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Throwing spatial light: on topological explanations in Gestalt psychology

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Abstract

It is a well-known fact that mathematics plays a crucial role in physics; in fact, it is virtually impossible to imagine contemporary physics without it. But it is questionable whether mathematical concepts could ever play such a role in psychology or philosophy. In this paper, we set out to examine a rather unobvious example of the application of topology, in the form of the theory of persons proposed by Kurt Lewin in his *Principles of Topological Psychology*. Our aim is to show that this branch of mathematics can furnish a natural conceptual system for Gestalt psychology, in that it provides effective tools for describing global qualitative aspects of the latter's object of investigation. We distinguish three possible ways in which mathematics can contribute to this: explanation, explication (construed in the spirit of Carnap) and metaphor. We hold that all three of these can be usefully characterized as *throwing light on* their subject matter, and argue that in each case this contrasts with the role of explanations in physics. Mathematics itself, we argue, provides something different from such explanations when applied in the field of psychology, and this is nevertheless still cognitively fruitful.

Keywords Topological explanations, explications, metaphors · Kurt Lewin · topological psychology · topology · Gestalt psychology

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1 Introduction

The question of the role of mathematics in science has been extensively discussed, as mathematics plays a crucial role there. Even so, what this role really amounts to remains an object of live philosophical debate. Topological explanations, in particular, have received increasing attention in recent years (e.g. in medicine, biology and the neural sciences).[1] Taking this ongoing discussion as a starting point, the present authors will address the problem of how mathematical notions, especially topological ones, can make a positive contribution to understanding in respect of Gestalt psychology.

Our principal subject matter will be furnished by the ideas of Kurt Lewin, taken from his book *Principles of Topological Psychology* (Lewin 1936), in which that author proposed a topological model of persons. According to this model, a person can be represented as a specific region in the space of life. Between the space of life and the person falls a boundary, which is modelled by Lewin using a Jordan curve. Then, within the person construed as a certain mathematical space, Lewin distinguishes many more regions, ranging from boundary zones closely connected to their surroundings environment to more central inner-personal regions located much further away from the latter and so more difficult to reach from outside. The regions themselves are separated by boundaries: some are connected, forming strongly defined wholes that, in their final form, take on the form of *Gestalts*, and some are connected by paths, these being the connections captured in a Jordan curve. They also influence each other, creating a dynamic system of energy flows.

Even from such a cursory account of Lewin's ideas as the above, one can discern the importance of the role played by such spatial concepts as *space*, *boundary*, *connection*, *change*, *Jordan curve* and *path*. These are hardly methods and concepts of the statistical kind that we have become accustomed to in modern psychology. Rather, they are qualitative concepts of the sort studied in topology—a branch of contemporary mathematics. Topology essentially explores that which remains unchanged during continuous transformations. A circle and a square on a plane count as the same object for a topologist, in that a circle can easily be continuously transformed into a square without tearing it apart or joining together its otherwise discrete parts: one may just imagine it as being made of rubber. Topology, then, is the geometry of rubber—metaphorically speaking! It is thus clear that shape is not important here. But in that case, what is? Well, in topological terms, the common properties of a circle and a square are *qualitative* properties: for example, the fact that both a circle and a square divide a plane into two parts consisting of an inside and an outside, or that their dimension also persists unchanged through continuous transformations. Put simply, all these are topological properties.

Our proposal will be that mathematics can indeed make genuine contributions to understanding certain notions in theoretical psychology at a fundamental conceptual level, and that these will carry with them a deep philosophical significance that itself then calls for thorough analysis from within that discipline. Our hope is that the present paper will also assist those seeking to understand in a more general way how mathematics enters into scientific theorizing, and what roles it can play—especially in terms

[1] Cf. Kostić (2018a) (Introduction to a special issue of *Synthese* on topological explanations) for an overview. See also Kostić (2018b) for a more detailed disscussion.

of the thought that the latter can be not just *explanatory*, but also *explicatory*. With such goals in mind, we shall describe the general phenomenon of the conceptual and cognitive contribution of mathematics to this subject matter by invoking, as suitably illustrative, the concept of *throwing light on something*. Mathematics, and in particular topology, has the potential to serve as a common unifying framework for diverse approaches (notably phenomenological and neuroscientific ones), in that it focuses on the very structure of the problems being addressed, as distinct from details pertaining to particular mechanisms and more localized features.

The paper falls into three main parts. We begin in Section 2 with a presentation of the topological way of thinking as manifested in Lewin's own approach to the issue of personhood. We discuss examples of the use of spatial concepts in Lewin, as well as briefly examining what topology itself is. In order to furnish a proper context of understanding, we also point to the role of topology in formal ontology, which constituted a fundamental science for Husserl. Section 3 then addresses the role of mathematical explanation in general, touching upon (a) explanatory, (b) explicatory, and (c) heuristic-metaphorical roles played by mathematics in the context of science. In Section 4, using ideas from Kazimierz Twardowski and Tarja Knuuttila,[2] we introduce the concept of *throwing light on something*, and juxtapose this with both explanation and metaphor. As an example of explication, we present the notion of the centre of a person as construed by Lewin, and as an example of metaphor, we put forward his conception of schizophrenia. Our thesis will be that both explication and metaphor are only possible in these contexts because Lewin has shed a distinctively spatial light on personhood. Finally we, present our overall conclusions.

2 The topological mode of thinking in Lewin's approach to personhood

2.1 The problematic nature of mathematical explanations in psychology

Generally speaking, the question of what persons amount to locates itself on the border between psychology and philosophy, and specifically philosophical uses of topology could prove especially inspiring and important for our current investigation. Following Lewin (one of the most prominent psychologists of the twentieth century[3]), we shall be viewing this problem from an essentially non-reductive phenomenological perspective. Our focus will therefore be on trying to understand certain psychological notions as they show up in their own right (without engaging in any form of hypothetical explanatory reduction invoking underlying causes, no matter whether these be

[2] We wish to thank an anonymous reviewer for directing our attention to (Knuuttila 2017). This paper allowed us to better define the process of *casting light on something*.

[3] Kurt Lewin was ranked 18th on the list of the most prominent psychologists of the twentieth century drawn up by the Review of General Psychology—the journal of the American Psychological Association. "Eminence was measured by scores on 3 quantitative variables and 3 qualitative variables. The quantitative variables were journal citation frequency, introductory psychology textbook citation frequency, and survey response frequency. The qualitative variables were National Academy of Sciences membership, election as American Psychological Association (APA) president or receipt of the APA Distinguished Scientic Contributions Award, and surname used as an eponym" (Haggbloom et al. 2002, p. 139).

neurophysiological, biological, chemical or physical), and on capturing the possible role that mathematics might be found to play therein.

Certainly, we should not expect to see such spectacular applications of mathematics in psychology as in physics, where powerful mathematical techniques are deployed to unify extensive and wide-ranging results, predict phenomena, and arrive at precise and empirically testable quantitative characterizations. In this regard, it seems reasonable to think that each field of science will inevitably afford its own distinctive level of accuracy. To be sure, the theory of persons does not present itself as an obvious candidate when it comes to the debate about the epistemic role and strengths of mathematics, but Lewin's approach to persons is directly dependent on topological (i.e. mathematical) concepts. Lewin also employs some equivalents of other scientific concepts, such as *field* (he is, in fact, widely known for his *field theory*), *force*, *energy* and *permeability*—though topological concepts are particularly important for him in the context of the problem of mathematical explanations. Pursuing such an unobvious application, then, may hopefully contribute to enriching our understanding of the wider problem of the epistemic significance of mathematics, even as it opens up novel perspectives on how mathematical concepts can be applied to an otherwise seemingly non-mathematizable approach in psychology.

2.2 The spatial mode of thinking in Lewin's theory of persons

When we think about persons in a psychological context, temporal aspects show up as important in many respects. To think of consciousness is to think of something distinguished by its own specific temporality, in that our mental acts themselves unfold in time. However, Lewin claimed that what determines the structure of the person are not temporal considerations, but rather *spatial* ones. The person is not a simple and homogeneous object, and as such possesses parts—parts which in turn exhibit a spatial character.

Lewin offers some examples of spatial representations within real, physical space (Lewin 1936, p. 41–50). The first concerns a prisoner and a cell. The prisoner is separated from what is outside their cell, or from what is outside the prison. The space of their bodily movement is confined to just what can occur within the space of the prison (the cell, corridor and courtyard), and as such is strictly limited. However, they may well think about what is going on outside the prison—about their family and friends—and may meet up with their family. Thus, it may be said that as long as they are thinking about something outside of the prison, the space of their mental and social life is not as limited as the space of their bodily movement: “The solidity of the boundary of the prison is different for bodily, for social, and for mental locomotions” (Lewin 1936, p. 44).

The notion of space appears in several contexts in Lewin's theory, and one of its more important applications takes the form of the notion of a *life space* (Lewin 1936, p. 216). This is the totality of possible events capable of determining the behaviour of a person. It takes in both the person and their environment. The first approximation of a person we can have will be that of an undifferentiated region in such a life space. However, as we mentioned earlier, a person is not a simple and homogeneous object: it contains many parts, which Lewin calls *regions*—so we might *model* the person as a system of such regions.

For example, if a person's dream is fulfilled, some of its other parts will change, while some will remain untouched; if one becomes a parent due to an expected child being born, this might not affect one's work efficiency, but could have a major impact on one's family life. These can be linked by dynamic relationships: certain changes in one region may or may not cause changes in another one. Taking into account all possible changes to regions caused by a change of state in one of them, Lewin arrived at a certain mathematical structure: every region comes with a set of neighbouring ones—i.e. those which could, possibly, be influenced by it. Lewin refers to the system of such changes as one of *dynamic dependency* (Lewin 1936, p. 168), and formulates the following definition:

> *a* and *b* are parts of a connected region (region of influence) if a change of state of *a* results in a change of state of *b*. (Lewin 1936, p. 169)

This, of course, is not unconnected with his clarification of the well-known concept of *Gestalt*:

> Gestalt: a system whose parts are dynamically connected in such a way that a change of one part results in a change of all other parts. (Lewin 1936, p. 218)

2.3 The person as a topological space

Different types of change will result in one and the same region's exerting an influence on different groups of regions, where this fact can then serve as a basis for defining the topology of the person as the entire system of regional dependencies associated with all possible changes of this sort. In topological terms, this is a *neighbourhood system*, and it is thus that a person comes to be represented as a topological space. In Lewin's words:

> The concepts which we have developed in this book concern the whole psychological life space, that is, person and environment. They allow treatment of all problems of position and connections of the life space and its parts. They are applicable to quasi-physical as well as to quasi-social and quasi-conceptual facts. By means of these concepts one can represent the structural changes of person and environment and all kinds of locomotion. To a certain extent one can also deal with those problems of psychology that are dynamic in a narrower sense of the word, for instance the friction of a region, the solidity of a barrier with respect to locomotions, the degree of separation of regions with respect to communications of different kinds, and the degree of wholeness (dynamic Gestalt) of systems of the environment and the person. Finally one can treat certain problems of tension and changes of state of regions, for instance the liquidity or solidity of a region. (Lewin 1936, p. 204–205)[4]

[4] In (Kirchhoff, Kiverstein 2019) the authors determine a boundary for the mind by using the Markov blanket formalism. It is interesting to observe that the Markov blanket formalism is also used for analyzing topological properties of certain causal networks.

Space in mathematics may consist of any type of objects: stretched lines, planes, sequences, functions, and also regions of the person. The space can be simple or highly tilted, compressed or stretched, flat or multi-dimensional, metrizable or not, etc. The general concept of a space turns out to be most important for Lewin, not only in mathematics and the natural sciences, but also in psychology itself.

Other examples given by Lewin include the use of spatial and topological concepts as real psychological concepts: the extent of what a child is allowed and what their own abilities permit them to do, the freedom to move around in a social space of rich and of poor individuals, approaching some goal or, say, fleeing from another person. If the goal of a child, say, is to become a physician, then "the 'path' to this goal […] leads through definite stages: college-entrance examinations […], college […], medical school […], internship […], establishing a practice" (Lewin 1936, p. 48).[5]

2.4 Topology and Husserlian formal ontology

Topology is a far-reaching generalization of geometry. A broad-brush approximation could be given by saying that its subject-matter consists of those properties that remain unchanged under continuous transformations. Consider a rubber ball and its continuous transformations, such as squeezing, stretching, twisting, etc.—but not tearing, gluing or drilling holes. What will remain untouched by such transformations? The volume of the ball and its shape will change. But that it (1) is in one piece, (2) is three-dimensional, (3) has no holes, and (4) has an interior, an exterior and a boundary—all these are features that will not change under continuous transformation. All of them are examples of properties explored by topology. Topology is a far-reaching generalization of these intuitions from the "geometry of rubber", and examples of topological invariants (topological properties) studied in a general kind of way are, for example, *connectedness* (and its variants, such as *path-connectedness* or *local connectedness*), *compactness* (and its variants), *separability*, *density*, and many others. (It is of interest to many non-mathematicians because its subjects are qualitative properties, rather than the quantitative ones with which mathematics is commonly but erroneously identified). Thus, from a topological point of view, a square and a triangle are the same object. They differ in shape, but preserve other important properties: for example, both of them divide a plane into two connected parts: an outside and an inside. This is, in fact, a qualitative property. A well-known joke about topologists describes them as people who cannot distinguish a coffee mug from a donut, since one can be continuously transformed into the other.

Topology might well be regarded as the science of general aspects of space as such. That is why it is a kind of formal ontology in Husserl's sense—in that it can be realized across multiple fields where spatial aspects are under investigation. It permeates all of modern mathematics, and also serves as an indispensable tool for physics, chemistry, biology and many other areas of science. It also comes as no surprise, we think, that

[5] The notion of social space is one of the concepts of social neuroscience. And spatial cognition might be relevant for social navigation. The problem is investigated in (Tavares et al. 2015), and according to the authors "The results predict that an impaired geometric representation of social space in the hippocampus may accompany social dysfunction across psychiatric populations." (Tavares et al. 2015, p. 240).

topology shows up as inherently present in psychology and philosophy,[6] and in particular phenomenology.

One of the places where phenomenology meets topology is the part-whole theory of Husserl, presented in his *Third Investigation*, which is entitled *On the Theory of Wholes and Parts* (Husserl 2001, p. 1–46). Husserl constructs a general theory for this, in which the basic concept is that of *foundation*: "If a law of essence means that an *A* cannot as such exist except in a more comprehensive unity which connects it with an *M*, we say that an *A as such requires foundation by an M* or also that *an A as such needs to be supplemented by an M*" (Husserl 2001, p. 25). If an object is founded upon a certain moment, then we can say that this object is non-independent. Husserl examined the relation of non-independence very carefully, and discovered many things about it. For example, he noted that this relationship is a transitive one, in that "[a] non-independent part of a non-independent part is a non-independent part of a whole" (Husserl 2001, p. 26). If colour saturation is a non-independent part of colour, and colour is a non-independent part of a whole, then colour saturation is a non-independent part of that whole. Along with this relation, he introduced a number of formal-ontological concepts of fundamental importance: *moment*, *abstractum*, *whole*, *unity*, *concretum*, *piece* (*portion*), *thing*, etc. All these concepts were then taken up and used by his students. Roman Ingarden made use of Husserl's considerations pertaining to the phenomenon of *independence* in his *Controversy over the Existence of the World* (Ingarden 2013), in particular in his existential ontology.

For a contemporary mathematician studying Husserl's writing, it is fairly clear that behind his investigations lie topological ideas. This has been shown directly by Kit Fine (2006), who has employed topology to formalize Husserl's theory of part-whole relations. It is also significant that Husserl himself indicated that his theory stood in need of formalization, and it turns out that Husserl would have had some reasons of his own to be mindful of topology, as Smith explains:

> That Husserl was at least implicitly aware of the topological aspect of his ideas, even if not under this name, is unsurprising given that he was a student of the mathematician Weierstrass in Berlin, and that it was Cantor, Husserl's friend and colleague in Halle during the period when the *Logical Investigations* were being written, who first defined the fundamental topological notions of open, closed, dense, perfect set, boundary of a set, accumulation point, and so on. Husserl consciously employed Cantor's topological ideas, not least in his writings on the general theory of (extensive and intensive) magnitudes which make up one preliminary stage on the road to the third *Investigation*. (Smith 1994)

3 The possible roles of mathematics

We can distinguish many different more or less precisely given senses in which mathematics can contribute to understanding in some particular area or other. This

[6] Examples of the use of topology in philosophy can be found in Fine (2006), Kaczmarek (2019a, b), Mormann (1995, 1997, 2013), Schulte and Cory (1996), Skowron (2017) and Smith (1994).

applies to science, and to philosophy as well—in that many philosophical problems are analysed in contexts informed by technically specialized results. Such findings (i.e. notions, theories, etc.) do not solve philosophical problems, but they do often allow us to view the latter from new perspectives and gain new insights—so that we can, in connection with this, speak of "seeing a problem in the light of certain results". Of course, this expression is somewhat vague and indefinite—but we might nevertheless wish to claim that it captures the kind of relationship that obtains between some mathematical results, definitions, or theories on the one hand, and some philosophical notions or questions on the other. Few people will claim, for instance, that the model-theoretic definition of satisfaction (Tarski) resolved the problem of the nature of truth once and for all, but it certainly sheds some light on this topic. So the use of mathematical notions and theories can serve to enhance our understanding of a given domain. Also, more particularly, it may contribute to the latter by enhancing the expressive power of our language.

We might wish to think of the following as roles potentially played by mathematics in relation to science generally and psychology in particular:

1 Explanation;
2 Explication;
3 Metaphor.

These three are different, but have something in common in that they all fall under the general category of *throwing light on something*. Moreover, two essentially distinct factors can be distinguished here: a passive and an active one. The passivity of casting light on something consists in our uncovering global structural similarities between entire fields without adding anything, while its active element is what this illumination itself brings to the subject matter under consideration. To use some illustrative language, when we throw light on a thing, we illuminate a certain side of the whole object, pointing to what was not previously visible. Displaying a certain side instead of another one, we emphasize its importance. We then focus our attention on the content that we had not previously distinguished. Moreover, it is clear that if we throw—metaphorically speaking—a red light, then the object will be seen as red, if a green light, then the object will be seen as green, and so on. The casting of light therefore adds new content to the object, which the object did not previously possess. This is what we actively and creatively contribute to the object when we throw light on it.[7]

In particular, topology contributes to our understanding of psychological concepts and problems, as it provides fruitful methods for conceptualizing certain vaguely understood notions. Moreover, even if it should turn out that we cannot directly translate topological notions and theorems into psychological language, they can still be heuristically useful and inspiring, and encourage us to formulate new psychological hypotheses, thanks to the distinctively spatial light that has been shed on them.

[7] The process of throwing light can be seen as a process of increasing conceptual order. If we put it that way, the process of throwing light can be added to the list of mechanisms increasing the conceptual order catalogued by Edwin Hutchins (2012).

3.1 Mathematical explanations in science

The role of mathematical explanation in the natural sciences has received much attention, especially recently. We all would like to know whether mathematics itself has some inherent explanatory power (when applied to physics, or in relation to science in general), or whether it only serves as a means of capturing and expressing some (causal) relationships between physical phenomena. According to many authors, there are examples of such explanations in science: i.e. examples of explanations where it is not scientific laws, but rather truths of mathematics (theorems) that are doing the real explanatory work. A familiar and much-discussed example is provided by the bridges of Königsberg: the fact that they cannot be crossed is explained by a simple theorem in graph theory, not by citing any physical properties of the system of bridges. Another frequently considered example is the periodical life-cycle of cicadas (13 and 17 years), introduced by Baker (2005): it is a striking fact that these cycles are prime numbers, so the claim is that to explain this phenomenon we must take note of the properties of prime numbers. (Hence, the explanation has a purely mathematical character, rather than a biological—or, more generally, a causal—one). An interesting example of a relevant biological phenomenon is the hexagonal structure of honeycombs, it being claimed that the proper explanation for this is provided by the Honeycomb theorem (Hales 2000, 2001) stating that hexagonal tiling is optimal with respect to total perimeter length. (Thus, returning to biology, the bees minimize the amount of wax needed for their work, where this proves evolutionarily advantageous.) Examples in physics, moreover, are abundant. Lange (2013) discusses the phenomenon of a pendulum's possessing exactly four equilibria, concluding that this is explained by a topological theorem on functions defined on a torus. The Borsuk-Ulam theorem,[8] meanwhile, offers a mathematical explanation of the fact that there are always two antipodal points on the surface of the Earth, where temperature and pressure are the same (cf. Baker 2005, 2009; Baker and Colyvan 2011). Hence, the topic has been much discussed, and the problem of whether there really are mathematical explanations in science, and what makes them so special, remains a pressing one.[9]

For our purposes, the notion of *topological explanation* is of particular interest, given the prominent role that topological notions play in Lewin's approach. Indeed, it has become an object of increasing attention in medicine, (evolutionary) biology and neuroscience. According to Kostić:

> [T]opological explanations explain by reference to structural or mathematical properties of the system (e.g. graph-theoretical properties, topological features, or properties of mathematical structures in general), and abstract away from the details of particular causal interactions or mechanisms. (Kostić 2018a, p. 2).

Many such investigations focus on topological graph theory—an approach (initiated in Watts and Strogatz (1998)) that explores the topological properties of graphs and

[8] The theorem states that for any continuous function f from a sphere into R^2, there will be two antipodal points x and y, such that the value $f(x) = f(y)$.

[9] The literature on this is vast, so that even just giving a general overview would exceed the scope of this study.

networks. What is distinctive here is that the explanatory input itself comes from mathematics, and especially topology.

In our view, it is the concern with structural properties that makes the topological account especially attractive, as topology can provide a conceptual basis for identifying some very general features of our perceptual (or, in general, phenomenal) experience on the one hand, and of the neurophysiology of the brain on the other.[10]

3.2 Mathematical explications in science

This notion of mathematical explication has been extensively discussed, and we shall focus here on Carnap's account, which can be taken as paradigmatic.[11] He describes the function of explicatory notions in the following, general way:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an explication for, the earlier concept. (Carnap 1947, p. 7–8)

In fact, a large part of philosophical analysis and scientific work can be viewed as being engaged in procedures for furnishing such explications. (See Brun (2016) for more examples and analysis). These can take various different forms, and one of the possibilities is to do this by formulating postulates. As an example, consider those of quantum mechanics. These are not meant to furnish a framework for testing that would enable one to actually *grasp* the ontological nature of quantum states—in fact, they might even be viewed as *metaphysically agnostic*. Yet they provide us with some formal counterparts of the notions involved, and explain how these should be incorporated into the system of scientific concepts. A topological explication of psychological concepts might equally be viewed in this way, in that it aims to capture the abstract structure of things rather than any specific causal dependencies.

3.3 Mathematical metaphors and their heuristic role

The general problem of metaphors in science, not to mention metaphors in general, is a vast subject, so we must be highly selective in our treatment here. We shall focus on some specific accounts in the philosophy of mathematics that make use of this notion. One example is the conception of Yablo (2002a, b), who views

[10] Duch (2018, p. 15) discusses the relationship between psychology and neurophysiology, and mentions some neurophysiological phenomena, notably claiming that "noise in the system and new stimuli may push the system out of the attractor basins. This process has been described in Lewin theory in terms of psychological forces that act on life energy field changing its state".

[11] Brun (2016) offers an interesting survey and analysis, mentioning many examples of such explicatory concepts (such as *knowledge*, *valid inference*, *justice*, etc.). In addition, Leitgeb (2013, p. 271) points out "three paradigm case examples from the 1930s, 1950s, and 1970s, respectively: Alfred Tarski's explication of *truth*, Carnap's own explication of *confirmation of hypotheses by evidence*, and Ernest Adams's explication of the *acceptability of conditionals*".

mathematics as a kind of *make-believe game*—in that our use of it resembles a game as and when we pretend to believe certain claims. Basing his theory on Walton's account of metaphor, the aforementioned author has put forward some fairly strong claims concerning the similarity between mathematical sentences and metaphorical utterances. According to Walton, the employment of make-believe "is useful for articulating, remembering and communicating facts" (Walton 1993, p. 40). An important notion here is the *principle of generation*—which is a kind of bridging law linking internal facts within the make-believe game with reality. For a simple example, consider what happens when children play, and one of them is the king of France: a claim such as *The king of France is injured* will show up as being perfectly intelligible within the realm of pretence, even though there really is no such king (and the supposed king is obviously not in fact injured).

Spivey's account presented in his book *The Continuity of Mind* (Spivey 2007) is an interesting example of a *metaphorical modelling*. The author recognises that the three systems originally treated as distinct: perception, cognition, and action are not separated, but overlap and form one continuous mental flow. The notion of metaphor is used frequently: "You will find throughout this book that I tend to mix my metaphors between dynamical systems and information processing terminologies." (Spivey 2007, p. 138). His account of mental phenomena is based on notions from dynamical systems theory—phase space, trajectory, attractor, basin of attraction etc., which also have a *par excellence* topological character. They are used to express novel claims about mental phenomena, but their use might be considered to be metaphorical, even in the case of the fundamental notion of the phase space. Spivey gives some metatheoretic discussion concerning the use of metaphors, indicating that metaphors are used when the target domain is too complex—and:

> we can import a richer understanding of other similar and somewhat simpler source domains, for example, dynamical systems theory and attractor networks, to provide descriptions and explanations of how the mind might function. Done properly, this requires a cyclic interplay between empirical predictions made by the metaphor/model, and results of those empirical tests being used to improve the metaphor/model. (Spivey 2007, p. 33)

Spivey's aim is to produce a model which also has predictive power and is not just a metaphorical description. Nevertheless, mathematical metaphors play an important role in his approach.

4 Throwing light on something: Discussion of the concept

In his analysis of the structure of persons, Lewin himself made use of several topological notions, including *regions' boundaries*, *boundary zones*, *connected regions*, *separated regions*, *paths*, *dimensions*, and so on. He identified some important structural properties of persons that resemble structural topological properties. Exploring further levels of this same analogy, he made it possible to formulate some claims concerning the nature or structure of persons, and in this sense we can say that he threw *new*, and distinctively *spatial*, light on the question of what a person is. Throwing new light on something, thus construed, means uncovering structural (i.e. global) properties that are shared with some other, unexpectedly comparable domain, where this allows us to identify a previously unnoticed property of the original one. Such a property must, of course, bring something significantly new to our understanding—otherwise, there would be no new light and little reason to attach a positive value to this procedure.

Lewin noted how certain topological notions can be applied in order to describe properties of persons, and some of his intuitions have been experimentally confirmed (for a discussion see (Roeckelein 1998, p. 305)). For example, if two regions are separated, they cannot be connected, and if one region of a person belongs to a higher dimension than some others, this will mean that its boundaries also belong to that higher dimension. It is important that showing (exhibiting) something in a new light is a global process that takes with it the entire network of connections: we might say that it is not possible to throw new light on just a very small part or aspect of any given issue—a certain minimum of engagement on our part is required. It might lead to making some sort of highly significant conceptual contribution.[12]

Casting, throwing or—if one prefers—shedding new light on something involves taking over connections from some other domain, and so is a creative process. This is as much as to say that it creates some new content pertaining to a previously unmanifested subject-matter. In his theory of cognition, Kazimierz Twardowski (1977, p. 79) claimed that there are no absolutely adequate representations: i.e. that we are always losing something from the content of the object and adding something to it. Therefore, this is not only a process of finding certain forms in a given object that are actually there within it, but also a matter of creatively inserting (or even imposing) a certain content into (or onto) the object under investigation.[13] In the case of Lewin's theory of persons, the creative input consists in the introducing of a *spatial* light and accompanying topological notions, where these allow us to exhibit important connections within the life of the person.

Throwing light on an object is not a completely arbitrary process. It is to cast a specific light on a specific object for a specific purpose. A novel representation of an object, such as is created after it has been illuminated anew, is a representation serving a predetermined purpose. This representation is, in fact, an epistemic artefact that can be transformed, developed, or even used for an entirely different purpose than its originally intended one (cf. Knuuttila 2017). Knuuttila has proposed that the models used in science should be considered epistemic artefacts of a specific kind: ones that enable both the creation of new knowledge and its embodiment (by means of appropriate media)—together with its transfer. Hence, on the basis of her ideas, it can be said that throwing new light on an object is tantamount to creating a new epistemic artefact that, though not itself absolutely cognitively transparent, nevertheless determines (for

[12] This notion is not perfectly clear, but it captures an important aspect of the use of mathematical methods in philosophy and science. We cannot say, for instance, that metalogical limitative theorems (Gödel's theorems, the Skolem-Löwenheim theorems, the unsolvability of certain problems (e.g. the halting problem), and many others) apply to the problem of our epistemological limitations in a direct way, or that they can be straightforwardly applied to generate a ready solution. But they do nevertheless shed new light on the subject, and are of essential value to the discussion.

[13] Hence, this is—generally speaking—a point of view that might well be accepted by Kantians, as Kant's critical philosophy paid attention to the creative aspect of cognition.

instance, through representational embodiments) what can be conceived. It thus determines a new horizon of possibilities for thinking about the object on which it falls.

Aiming to get to know some object ever more effectively, we survey it from an ever closer position. Just as we want to take a picture from a distance, but do not want to lose any details, so we often use our camera's zoom function. This allows us to zoom in, focusing on a smaller part of the previous image and seeing more details as a result. But it also allows us to zoom out and display a wider picture with less detail. The operations of throwing light on something and zooming in and out are different, though strongly linked.[14] Lighting contains an active factor that introduces new content to the object being illuminated. Lewin, by casting a *spatial* light on personhood, has somehow added something to our understanding of persons that we had not seen before. Nevertheless, after becoming acquainted with Lewin's concept, it seems quite natural for persons to be thought of as constituting separate regions in the space of life. But it is only so, *given* Lewin's already having shed light on it of this distinctively spatial sort. Once it has been illuminated as an object, such zooming in and out is possible, but otherwise not. Hence, any zooming in and out presupposes the existence of the light thrown upon the object, and it can therefore be said—to use Husserlian ontological language—that zooming in and out is an operation *founded on* the latter. The casting of light is thus what is ontologically prior, and it is only this that makes it possible to zoom in and out.

4.1 Mathematical explanations in (Gestalt) psychology?

One important feature of topological explanations (in contrast, say, to mechanistic ones) is that they allow us to abstract from the causal structure of the underlying mechanism involved and concentrate instead on its formal mathematical features, which themselves are taken to furnish explanatory input. Consequently, it is not necessary to trace the underlying (neurophysiological, etc.) mechanism, and what matters instead is that we furnish a good mathematical conceptualization and description. If we agree that this also constitutes *explanation* in psychology, then we can treat the presence of topological notions in Lewin's account as fulfilling an explanatory role.[15]

Nevertheless, we acknowledge that this might be viewed as problematic, given that the examples of mathematical explanation in physics that are usually discussed involve a direct application of mathematical theorems (and not just a formalization of informally disclosed notions). There, what we see is that theorems are important ingredients of the *explanans*. The difficulty is that nothing similar can be observed in the case of *topological psychology*.

Here, in fact, we encounter a more general problem pertaining to the possible explanatory role of mathematics in psychology. It is fairly indisputable that there are vast differences between physics and psychology in its present state (not to mention

[14] We wish to thank an anonymous reviewer for indicating the relevance to our research of the metaphor of zooming in and out.

[15] Cf. the opinion expressed in Darrason (2018, p.147), who gives an overview of topological explanations within medical genetics and network medicine, claiming that "topological explanations in network medicine can help solving the conceptual issues that pure mechanistic explanations of the genetics of disease are currently facing". Here, the impact on *conceptual* issues is stressed. And in this sense topology (and mathematics) can obviously contribute to a better understanding.

psychology as it was in the 1930s), where these make the application in psychology of such complex mathematical methods as we see employed in physics virtually impossible. It seems obvious that we cannot expect a predictive role to be fulfilled in any comparably strict sense to what goes on in physics. Therefore, our position is that mathematics, at least for the present, cannot play an *explanatory* role in psychology in any sense similar to that which it performs in physics.[16] In the latter, we can find natural areas of application for highly non-trivial forms of mathematics (examples abound!), but it is hard to imagine how a theorem on, say, the spectra of operators on Hilbert spaces could have any plausible bearing on psychological representations or be needed to explain some psychological phenomenon or other. It is quite probably the case that the range of mathematical theorems that might be interpreted as directly expressing non-trivial psychological theses (or even as having plausible psychological counterparts) is rather limited.

But regardless of how one construes the notion of *mathematical explanation in psychology*, mathematics (and topology, in particular) can be of great epistemic value to psychology in other ways: one important role is that of providing a framework for conceptualization, where mathematics can offer just the tools we need. So, even if it has only very limited explanatory power, it can still have great *explicatory power*, as well as performing an important heuristic role.[17]

Our expectation, then, is that the use of topological notions will chiefly enhance the possibilities for arriving at and formulating interesting claims about the structure of persons by helping us to explicate concepts (and by furnishing inspiring metaphors). In particular, we think that such notions are very well suited to expressing some of the distinctive intuitions and proposals of the *Gestalt* approach, as many of the topological notions concern global properties of systems, which cannot be *retrieved* from just their local features. Moreover, what makes these contributions possible is itself an intriguing philosophical question to ask.

4.2 An example of explication: The Centre of the person

In Lewin's theory, persons are not mere simples: they exhibit internal structure, and we must therefore seek adequate conceptual tools in order to make the differentiations needed for grasping the latter. Our proposed example concerns the intuitive concept of the *core* or *centre* (or perhaps, even, the *heart*) of the person. We have an everyday experience that certain parts of our personality are more important than others. The most important parts form the core of the person—they are its centre. It is necessary to clarify these intuitive concepts. Lewin undertakes this clarification, which is de facto an explication in the sense of Carnap; however, explication of the latter sort is only possible in the wake of the construction of a global analogy encompassing all of its

[16] Of course, there are advanced mathematical models in use in mathematical psychology: e.g., when we try to describe risk aversion quantitatively, or focus on neurophysiologial *hard data*, or when statistical correlations are being analysed. But our own focus here is rather on theoretical descriptions of certain fundamental questions, and as was mentioned before, we are approaching the theory of persons from a non-reductive perspective.

[17] In Kuś and Wójtowicz (in preparation) the role of the theory of dynamical systems in embodied cognition is investigated, with similar conclusions: their explanatory role is very limited, while they might play an fruitful heuristic role, enhancing understanding of the problem.

consequences, especially as regards our taking over of some network of conceptual connections.[18] Thus, what we are dealing with here is a process of throwing light on the subject matter in question, where this is what makes such explication possible—much like the metaphor of zooming in or out.

Lewin offers the following general description of the structure of the person:

> The person is to be represented as a connected region which is separated from the environment by a Jordan curve. Within this region there are part regions. One can begin by distinguishing as such parts the 'inner-personal' regions (*I*) from the motor and perceptual region (*M*). The motor and perceptual region has the position of a boundary zone between the inner-personal regions and the environment […]. (Lewin 1936, p. 177)

The motor and perceptual regions include speech, hearing, seeing, etc. They are responsible for contact with what is outside the person. Hidden more deeply, though, there are also more central parts of the person. To reach these, one must penetrate further regions. Within the inner-personal regions Lewin distinguishes inner-personal peripheral parts of *I* that are closer to motor-perceptual regions, and more central parts of *I* that are further from the perceptual parts. But how might one distinguish these central parts without resorting to metaphors, so that one employs instead the sort of strict language that will also prove useful when designing psychological experiments? Here is Lewin's answer:

> It is of great general importance whether a psychological process belongs to more central or to more peripheral strata. Dembo's experimental investigations […] on anger have shown the significance of this factor for emotions. If only peripheral strata of the person are touched, manifestations of anger occur more easily. The outbreaks of anger are then more superficial. If more central strata are involved an open outbreak of affect is more rare. (Lewin 1936, p. 180)

It thus turns out that manifestations of anger show how deeply located the affected part of the person is. But where does such a connection come from? Lewin responds to this as follows:

> Indeed the boundary zone between the central strata […] and the environment (E) is stronger than the boundary zone between the peripheral strata (p) and the environment. (Lewin 1936, p. 180)

Furthermore:

> The peripheral strata come more easily into connection with the motor region to which they lie closer. Therefore expression usually occurs more readily when

[18] Carnap (1950, p. 7) issues the following demand: "The characterization of the explicatum, that is, the rules of its use (for instance, in the form of a definition), is to be given in an exact form, so as to introduce the explicatum into a well-connected system of scientific concepts".

> events of more peripheral strata are concerned. One speaks about personal matters only under special circumstances. (Lewin 1936, p. 180)

The notion to be explicated here (the *explicandum*) is *the centre of the person*. As an *explicatum*, Lewin uses the notion of *parts of the person that lie far from the motor-perceptual region*. These two exhibit genuine similarity, as required by Carnap. The *explicatum* is embedded in a well-connected system of scientific concepts: namely, the construction corresponding to the overall framework of Lewin's topological psychology. More precisely, it figures in the psycho-empirical nomological claim that *if more central strata are involved, an open outbreak of affect will be rarer*. Of course, the *explicatum* cannot itself be overly complex—it should rather be a clear and relatively simple concept. So Carnap's demands for being an adequate *explicatum* are fulfilled.

It is worth noting, moreover, that this explication of the concept of the centre of the person allows for new experiments to be designed, and permits us to describe states of a given person regardless of whether they happen to be affected by some sort of tension or not (for details, see Lewin 1936, p. 181). It also affords a description of the current state of the person that may then count as a part of their self-knowledge. Thus, the procedure maybe said to be cognitively fruitful (where this is considered an important and desirable feature). Of course, the behaviour of a person cannot be predicted with the degree of precision characteristic of the natural sciences when predicting how material objects will behave—but that is because persons *per se* only allow for so much accuracy. What really counts as a major cognitive and theoretical gain here, anyway, is not any increase in predictive power, but rather the conceptual insights that become possible when we apply the language and concepts of topology (and of mathematics more generally).[19]

To sum up:

1. Lewin notes that a person has regions, and that these depend dynamically on one another.
2. He also observes the structure of these regions, and describes this using spatial concepts such as space, boundary, connection, separation, etc.
3. Because a formal system for such concepts has already been developed by topologists, Lewin is able to employ topology to formulate, formalize and expand upon his original spatial intuition.
4. In this way, he constructs a global analogy between a person and topological space; this allows him to offer explications (in the sense of Carnap) for many concepts—in particular, the intuitive concept of a person's centre.
5. We might therefore say that in this way he throws new, and *spatial*, light on the problem of the structure of personhood.

Lewin also proffered explications of other notions, describing such properties of persons as the harmoniousness of their character (Lewin 1936, p. 187), their being

[19] We do not mean to suggest that the introduction of topological notions will increase predictive power in any direct and immediate or entirely straighforward fashion: rather, first and foremost, it will contribute positively to a process of theoretical reflection of a highly general (and possibly metaphysical, or metaphilosophical) character.

affected by tension (ibidem, p. 181), their degree of differentiatedness and integratedness (ibidem, p. 182–187), and others. All of these explications are *spatial* and *topological* in nature.

4.3 An example of metaphor: Dimensions of the person and schizophrenia

Throwing, casting or shedding light on something is an operation that consists in our putting a given subject matter forward for consideration in a novel global context, albeit for specific purposes. It makes visible some previously neglected aspects—ones that permeate the entirety of the phenomenon. The explicatory significance, in turn, comes from the fact that we manage to precisely define our intuitive concepts in locally specified terms via an appeal to certain already more precise concepts furnished by an existing conceptual field (in our case, topology)—hence the thought that this corresponds to the general phenomenon of *throwing light on something*. We will now interpret the use of metaphor as offering a less strict version of explication thus construed.[20] Metaphor, on such a reading, does not consist in a global approach to the subject matter under consideration, but rather in one's constructing looser local analogies without taking over an entire network of conceptual connections. Lewin himself also sought to develop several such metaphors, including one relating to schizophrenia, which we present below.

Furthermore, the heuristic role performed in such cases may be different from that of simply explicating a pre-existing psychological concept. An explication is—broadly speaking—a mapping $\Phi{:}P \rightarrow T$, from the class of psychological notions P onto the class of topological notions T. And certainly, the class T is richer than P. But if we have such a (even informally) defined mapping, we might also then think up a procedure for re-translating topological notions back into psychology. This might allow us to formulate some quite new ideas, concepts and hypotheses: i.e. we could take topological notions, concepts, results and theorems, and explore them to see whether they inspire us to formulate some novel psychological ideas. So—still informally speaking—we can conceive of some sort of re-translation, in the sense of a mapping $\Psi{:}T \rightarrow P^*$.[21] And if such a heuristic procedure were indeed to prove productive, this would mean we have enriched our initial class of psychological concepts P to obtain an (enriched) P^*.

One fact of importance is that in the target domain T (topology) there are many conceptual connections available, and *immersing* the domain P into T enables us to make use of these pre-existing connections. For this reason, the topology resembles the formal ontology of Husserl's *Logical Investigations*. If the mapping $\Psi{:}T \rightarrow P^*$ is plausible, then some topological concepts and facts might prompt us to explore certain novel

[20] At least, this is how we shall make use of the notion of *metaphor* in this paper. Of course, there is a vast literature concerning this, including as it pertains to science and mathematics. However, a serious discussion of the general problem of metaphor would exceed the scope of the present article. Here we shall only mention the position of Lakoff (2012), as it to some extent supports our view concerning the role of mathematical metaphors.

[21] We are not employing the symbols $\Phi{:}P \rightarrow T$ and $\Psi{:}T \rightarrow P^*$ in any formal sense here (as P is obviously not formalized). $\Phi{:}P \rightarrow T$ is rather a paraphrase (of an explicatory sort), and $\Psi{:}T \rightarrow P^*$ is also to be understood in an informal way. We might also characterize the procedure in question as an instance of "rational reconstruction of a theory"—an expression also employed by Carnap himself. However, we shall not go into details about this here. Hutchins (2012, p. 319-320) calls these types of cognitive operations "mappings across conceptual spaces".

psychological concepts and hypotheses, which in turn may reveal some previously hidden facts—and, in particular, could also enable us to reveal some facts latent within the initial formulation of the theory of personhood. As Heinrich Hertz famously stated:

> One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers […]. (quoted in Bell (1937))

Even if we consider this no more than a rhetorically expressive exclamation (as opposed to a philosophical thesis), it describes an interesting phenomenon and would seem to be applicable to the situation in question: if the representation $\Phi{:}P{\to}T$ is plausible, then indeed, topology (T) is able to disclose some new information that could not have been discovered within psychology (P) itself. So "explications give rise to a feedback effect" (Brun 2016, p. 1237).

If we view the person as modelled by a family of regions, then we can ask what the properties of the boundary of these regions are. Is the topological fact that the boundary has a lower dimension than the regions somehow psychologically suggestive? Quite possibly not, and probably in the case of a large majority of topological concepts there will be no plausible psychological counterparts. Nevertheless, in some cases there may well be. For example, could it be that the topological concept of retraction will somehow contribute to our understanding of someone's being in a state of shock? Hence, the cognitive fruitfulness of topology within psychology can assume a number of forms. It is especially worth noting that even the basic principles of *Gestalt* psychology (or at least some of them) exhibit a highly topological flavour: notions such as *similarity*, *closure*, *good form* (*Prägnanz*), *proximity* and *region* can all be defined within topology.[22]

If we represent the person as a topological space, we will want to describe the properties of this space: in particular, we might investigate its dimensionality. It is well known that this is an invariant of homeomorphisms—which means it is a topological property. We might (after making the necessary identifications) see fit to speak of the dimensions of persons (or perhaps, rather, of their regions). However, it would be implausible to claim, say, that a person has seven dimensions—and perhaps it is not even possible to talk about a person's (or the person's parts') dimensions in a strict sense. Only metaphors remain. Lewin was struggling with these problems (Lewin 1936, p. 200–202), and he finally suggested that dimensionality should be understood in terms of degrees of reality.

Some human activities are certainly real, and impact strongly on our environment. In fact, Lewin himself understood psychological reality in terms of its efficaciousness. Cycling is real: it brings with it a real change in the position of the body. Nevertheless, the human personality includes dreams, goals, fantasies, inventions, delusions, and so on. All of these lead, in one way or another, to what is unreal. Our mental reality falls into degrees, and Lewin proposed representing higher degrees of unreality as

[22] Analysing Lewin's overall approach, Duch (2018, p. 12) observes that "[h]e has introduced several new constructs, such as valence, action research, sensitivity training, group dynamics, mind as a complex energy field, behavior as a change in the state of this field, regions, life space, forces and tension, equilibrium states. Complex energy field can be presented in the language of dynamical systems".

successive dimensions of the person—dimensions, of course, in a rather metaphorical sense. The higher the personality dimension we are considering, the more fluid will be the structure of the person. As he puts it: "the boundaries between person and environment are less clear and the structure of the environment depends to a greater extent on the needs of the person" (Lewin 1936, p. 199–200).

After analysing the dimensionality of personality and its problematic nature, Lewin, in passing, constructs a metaphor for schizophrenia, which can be represented as "two dynamically relatively separated regions which belong to different levels of reality" (Lewin 1936, p. 201). It can be assumed that this metaphor was dictated by his construction of a global analogy between persons and topology, rather than being something the author had specifically intended. We may speculate that it is as if topology (together with the other scientific terms used by Lewin) had somehow *imposed* this metaphor on him. The reader should recall that we mentioned (in an informal way) two procedures: $\Phi{:}P \rightarrow T$ (as a kind of explicatory paraphrase), and $\Psi{:}T \rightarrow P^*$ (as some kind of retranslation and reinvention). Using these symbols, we can say that this way of viewing schizophrenia was not present within the original P, but emerged within the enriched P^*. In this way, it became possible for some interesting new claims to be put forward.[23]

Of course, the notion of *topological dimensionality* should be handled with great care, if we intend to use it to describe psychological phenomena. It is really hard to imagine a plausible psychological counterpart (i.e. within P^*) of the notion of, say, a locally compact space with an uncountably infinite number of dimensions. Perhaps only very few abstract topological notions can serve as an inspiration for psychological investigations. (Lewin was certainly well aware of that). Even so, in everyday language we do speak vaguely and loosely of "dimensions" in a range of contexts, and we need a method for understanding this. We might even try to explicate, in formal terms, claims such as "this dimension of somebody's personality is reduced/twisted/fractional". Even if we consider such claims vague and merely metaphorical, we might attempt to exploit this metaphor and try to "extract" some new ideas from our precise conceptual system.[24] At this point, it should also be recalled that throwing light on something is not without a purpose—that, as Knuuttila (2017) has noted, all modelling in science is intentional, and is carried out with some aim in mind. Certainly, Lewin's goal was not to build a theory of persons that would be as strict as the research conducted in modern mathematics; his aspiration was rather to make sense of the phenomenon of persons and to confirm this understanding through experimentation.

5 Conclusion

Lewin observed that the conceptual system of topology is well suited to conceptualizing the notion of personhood. In particular, he noted that we can view the person as having certain parts, that there are certain relations between these parts, that these parts

[23] The situation is to some extent similar to that of category theory; see Kuś et al. (2019) and Wójtowicz (2019) for discussion of this.

[24] Hempel famously described mathematics as a *theoretical juice extractor*. We might adopt such an analogy, viewing topology as such a kind of *spatial juice extractor* that provides methods for conceptualizing the field while also furnishing new insights and ideas.

can be either separated from each other or connected, that they have boundaries, etc. As a result, he was able to construct a formal spatial ontology of persons, distinguishing many spatial forms that can serve as the subject of topological research. In fact, he took over many conceptual elements from topology, which led him to distinguish new aspects and features of persons. This, moreover, was a creative contribution, and one that he made by *casting topological light* on the subject matter in question. Without the use of topological concepts, he would not have been in a position to perceive and describe many of the important relations that obtain between parts of a person. He would also not have been able to construct his distinctive metaphors and explications. In his work, the use of topology thus contributed to the theoretical endeavour of understanding the person in an important way, and so we can also state more specifically that he *threw spatial light on the problem of the structure of persons*.

It can reasonably be argued that mathematics plays an explanatory role in physics, but it would be difficult to assign a similar role to it in the context of psychology—especially if we opt to construe it in a theoretical and non-reductionist way. Yet, in spite of this, mathematics can still play a very fruitful cognitive role here. Lewin's theory provides us with a telling example that confirms this point. Mathematics is employed there to arrive at fruitful explications of psychological concepts and create inspiring metaphors. An array of topological concepts makes possible a perspicuous conceptualization of a set of important psychological notions, embedding these in more precise language, and this in turn means that we take over, in one way or another, a ready-made system of conceptual relations. Lewin's topological psychology is thus also an instructive case for formal philosophy (cf. Leitgeb 2013). There are several reasons why this is so. First of all, within a single conceptual system we witness him using multiple conceptual schemes, including ones taken from the fields of physics and mathematics. Secondly, he employs topological concepts, even though in his time—as now—formal philosophy was dominated by logic. Thirdly, his results have had, and continue to have, a huge impact on psychology and related fields. Fourthly, he modelled psychological phenomena using qualitative topological concepts rather than quantitative statistical tools. Fifthly, he first uncovered some recurring spatial schemes in the form of structural spatial resemblances, and only on the basis of the resulting analogies (which were unclear and difficult to express) constructed his model of persons. It was precisely because of the fact that he saw analogies in such distant areas as topology and psychology that he was able to achieve such a far-reaching influence: what he accomplished in this regard cannot be found elsewhere—in any known schema of formal philosophy.

Our view, then, is that Lewin's work furnishes a most interesting illustration of a more general phenomenon: namely, the fruitfulness of applying mathematical concepts when seeking to make sense of methodological and philosophical problems. What makes it particularly engaging is its use of topology, and of certain highly general spatial notions in the context of a theory of personhood: that surely constitutes a strikingly original approach. Moreover, the rising importance of topological explanations in science generally and cognitive science in particular shows that his intuitions turned out to possess a certain brilliance, even if they could not be fully developed back in 1936, given the state of the discipline then.

Finally, let us remark that the topological way of thinking is particularly well suited to *Gestalt* psychology, and has the potential to enhance interdisciplinary work and further the integration of diverse approaches, in that topological explanations are formulated in terms of properties of an abstract underlying structure, rather than through identifying, and translating into spatial form, underlying causal mechanisms. It is mathematics and mathematical dependencies that are taken to be of importance, not some *physical implementation*. Hence, this approach can also be regarded as most attractive from a phenomenological point of view.

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