n\rangle \in f(w)$ whenever $a_i \sim_w b_i$ (for $1 \leq i \leq n$). A *variable assignment* g over a given model $\mathfrak{M} = \langle W, R, D, d, \sim, I\rangle$ is a function taking each first-order variable v to a member $g(v) \in D$, and taking each n-adic second-order variable V to an n-plural intension $g(V)$. When θ is either first-order or second-order variable $I_g(\theta) = g(\theta)$.

Over this class of models, one can introduce a natural semantics to specify appropriate notions of truth, validity and consequence. To handle the Vlach operators, the semantics assigns each formula a subset of W relative to a variable assignment and a sequence of worlds, which may be thought of as the proposition that formula expresses relative to that variable assignment and sequence.

Definition When W is a set, let $W^{<\omega}$ be the set of finite sequences of elements of W. In what follows, let $\mathfrak{M} = \langle W, R, D, d, \sim, I\rangle$ be an arbitrary model, s be an arbitrary sequence in $W^{<\omega}$, and g be an arbitrary variable assignment for $\mathfrak{M}$. The *interpretation function* over $\mathfrak{M}$ is defined recursively as follows. (In these clauses, $R(w) = \{u : Rwu\}$, and $s^w = s^\frown\langle w\rangle$ where $\langle x_1, \dots, x_n\rangle^\frown\langle y_1, \dots, y_m\rangle = \langle x_1, \dots, x_n, y_1, \dots, y_m\rangle$. Moreover, Φ is an n-adic predicate constant or variable (for $n \geq 1$), $v, v_1, \dots, v_n$ are first-order variables, V is a second-order variable, and ϕ and ψ are formulas.)

$[\![\Phi v_1 \dots v_n]\!]^s_g = \{w \in W : \langle I_g(v_1), \dots, I_g(v_n)\rangle \in I_g(\Phi)(w) \subseteq d(w)^n\}$;

$[\![v_1 = v_2]\!]^s_g = \{w \in W : I_g(v_1) \sim_w I_g(v_2)\}$;

$[\![\neg\phi]\!]^s_g = W/[\![\phi]\!]^s_g$;

$[\![\phi \wedge \psi]\!]^s_g = [\![\phi]\!]^s_g \cap [\![\psi]\!]^s_g$;

$[\![\Box\phi]\!]^s_g = \{w \in W : R(w) \subseteq [\![\phi]\!]^s_g\}$;

$[\![\forall v\phi]\!]^s_g = \{w \in W : w \in [\![\phi]\!]^s_{g[a/v]}$ for all $a \in d(w)\}$;

$[\![\forall V\phi]\!]^s_g = \{w \in W : w \in [\![\phi]\!]^s_{g[f/V]}$ for all n-plural intensions f such that $f(w) \subseteq d(w)^n\}$;

$[\![EX]\!]^s_g = \{w \in W : g(X)(w) \subseteq d(w)^n\}$;

$[\![\uparrow\phi]\!]^s_g = \{w \in W : w \in [\![\phi]\!]^{s^w}_g\}$

$[\![\downarrow\phi]\!]^{\langle\rangle}_g = [\![\phi]\!]^{\langle\rangle}_g$

$[\![\downarrow\phi]\!]^{s^u}_g = \{w \in W :$ there is some $p \subseteq W$ s.t. $u \in p$ and $R(w) \cap p = R(w) \cap [\![\phi]\!]^s_g\}$

A formula ϕ is *true at w in $\mathfrak{M}$ relative to s and g* ($\mathfrak{M}, w \models^s_g \phi$) iff $w \in [\![\phi]\!]^s_g$. A set of formulas Γ is *true at w in $\mathfrak{M}$ relative to s and g* ($\mathfrak{M}, w \models^s_g \Gamma$) iff for every $\gamma \in \Gamma$, $\mathfrak{M}, w \models^s_g \gamma$. A formula or set of formulas Θ is *valid in $\mathfrak{M}$* ($\mathfrak{M} \models \Theta$) iff it is true at all $w \in W$ on every variable assignment and sequence. A formula or set of formulas Θ is *valid* ($\models \Theta$) iff it is valid in every model. A formula or set of formulas Θ *entails* a formula ϕ ($\Theta \models \phi$) just in case for every model $\mathfrak{M} = \langle W, R, D, d, \sim, I\rangle$, every $w \in W$, every $s \in^{<\omega}$, and every assignment g for $\mathfrak{M}$, if Θ is true at w in $\mathfrak{M}$ relative to s and g, so is ϕ.

As in the model theory used by Williamson, auxiliary necessitism is true in some of the new models and auxiliary contingentism is true in others, which recreates the mild neutrality between the two views. Moreover the new model theory respects the being constraint. In contrast to Williamson's model theory, however, the current class of models does not validate the B axiom or the necessity of distinctness. As a result, Constancy and Non-Distortion are also invalidated. I shall now explain why this is the case by examining the clauses for the second-order quantifiers and the Vlach operators.

To begin with the former, the clause for the second-order quantifiers now captures the idea that pluralities are merely *rigid* in the sense that whatever is one of some things is necessarily one of those things provided it and those things exist, and that plural membership is modally non-increasing. This is captured by the following valid principle in the 1-adic case (the generalisation to the arbitrary n-adic case is formalised in the obvious way):

Rigidity $\Box\forall X(\forall x(Xx \rightarrow \Box(Ex \wedge EX \rightarrow Xx)) \wedge$
$\forall Y(\forall x(Xx \rightarrow \Box(Ex \wedge EY \rightarrow Yx)) \rightarrow \Box\forall x(Xx \rightarrow (EY \rightarrow Yx))))$

Put informally, Rigidity allows some thing which is not one of some things to be possibly one of those things provided it and those things exist, but only on the condition that it becomes identical with one of the initial things at the possibility in question. With mere rigidity, one loses the argument for the necessity of distinctness from Constancy.

A similar point applies to the semantic clause for the ↓ operator. Although this clause is unfamiliar, the idea it captures is quite natural.[20] One way to think of actuality operators and ↓ operators is that they are, like other modal operators, just certain properties of propositions. Indeed in the familiar setting of Williamson's S5 they may be thought of as *constant* properties of propositions: if a proposition has that property, it is necessary that it has that property; and if a proposition does not have that property, it is necessary that it does not have that property. (Consider the principle: whatever is (not) actually the case is necessarily (not) actually the case.) This assumption of constancy, however, is responsible for validating the necessity of distinctness, as should be clear from the argument for the necessity of distinctness from Non-Distortion. To avoid this consequence, a natural thought is to treat actuality operators and ↓ operators as merely *rigid* properties of propositions: properties of propositions whose instances are modally non-decreasing (provided they exist) and non-increasing. With mere rigidity, a proposition which is not one of such a propositional operator's instances is permitted to possibly be one of the operator's instances but only if it is identical with one of the operator's initial instances in the possibility in question. Intuitively a rigid property may 'acquire' new instances but only via collapses of identity amongst instances and previous non-instances of it. Since, in effect, the current model theory treats propositional identity as necessary equivalence, this conception motivates the semantic clause used above. Put less formally, it states that $\downarrow\phi$ is true at a world w relative to a world-sequence s^v and a variable assignment g just in case, at w, the proposition expressed by ϕ relative to s and g is identical with a proposition which was true at u.

A more technical way of appreciating this point is by observing that, in place of Non-Distortion, the Vlach operators are only required to be non-forgetful in the sense that whatever is actually the case is necessarily actually the case.

No Forgetting $\Box(\phi \rightarrow \uparrow\Box\downarrow\phi)$

Nonetheless the Vlach operators may intuitively 'enrich' how actuality stands. To put the point informally, at a given world w a proposition is allowed to be not actually the case but possibly actually the case provided that, at the possibility in question, it becomes necessarily equivalent with a proposition which is the case at w. To describe an example of this informally, suppose a world u is possible from a world w but not vice-versa. Clearly, at w the impossible proposition (in model theoretic terms, the empty set) is not the case. However it is possible that the 'world-proposition' of w—the proposition true only at w—is impossible, since w is impossible from u. Thus at w it is possible that the impossible proposition is necessarily equivalent with the world-proposition of w, and so, by the semantics, at w it is possible that the impossible proposition is actually the case. Yet this is exactly what ought to be

[20] The following treatment of Vlach operators is developed further in the context of a modal relational type theory in Roberts (forthcoming). Additionally, in Sect. 4 below, I further discuss this conception of Vlach operators in relation to one of the central results from Sect. 3 (Theorem 8).

the case at w, since it is impossible from u which is possible from w. Although the Vlach-operators behave non-standardly, then, their behaviour is suited to the ND-free setting.[21]

Since the semantics for the Vlach operators is unfamiliar, it helps to see these results in more detail:

Theorem 4 $\vDash \Box(\phi \rightarrow \uparrow\Box\downarrow\phi)$

Proof Let $\mathfrak{M} = \langle W, R, D, d, \sim, I\rangle$ be an arbitrary model and consider arbitrary $w \in W$, $s \in W^{<\omega}$ and variable assignment g. Suppose $w \in [\![\phi]\!]^s_g$. Then consider any u such that Rwu. To show that $u \in [\![\downarrow\phi]\!]^{sw}_g$ let $p = [\![\phi]\!]^s_g$. By supposition $w \in p$, and it is trivial that $R(v) \cap p = R(v) \cap [\![\phi]\!]^s_g$ for any $v \in W$. Thus $u \in [\![\downarrow\phi]\!]^{sw}_g$ and so $w \in [\![\uparrow\Box\downarrow\phi]\!]^s_g$. □

Theorem 5 $\nvDash \Box(\neg\phi \rightarrow \uparrow\Box\neg\downarrow\phi)$

Proof Consider the formula $\Box(\neg x = y \rightarrow \uparrow\Box\neg\downarrow x = y)$, which is an instance of the schema under consideration. Let $\mathfrak{M} = \langle\{0,1\}, \leq, \{0,1\}, d, \sim, I\rangle$ be the following model:

$$d(i) = \{0,1\} \text{ for all } i \in \{0,1\};$$
$$i \sim_0 j \text{ iff } i = j \text{ for } i,j \in \{0,1\};$$
$$i \sim_1 j \text{ iff } i,j \in \{0,1\}, \text{ and}$$
$$I(F)(i) = \emptyset \text{ for all non-logical predicate constants } F \text{ and all } i \in \{0,1\}.$$

Observe that $[\![\neg x = y]\!]^{\langle\rangle}_g = \{0\}$ when $g(x) = 0$ and $g(y) = 1$. Moreover, $\{1\} \cap \{0,1\} = \{1\} = \{1\} \cap [\![x = y]\!]^{\langle\rangle}_g$. Thus since $0 \in \{0,1\}$, by the semantics $1 \in [\![\downarrow x = y]\!]^{\langle 0\rangle}_g$ and so $0 \notin [\![\Box\neg\downarrow x = y]\!]^{\langle 0\rangle}_g$, hence $0 \notin [\![\uparrow\Box\neg\downarrow x = y]\!]^{\langle\rangle}_g$. □

Theorem 5 assures us that the semantics achieves its intended purpose of invalidating the principle about the Vlach operators from which the necessity of distinctness can be derived. Notice also that the proof of Theorem 5 illustrates how the necessity of Leibniz's Law, the non-forgetfulness of actuality, and the contingency of distinctness generate counterexamples to the idea that whatever is *not* actually the case is necessarily not actually the case. For suppose that a and b are actually distinct but possibly identical. At the possibility where a and b are identical, it is still actually the case that a is identical with a (since actuality is non-forgetful). Therefore by the necessity of Leibniz's Law, at that possibility it is actually the case that a is identical with b. So, at the initial world, even though the proposition that a is identical with b is not actually the case, it is possibly actually the case.

[21] See Bacon (2018, Sect. 5.4) and Dorr et al. (2021) for similar treatments of actuality operators in ND-free settings.

A final way to get a handle on the new clause for the ↓ operator is to observe how it can be simplified. Thinking informally, one way of stating the clause is that, provided world u is possible from world w, a formula $\downarrow\phi$ is true at w relative to a sequence of worlds whose final member is u when ϕ is true at u (relative to the sequence which results from deleting u as the final member); but when u is impossible from w, $\downarrow\phi$ is simply true at w relative to the former sequence of worlds. In slogan form, if the actual world is impossible from a given world w, then at w everything whatsoever is actually the case. Intuitively this is correct because if the actual world is indeed impossible from a given world, then the actual truths should not possibly all be true together from that world, and hence they should collectively necessitate any proposition whatsoever. This allows one to appreciate that the second clause for the ↓ operator can be stated more simply:

$$[\![\downarrow\phi]\!]_g^{s^u} = \{w \in W : \text{if } Rwu \text{ then } u \in [\![\phi]\!]_g^s\}$$

Helpfully, this allows one to see that over a natural subclass of S5 models in which accessibility is universal the standard clause for the ↓ operator may be recovered:

Theorem 6 *When $\mathfrak{M} = \langle W, R, D, d, \sim, I\rangle$ is a model in which $R = (W \times W)$ one may derive the following semantic clause:*

$$[\![\downarrow\phi]\!]_g^{s^u} = \begin{cases} W, & \text{if } u \in [\![\phi]\!]_g^s \\ \emptyset, & \text{otherwise} \end{cases}$$

Proof Routine. □

Theorem 6 demonstrates that the new clause for the ↓ operator satisfies a constraint which a correct treatment of the operator ought to do.

3 A new challenge

It is routine to check that the new model theory sustains Williamson's initial result that every formula A is mapped to a neutral formula $[A]^{\text{CON}}$ which is equivalent to A by the auxiliary contingentist's lights (i.e. Theorem 1).[22]

Theorem 7 $\text{AuxCon} \vDash \text{A} \leftrightarrow [\text{A}]^{\text{CON}}$ *for all formulas A.*

[22] For this result we extend the mapping $[.]^{\text{CON}}$ with the natural clause for the new primitive predicate for second-order existence: $[EX]^{\text{CON}} = EX \wedge X \leq C$.

Nonetheless the new model theory fails to sustain Williamson's other result that every FO formula A is mapped to a neutral formula $[A]^{\text{NEC}}$ which equivalent to A by the auxiliary necessitist's lights (Theorem 2). Indeed in the current setting one can significantly strengthen Williamson's primary limitative result that there is a formula A for which there is no neutral formula B which is equivalent to A by the auxiliary necessitist's lights (Theorem 3).[23] Specifically, this limitative result now holds with respect to FO formulas.

Theorem 8 *There is a FO formula A for which there is no neutral formula B such that*:

$$\text{AuxNec} \vDash A \leftrightarrow B.$$

The argument for Theorem 8 will occupy the remainder of this section, but the core idea is quite intuitive. A key part of the problem for the contingentist is that they are no longer able to 'simulate' necessitist first-order quantification with the use of Vlach operators. As I shall bring out, this is because Vlach operators may now 'enrich' how actuality stands, so some proposition which is false may be possibly actually true. Furthermore, there are particular necessitist claims about possible collapses of identity between chunky and non-chunky individuals which will no longer have any neutral analogue as far as the auxiliary necessitist is concerned. The contingentist is thus unable to extract any 'cash-value' or 'kernel of truth' from these simple first-order modal claims.

Let me begin with the observation that contingentists can no longer 'simulate' necessitist first-order quantification with the use of Vlach operators. Intuitively, one reason this occurs is that the auxiliary necessitist may regard some simple first-order existentially quantified statement as false but recognise that each of its instances are possibly actually necessary truths because of a possible distortion of how things actually stand. This is brought out in the argument for the following result.

Theorem 9 $\text{AuxNec} \nvDash \exists x Cx \leftrightarrow \uparrow\Diamond\exists x(Cx \wedge \downarrow Cx)$

Proof Consider the model $\mathfrak{M} = \langle \{0,1\}, \leq, \{0\}, d, \sim, I\rangle$ in which:

$d(i) = \{0\}$ for all $i \in \{0,1\}$;

$i \sim_k j$ iff $i = 0 = j$ for all $k \in \{0,1\}$,

$I(C)(0) = \emptyset$;

$I(C)(1) = \{0\}$, and

$I(F)(i) = \emptyset$ for all other non-logical predicate constants F and all $i \in \{0,1\}$.

[23] It is worth noting that, in the current setting, AuxNec is consistent with failures of some instances of the Barcan formula, which some necessitists may wish to retain (although see the discussion of necessitism in Dorr et al. (2021, Chap. 1) and the view of broad necessity developed in Bacon (2018)). However even if one strengthened AuxNec with the addition of all instances of the Barcan formula, the arguments of this section would still work; indeed all of the models of AuxNec I consider below use a 'constant domain' and validate all instances of the Barcan formula.

It is easy to verify that $\mathfrak{M} \models$ AuxNec. Moreover observe that $[\![Cx]\!]^s_{g[0/x]} = \{1\}$ for arbitrary s. Thus clearly $0 \notin [\![\exists x Cx]\!]^s_g$ for arbitrary s and g. However, $0 \in [\![\uparrow\Diamond\exists x(Cx \wedge \downarrow Cx)]\!]^s_g$ for arbitrary s and g, since $1 \in [\![\downarrow Cx]\!]^{s^0}_{g[0/x]}$ because $0 \in \{0, 1\}$ and $\{i \in \{0, 1\} : 1 \leq i\} \cap \{0, 1\} = \{1\} = \{i \in \{0, 1\} : 1 \leq i\} \cap [\![Cx]\!]^s_{g[0/x]}$. □

Although this constitutes a significant departure from the setting of S5, it is a departure which ought to be expected in a B-free and ND-free setting. As mentioned previously, if a world *u* is possible from the actual world *w* but not vice-versa, then at *u* every proposition is actually the case. So auxiliary necessitists should not expect that a claim of the form 'there is an F' to be materially equivalent to a claim of the form 'possibly, there is a chunky thing which is actually F': if there are possibilities where the actual world is impossible, at those possibilities any proposition will actually be the case.

It is a corollary of Theorem 9 that $\exists x Cx$ is a FO formula which is not equivalent (given AuxNec) to $[\exists x Cx]^{\text{NEC}}$ in the new background logic. Thus the analogue of Williamson's Theorem 2 with respect to the new background logic fails. Nevertheless this does not yet establish Theorem 8, the more general result that there are FO formulas for which there is *no* neutral equivalent (given AuxNec) in the new background logic. I now turn to that result.

Consider the assertion that any chunky thing and any non-chunky thing are possibly identical whilst non-chunky:

$$\textbf{Collapse} \quad \forall x(Cx \to \forall y(\neg Cy \to \Diamond(\neg Cy \wedge x = y)))$$

Claims of this form seem to draw perfectly intelligible modal distinctions. For example, consider a speech such as the following:

> Harry, a human, grew from a sperm *s* and an egg *e*. But there is also a merely possible human, Henry, who could have grown from a sperm *s* and an egg e^* distinct from *e* (but produced by the same person). Nevertheless, had *e* been identical with e^* despite never being fertilised, Harry and Henry would have been one and the same merely possible human.[24]

The hypothesis offered in this speech is that collapses of identity amongst the concrete induce collapses of identity amongst the merely possible. One might disagree with this hypothesis; but to disagree with it involves recognising it as intelligible, which is all that the example requires. Indeed, given a metaphysics on which there are failures of ND, it seems natural to want to engage with such hypotheses about how collapses of identity might ramify. Claims like Collapse thus appear to draw genuine, intelligible modal distinctions, just like the second-order sentence (4)

[24] This example is inspired by the example from Fritz and Goodman (2017, p. 1064) of Richard Dawkins's claim that "[t]he potential people who could have been here in my place but who will in fact never see the light of day outnumber the sand grains of Arabia. [...] We know this because the set of possible people allowed by our DNA so massively exceeds the set of actual people." One can imagine this being followed up with the claim that two merely possible people might have one and the same merely possible person, had our DNA allowed for drastically fewer possible people.

which the auxiliary necessitist deemed to have no neutral equivalent (in the previous background logic).

There are various models of auxiliary necessitism in which Collapse is true, and various models in which this claim is false. To take some examples, consider the following pair of three-world models $\mathfrak{M} = \langle \{0,1,2\}, \leq, \{0,1\}, d, \sim, I \rangle$ and $\mathfrak{M}' = \langle \{0,1,2\}, \leq, \{0,1\}, d, \sim', I' \rangle$ where:

$d(i) = \{0,1\}$ for all $i \in \{0,1,2\}$;

$i \sim_k j$ iff $i = j \in \{0,1\}$ when $k \in \{0,1\}$, and $i, j \in \{0,1\}$ when $k = 2$;

$i \sim'_k j$ iff $i = j \in \{0,1\}$ when $k = 0$, and $i, j \in \{0,1\}$ when $k \in \{1,2\}$;

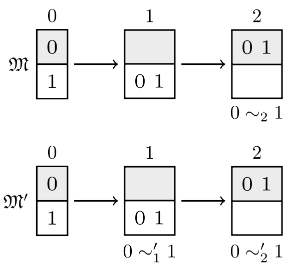
$I(C)(0) = I'(C)(0) = \{0\}$

$I(C)(1) = I'(C)(1) = \emptyset$

$I(C)(2) = I'(C)(2) = \{0,1\}$

$I(F)(i) = I'(F)(i) = \emptyset$ for all other non-logical predicates F and all $i \in \{0,1,2\}$

This pair of models can be represented diagrammatically. In these diagrams, worlds are represented by the labelled rectangular nodes; the shaded section of a node represents the extension of the predicate 'C' at that world; relevant instances of the extension of the identity predicate at a world are specified below each world node using the function the model uses to interpret the identity predicate, and arrows represent accessibility amongst distinct worlds (accessibility from a world to itself is always suppressed).



It is easy to verify that Collapse is valid in $\mathfrak{M}'$ but not valid in $\mathfrak{M}$. After all, $\mathfrak{M}, w \nvDash^s_g \Diamond(\neg Cy \wedge x = y)$ when $g(x) = 0$, $g(y) = 1$ and w and s are arbitrary.

However one can also show that $\mathfrak{M}$ and $\mathfrak{M}'$ agree on all neutral formulas, in the sense that a neutral formula is true at one world in $\mathfrak{M}$ (with respect to an arbitrary sequence and assignment) iff it is true at the same world in $\mathfrak{M}'$ (with respect to the same sequence and assignment). From that, however, it follows that no neutral formula is equivalent with Collapse by the auxiliary necessitist's lights. To show that $\mathfrak{M}$ and $\mathfrak{M}'$ agree on all neutral formulas in this sense, one can adapt a technique from Williamson (2010). One begins by defining the *restriction* of a model, which intuitively is the model which results from restricting attention to what is chunky according to the initial model.

Definition (Restriction). For $\mathfrak{M} = \langle W, R, D, d, \sim, I \rangle$, its *restriction* $\mathfrak{M}^c = \langle W, R, D, d^c, \sim^c, I^c \rangle$, where:

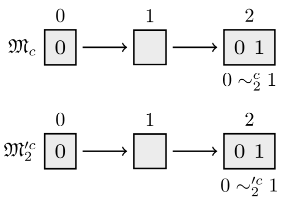
$$d^c(w) = I(C)(w)$$
$$\sim^c_w = \sim_w \cap I(C)(w)^2$$
$$I^c(F)(w) = I(F)(w) \cap I(C)(w)^n, \text{ for } n\text{-adic non-logical predicates } F$$

With this definition to hand, one can establish a useful correspondence between the neutral formulas true at a given world in a model (relative to a variable assignment and sequence) and the formulas true at the same world in that model's restriction (relative to the same variable assignment and sequence).[25]

Lemma 10 $w \in [\![[A]]\!]^s_{\mathfrak{M},g}$ *iff* $w \in [\![A]\!]^s_{\mathfrak{M}^c,g}$

Proof By a routine induction on the complexity of formulas; see Proposition 1.1 of Williamson (2010) for what is essentially the required argument. □

However the restriction of $\mathfrak{M}$ is the just same model as the restriction of $\mathfrak{M}'$. This identity can be visualised in diagram form as follows (since the world domain of a restriction is identical with the extension of 'C' at that world in the restriction, these diagrams associate no unshaded section with any node):



Thus by Lemma 10 we can conclude the desired corollary that $\mathfrak{M}$ and $\mathfrak{M}'$ agree on all neutral formulas.

Corollary 11 $w \in [\![[\phi]]\!]^s_{\mathfrak{M},g}$ iff $w \in [\![[\phi]]\!]^s_{\mathfrak{M}',g}$ *for all formulas* ϕ.

Since $\mathfrak{M}$ and $\mathfrak{M}'$ are models of AuxNec which disagree on the truth of Collapse, Theorem 9 follows from this corollary: Collapse is an FO formula for which there is no neutral formula B such that AuxNec $\vDash B \leftrightarrow$ Collapse. In other words, there are very simple first-order formulas from which the contingentist is unable to extract any kernel of truth or cash-value.

Let me next take a more complex example, but one with more relevance to metaphysics. Consider the following speech:

> There are two similar but distinct lumps of clay, c and c', which are never moulded into statue form. However they easily could have been: there is a possible statue which one could have made from c, and a different possible statue

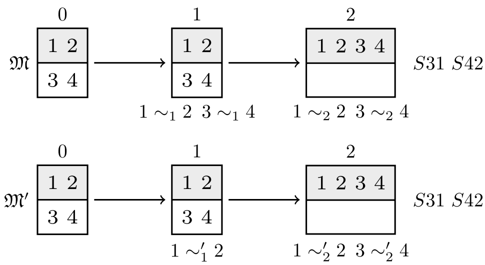
[25] Note that any world-sequence and variable assignment for a given model are also world-sequences and variable assignments for that model's restriction, since models share their space of worlds and outer domain with their restrictions.

which one could have made from c'. But had c and c' been identical despite still not being moulded into statue form, those possible statues would have been the same too.

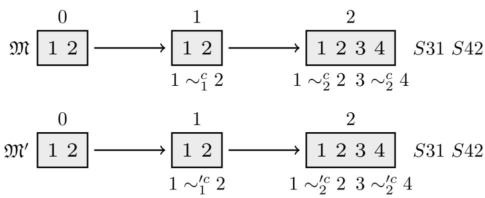
Again this speech appears to articulate a perfectly intelligible hypothesis which draws a genuine a modal distinction. For those who embrace failures of the necessity of distinctness, there should be a desire to be able to draw such distinctions and ask with which side of the distinction one's metaphysics of identity agrees. Yet there is no neutral equivalent of this hypothesis given auxiliary necessitism. To see why, observe that one can formalise the hypothesis as follows (where 'S' is read as the binary predicate 'is a statue made of'):

Statues $$\exists x_1 \ldots \exists x_4 \Big(\bigwedge_{1 \leq i \neq j \leq 4} x_i \neq x_j \wedge Cx_1 \wedge Cx_2 \wedge \neg Cx_3 \wedge \neg Cx_4 \wedge \Diamond Sx_3x_1 \wedge \Diamond Sx_4x_2 \wedge \Box(x_1 = x_2 \wedge \neg Cx_3 \wedge \neg Cx_4 \rightarrow x_3 = x_4) \Big)$$

As with Collapse, there are models of auxiliary necessitism in which Statues is true and models in which it is false. For example, consider the following pair of three-world models $\mathfrak{M}$ and $\mathfrak{M}'$ (which I shall now just specify in diagram form with the additional convention that 'Sab' indicates that the pair $\langle a, b \rangle$ belongs to the extension of 'S' at the given world):



The sentence Statues is true at world 0 in $\mathfrak{M}$ relative to any world-sequence and assignment; however it is not true at any world in $\mathfrak{M}'$ relative to any world-sequence and assignment. Nevertheless models $\mathfrak{M}$ and $\mathfrak{M}'$ have the same restriction, which is depicted in the following diagram:



By Lemma 10 we may conclude that world 0 in $\mathfrak{M}$ agrees with world 0 in $\mathfrak{M}'$ on all neutral formulas, despite Statues varying in truth-value across world 0 in those models. Thus Statues is another FO formula which the auxiliary necessitist will deem to have no neutral equivalent.

The philosophical significance of these results is that they strengthen Williamson's challenge to contingentists.[26] To adapt a remark from Williamson, the results demonstrate that the necessitist can draw *first-order* modal distinctions whose genuineness contingentists can neither plausibly deny nor explain on their own terms. In other words, in the ND-free setting the contingentist's inability to draw genuine modal distinctions is deeper and more extensive than it was in the context of S5. However let me be clear that I take neither form of the challenge to constitute a decisive argument against contingentism. That contingentists are unable to draw genuine second-order distinctions in the context of S5 signals that their theory is impoverished, but it does not imply that the theory stands refuted. Similarly, I take my results to strengthen that signal significantly, but not turn it into a refutation. Rather, they constitute further evidence that there is an asymmetry in the ability to articulate genuine modal distinctions between necessitism and contingentism—an asymmetry which breaks in favour of necessitism. And the greater the variety of genuine modal distinctions which contingentists are unable to draw, the harder it is for contingentists to dismiss the worry that their theory is inadequate for the purpose of describing modal reality.

Additionally, the fact that this asymmetry extends to first-order modal discourse is extremely pertinent. For contingentists have reacted to Williamson's challenge by introducing powerful expressive resources designed to simulate necessitist second-order quantification. For example, Williamson (2013, Chap. 7, Sect. 7) discusses an adaptation of the proposal in Fine (1977) to use highly infinitary resources to simulate 'possibilist' second-order quantification.[27] Yet introducing such expressive resources will not help contingentists remedy the issue I have highlighted at the first order. Indeed this Finean proposal makes essential use of Vlach operators; but we have seen that the standard way of exploiting such resources to simulate even necessitist *first*-order quantification fails in the ND-free setting. Thus my results narrow the space of solutions available to contingentists in the face of the challenge.

4 Robustness of the challenge

How might contingentists who are set on meeting this challenge proceed? A natural line of response would be for them to scrutinise the robustness of the challenge. For example, they might ask whether Theorem 8, my central limitative result, extends to more expressive formal languages such as those which admit not just second-order quantification but higher-order quantification more generally. In that setting they might attempt to identify an alternative, perhaps more natural, auxiliary necessitist theory which secures over a suitable class of models a neutral equivalent for every formula of the language. Williamson (2013, pp. 364–365) himself recommends

[26] In what follows I connect my results with Williamson's reflections on the significance of his own challenge (see especially Williamson 2010, pp. 711–712; Williamson 2013, pp. 364–365).

[27] Williamson argues that it is not legitimate for contingentists to use these resources, and that the resources themselves are philosophically suspect. Fine (2016) and Williamson (2016b) debate these issues further.

that the necessitist be circumspect about the significance of Williamson's limitative result for exactly these reasons, although of course he registers his optimism for the necessitist cause.

Some of these questions are particularly pertinent in the wake of my own challenge. I emphasised that the new semantic clause for the $\downarrow$ operator was motivated by a conception of that operator as a certain property of propositions. More specifically, I suggested that its new clause was motivated by a conception of it as a 'rigid' property of propositions, a property of propositions whose instances are modally non-decreasing (provided they exist) and non-increasing. This conception of the operator, and talk of 'properties of propositions' more generally, is naturally studied in a higher-order setting which admits propositional variables, predicate variables which concatenate with propositional variables to produce formulas, and quantification into the position of those variables. The pertinent question then becomes whether my limitative result extends to such a setting.

Although it would be difficult to investigate that question in suitable technical depth here, it is worth reflecting on one issue which can be studied in the higher-order setting. In that setting, with propositional quantification (and a notion of propositional identity) one can ask whether it is contingent which propositions exist. Assuming that it would be inappropriate to formulate the challenge over a class of models which validates propositional necessitism, one would require a class of variable domain Kripke models in which the domain of propositions varies from world to world. This would affect the clause for the $\downarrow$ operator. For, put informally, that clause stated that $\downarrow\phi$ is true at a world relative to a world-sequence s^v and variable assignment g just in case, at w, the proposition expressed by ϕ relative to s and g is identical with a proposition which is true at u. However if it is contingent which propositions exist, then at w there might exist a proposition which is true at u despite not itself existing at u. Put even less formally, if one thinks of $\downarrow$ operators as rigid properties of propositions in this setting, it might be possible for there to exist a new proposition which is possibly identical with one of the old propositions to which $\downarrow$ applies.

Here is an intuitive, simplified example of this phenomenon[28]:

> Suppose that actually there exists exactly one individual, *a*, and necessitism is true. Yet suppose further that it is possible for things to be exactly as they are except there is a new individual, *b*, which is distinct from *a*. There is thus the new (false) proposition *a is identical to b*. Nevertheless suppose also that at *that* possibility it is possible for *a* and *b* to be identical, in which case the proposition *a is identical to b* would just be the proposition *a is identical to a*.[29]

[28] Thanks to an anonymous reviewer for suggesting this example.

[29] This is underwritten by a Leibniz's Law argument in a classical higher-order logic. For by Leibniz's Law, if *a* is *b*, then if the proposition *a is such that it is a* just is the proposition *a is a*, then the proposition *b is such that it is a* just is the proposition *a is a*. Given that the proposition *a is such that it is a* is the proposition *a is a* by classical higher-order logic, it follows by classical higher-order logic that the proposition *b is a* is the proposition *a is a*.

This style of example is doubly relevant because it suggests that even if the actual world in a variable domain Kripke model is not accessible from a given world *w*, a correct treatment of the actuality operator need not reckon that, at *w*, everything whatsoever is actually the case. For at *w* there might be a 'new' possibly true world-proposition with which the actual world-proposition is possibly identical. In that case, an instance of non-symmetric accessibility amongst worlds need not induce contingency in which propositions are true. To put it intuitively, even if the actual world is no longer accessible, the actual world-proposition need not strictly imply absurdity if there is a 'new' possible world-proposition which 'describes' that things are exactly as they actually are.

Would a more general semantic clause for the ↓ operator which allows for this sort of case disrupt the argument for the new limitative result (Theorem 8)? It is natural to conjecture that it would not, primarily due to the fact that the models of auxiliary necessitism used in that argument have a constant domain function: the models did not involve the possibility of new individuals. In other words, they validate all instances of the Barcan formula for individuals; thus their most natural extensions to a higher-order setting would validate all instances of the Barcan formula for propositions too.[30] In such constant domain models, the new envisaged clause for the ↓ operator would be equivalent to the original clause used in the argument for Theorem 8, so the argument for Theorem 8 would be preserved. This indicates that the new limitative result has a welcome degree of robustness. Although it still pays to be circumspect about the overall robustness of the result, it seems safe to assume that contingentists require an alternative proposal if they are to meet the challenge.

5 Conclusion

I have argued that contingentists face a particularly concerning problem when their view is considered within a natural setting in which the necessity of distinctness fails. In this setting, contingentists cannot 'simulate' necessitist first-order quantification via their usual method of Vlach operators. Moreover there are quite simple first-order modal claims from which contingentists are unable to extract any kernel of truth or cash-value. The central philosophical upshot is that even contingentists face a particularly difficult challenge on natural ND-free pictures which are serious competitors for the correct metaphysics of modality.

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[30] Even some of the metaphysics of modality discussed in n. 17, which predict a dramatic failure of the necessity of distinctness, are consistent with all instances of the Barcan formula for individuals. For example, the extreme anti-essentialist view discussed in n. 17 guarantees that arbitrary pairs of individuals are possible identical with one another; but that is perfectly compatible with the truth of all instances of the Barcan formula for individuals.

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