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ERRATUM

Erratum to: Antimatroids and Balanced Pairs

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In the paper "Antimatroids and Balanced Pairs", published by the first author in *Order* 31(1):81–99, 2014, the proof of Lemma 12 is incorrect, but can be repaired, as was observed by V. Wiechert, with an argument similar to that in the incorrect proof.

Lemma 12 *Let x be an initial element of a set of orderings Ω that is preceded in relation $\prec$ by $i \geq 2$ other initial elements, and suppose that relation $\prec$ is total in Ω. Then $\bar{r}(x) > (2i + 5)/3$.*

Proof The expected rank of x is one plus the expected number of elements of Ω that precede x in a uniformly random ordering. If S is the set of initial elements that precede x in relation $\prec$, then the expected contribution of S to the number of elements that precede x in a random ordering is

$$\sum_{y \in S} \Pr[y < x] > \frac{2i}{3}.$$

Adding the one unit for the fact that the rank is one even when there are zero elements preceding x, we obtain a total of $(2i + 3)/3$.

To obtain the remaining 2/3 of a unit in the expected rank given in the statement of the lemma, let y be the immediate predecessor of x in $\prec$, and let z be the immediate predecessor

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of y in $\prec$ (this is where we use the assumption that $i \geq 2$). Let E_1 be the event that y precedes x in an ordering and that at least one non-initial element occurs between y and x in the word. Let E_2 be the event that x precedes y and there is at least one non-initial element between them. Finally, let E_3 be the event that there are only initial elements between x and y in an ordering. Then y precedes x in all the orderings of E_1, none of the orderings of E_2, and half of the orderings in E_3. Since $y \prec x$, we obtain

$$\Pr[E_1] + \frac{1}{2}\Pr[E_3] > \frac{2}{3}.$$

Doubling this inequality and subtracting the equation $\Pr[E_1]+\Pr[E_2]+\Pr[E_3] = 1$ (coming from the fact that these three events cover all orderings in Ω) gives the equivalent form

$$\Pr[E_1] - \Pr[E_2] > \frac{1}{3}.$$

Now let F be the event that z precedes y in an ordering and that at least one non-initial element occurs between z and y in the word. By similar analysis to the above we get $\Pr[F] > 1/3$. We split F into the event F^- in which x precedes the rightmost non-initial element between y and z, and the event F^+ in which x does not precede this element; thus

$$\Pr[F^-] + \Pr[F^+] > \frac{1}{3}.$$

Because F^- is a subset of event E_2, we also have the inequality $\Pr[F^-] \leq \Pr[E_2]$.

Now let us count the non-initial predecessors of x. Whenever E_1 occurs we have a non-initial predecessor of x that is a successor of y. Whenever F^+ occurs we have a non-initial predecessor of x that is not a successor of y. Thus, these two sets of non-initial predecessors are disjoint, and by linearity of expectation their expected number is

$$\Pr[E_1] + \Pr[F^+] > \Pr[E_1] + \frac{1}{3} - \Pr[F^-] \geq \Pr[E_1] + \frac{1}{3} - \Pr[E_2] > \frac{2}{3}.$$

Together with the contribution of the initial elements to the expected rank of x, this gives a lower bound of $(2i + 5)/3$ on the expected rank, as stated. □