



The explanation of logical theorems and reductive truthmakers

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Abstract This paper first identifies several plausible desiderata on satisfactory explanations of logical theorems, shows that ordinary grounding explanations cannot satisfy them and argues that there is reason to believe that no alternative grounding explanations of logical theorems can be given. It then develops an alternative explanation of logical theorems based on Yablo’s (Aboutness, Princeton University Press, Princeton, 2014) idea of reductive truthmaking. The resulting proposal invokes instances of reductive truthmaking that bear an interesting structural similarity to the notion of zero-ground, in virtue of which it is able to satisfy the identified desiderata.

Keywords Explanation of logical theorems · Reductive truthmakers · Zero-grounding · Explanation by essence · Explanation by status · Empty-base explanation

1 Introduction

The purpose of this paper is to explore how logical theorems may be explained. In Sect. 2 I argue that their ordinary grounding explanations do not appear to be completely satisfactory and identify desiderata for more satisfactory explanations. In Sect. 3 I consider and criticize two proposals for extraordinary grounding explanations: grounding explanation by status and explanation by zero-ground. In Sect. 4 I then offer several ways in which we might deal with the apparent failure of grounding to provide satisfactory explanations of logical theorems. In Sect. 5 I introduce the notion of an *empty-base explanation* and argue that empty-base

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explanations that do not involve grounding could satisfy the desiderata without running into the problems that confront grounding explanations. In Sect. 6 I attempt to implement this idea by developing Yablo's (2014) account of reductive truthmaking. I anticipate an objection in Sect. 7 and conclude in Sect. 8.

2 Ordinary grounding explanations and why they might be unsatisfactory

Our question is how logical theorems such as $P \vee \neg P$, which will be our schematic example, can be explained.¹ More specifically, our goal is to answer why P for every logical theorem P —hence, to answer why $P \vee \neg P$ (for instance, why the sun is shining or it is not the case that the sun is shining). Since, as we may assume, the desired explanation is not a causal one, it is natural to turn to grounding explanation.² Indeed, a standard kind of grounding explanation for logical theorems is readily available: For example, the logic of 'because' in Schnieder (2011) and the logic of ground in Fine (2012) specify grounds for logical theorems of classical first order logic.

According to these proposals, the grounds of a logical theorem are propositions that correspond to some of the atomic formulae (or negations thereof) into which the formula that expresses the logical theorem can be decomposed, namely those that make the logical theorem true—ground it—on a given occasion. For example, since in general a disjunction is grounded in its true disjunct, a logical theorem of the form of $P \vee \neg P$ is grounded in and hence can be explained by its true disjunct, with the corresponding because-claim being ' $P \vee \neg P$ because P ' or ' $P \vee \neg P$ because $\neg P$ ', depending on whether P or $\neg P$ is true.³ Call this kind of explanation the 'ordinary grounding explanation'. To give a concrete example, given that the sun is shining, [the sun is shining or it is not the case that the sun is shining] is fully grounded in [the sun is shining].⁴ Correspondingly, given that the sun is shining, the sun is shining or it is not the case that the sun is shining because the sun is shining.

¹ For the sake of convenience, unless stated otherwise, schematic letters and formulae like ' P ' and ' $P \vee \neg P$ ' will be used both in sentence position and to (schematically) refer to the corresponding propositions.

² Grounding is a notion of metaphysical priority closely related to explanation that has gained much attention in recent years. For accounts of this notion see for example Rosen (2010), Fine (2012), and the introduction by Correia and Schnieder (2012). I take grounding relations to underwrite corresponding explanations such that (typically, at least) if P grounds Q , a (grounding) explanation of Q in terms of the ground P exists.

³ I will assume here that the truth of the grounding claim is sufficient for the truth of the corresponding because-claim.

⁴ I use square brackets to refer to the proposition expressed by the sentence within the brackets. For the sake of convenience I sometimes use a predicational idiom of grounding and assume grounding to relate propositions; no commitment as to the nature of grounding's relata, if any, and concerning operatorational versus predicational views of grounding is intended by this. Cf. Correia and Schnieder (2012, 3.1) for the distinction between these two views.

As for instance Schnieder (2011, pp. 457f.) observes, these ordinary grounding explanations may not seem completely satisfactory. For example, one might have thought that logical theorems are a good candidate for truths that possess some sort of special, perhaps somehow particularly good, explanation. Some desiderata that may be the source of this idea are that a satisfactory explanation of logical theorems should

- 1 somehow also account for their necessity or their status as logical theorems.
- 2 be modally stable in that it holds with necessity.
- 3 give rise to no, or just very few, or not especially pressing further why-questions.
- 4 be compatible with certain non-classical logics in which e.g. $P \vee \neg P$ can be true without either of its disjuncts being true.

It is clear that ordinary grounding explanations do not satisfy these desiderata: An ordinary grounding explanation of, say, $P \vee \neg P$ in terms of its true disjunct gives rise to the question why this disjunct obtains and hence to all the why-questions that an explanation of that disjunct gives rise to. Since the disjunct is arbitrary, no special explanatory status with respect to what further why-questions arise in that fashion seems available for $P \vee \neg P$, if it only has an ordinary grounding explanation. There also appears to be no sense in which the ordinary grounding explanation could account for the necessity of $P \vee \neg P$ or its status as a logical theorem. After all, $P \vee \neg P$ has the same kind of ordinary grounding explanation as any true disjunction. Since the ordinary grounding explanation of a disjunction proceeds through its disjuncts, it also fails to be modally stable: If P is contingent, $P \vee \neg P$ will be grounded in P if P is true, and in $\neg P$ otherwise. For the same reason the fourth desideratum fails for ordinary grounding explanations: If $P \vee \neg P$ is to be explained in a setting in which it can be true without either P or $\neg P$ being true, then ordinary grounding explanations will not do.⁵

Before we continue, let me note that it is not clear that there must be explanations for logical theorems that satisfy (one or more of) the desiderata—for example, perhaps the status of logical theorems as logical theorems can be accounted for by explaining not the theorems themselves, but rather explaining why they are logical theorems. It is also not clear that necessary truths should require necessary explanations if it is necessary that they do have an explanation, just not the same in every possible circumstance. Nevertheless, I will take the dissatisfaction with the ordinary grounding explanations as a datum and attempt to find (additional) alternative explanations for logical theorems that satisfy the desiderata just identified.

⁵ I consider the fourth desideratum to be weaker than the first three, but I take these to be compelling on their own in any case.

3 A problem for extraordinary grounding explanations

On reflection, two proposals for alternative grounding explanations of logical theorems readily come to mind: First, there is the idea that logical theorems might somehow be grounded in (and thereby explained by) propositions expressing their status as logical theorems, their being logical or metaphysical laws or their being part of certain essences. Call these proposed explanations ‘explanations by status’ and the mentioned status-expressing propositions ‘status propositions’.⁶ The corresponding because-claims would then have the form ‘ $(P \vee \neg P)$ because $\blacksquare(P \vee \neg P)$ ’, where ‘ \blacksquare ’ is a placeholder for the respective status expressing operator.

According to the second proposal that comes to mind, logical theorems are zero-grounded. Let me say a little bit about the notion of zero-ground: Normally, metaphysical grounding is taken to be a relation (or at least something approximately like a relation) between a plurality of propositions or facts, the *grounds*, and a single proposition or fact, the *grounded* proposition/fact or *groundee*. Zero-grounding is a limiting case of grounding in which the set of grounds is empty. A zero-grounded proposition or fact is grounded and not ungrounded, but it does not require any propositions or facts to ground it—it is grounded in zero propositions/facts.⁷

More precisely, if we assume grounding statements to have the form ‘ $\Gamma < \phi$ ’, then since in the case of zero-grounding statements, the ‘ Γ ’ stands for an empty plurality of grounds, statements of zero-grounding have the form ‘ $< \phi$ ’. Alternatively we might express zero-grounding using sentences of the form ‘ $\emptyset < \phi$ ’. As for the corresponding because-statements, we can adopt a similar convention and use ‘ \emptyset ’ to stand for the empty set of grounds, which gives us ‘ $P \vee \neg P$ because \emptyset ’. Somewhat tongue-in-cheek, we could alternatively take and adapt the natural language expression ‘just because’, giving us ‘ $P \vee \neg P$ just because’.⁸

Intuitively, at least, both proposals promise to scratch an explanatory itch that the ordinary grounding explanations do not address: They do, in some sense, account for the special status of logical theorems, they are necessary, they satisfy the alternative-logics desideratum, and at least explanation by status has by some been considered to be an—in some way or other—especially good kind of explanation.⁹

One way to spell out the latter point is to focus on the idea that logical theorems are grounded in propositions that express their status as essential truths and to adopt Dasgupta’s (2014) idea that such propositions are explanatorily autonomous, i.e. not in need of any explanation. The grounding explanation of logical theorems in

⁶ A related conceivable option that I will not address is that while logical theorems cannot in general be explained by propositions expressing *their* status, they can be explained by other status propositions.

⁷ The notion of zero-ground has been introduced by Fine (2012, pp. 47f.). One prominent application of the notion is Litland’s (2017) account of the grounds of ground, but see also Muñoz (2020) and De Rizzo (2020).

⁸ I discuss the nature of this explanatory proposal further in Sect. 5.

⁹ This idea is common in the literature on why there is anything at all, in which explanation by necessary status is often taken to be of particular explanatory value. Cf., e.g., Goldschmidt (2013).

question would then be particularly good because according to it, logical theorems are grounded in propositions which themselves do not require any further explanation. Quite similarly, the zero-grounding proposal promises particularly good explanations of logical theorems in that the relevant explanatory candidates do not involve any grounds of logical theorems at all for which further explanations could be demanded.

That the proposals promise to satisfy the other desiderata can be seen as follows. First, the proposal for explanation by status accounts for the special status of logical theorems by employing that very status in explaining logical theorems. This status can then, so to speak, be read off these explanations of logical theorems. The desideratum for modal stability is satisfied by the proposal for explanation by status because the status propositions are necessary and the (at least in this context) eminently plausible principle that grounding is non-contingent, according to which if propositions Γ together ground Q , then necessarily, if all propositions Γ are the case, then the Γ together ground Q .¹⁰

According to the zero-grounding proposal on the other hand, logical theorems are grounded in the empty plurality of grounds. Since all propositions in the empty plurality of grounds are necessarily the case, this, together with the principle that grounding is non-contingent, also results in logical theorems being necessarily zero-grounded. In the same way, the necessary status of logical theorems can be read off their proposed zero-grounding explanations, whereby this proposal also satisfies the first desideratum. Moreover, the special status of being zero-grounded itself can be read off the proposed zero-grounding explanations: According to the proposal, logical theorems do not only logically follow from zero premises, they are also grounded in and hence explained by zero premises.

Finally, the alternative-logics desideratum can be satisfied by both proposals simply because they offer grounds for logical theorems that obtain even if we assume that, e.g., $P \vee \neg P$ obtains without either P or $\neg P$ obtaining.

One drawback of these two proposals for extraordinary grounds of logical theorems is that they both conflict with Fine's (2012, pp. 63f.) attractive account of the logic of ground, according to which conjunctions can only be grounded via their conjuncts and disjunctions can only be grounded via their (true) disjuncts.¹¹ According to this assumption, our example $P \vee \neg P$ can also *only* be grounded via its true disjunct. Since the alternative grounds proposed above are not in general either the true disjunct of $P \vee \neg P$, nor do they ground it, these proposals are ruled out by the present assumption. Note in particular that this is also true for the zero-grounding proposal: According to the assumption, $P \vee \neg P$ can only be zero-grounded if one of its disjuncts is zero-grounded. But of course, only in very specific

¹⁰ For the principle cf. Correia and Schnieder (2012), pp. 21ff.

¹¹ More specifically, the logic of Fine (2012) captures this idea by postulating elimination rules for the impure logic of ground, for instance the rule $\vee E$. But the insight is more general than this implementation, it is for example also contained in Fine's (2017b) account of grounding in terms of truthmaking.

instances will the true disjunct of a disjunction be zero-grounded (or grounded in the relevant status-expressing proposition).¹²

4 What to do?

Let us consider some possible reactions to this difficulty:

- 1 Accept that despite intuitive appearance to the contrary, an explanation of logical theorems that does not proceed via the ordinary grounding explanation cannot be had. Additionally, it might be argued against the need for any additional explanation.¹³
- 2 Change the target: Perhaps a more satisfactory explanation can only be had for a proposition in the vicinity of $P \vee \neg P$. One salient candidate would be to explain why certain status propositions obtain, such as the propositions that it is a logical theorem, a necessary truth or a metaphysical law that $P \vee \neg P$ (rather than explaining why $P \vee \neg P$).
- 3 Revise the logic of ground to allow for more diverse—extraordinary—grounds for logical theorems.
- 4 Find a different explanatory notion that allows for a more satisfactory explanation of logical theorems than grounding does.

I have some reservations with respect to the first three options: First, it seems that we should only, despite appearance, accept that no more satisfactory explanation can be had and try to explain away the need for a better explanation, if indeed no alternative candidate is available. As a matter of fact, such a candidate may be available, as I will argue below. With respect to the second option I have a similar reservation: While it is an interesting question what, if anything, explains truths expressible by sentences of the form ‘It is a logical theorem that ...’, I first want to investigate whether a more satisfactory explanation of logical theorems themselves can be found.

With respect to a revision of the logic of ground I have the following reservations: First, the logic as it is is neat and somewhat intuitively motivated. Second, there is some reason to suspect that if we try to change the principles of the logic of ground, we end up talking about different propositions involving different operators and nothing has been won with respect to our original question. The thought is this: According to Fine (2017a), propositions can be defined in terms of their exact truthmakers. But to postulate an extraordinary ground of a logical

¹² Cf. Glazier (2017) for more discussion and an alternative approach to explanation by essence.

¹³ A notable variant of this reaction would be to suggest that while no more satisfactory explanations *why* of logical theorems can be had, perhaps other kinds of explanations *wh-* such as explanations *what* can be had. For a recent application of the distinction between explanation *why* and explanation *what* in metaphysics see Skiles (2019).

theorem P in addition to its ordinary grounds is, in effect, to change its set of exact truthmakers.¹⁴ So it seems that we would be dealing with two propositions: The proposition P_1 that only has the ordinary grounds and associated exact truthmakers, and the proposition P_2 that additionally has the extraordinary ground and associated exact truthmakers. But what we were interested in was not an explanation of P_2 , but an explanation of P_1 .

Here, it could be objected that our goal was to find satisfactory explanations for propositions such as [the sun is shining or it is not the case that the sun is shining] and that Fine's theory is simply mistaken about what truthmakers this proposition has. Nevertheless, the following problem remains even if we admit that [the sun is shining or it is not the case that the sun is shining] has extraordinary grounds and associated truthmakers. For what about the proposition, call it P_1 , that according to the objection Fine mistakenly identified with [the sun is shining or it is not the case that the sun is shining], and which shares all truthmakers with this latter proposition except those required for its having extraordinary grounds? Plausibly, this proposition is also a logical truth for which we would like to have a satisfactory explanation, yet by assumption it cannot have extraordinary grounds. Of course, this argument could be resisted by denying that propositions like P_1 exist, but it is not clear to me on which basis.¹⁵

Third, if we revise the logic of ground to be more permissive, logical theorems will have ordinary grounds (those they had all along) and extraordinary grounds (those that are required for the more satisfactory explanations of logical theorems). Then the question arises how extraordinary grounds can be characterized and how the difference between ordinary and extraordinary grounds can be accounted for. In the remainder of this paper, I will primarily pursue the fourth option and try to characterize an explanatory relation on the basis of Yablo's (2014) thoughts about *reductive truthmaking* that allows for a more satisfactory explanation of logical theorems than grounding does. As will become clearer later, most of what I am going to say can alternatively be understood as realizing the third option by conceiving of the newly characterized explanatory notion as a special case of grounding.

5 Empty-base explanation

But before we turn to the proposal proper, let us take a step back and consider the zero-grounding proposal once more. I believe that explanations by zero-ground belong to a peculiar kind of explanation that can be characterized as follows. As we will see, it is the structure of this kind of explanation that makes it particularly apt to satisfy our desiderata for explanations of logical theorems.

¹⁴ See Fine (2017b) on the definition of grounding in terms of exact truthmakers, which, like Fine (2012), captures the idea that disjunctions can only be grounded via their true disjuncts.

¹⁵ See also footnote 19.

Following Schaffer (2017) we can observe that explanations in general consist of three components: First, that which is to be explained—the explanandum or explanatory *result* that P , e.g. that a rose r is red. Second, the explanatory *base*—a set of reasons why P , such as the proposition that r is crimson. Third, an explanatory *link* that connects the base with the explanandum, such as a law of nature or a fact that involves an explanatory notion like causation or grounding, e.g. the fact that r 's being crimson grounds r 's being red. The explanatory base and the explanatory link together make up what is often called 'explanans'. Thus, ordinary grounding explanations (involving a single ground) have this structure:

Base: P
 Link: $P < Q$
 Result: Q

This structure is mirrored by because-claims whose left-hand side expresses the explanatory result, whose right-hand side expresses an explanatory base (or the corresponding reasons why the explanatory result obtains), and which themselves correspond to explanatory links.¹⁶

In ordinary explanations, base and link work together to explain the result, but cases of zero-ground are different. In such cases, there is a proposition that is grounded and a proposition that serves as the grounding-link, but no proposition that does the grounding. Here, the grounding-link does the explaining on its own, without requiring help from any grounds. Hence, the corresponding explanation has this structure:

Base: /
 Link: $< Q$
 Result: Q

In such cases, the explanatory base is empty. Let us therefore call explanations of this kind 'empty-base explanations'. As for the corresponding because-claims that may express empty-base explanations, we use the proposal made in Sect. 3 and employ '... because \emptyset ' or '... just because'.

As I have argued in Sect. 3, the structure of zero-grounding explanations is suitable to satisfy the desiderata for explanations of logical theorems. But it is now clear that it is more generally the case that the structure of empty-base explanation allows for the satisfaction of the desiderata:

Just like the zero-grounding proposal, empty-base explanations more generally promise particularly good explanations of logical theorems in that the relevant explanatory candidates do not involve any reasons why logical theorems obtain for which further explanations could be demanded. According to the empty-base proposals in general, logical theorems are explained in an empty plurality of propositions (i.e. reasons why the relevant logical theorem obtains). Since all

¹⁶ For defenses of these assumptions about 'because' see Schnieder (2011) and Skow (2016).

propositions in the empty plurality are necessarily the case, this, together with an assumption to the effect that the relevant explanatory notion is non-contingent (understood in analogy to the principle of non-contingency of grounding assumed above), also results in logical theorems being necessarily empty-base explained.

Likewise, the necessary status of logical theorems can be read off their proposed empty-base explanations, whereby this proposal also satisfies the first desideratum. Moreover, the special status of being empty-base explained itself can be read off the proposed empty-base explanations: According to the proposal, logical theorems do not only logically follow from the empty set of premises, they are also explained by this empty set of reasons. Lastly, the alternative-logics desideratum can be satisfied by empty-base explanations in general, because such explanations can provide their reasons for logical theorems (i.e. none) even if we assume that, say, $P \vee \neg P$ can be true without either disjunct being true.

So, logical theorems seem to be suitable candidates for empty-base explainability, but if we follow the previous sections, not for zero-groundability. Thus, our question is whether we can find an alternative explanatory relation that provides us with an empty-base explanation of logical theorems. Here, the most salient idea is perhaps to look again at the proposal that logical theorems can be explained by propositions that express their having a certain status. Indeed, I believe that proposals for explanation by status can be understood as empty-base explanations in which the status proposition plays the role of an explanatory link (rather than ground) that can explain the corresponding explanandum on its own, without requiring help from anything in the explanatory base.

To get a grasp of the idea, consider the proposal that metaphysical laws or certain essential truths can play the role of explanatory link in certain explanations.¹⁷ Given this thought, there are explanations that have the following form (let ‘■’ stand for the metaphysical law or essence operator and let ‘ \rightarrow ’ express a suitable conditional):

Base: P
 Link: ■($P \rightarrow Q$)
 Result: Q

Now, using $P \vee \neg P$ as our example and ■($P \vee \neg P$) as a placeholder for a proposition expressing its essential or metaphysical-law status, it is natural to suggest that ■($P \vee \neg P$) does not figure in the base of an explanation of $P \vee \neg P$ (where the link is a grounding fact that connects ■($P \vee \neg P$) with $P \vee \neg P$) but is the link of an empty-base explanation of $P \vee \neg P$ (note the structural similarity to the case of zero-grounding):

Base: /
 Link: ■($P \vee \neg P$)
 Result: $P \vee \neg P$

¹⁷ For a defense of this idea see Kment (2014).

The next step in the development of this proposal would be to provide a characterization of the corresponding explanatory relation suitable to avoid the following obstacles: For the case of metaphysical laws, an account is needed according to which they are sufficiently distinct from grounding—namely such that there are explanations involving metaphysical laws as links that do not correspond to grounding explanations, otherwise the metaphysical law of the form ‘ $\blacksquare(P \vee \neg P)$ ’ would threaten to have a corresponding zero-grounding fact, which is the very thing that we set out to avoid. For the case of essence, one might think that certain essential conditionals can play the role of explanatory links, but this proposal would have to be properly developed; additionally, just as with metaphysical laws, the resulting explanatory notion would have to be sufficiently distinct from grounding. For example, Kment (2014, p. 164) can be understood as claiming that for every explanation e with a metaphysical law or essential conditional as link, there is a corresponding grounding fact that holds between the elements of the base and the result of e . Given this assumption, it would be plausible that an metaphysical law or essential truth of the form ‘ $\blacksquare(P \vee \neg P)$ ’ possesses a corresponding zero-grounding statement.¹⁸

Importantly, this problem of corresponding grounding claims only arises if we want to find an explanatory notion that is truly distinct from grounding. If we on the other hand aim to characterize a range of special, extraordinary cases of (zero)-grounding, no such problem arises. In the future, I hope to further argue that explanations by status should indeed be understood as empty-base explanations and to develop the project outlined in the previous paragraph. Presently, let me instead turn to Yablo’s idea of *reductive truthmaking* and show that it provides us with an explanatory notion that affords empty-base explanations of logical theorems.

6 Explanation by reductive truthmakers

Yablo (2014, ch. 4) distinguishes two conceptions of truthmakers, the recursive conception, and the reductive conception. Here, recursive truthmaking approximately corresponds to our notion of grounding. In particular, disjunctions like $P \vee \neg P$ are recursively made true by the fact that corresponds to its true disjunct. As an alternative to recursive truthmaking, Yablo proposes a notion of reductive truthmaking. Here he is motivated by intuitions like the following:

A disjunction is true [...] because of a fact that verifies one disjunct, or a fact that verifies the other. This does not seem to exhaust the options. Why not a fact that ensures that one disjunct or the other is true, without taking sides? (Yablo (2014, p. 60))

¹⁸ Some discussion on principles connecting grounding and essence is available in the literature, see for example Correia (2013) and Correia and Skiles (2017), but note that it is at least not obvious that these proposals indeed lead to the problem just described.

Consider next a conditional $P \wedge Q \rightarrow P \wedge Q \wedge R$. It owes its truth, on the recursive conception, either to a fact that falsifies P , or a fact that falsifies Q , or a fact that verifies $P \wedge Q \wedge R$. Why not a fact [(like the fact that R)] that blocks the *combination* of $P \wedge Q$ true, $P \wedge Q \wedge R$ false, without pronouncing on the components taken separately? (Yablo (2014, *ibid.*))

To capture these intuitions, Yablo (2014, p. 61) proposes the following notion of reductive truthmakers, defined via his notion of a minimal model.¹⁹

(Minimal model) m is a minimal model of ϕ iff._{def.} m is a partial valuation of the language of ϕ that verifies ϕ and no proper subvaluation of m verifies ϕ .

(Reductive truthmakers) ϕ 's reductive truthmakers (falsmakers) are its minimal models (countermodels), or the associated facts.

This idea needs some amendment and explication: First, the definition of a minimal model has to be fixed: According to Yablo (*ibid.*), the formula $P \rightarrow (P \wedge Q)$ has as a minimal model the partial valuation that assigns truth to Q . But it is not clear how such a valuation verifies $P \rightarrow (P \wedge Q)$, since the truth-conditions for this formula require P to be false or $P \wedge Q$ to be true. But the truth-conditions for these in turn are not satisfied in the proposed model. This problem can be solved by adopting the following definitions:

(Minimal model*) m is a minimal model of ϕ iff._{def.} m is a partial valuation of the language of ϕ such that all its *supplementations* verify ϕ and no proper subvaluation of m is such that all its supplementations verify ϕ .

(Supplementation) A supplementation m^* of a partial valuation m of a language is a (full) valuation of the language such that m is a subvaluation of m^* .

Second, we need to clarify what the facts that are associated with a minimal model are. Here, we only look at a propositional language, so a minimal model (*viz.* partial valuation of the language) is a partial truth-value assignment to atomic formulae. I will further assume that every atomic formula ϕ expresses exactly one state of affairs, and I shall say that such a state of affairs *obtains according to a model* iff the model assigns truth to ϕ . We can then stipulate that the facts that are

¹⁹ An interesting alternative option to treat Yablo's cases would be to determine how Fine's truthmaker semantics would have to be revised to capture these cases. It probably is possible to capture the first case by allowing certain additional truthmakers for disjunctions. Consider for example the disjunction $(P \wedge Q) \vee (\neg P \wedge Q)$. For this particular case, the additional truthmakers of $(P \wedge Q) \vee (\neg P \wedge Q)$ would be the truthmakers of Q , and these are part of the truthmakers of both disjuncts. Interestingly, the second case seems to differ from the first in this respect: If we conceive of the conditional as the disjunction $\neg(P \wedge Q) \vee (P \wedge Q \wedge R)$, we can see that the truthmakers of R that would have to be added to capture Yablo's idea need not be part of the truthmakers of the first disjunct. Additionally, as mentioned in Sect. 4, a rationale would have to be found why this does not leave the original propositions defined by Fine without satisfactory explanations.

associated with a minimal model are the states of affairs that (1) obtain according to the model, and (2) that do in fact obtain.²⁰ An analogous definition can be given for falsmakers and countermodels.

Furthermore, we will use Yablo's convention to refer to states of affairs and facts: p is the state of affairs or fact associated with the formula P and \bar{p} is the state of affairs or fact associated with the formula $\neg P$. Yablo further refers to models by the set of simple states of affairs that obtain according to the model. For example, a model that only assigns truth to P can be referred to using ' $\{p\}$ '.²¹

Third, Yablo sometimes talks as if all the minimal models of a formula themselves are the truthmakers of that formula. Alternatively, we can give a corresponding (perhaps more perspicuous) definition, according to which the states of affairs that obtain according to a minimal model are the reductive truthmakers:

(Reductive truthmakers_{NF}) ϕ is *reductively made true*_{NF} by states of affairs Γ , iff there is a minimal model m of ϕ such that the Γ are the states of affairs that obtain according to m .

This notion of reductive truthmaking is non-factive: it defines a relation between states of affairs and formulae irrespective of whether or not the states of affairs obtain or the formulae are true. In addition to this non-factive notion, we need a factive notion of reductive truthmaking: According to an intuitive understanding of 'making true', only facts can make anything true. For example, $P \rightarrow (P \wedge Q)$ has both $\{\bar{p}\}$ and $\{q\}$ as minimal models, but of course it might be true without either Q being true (namely if $\neg P$ is true) or $\neg P$ being true (namely if Q is true). While both minimal models contain reductive truthmakers in the non-factive sense, we also want a notion to express what *actually* makes the formula in question true. Moreover, the explanatory relation that we want to define using reductive truthmaking, and the notion of 'because' are factive: If Q is not even true, it surely cannot explain why $P \rightarrow (P \wedge Q)$. Therefore, we define a factive notion of reductive truthmaking (to be used in the following unless stated otherwise) like this:

(Reductive truthmakers_F) ϕ is *reductively made true*_{NF} by facts Γ , iff there is a minimal model m of ϕ such that the Γ are the facts associated with m .

So far, we have followed Yablo in defining a notion of truthmaking for *formulae* or *sentences*. To obtain a corresponding notion for *propositions*, we assume that a proposition P is associated with a minimal model m iff there is a sentence S that expresses P and m is a minimal model of S , as defined above. Accordingly, we define that P is reductively made true by states of affairs Γ , iff there is a sentence S that expresses P and S is reductively made true by the states of affairs Γ .

²⁰ If we want to assume that facts are distinct from states of affairs that obtain, then we can say that the facts that are associated with a minimal model are the facts that correspond to the states of affairs that obtain according to the model, and that do in fact obtain.

²¹ Note that we could alternatively omit reference to truth from the definition of a model and let the model assign states of affairs and specify whether they obtain according to the model or not.

With respect to Yablo's motivational examples, the above definitions yield the following results:

- $(P \wedge Q) \vee (P \wedge \neg Q)$ has $\{p\}$ as a minimal model. If p obtains, then $(P \wedge Q) \vee (P \wedge \neg Q)$ is reductively made true by p .
- One of the minimal models of $P \wedge Q \rightarrow P \wedge Q \wedge R$ is $\{r\}$. If r obtains, then $P \wedge Q \rightarrow P \wedge Q \wedge R$ is reductively made true by r .

We can now look at what the proposal says about logical theorems, for example $P \vee \neg P$:

- $P \vee \neg P$ has $\{\}$ as a minimal model. This holds for every logical theorem.

Here, ' $\{\}$ ' refers to the empty model which makes no truth-value assignment. Above we said that the reductive truthmakers of a proposition are the facts that are associated with its minimal models. We can correspondingly say that for a proposition P and a minimal model m of P , P is reductively made true by the facts that are associated with its minimal model m . Consequently, since no facts are associated with the empty minimal model $\{\}$, logical theorems such as $P \vee \neg P$ are reductively made true by zero facts, i.e. the empty plurality of facts.

We have now already arrived at a situation and instance of reductive truthmaking that is clearly reminiscent of zero-grounding – namely reductive truthmaking by zero facts. Some more work needs to be done to arrive at a corresponding kind of empty-base explanation of logical theorems. We do this as follows:

First, we assume that for every state of affairs that obtains according to a minimal model, there is a corresponding proposition that has this state of affairs and no other as a (non-factive) reductive truthmaker, and we say that such a proposition *expresses* its (non-factive) reductive truthmaker. We then define explanation by reductive truthmaking:

(Explanation by reductive truthmaking) For every true proposition P with associated minimal model m , propositions Γ explain P by reductive truthmaking iff the Γ express the reductive truthmakers associated with m , and P does not itself express one of its reductive truthmakers.

For the limiting case in which P is made true by zero facts, we can then say that the empty plurality Γ “expresses” the reductive truthmakers of P , i.e. none. Now since P is made true by zero facts, there is no reductive truthmaker of P that P could express, thus we can say that P is explained (via reductive truthmaking) by the propositions Γ , viz. zero propositions. Under the assumption that explanation via reductive truthmaking so construed corresponds to because-claims, we can state this more succinctly in terms of ‘because’: P is empty-base explained and P holds just because.

Now, the proposal yields the following because-claims:

$(P \wedge Q) \vee (P \wedge \neg Q)$ because P , if P is true.²²
 $P \vee \neg P$ just because.²³

$P \vee \neg P$ and logical theorems in general can be empty-base-explained in this fashion because they are reductively made true by zero facts. As explained in Sect. 4, we can use ‘just because’ to express empty-base-explanations, so for every logical theorem ϕ , we obtain the result that ϕ just because.

At this point, one might perhaps worry whether what we have characterized so far is really an *explanatory* relation that underwrites because-claims and affords explanations why. Note at the outset that it is not quite clear what would constitute a satisfactory response to this worry. I will simply provide some considerations in support of our relation being explanatory.

First, let us see whether the relation satisfies some formal features that explanatory relations are often assumed to possess: The relation satisfies irreflexivity because of the requirement that a proposition P can only be explained (via reductive truthmaking) by the propositions Γ that express the reductive truthmakers corresponding to a minimal model m of P , if P does not itself express one of its reductive truthmakers. The relation satisfies asymmetry for similar reasons: Suppose P explains Q by reductive truthmaking. Then P expresses a reductive truthmaker of Q , say p . According to our assumptions, for Q to in turn explain P by reductive truthmaking, Q must express a reductive truthmaker of P , say q . But by our definition of what it is to express a reductive truthmaker, P has just the single reductive truthmaker that it expresses, so $p = q$. But then Q expresses p , which is its own reductive truthmaker, so according to (Explanation by reductive truthmaking), if P explains Q by reductive truthmaking, then Q does not explain P by reductive truthmaking.

The requirement of transitivity is satisfied because the explanatory structure that results from the proposal is somewhat flat: Propositions corresponding to complex logical formulae are directly explained by propositions corresponding to atomic formulae (or their negations), and in the case of logical theorems, they are empty-base explained. Thus, the situation does not arise in which, for example, an atomic formula P explains a complex logical formula Q , which in turn explains a further complex logical formula R , such that the question could arise whether P explains R .²⁴ While this can cover the logical cases we are considering here, it is in general a

²² Note that $(P \wedge Q) \vee (P \wedge \neg Q)$ is partially grounded in P , if P is true. Therefore, the ordinary grounding account already allows that $(P \wedge Q) \vee (P \wedge \neg Q)$ partially because P . The present proposal on the other hand allows that $(P \wedge Q) \vee (P \wedge \neg Q)$ (fully) because P .

²³ Or, alternatively, ‘ $P \vee \neg P$ because \emptyset ’.

²⁴ One might wonder if the flatness of the explanatory structure is not implausible. For instance, given that P fully explains $(P \wedge Q) \vee (P \wedge \neg Q)$, one might think that also $P \vee \neg P$ fully explains $((P \vee \neg P) \wedge Q) \vee ((P \vee \neg P) \wedge \neg Q)$. But as it stands, the proposal does not deliver this result. As a referee for this journal has pointed out, the present approach also has trouble handling the generalization to infinitary non-modal propositional logic, for it relies on the assumption that any formula with models has minimal models (i.e. minimal partial valuations): Consider a countably infinite set S of semantically independent atomic formulas $\{P_0, P_1, P_2, \dots\}$ and a formula *INF* that in effect says that S has infinitely

question for further investigation whether and if so, how the proposal extends to non-logical cases.

A third, broadly formal feature that explanatory relations are sometimes argued to have is what Yablo (2014, pp. 47f.) calls proportionality. But as Yablo shows, this observation may even identify a particular strength of the reductive truthmaking proposal, since reductive truthmakers seem to have an especially good claim to proportionality compared to ordinary grounds:

Truthmakers, like causes, should not be overladen with extra detail. [...] [Truthmakers] should [...] not incorporate irrelevant extras, in whose absence we'd still have a guarantee of truth. (Yablo (2014, p. 48))

There thus appears to be a kind of explanatory relevance that is captured by the new notion that is not captured by grounding.

Finally, our proposal captures intuitively appropriate explanatory proposals that otherwise would remain uncaptured; we should not forget that with respect to logical theorems, the proposal from reductive truthmaking is supposed to deliver the desired alternatives to grounding explanations. So, let us make explicit how explanation by reductive truthmaking indeed provides more satisfactory explanations of logical theorems than grounding explanation. As we have seen, is not completely straightforward to spell out how in what respect the ordinary grounding explanations seem to be lacking. Yet, explanation by reductive truthmaking provides logical theorems with empty-base explanations with all their special explanatory features that I have been mentioned above.

Here, recall once more the four desiderata for explanations of logical theorems identified in Sect. 3: accounting for the status as necessary truths or logical theorems, modal stability, not giving rise to further (or just very few or not very pressing) why-questions, and compatibility with certain non-standard logics. Satisfaction of the first desideratum might be witnessed by the following reasoning: According to the proposal, logical theorems are explained in the empty set of facts. Necessarily, all facts in this set obtain. Under the assumption that explanation by reductive truthmaking transmits necessity, the necessity of logical theorems follows. Likewise, the explanation is modally stable: Whatever may be the case, logical theorems can be explained in the empty set of facts. Like every empty-base explanation, the explanatory proposal at hand does not involve reasons why its explanandum obtains and hence does not give rise to corresponding demands for further explanations. The empty-base proposal is moreover (given small adjustments) compatible with at least some logical settings in which $P \vee \neg P$ can be true without either of its disjuncts being true: In such a case, an ordinary grounding explanation is unavailable, but $P \vee \neg P$ can still be empty-base explained by reductive truthmaking. For example, in a supervaluationist setting, we can define minimal models as follows:

Footnote 24 continued

many true members, e.g. an infinite disjunction of infinite conjunctions of each infinite subset of S . Then INF has models but no minimal models, since any model of INF can be reduced by dropping its assignment of a truth-value to one member of S . I leave to future research the questions of how forceful these objections are, and whether the proposal can be amended in such a way as to meet them.

(Minimal model*_{SV}) m is a minimal model of ϕ iff_{def.} m is a partial *supervaluation* of the language of ϕ such that all its *supplementations* verify ϕ and no proper *subsupervaluation* of m is such that all its supplementations verify ϕ .

(Supplementation_{SV}) A supplementation m^* of a partial *supervaluation* m of a language is a (full) *supervaluation* of the language such that m is a *subsupervaluation* of m^* .

Here, a (full) *supervaluation* is a set of (full) classical valuations and a partial *supervaluation* is a set of partial classical valuations. Moreover, we define that m is a *subsupervaluation* of m^* iff every classical valuation in m is a subvaluation of a classical valuation in m^* . According to these definitions, $P \vee \neg P$ has an empty minimal model. In the supervaluationist setting, $P \vee \neg P$ can be (super-)true without either P or $\neg P$ being (super-)true. Because it has an empty minimal model, $P \vee \neg P$ is (reductively) made true by zero facts in this case as well.

Given these considerations, explanation by reductive truthmaking appears to be promising with respect to our goal of finding more satisfactory explanations of logical theorems. While I am inclined to treat the developed notion as distinct from grounding, we could (as mentioned in Sect. 4) alternatively conceive of it as a special case of grounding and revise the logic of ground accordingly such that, for instance, a disjunction may be grounded via its disjuncts, or it may be grounded in propositions that express its reductive truthmakers.²⁵

7 Anticipating an objection

Let me anticipate one objection: According to the proposal, some explanatory claims arise that, in a certain light, may seem problematic: For example, suppose that $\neg P$ and Q are the case. Then according to the above proposal it is the case that (1) Q explains why $P \rightarrow (P \wedge Q)$ and (2) that Q explains why $\neg P \vee (P \wedge Q)$ (and analogously for the corresponding because-claims). This can appear intuitively problematic: It can seem that in some sense for Q to explain why, e.g., $\neg P \vee (P \wedge Q)$, Q has to *ensure* that $\neg P \vee (P \wedge Q)$ is being the case. But one may wonder how Q can achieve this, if not together with P . Yet, as stipulated, P is not the case and thus Q cannot ensure that $\neg P \vee (P \wedge Q)$ is the case.²⁶

I propose to respond to this worry by taking a closer look at the notion of ensurance involved in the objection: Apparently, it is closely tied up with grounding, or perhaps it is indeed the notion of grounding. But then the objection appears to miss its mark: Presently, we are trying to find and characterize a different kind of explanatory relation that is distinct from grounding and hence must not assess the explanatory proposals it occurs in in the same way in which we assess grounding explanations. In response to the objection we can then claim that the intuitive doubts arise because of an assessment

²⁵ As mentioned in footnote 11, one rule that would have to be changed is the elimination rule for disjunction.

²⁶ An analogous problem arises for Yablo's example $P \wedge Q \rightarrow P \wedge Q \wedge R$.

of the explanatory proposals as grounding explanations, while in fact they are a different kind of explanation that does not involve grounding.

For this defense to be successful, we should be able to show that the explanatory proposals in question need not appear to be intuitively dubious. Here, talk of ensurance can actually help: While there is a sense of ‘ensurance’ in which the above ensurance-claims hold, there surely is another (not merely modal) sense in which Q alone *does* ensure that $\neg P \vee (P \wedge Q)$: After all, given Q , whether P or $\neg P$ turns out to be the case can appear, in a sense, irrelevant to whether $\neg P \vee (P \wedge Q)$ obtains or not— Q alone already does the job. From this point of view, the intuitive doubts should dissolve. Here, recall also Yablo from above:

Consider next a conditional $P \wedge Q \rightarrow P \wedge Q \wedge R$. It owes its truth, on the recursive conception, either to a fact that falsifies P , or a fact that falsifies Q , or a fact that verifies $P \wedge Q \wedge R$. Why not a fact that blocks the *combination* of $P \wedge Q$ true, $P \wedge Q \wedge R$ false, without pronouncing on the components taken separately? (Yablo (2014, p. 60))

The rhetorical question here invokes the intuition that there is indeed a sense of making true (or ensuring the truth) according to which a fact r (corresponding to R) makes true (ensures the truth of) $P \wedge Q \rightarrow P \wedge Q \wedge R$, even if $P \wedge Q$ is false. This is the sense we set out to capture in the previous section.

8 Conclusion

Let us recapitulate: I have first (Sect. 2) identified several desiderata for explanations of logical theorems that their ordinary grounding explanations fail to satisfy. I have then (Sect. 3) argued that certain extraordinary grounding explanations according to which logical theorems are zero-grounded or grounded in propositions expressing their lawlike or essential status are promising in regard to the identified desiderata, but in tension with certain plausible assumptions about the logic of ground. Next (Sect. 4) I have suggested to find an alternative explanatory notion to explain logical theorems (or alternatively conceive of the newly characterized notion as a special instance of grounding). I have then (Sect. 5) argued that it is the structure of *empty-base explanations* that is apt to satisfy the desiderata for explanations of logical theorems. After having suggested that explanations by status might be fruitfully understood as empty-base explanations and pointing out some tasks for a further development of this proposal, I have then (Sect. 6) developed Yablo’s idea of reductive truthmaking to yield an empty-base explanation of logical theorems by reductive truthmaking that satisfies the desiderata identified in the beginning. While I prefer to conceive of the characterized notion as distinct from grounding, I have alternatively suggested to treat it as a special case of grounding and to revise the logic of ground accordingly. Finally (Sect. 7), I answered an anticipated objection to the proposal.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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