Optical and Quantum Electronics (2024) 56:834
https://doi.org/10.1007/s11082-024-06579-1

Conformable modeling of normalization and recursional electromagnetic fields of spacelike magnetic curves

Talat Körpinar[1] · Zeliha Körpinar[2] · Hatice Özdemir[1]

Received: 29 September 2023 / Accepted: 5 February 2024 / Published online: 29 March 2024

Abstract
In this paper, we investigate spacelike magnetic curves according to Bishop frame. Firstly, we present conformable derivatives of Lorentz magnetic fields of these magnetic curves. Moreover, we calculate the conformable derivatives of the normalization and recursional electromagnetic vector fields. Finally, we give conformable energies of normalization and recursional electromagnetic fields related to spacelike magnetic curves.

Keywords Spacelike magnetic curve · Conformable derivative · Bishop frame · Energy

1 Introduction

Optical applications of differential fractional approaches have been used for defining electric and magnetic phases in electromagnetic structures. These approaches generally are used to define the magnetic flux frequency circulations within the structures. Electromagnetic vector fields are presented by model and work of electromagnetic flow with fractional differential structures (de Andrade 2006; Maluf and Faria 2008; Körpinar and Körpinar 2021a, b; Körpınar et al. 2021a, b).

Associated curves bring important geometric definitions to fields of differential geometry, physics, and mathematics in explanation of the behavior of curves and surfaces and in work of particle motion in ordinary space. Also, concepts such as magnetic curves, Bertrand curves, spherical curves, and involute-evolute curve pairs have been widely investigated in fields of differential geometry (do Carmo 1976; Bishop 1975; Bukcu and Karacan 2008; Bükcü and Karacan 2009a; Karacan and Bukcu 2007a; Bükcü and Karacan 2009b; Karacan and Bukcu 2007b, 2008; Maluf and Faria 2008; Körpınar et al. 2021c).

✉ Hatice Özdemir
haticeozdmir49@gmail.com

Talat Körpinar
talatkorpinar@gmail.com

Zeliha Körpinar
zelihakorpinar@gmail.com

[1] Department of Mathematics, Muş Alparslan University, 49250 Muş, Turkey

[2] Department of Administration, Muş Alparslan University, 49250 Muş, Turkey

There is no concrete calculation of entropy and energy in Minkowski spacetime in literature research. Diverse efforts have been made to define the energy using local-quasi concepts. But definitions of energy don't match with each other all time and they are not suitable to spaces of anti de Sitter and de Sitter type (Hayward 1994; Martinez 1994; Epp 2000). It can be started with using local method to make any advancement on the notion of energy in this spacetime. Therefore, it can be said that one of most effective ways to achieve this method is to use geometric properties of particle moving in Minkowski spacetime. Moreover, it is examined that calculation of energy of a specific particle in many spacetimes has various applications (Körpinar 2014; Körpinar and Turhan 2015; Körpinar 2018; Körpinar and Demirkol 2017). Geometrical energy examined its ordinary optical geometric propriety for instances of geometrical physics. Also, diverse methods of geometric phases of energy and fractional magnetic flux for physical energy areas are presented and are computed F-W derivative on $\mathbb{S}_1^2$ by authors (Körpinar and Demirkol 2018; Körpinar 2022).

This work is organized as follows. Firstly, we have given basic definitions of Bishop frame equations for different type of spacelike magnetic curves in space. Then, we have presented a geometrical analysis of conformable energy for electromagnetic vector fields. In the following section, we have computed the conformable derivatives of the normalization and recursional electromagnetic vector fields. Finally, we have given conformable energies of normalization and recursional electromagnetic fields related to spacelike magnetic curves.

2 Preliminaries

Let $\delta : I \to \mathbb{E}^3$ be unit speed magnetic curve. Thus, conformable derivatives according to $\{\mathbf{t}, \mathbf{n_1}, \mathbf{n_2}\}$ Bishop frame are as

$$\begin{aligned} \nabla_{\delta'} \mathbf{t} =& v^{1-\sigma} k_1 \mathbf{n}_1 + v^{1-\sigma} k_2 \mathbf{n}_2, \\ \nabla_{\delta'} \mathbf{n}_1 =& - v^{1-\sigma} k_1 \epsilon_{\mathbf{n}_1} \mathbf{t}, \\ \nabla_{\delta'} \mathbf{n}_2 =& - v^{1-\sigma} k_2 \epsilon_{\mathbf{n}_2} \mathbf{t}. \end{aligned} \tag{1}$$

Here the vector product is as follows

$$\epsilon_{\mathbf{t}} \mathbf{t} = \mathbf{n}_1 \times \mathbf{n_2}, \ \epsilon_{\mathbf{n}_1} \mathbf{n_1} = \mathbf{n_2} \times \mathbf{t}, \ \epsilon_{\mathbf{n}_2} \mathbf{n_2} = \mathbf{t} \times \mathbf{n_1}.$$

and where is $\epsilon_{\mathbf{X}} = g(\mathbf{X}, \mathbf{X})$.

♦*Let β spacelike magnetic curves according to Bishop frame. Thus, Lorentz equations of magnetic fields are given as*

$$\begin{aligned} \nabla_{\delta'} \mathbf{t} =& \Pi(\mathbf{t}) = \mathbf{G} \times \mathbf{t}, \\ \nabla_{\delta'} \mathbf{n}_1 =& \Pi\left(\mathbf{n}_1\right) = \mathbf{G} \times \mathbf{n}_1, \\ \nabla_{\delta'} \mathbf{n}_2 =& \Pi\left(\mathbf{n}_2\right) = \mathbf{G} \times \mathbf{n}_2. \end{aligned} \tag{2}$$

The first equation is named $\mathbf{t} -$ *spacelike magnetic curve*; *the second equation is named* $\mathbf{n}_1 -$ *spacelike magnetic curve*; *the third equation is named* $\mathbf{n}_2 -$ *spacelike magnetic curve.*

♦Lorentz magnetic fields of $\mathbf{t} -$ *spacelike magnetic curve are given as*

$$\begin{aligned} \Pi(\mathbf{t}) &= v^{1-\sigma}k_1\mathbf{n}_1 + v^{1-\sigma}k_2\mathbf{n}_2, \\ \Pi(\mathbf{n}_1) &= -v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{t} + \rho\epsilon_{\mathbf{n}_2}\mathbf{n}_2, \\ \Pi(\mathbf{n}_2) &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{t} - \rho\epsilon_{\mathbf{n}_1}\mathbf{n}_1, \\ \mathbf{G} &= \rho\mathbf{t} - v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_1 + v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{n}_2, \end{aligned} \tag{3}$$

where $\rho = g(\Pi(\mathbf{n}_1), \mathbf{n}_2)$ *is conformable differential potential.*

⧫Lorentz magnetic fields of $\mathbf{n}_1 -$ *spacelike magnetic curve are presented as*

$$\begin{aligned} \Pi(\mathbf{t}) &= v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + \epsilon_{\mathbf{n}_2}\mu\mathbf{n}_2, \\ \Pi(\mathbf{n}_1) &= -v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{t}, \\ \Pi(\mathbf{n}_2) &= -\mu\mathbf{t}, \\ \mathbf{G} &= -\mu\mathbf{n}_1 + v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{n}_2, \end{aligned}$$

where $\mu = g(\Pi(\mathbf{t}), \mathbf{n}_2)$ *is conformable differential potential.*

⧫ Lorentz magnetic fields of $\mathbf{n}_2 -$ *spacelike magnetic curve are obtained as*

$$\begin{aligned} \Pi(\mathbf{t}) &= \gamma\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_2, \\ \Pi(\mathbf{n}_1) &= -\gamma\mathbf{t}, \\ \Pi(\mathbf{n}_2) &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{t}, \\ \mathbf{G} &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_1 + \gamma\mathbf{n}_2, \end{aligned}$$

where $\gamma = g(\Pi(\mathbf{t}), \mathbf{n}_1)$ *is conformable differential potential.*

3 Conformable normalization and recursional functions of spacelike magnetic curves

Let $\{\mathbf{t}, \mathbf{n}_1, \mathbf{n}_2\}$ be the Bishop frame, and conformable derivative system is

$$\begin{aligned} D_\sigma\mathbf{t} &= v^{1-\sigma}k_1\mathbf{n}_1 + v^{1-\sigma}k_2\mathbf{n}_2, \\ D_\sigma\mathbf{n}_1 &= -v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{t}, \\ D_\sigma\mathbf{n}_2 &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{t}. \end{aligned}$$

Here $\mu = \det(\delta, \mathbf{t}, \nabla_v\mathbf{t})$.

Normalization operator of $\mathbb{V} = a_0\mathbf{t} + a_1\mathbf{n}_1 + a_2\mathbf{n}_2$ is

$$\mathbb{N}\mathbb{V} = \mathbf{I}_\sigma^a(a_1\epsilon_{\mathbf{n}_1}k_1v^{1-\sigma} + a_2\epsilon_{\mathbf{n}_2}k_2v^{1-\sigma})\mathbf{t} + a_1\mathbf{n}_1 + a_2\mathbf{n}_2.$$

Recursional operators $\mathcal{R}\mathbb{V}$ and $\mathcal{R}^2\mathbb{V}$ are

$$\mathcal{R}\mathbb{V} = -\mathbb{N}(\mathbf{t} \times D_\sigma\mathbb{V}),$$

and

$$\mathcal{R}^2\mathbb{V} = -\mathbb{N}(\mathbf{t} \times D_\sigma\mathcal{R}\mathbb{V}).$$

♠*The Lorentz magnetic fields of* $\mathbf{t} -$ *spacelike magnetic curve are obtained as*

$$\begin{aligned}\Pi(\mathbf{t}) &= v^{1-\sigma}k_1\mathbf{n}_1 + v^{1-\sigma}k_2\mathbf{n}_2,\\ \Pi(\mathbf{n}_1) &= -v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}\mathbf{t} + \rho\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\ \Pi(\mathbf{n}_2) &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n_2}}\mathbf{t} - \rho\epsilon_{\mathbf{n_1}}\mathbf{n}_1,\\ \mathbf{G} &= \rho\mathbf{t} - v^{1-\sigma}k_2\epsilon_{\mathbf{n_2}}\mathbf{n}_1 + v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}\mathbf{n}_2.\end{aligned} \tag{4}$$

Conformable derivatives of $\mathbf{t}-$ *spacelike magnetic curve* Lorentz magnetic fields are

$$\begin{aligned}D_\sigma\Pi(\mathbf{t}) =& -((v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n_1}} + (v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n_2}})\mathbf{t}\\ &+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\mathbf{n}_1 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\mathbf{n}_2,\\ D_\sigma\Pi(\mathbf{n}_1) =& -(v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_1}}v^{1-\sigma}k_1) + \rho v^{1-\sigma}k_2)\mathbf{t}\\ &- \epsilon_{\mathbf{n_1}}(v^{1-\sigma}k_1)^2\,\mathbf{n}_1 + (v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n_2}}) - \epsilon_{\mathbf{n_1}}v^{2-2\sigma}k_1k_2)\mathbf{n}_2,\\ D_\sigma\Pi(\mathbf{n}_2) =& (-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}v^{1-\sigma}k_2) + \rho v^{1-\sigma}k_1)\mathbf{t}\\ &- (v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n_2}} + v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n_1}}))\mathbf{n}_1 - \epsilon_{\mathbf{n_2}}(v^{1-\sigma}k_2)^2\mathbf{n}_2,\\ D_\sigma\mathbf{G} =& v^{1-\sigma}\frac{d\rho}{dv}\mathbf{t} + (\rho v^{1-\sigma}k_1 - v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}v^{1-\sigma}k_2))\mathbf{n}_1\\ &+ (\rho v^{1-\sigma}k_2 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}))\mathbf{n}_2.\end{aligned}$$

Normalization of these spacelike curves of Lorentz forces are

$$\begin{aligned}\mathbb{N}\Pi(\mathbf{t}) =& \mathbf{I}_\sigma^a(\epsilon_{\mathbf{n_1}}(k_1v^{1-\sigma})^2 + \epsilon_{\mathbf{n_2}}(k_2v^{1-\sigma})^2)\mathbf{t}\\ &+ v^{1-\sigma}k_1\mathbf{n}_1 + v^{1-\sigma}k_2\mathbf{n}_2,\\ \mathbb{N}\Pi(\mathbf{n}_1) =& \mathbf{I}_\sigma^a(\rho k_2v^{1-\sigma})\mathbf{t} + \rho\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\ \mathbb{N}\Pi(\mathbf{n}_2) =& -\mathbf{I}_\sigma^a(\rho k_1v^{1-\sigma})\mathbf{t} - \rho\epsilon_{\mathbf{n_1}}\mathbf{n}_1,\\ \mathbb{N}\mathbf{G} =& c_0\mathbf{t} - v^{1-\sigma}k_2\epsilon_{\mathbf{n_2}}\mathbf{n}_1 + v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}\mathbf{n}_2,\end{aligned}$$

where c_0 is constant of normalized function.

Thus, we get

$$\begin{aligned}D_\sigma\Pi(\mathbf{t}) =& -((v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n_1}} + (v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n_2}})\mathbf{t}\\ &+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\mathbf{n}_1 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\mathbf{n}_2,\\ D_\sigma\Pi(\mathbf{n}_1) =& -(v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_1}}v^{1-\sigma}k_1) + \rho v^{1-\sigma}k_2)\mathbf{t}\\ &- \epsilon_{\mathbf{n_1}}(v^{1-\sigma}k_1)^2\,\mathbf{n}_1 + (v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n_2}}) - \epsilon_{\mathbf{n_1}}v^{2-2\sigma}k_1k_2)\mathbf{n}_2,\\ D_\sigma\Pi(\mathbf{n}_2) =& (-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}v^{1-\sigma}k_2) + \rho v^{1-\sigma}k_1)\mathbf{t}\\ &- (v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n_2}} + v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n_1}}))\mathbf{n}_1 - \epsilon_{\mathbf{n_2}}(v^{1-\sigma}k_2)^2\mathbf{n}_2,\\ D_\sigma\mathbf{G} =& v^{1-\sigma}\frac{d\rho}{dv}\mathbf{t} + (\rho v^{1-\sigma}k_1 - v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}v^{1-\sigma}k_2))\mathbf{n}_1\\ &+ (\rho v^{1-\sigma}k_2 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}))\mathbf{n}_2,\end{aligned}$$

and we have

$$\mathbf{t}\times D_\sigma\Pi(\mathbf{t}) = -v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\epsilon_{\mathbf{n}_2}\mathbf{n}_2,$$
$$\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_1) = -(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}) - \epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2)\epsilon_{\mathbf{n}_1}\mathbf{n}_1 - \epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n}_2}\mathbf{n}_2,$$
$$\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_2) = -\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + (v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2} + v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))\epsilon_{\mathbf{n}_2}\mathbf{n}_2,$$
$$\mathbf{t}\times D_\sigma\mathbf{G} = -(\rho v^{1-\sigma}k_2 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))\mathbf{n}_1 + (\rho v^{1-\sigma}k_1 - v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))\mathbf{n}_2,$$

moreover, we have

$$\begin{aligned}
\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{t})\big) =& \mathbf{I}^a_\sigma(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2) \\
&+ v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1))\mathbf{t} - v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\epsilon_{\mathbf{n}_1}\mathbf{n}_1 \\
&+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\epsilon_{\mathbf{n}_2}\mathbf{n}_2, \\
\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_1)\big) =& -\mathbf{I}^a_\sigma(k_1(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}) \\
&- \epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2) + v^{3-3\sigma}k_2k_1^2\epsilon_{\mathbf{n}_1})\mathbf{t} - (v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}) \\
&- \epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2)\epsilon_{\mathbf{n}_1}\mathbf{n}_1 - \epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n}_2}\mathbf{n}_2, \\
\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_2)\big) =& \mathbf{I}^a_\sigma(v^{3-3\sigma}k_1{k_2}^2\epsilon_{\mathbf{n}_2} \\
&+ v^{1-\sigma}k_2(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2} + v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1})))\mathbf{t} \\
&- \epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + (v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2} \\
&+ v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))\epsilon_{\mathbf{n}_2}\mathbf{n}_2, \\
\mathbb{N}\big(\mathbf{t}\times D_\sigma\mathbf{G}\big) =& \mathbf{I}^a_\sigma(-v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}(\rho v^{1-\sigma}k_2 \\
&+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})) + v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}(\rho v^{1-\sigma}k_1 \\
&- v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2)))\mathbf{t} - (v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1} \\
&+ \rho v^{1-\sigma}k_2))\mathbf{n}_1 + (\rho v^{1-\sigma}k_1 - v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))\mathbf{n}_2.
\end{aligned}$$

Recursional operators of $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2)$ and $\mathbf{G}$ magnetic fields are

$$
\begin{aligned}
\mathcal{R}\Pi(\mathbf{t}) =& - \mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{t})\big) = -\mathbf{I}_\sigma^a(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2)\\
&+ v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1))\mathbf{t} + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\epsilon_{\mathbf{n}_1}\mathbf{n}_1\\
&- v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\Pi\big(\mathbf{n}_1\big) =& - \mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi\big(\mathbf{n}_1\big)\big) = \mathbf{I}_\sigma^a(k_1(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2})\\
&- \epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2) + v^{3-3\sigma}k_2k_1^2\epsilon_{\mathbf{n}_1})\mathbf{t} + (v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2})\\
&- \epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2)\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + (v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\Pi\big(\mathbf{n}_2\big) =& - \mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi\big(\mathbf{n}_2\big)\big) = -\mathbf{I}_\sigma^a(v^{3-3\sigma}k_1{k_2}^2\epsilon_{\mathbf{n}_2}\\
&+ v^{1-\sigma}k_2(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2} + v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1})))\mathbf{t}\\
&+ \epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n}_1}\mathbf{n}_1 - (v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\\
&+ v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\mathbf{G} =& - \mathbb{N}\big(\mathbf{t}\times D_\sigma\mathbf{G}\big) = -\mathbf{I}_\sigma^a(-v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}(\rho v^{1-\sigma}k_2\\
&+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})) + v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}(-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2)\\
&+ \rho v^{1-\sigma}k_1))\mathbf{t} + (\rho v^{1-\sigma}k_2 + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))\mathbf{n}_1\\
&- (\rho v^{1-\sigma}k_1 - v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))\mathbf{n}_2.
\end{aligned}
$$

♠ *The Lorentz magnetic fields of* $\mathbf{n}_1$ *− spacelike magnetic curve are*

$$
\begin{aligned}
\Pi(\mathbf{t}) =& v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{n}_1 + \epsilon_{\mathbf{n}_2}\mu\mathbf{n_2},\\
\Pi\big(\mathbf{n}_1\big) =& - v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{t},\\
\Pi\big(\mathbf{n}_2\big) =& - \mu\mathbf{t},\\
\mathbf{G} =& - \mu\mathbf{n}_1 + v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}\mathbf{n_2}.
\end{aligned}
$$

Conformable derivatives of magnetic fields are

$$
\begin{aligned}
D_\sigma\Pi(\mathbf{t}) =& - ((v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})^2 + \mu v^{1-\sigma}k_2)\mathbf{t}\\
&+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})\mathbf{n}_1 + v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu)\mathbf{n}_2,\\
D_\sigma\Pi\big(\mathbf{n}_1\big) =& - v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})\mathbf{t}\\
&- v^{2-2\sigma}k_1^2\epsilon_{\mathbf{n}_1}\mathbf{n}_1 - v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
D_\sigma\Pi\big(\mathbf{n}_2\big) =& - v^{1-\sigma}\frac{d\mu}{dv}\mathbf{t} - \mu k_1v^{1-\sigma}\mathbf{n}_1 - \mu v^{1-\sigma}k_2\mathbf{n}_2,\\
D_\sigma\mathbf{G} =& (\mu\epsilon_{\mathbf{n}_1}k_1v^{2-2\sigma} - k_1k_2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}v^{2-2\sigma})\mathbf{t}\\
&- v^{1-\sigma}\frac{d\mu}{dv}\mathbf{n}_1 + v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})\mathbf{n}_2.
\end{aligned}
$$

Normalizations of Lorentz fields are

$$\begin{aligned}\mathbb{N}\Pi(\mathbf{t}) =&\mathbf{I}_{\sigma}^{a}((k_1 v^{1-\sigma})^2\\&+\mu v^{1-\sigma}k_2)\mathbf{t}\\&+v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}\mathbf{n}_1+\epsilon_{\mathbf{n_2}}\mu\mathbf{n_2},\\\mathbb{N}\Pi(\mathbf{n}_1) =&c_1\mathbf{t},\\\mathbb{N}\Pi(\mathbf{n}_2) =&c_2\mathbf{t},\\\mathbb{N}\mathbf{G} =&\mathbf{I}_{\sigma}^{a}(-\mu\epsilon_{\mathbf{n_1}}k_1v^{1-\sigma}\\&+v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n_1}}\epsilon_{\mathbf{n_2}})\mathbf{t}\\&-\mu\mathbf{n}_1+v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}\mathbf{n_2},\end{aligned}$$

where c_1, c_2 are constants of normalized function.

Also, we have

$$\begin{aligned}\mathbf{t}\times D_{\sigma}\Pi(\mathbf{t}) =&-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}\mu)\epsilon_{\mathbf{n_1}}\mathbf{n}_1+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}})\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\\mathbf{t}\times D_{\sigma}\Pi(\mathbf{n}_1) =&v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n_2}}\epsilon_{\mathbf{n_1}}\mathbf{n}_1-v^{2-2\sigma}k_1^2\epsilon_{\mathbf{n_1}}\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\\mathbf{t}\times D_{\sigma}\Pi(\mathbf{n}_2) =&\mu v^{1-\sigma}k_2\epsilon_{\mathbf{n_1}}\mathbf{n}_1-\mu k_1v^{1-\sigma}\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\\mathbf{t}\times D_{\sigma}\mathbf{G} =&-v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n_1}}v^{1-\sigma})\epsilon_{\mathbf{n_1}}\mathbf{n}_1-v^{1-\sigma}\frac{d\mu}{dv}\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\end{aligned}$$

and

$$\begin{aligned}\mathbb{N}(\mathbf{t}\times D_{\sigma}\Pi(\mathbf{t})) =&\mathbf{I}_{\sigma}^{a}(-k_1v^{2-2\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}\mu)+k_2v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}}))\mathbf{t}\\&-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n_2}}\mu)\epsilon_{\mathbf{n_1}}\mathbf{n}_1+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n_1}})\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\\mathbb{N}(\mathbf{t}\times D_{\sigma}\Pi(\mathbf{n}_1)) =&\mathbf{I}_{\sigma}^{a}(v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n_2}}-v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n_1}})\mathbf{t}\\&+v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n_2}}\epsilon_{\mathbf{n_1}}\mathbf{n}_1-v^{2-2\sigma}k_1^2\epsilon_{\mathbf{n_1}}\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\\mathbb{N}(\mathbf{t}\times D_{\sigma}\Pi(\mathbf{n}_2)) =&c_3\mathbf{t}+\mu v^{1-\sigma}k_2\epsilon_{\mathbf{n_1}}\mathbf{n}_1-\mu k_1v^{1-\sigma}\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\\\mathbb{N}(\mathbf{t}\times D_{\sigma}\mathbf{G}) =&\mathbf{I}_{\sigma}^{a}(-k_1v^{2-2\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n_1}}v^{1-\sigma})-v^{2-2\sigma}k_2\frac{d\mu}{dv})\mathbf{t}\\&-v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n_1}}v^{1-\sigma})\epsilon_{\mathbf{n_1}}\mathbf{n}_1-v^{1-\sigma}\frac{d\mu}{dv}\epsilon_{\mathbf{n_2}}\mathbf{n}_2,\end{aligned}$$

where is c_3 recursion function.

Recursional $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ magnetic fields are as following

$$\begin{aligned}
\mathcal{R}\Pi(\mathbf{t}) &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{t})\big) = -\mathbf{I}^a_\sigma(-k_1v^{2-2\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu)+k_2v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))\mathbf{t}\\
&+v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu)\epsilon_{\mathbf{n}_1}\mathbf{n}_1-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\Pi(\mathbf{n}_1) &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_1)\big) = -\mathbf{I}^a_\sigma(v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_2}-v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_1})\mathbf{t}\\
&-v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{2-2\sigma}k_1^2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\Pi(\mathbf{n}_2) &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_2)\big) = c_3\mathbf{t}-\mu v^{1-\sigma}k_2\epsilon_{\mathbf{n}_1}\mathbf{n}_1+\mu k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\mathbf{G} &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\mathbf{G}\big) = -\mathbf{I}^a_\sigma(-k_1v^{2-2\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})-v^{2-2\sigma}k_2\frac{d\mu}{dv})\mathbf{t}\\
&+v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{1-\sigma}\frac{d\mu}{dv}\epsilon_{\mathbf{n}_2}\mathbf{n}_2.
\end{aligned}$$

♠*The Lorentz magnetic fields of* $\mathbf{n}_2-$ *spacelike magnetic curve are*

$$\begin{aligned}
\Pi(\mathbf{t}) &= \gamma\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\Pi(\mathbf{n}_1) &= -\gamma\mathbf{t},\\
\Pi(\mathbf{n}_2) &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{t},\\
\mathbf{G} &= -v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_1+\gamma\mathbf{n}_2.
\end{aligned}$$

Conformable derivatives of Lorentz forces are

$$\begin{aligned}
D_\sigma\Pi(\mathbf{t}) &= -((v^{1-\sigma}k_2)^2+\gamma v^{1-\sigma}k_1)\mathbf{t}\\
&+v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})\mathbf{n}_1+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\mathbf{n}_2,\\
D_\sigma\Pi(\mathbf{n}_1) &= -v^{1-\sigma}\frac{d\gamma}{dv}\mathbf{t}-\gamma k_1v^{1-\sigma}\mathbf{n}_1-\gamma k_2v^{1-\sigma}\mathbf{n}_2,\\
D_\sigma\Pi(\mathbf{n}_2) &= -v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\mathbf{t}\\
&-v^{2-2\sigma}k_2k_1\epsilon_{\mathbf{n}_2}\mathbf{n}_1-v^{2-2\sigma}k_2^2\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
D_\sigma\mathbf{G} &= (v^{2-2\sigma}\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}k_1k_2-\gamma k_2\epsilon_{\mathbf{n}_2}v^{1-\sigma})\mathbf{t}\\
&-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\mathbf{n}_1+v^{1-\sigma}\frac{d\gamma}{dv}\mathbf{n}_2.
\end{aligned}$$

Normalizations of Lorentz fields are

$$\begin{aligned}
\mathbb{N}\Pi(\mathbf{t}) &= \mathbf{I}^a_\sigma(\gamma k_1v^{1-\sigma}+(k_2v^{1-\sigma})^2)\mathbf{t}\\
&+\gamma\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathbb{N}\Pi(\mathbf{n}_1) &= c_4\mathbf{t},\\
\mathbb{N}\Pi(\mathbf{n}_2) &= c_5\mathbf{t},\\
\mathbb{N}\mathbf{G} &= \mathbf{I}^a_\sigma(-v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\\
&+v^{1-\sigma}\gamma\epsilon_{\mathbf{n}_2}k_2)\mathbf{t}-v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}\mathbf{n}_1+\gamma\mathbf{n}_2,
\end{aligned}$$

where c_4, c_5 are constants of normalized function. Then, we get

$$\begin{aligned}
\mathbf{t}\times D_\sigma\Pi(\mathbf{t}) &= -v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_1) &=\gamma k_2v^{1-\sigma}\epsilon_{\mathbf{n}_1}\mathbf{n}_1-\gamma k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_2) &=v^{2-2\sigma}k_2^2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\mathbf{n}_1-v^{2-2\sigma}k_2k_1\mathbf{n}_2,\\
\mathbf{t}\times D_\sigma\mathbf{G} &=v^{1-\sigma}\frac{d\gamma}{dv}\epsilon_{\mathbf{n}_1}\mathbf{n}_1-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_2}\mathbf{n}_2,
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{t})\big) &=\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})+k_2v^{2-2\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))\mathbf{t}\\
&\quad -v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_1)\big) &=c_6\mathbf{t}+\gamma k_2v^{1-\sigma}\epsilon_{\mathbf{n}_1}\mathbf{n}_1-\gamma k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_2)\big) &=c_7\mathbf{t}+v^{2-2\sigma}k_2^2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\mathbf{n}_1-v^{2-2\sigma}k_2k_1\mathbf{n}_2,\\
\mathbb{N}(\mathbf{t}\times D_\sigma\mathbf{G}) &=\mathbf{I}_\sigma^a(\frac{d\gamma}{dv}v^{2-2\sigma}k_1-v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\mathbf{t}\\
&\quad +v^{1-\sigma}\frac{d\gamma}{dv}\epsilon_{\mathbf{n}_1}\mathbf{n}_1-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_2}\mathbf{n}_2,
\end{aligned}$$

where c_6, c_7 are constants of normalized potential.

Recursional operators of $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ magnetic fields are

$$\begin{aligned}
\mathcal{R}\Pi(\mathbf{t}) &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{t})\big) = -\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\\
&\quad +k_2v^{2-2\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))\mathbf{t}+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_1}\mathbf{n}_1-v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\Pi(\mathbf{n}_1) &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_1)\big) = c_6\mathbf{t}-\gamma k_2v^{1-\sigma}\epsilon_{\mathbf{n}_1}\mathbf{n}_1+\gamma k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2}\mathbf{n}_2,\\
\mathcal{R}\Pi(\mathbf{n}_2) &= -\mathbb{N}\big(\mathbf{t}\times D_\sigma\Pi(\mathbf{n}_2)\big) = c_7\mathbf{t}-v^{2-2\sigma}k_2^2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{2-2\sigma}k_2k_1\mathbf{n}_2,\\
\mathcal{R}\mathbf{G} &= -\mathbb{N}(\mathbf{t}\times D_\sigma\mathbf{G}) = -\mathbf{I}_\sigma^a(\frac{d\gamma}{dv}v^{2-2\sigma}k_1-v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\mathbf{t}\\
&\quad +v^{1-\sigma}\frac{d\gamma}{dv}\epsilon_{\mathbf{n}_1}\mathbf{n}_1+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_2}\mathbf{n}_2.
\end{aligned}$$

4 Conformable energies of spacelike magnetic curves

In this section, we compute the conformable energy characterized by Sasaki metric of normalization and recursion operator of spacelike magnetic curves according to Bishop frame.

Let be Riemann manifolds (M,ρ) and (s, h) magnetic energy of differentiable map $q:(M,\rho)\to(s,h)$. Sasaki metric energy is characterized as the following

$$\mathbb{E}(q)=\frac{1}{2}\int_M\sum h(dq(p_a),dq(p_a))\upsilon. \tag{5}$$

Here $\{p_a\}$ is base of ordinary space and υ is canonical type in M.

4.1 Conformable energy of normalized electromagnetic fields

In this part, the conformable energy of normalized $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2)$ and $\mathbf{G}$ electromagnetic fields are calculated. Also, geometrical and physical characterizations for energies of normalization operator of these electromagnetic fields are presented.

♠ *The energy of normalized* $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ *magnetic fields for* $\mathbf{t}-$ *spacelike magnetic curve are presented as following*

$$\begin{aligned} D_\sigma \mathbb{N}\Pi(\mathbf{t}) =&(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma})^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1))\mathbf{n}_1\\ &+(v^{1-\sigma}k_2\mathbf{I}^a_\sigma(\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma})^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2))\mathbf{n}_2,\\ D_\sigma \mathbb{N}\Pi(\mathbf{n}_1) =&k_1v^{1-\sigma}\mathbf{I}^a_\sigma(\rho k_2v^{1-\sigma})\mathbf{n}_1+(k_2v^{1-\sigma}\mathbf{I}^a_\sigma(\rho k_2v^{1-\sigma})+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,\\ D_\sigma \mathbb{N}\Pi(\mathbf{n}_2) =&-(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(\rho k_1v^{1-\sigma})+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))\mathbf{n}_1-k_2v^{1-\sigma}\mathbf{I}^a_\sigma(\rho k_1v^{1-\sigma})\mathbf{n}_2,\\ D_\sigma \mathbb{N}\mathbf{G} =&(c_0v^{1-\sigma}k_1-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\mathbf{n}_1+(c_0v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))\mathbf{n}_2, \end{aligned}$$

and we have conformable energy

$$\begin{aligned} \mathbb{E}(\mathbb{N}\Pi(\mathbf{t})) =&\frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma})^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1))^2\\ &+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2\mathbf{I}^a_\sigma(\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma})^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2))^2),\\ \mathbb{E}(\mathbb{N}\Pi(\mathbf{n}_1)) =&\frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma}\mathbf{I}^a_\sigma(\rho k_2v^{1-\sigma}))^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma}\mathbf{I}^a_\sigma(\rho k_2v^{1-\sigma})+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}))^2),\\ \mathbb{E}(\mathbb{N}\Pi(\mathbf{n}_2)) =&\frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(\rho k_1v^{1-\sigma})+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma}\mathbf{I}^a_\sigma(\rho k_1v^{1-\sigma}))^2),\\ \mathbb{E}(\mathbb{N}\mathbf{G}) =&\frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(c_0v^{1-\sigma}k_1-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))^2+\epsilon_{\mathbf{n}_2}(c_0v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))^2). \end{aligned}$$

♠ *The energy of normalized* $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ *magnetic fields for* $\mathbf{n}_1-$ *spacelike magnetic curve are given as following*

$$\begin{aligned} D_\sigma \mathbb{N}\Pi(\mathbf{t}) =&(v^{1-\sigma}k_1\mathbf{I}^a_\sigma((k_1v^{1-\sigma})^2+\mu v^{1-\sigma}k_2)\\ &+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))\mathbf{n}_1+(v^{1-\sigma}k_2\mathbf{I}^a_\sigma((k_1v^{1-\sigma})^2\\ &+\mu v^{1-\sigma}k_2)+v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu))\mathbf{n}_2,\\ D_\sigma \mathbb{N}\Pi(\mathbf{n}_1) =&k_1v^{1-\sigma}c_1\mathbf{n}_1+k_2v^{1-\sigma}c_1\mathbf{n}_2,\\ D_\sigma \mathbb{N}\Pi(\mathbf{n}_2) =&k_1v^{1-\sigma}c_2\mathbf{n}_1+k_2v^{1-\sigma}c_2\mathbf{n}_2,\\ D_\sigma \mathbb{N}\mathbf{G} =&(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}\\ &-\mu\epsilon_{\mathbf{n}_1}k_1v^{1-\sigma})-v^{1-\sigma}\frac{d\mu}{dv})\mathbf{n}_1+(v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-\mu\epsilon_{\mathbf{n}_1}k_1v^{1-\sigma}\\ &+v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2})+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))\mathbf{n}_2, \end{aligned}$$

and we obtain conformable energy

$$\begin{aligned}\mathbb{E}(\mathbb{N}\Pi(\mathbf{t})) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+(v^{1-\sigma}k_1\mathbf{I}_{\sigma}^{a}((k_1v^{1-\sigma})^2+\mu v^{1-\sigma}k_2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))^2\\&+(v^{1-\sigma}k_2\mathbf{I}_{\sigma}^{a}((k_1v^{1-\sigma})^2+\mu v^{1-\sigma}k_2)+v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu))^2,\\ \mathbb{E}(\mathbb{N}\Pi(\mathbf{n}_1)) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma}c_1)^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma}c_1)^2),\\ \mathbb{E}(\mathbb{N}\Pi(\mathbf{n}_2)) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma}c_2)^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma}c_2)^2),\\ \mathbb{E}(\mathbb{N}\mathbf{G}) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1\mathbf{I}_{\sigma}^{a}(-\mu\epsilon_{\mathbf{n}_1}k_1v^{1-\sigma}+v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2})-v^{1-\sigma}\frac{d\mu}{dv})^2\\&+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2\mathbf{I}_{\sigma}^{a}(-\mu\epsilon_{\mathbf{n}_1}k_1v^{1-\sigma}+v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2})+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))^2).\end{aligned}$$

♠ *The energy of normalized* $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ *magnetic fields for* $\mathbf{n}_2$ *− spacelike magnetic curve are presented as following*

$$\begin{aligned}D_{\sigma}\mathbb{N}\Pi(\mathbf{t}) =& (v^{1-\sigma}k_1\mathbf{I}_{\sigma}^{a}(\gamma k_1v^{1-\sigma}+(k_2v^{1-\sigma})^2)\\&+v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))\mathbf{n}_1+(v^{1-\sigma}k_2\mathbf{I}_{\sigma}^{a}(\gamma k_1v^{1-\sigma}\\&+(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,\\ D_{\sigma}\mathbb{N}\Pi(\mathbf{n}_1) =& k_1v^{1-\sigma}c_4\mathbf{n}_1+k_2v^{1-\sigma}c_4\mathbf{n}_2,\\ D_{\sigma}\mathbb{N}\Pi(\mathbf{n}_2) =& k_1v^{1-\sigma}c_5\mathbf{n}_1+k_2v^{1-\sigma}c_5\mathbf{n}_2,\\ D_{\sigma}\mathbb{N}\mathbf{G} =& (v^{1-\sigma}k_1\mathbf{I}_{\sigma}^{a}(-v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\\&+v^{1-\sigma}\gamma\epsilon_{\mathbf{n}_2}k_2)-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\mathbf{n}_1\\&+(v^{1-\sigma}k_2\mathbf{I}_{\sigma}^{a}(-v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}\\&+v^{1-\sigma}\gamma\epsilon_{\mathbf{n}_2}k_2)+v^{1-\sigma}\frac{d\gamma}{dv})\mathbf{n}_2,\end{aligned}$$

and we obtain conformable energy

$$\begin{aligned}\mathbb{E}(\mathbb{N}\Pi(\mathbf{t})) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}((v^{1-\sigma}k_1\mathbf{I}_{\sigma}^{a}(\gamma k_1v^{1-\sigma}+(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})))^2\\&+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2\mathbf{I}_{\sigma}^{a}(\gamma k_1v^{1-\sigma}+(k_2v^{1-\sigma})^2)+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))^2),\\ \mathbb{E}(\mathbb{N}\Pi(\mathbf{n}_1)) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma}c_4)^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma}c_4)^2),\\ \mathbb{E}(\mathbb{N}\Pi(\mathbf{n}_2)) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}(k_1v^{1-\sigma}c_5)^2+\epsilon_{\mathbf{n}_2}(k_2v^{1-\sigma}c_5)^2),\\ \mathbb{E}(\mathbb{N}\mathbf{G}) =& \frac{1}{2}\mathbf{I}_{\sigma}^{a}(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1\mathbf{I}_{\sigma}^{a}(-v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}+v^{1-\sigma}\gamma\epsilon_{\mathbf{n}_2}k_2)\\&-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))^2+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2\mathbf{I}_{\sigma}^{a}(-v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}+v^{1-\sigma}\gamma\epsilon_{\mathbf{n}_2}k_2)+v^{1-\sigma}\frac{d\gamma}{dv})^2.\end{aligned}$$

4.2 Conformable energy of recursional electromagnetic fields

In this part, the conformable energy of recursional $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2)$ and $\mathbf{G}$ electromagnetic fields are computed.

♠ *The energy of recursional* $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ *magnetic fields for* $\mathbf{t}-$ *spacelike magnetic curve are given as following*

$$\begin{aligned}
D_\sigma \mathcal{R}\Pi(\mathbf{t}) &= (-v^{1-\sigma}k_1\mathbf{I}^a_\sigma(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2)+v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1)) \\
&+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\epsilon_{\mathbf{n}_1}))\mathbf{n}_1+(-v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2) \\
&+v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1))-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\epsilon_{\mathbf{n}_2}))\mathbf{n}_2, \\
D_\sigma \mathcal{R}\Pi(\mathbf{n}_1) &= (k_1v^{1-\sigma}\mathbf{I}^a_\sigma(k_1(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2})-\epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2)+v^{3-3\sigma}k_2k_1^2\epsilon_{\mathbf{n}_1} \\
&+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2})-\epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2))\mathbf{n}_1+(k_2v^{1-\sigma}\mathbf{I}^a_\sigma(k_1(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}) \\
&-\epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2)+v^{3-3\sigma}k_2k_1^2\epsilon_{\mathbf{n}_1})+v^{1-\sigma}\frac{d}{dv}((v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}))\mathbf{n}_2, \\
D_\sigma \mathcal{R}\Pi(\mathbf{n}_2) &= -(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(v^{3-3\sigma}k_1{k_2}^2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}k_2(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1})) \\
&+v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n}_1}))\mathbf{n}_1-(k_2v^{1-\sigma}\mathbf{I}^a_\sigma(v^{3-3\sigma}k_1{k_2}^2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}k_2(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2} \\
&+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}\frac{d}{dv}((v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))\epsilon_{\mathbf{n}_2})\mathbf{n}_2, \\
D_\sigma \mathcal{R}\mathbf{G} &= (v^{1-\sigma}k_1\mathbf{I}^a_\sigma(-v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}(\rho v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}(\rho v^{1-\sigma}k_1 \\
&-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))+v^{1-\sigma}\frac{d}{dv}(\rho v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})))\mathbf{n}_1 \\
&+(v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}(\rho v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}(\rho v^{1-\sigma}k_1 \\
&-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))-v^{1-\sigma}\frac{d}{dv}(\rho v^{1-\sigma}k_1-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2)))\mathbf{n}_2,
\end{aligned}$$

moreover, we have conformable energy

$$\begin{aligned}
\mathbb{E}(\mathcal{R}\Pi(\mathbf{t})) =& \frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(-v^{1-\sigma}k_1\mathbf{I}^a_\sigma(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2)+v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1)) \\
&+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2)\epsilon_{\mathbf{n}_1}))^2+\epsilon_{\mathbf{n}_2}(-v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2) \\
&+v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1))-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1)\epsilon_{\mathbf{n}_2}))^2), \\
\mathbb{E}(\mathcal{R}\Pi(\mathbf{n}_1)) =& \frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}((k_1v^{1-\sigma}\mathbf{I}^a_\sigma(k_1(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2})-\epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2) \\
&+v^{3-3\sigma}k_2k_1^2\epsilon_{\mathbf{n}_1}+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2})-\epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2))^2+\epsilon_{\mathbf{n}_2}((k_2v^{1-\sigma}\mathbf{I}^a_\sigma(k_1(v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_2}) \\
&-\epsilon_{\mathbf{n}_1}v^{2-2\sigma}k_1k_2)+v^{3-3\sigma}k_2k_1^2\epsilon_{\mathbf{n}_1})+v^{1-\sigma}\frac{d}{dv}((v^{1-\sigma}k_1)^2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}))^2), \\
\mathbb{E}(\mathcal{R}\Pi(\mathbf{n}_2)) =& \frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(v^{3-3\sigma}k_1{k_2}^2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}k_2(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2} \\
&+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2)^2\epsilon_{\mathbf{n}_1})^2+\epsilon_{\mathbf{n}_2}(\Leftarrow k_2v^{1-\sigma}\mathbf{I}^a_\sigma(v^{3-3\sigma}k_1{k_2}^2\epsilon_{\mathbf{n}_2} \\
&+v^{1-\sigma}k_2(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}\frac{d}{dv}((v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}+v^{1-\sigma}\frac{d}{dv}(\rho\epsilon_{\mathbf{n}_1}))\epsilon_{\mathbf{n}_2})^2), \\
\mathbb{E}(\mathcal{R}\mathbf{G}) =& \frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1\mathbf{I}^a_\sigma(-v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}(\rho v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})) \\
&+v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}(\rho v^{1-\sigma}k_1-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))+v^{1-\sigma}\frac{d}{dv}(\rho v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})))^2 \\
&+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}(\rho v^{1-\sigma}k_2+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}(\rho v^{1-\sigma}k_1 \\
&-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2))-v^{1-\sigma}\frac{d}{dv}(\rho v^{1-\sigma}k_1-v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}v^{1-\sigma}k_2)))^2).
\end{aligned}$$

♠ *The energy of recursional* $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ *magnetic fields for* $\mathbf{n}_1-$ *spacelike magnetic curve are presented as following*

$$D_\sigma \mathcal{R}\Pi(\mathbf{t}) = (-v^{1-\sigma}k_1\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu) + k_2v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1}))$$
$$+ v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu)\epsilon_{\mathbf{n}_1}))\mathbf{n}_1 + (-v^{1-\sigma}k_2\mathbf{I}_\sigma^a(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2)$$
$$+ v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1)) - v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,$$
$$D_\sigma \mathcal{R}\Pi(\mathbf{n}_1) = (v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1} - k_1v^{1-\sigma}\mathbf{I}_\sigma^a(v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_2} - v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_1})))\mathbf{n}_1$$
$$+ (-k_2v^{1-\sigma}\mathbf{I}_\sigma^a(v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_2} - v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_1}) + v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_1^2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,$$
$$D_\sigma \mathcal{R}\Pi(\mathbf{n}_2) = (v^{1-\sigma}k_1c_3 - v^{1-\sigma}\frac{d}{dv}(\mu v^{1-\sigma}k_2\epsilon_{\mathbf{n}_1}))\mathbf{n}_1 + (v^{1-\sigma}k_2c_3 + v^{1-\sigma}\frac{d}{dv}(\mu k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2})\mathbf{n}_2,$$
$$D_\sigma \mathcal{R}\mathbf{G} = (-v^{1-\sigma}k_1\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma}) - v^{2-2\sigma}k_2\frac{d\mu}{dv}) + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})))\mathbf{n}_1$$
$$+ (-v^{1-\sigma}k_2\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma}) - v^{2-2\sigma}k_2\frac{d\mu}{dv}) + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d\mu}{dv}\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,$$

moreover, we have conformable energy

$$\mathbb{E}(\mathcal{R}\Pi(\mathbf{t})) = \frac{1}{2}\mathbf{I}_\sigma^a(1 + \epsilon_{\mathbf{n}_1}(-v^{1-\sigma}k_1\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu)$$
$$+ k_2v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})) + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(\epsilon_{\mathbf{n}_2}\mu)\epsilon_{\mathbf{n}_1}))^2$$
$$+ \epsilon_{\mathbf{n}_2}(-v^{1-\sigma}k_2\mathbf{I}_\sigma^a(-v^{2-2\sigma}k_1\frac{d}{dv}(v^{1-\sigma}k_2) + v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_1))$$
$$- v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_1\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}))^2),$$
$$\mathbb{E}(\mathcal{R}\Pi(\mathbf{n}_1)) = \frac{1}{2}\mathbf{I}_\sigma^a(1 + \epsilon_{\mathbf{n}_1}(-k_1v^{1-\sigma}\mathbf{I}_\sigma^a(v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_2}$$
$$- v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_1}) + v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_1k_2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}))^2$$
$$+ \epsilon_{\mathbf{n}_2}(-k_2v^{1-\sigma}\mathbf{I}_\sigma^a(v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_2} - v^{2-2\sigma}k_1^2k_2\epsilon_{\mathbf{n}_1})$$
$$+ v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_1^2\epsilon_{\mathbf{n}_1}\epsilon_{\mathbf{n}_2}))^2),$$
$$\mathbb{E}(\mathcal{R}\Pi(\mathbf{n}_2)) = \frac{1}{2}\mathbf{I}_\sigma^a(1 + \epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1c_3 - v^{1-\sigma}\frac{d}{dv}(\mu v^{1-\sigma}k_2\epsilon_{\mathbf{n}_1})^2$$
$$+ \epsilon_{\mathbf{n}_2}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})))^2),$$
$$\mathbb{E}(\mathcal{R}\mathbf{G}) = \frac{1}{2}\mathbf{I}_\sigma^a(1 + \epsilon_{\mathbf{n}_1}(-v^{1-\sigma}k_1\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})$$
$$- v^{2-2\sigma}k_2\frac{d\mu}{dv}) + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})))^2$$
$$+ \epsilon_{\mathbf{n}_2}(-v^{1-\sigma}k_2\mathbf{I}_\sigma^a(-k_1v^{2-2\sigma}\frac{d}{dv}(k_1\epsilon_{\mathbf{n}_1}v^{1-\sigma})$$
$$- v^{2-2\sigma}k_2\frac{d\mu}{dv}) + v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d\mu}{dv}\epsilon_{\mathbf{n}_2}))^2).$$

♠ *The energy of recursional* $\Pi(\mathbf{t}), \Pi(\mathbf{n}_1), \Pi(\mathbf{n}_2), \mathbf{G}$ *magnetic fields for* $\mathbf{n}_2 -$ *spacelike magnetic curve are given as following*

$$
\begin{aligned}
D_\sigma \mathcal{R}\Pi(\mathbf{t}) =&(-v^{1-\sigma}k_1\mathbf{I}^a_\sigma(-k_1v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\\
&+k_2v^{2-2\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_1}))\mathbf{n}_1\\
&+(-v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-k_1v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})+k_2v^{2-2\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))\\
&-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,\\
D_\sigma \mathcal{R}\Pi\left(\mathbf{n}_1\right) =&(v^{1-\sigma}k_1c_6-v^{1-\sigma}\frac{d}{dv}(\gamma k_2v^{1-\sigma}\epsilon_{\mathbf{n}_1}))\mathbf{n}_1\\
&+(v^{1-\sigma}k_2c_6+v^{1-\sigma}\frac{d}{dv}(\gamma k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2})\mathbf{n}_2,\\
D_\sigma \mathcal{R}\Pi\left(\mathbf{n}_2\right) =&(v^{1-\sigma}k_1c_7-v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_2^2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}))\mathbf{n}_1\\
&+(v^{1-\sigma}k_2c_7+v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_2k_1)\mathbf{n}_2,\\
D_\sigma \mathcal{R}\mathbf{G} =&(-v^{1-\sigma}k_1\mathbf{I}^a_\sigma(\frac{d\gamma}{dv}v^{2-2\sigma}k_1-v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\\
&-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d\gamma}{dv}\epsilon_{\mathbf{n}_1})))\mathbf{n}_1+(-v^{1-\sigma}k_2\mathbf{I}^a_\sigma(\frac{d\gamma}{dv}v^{2-2\sigma}k_1\\
&-v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_2}))\mathbf{n}_2,
\end{aligned}
$$

and we obtain conformable energy

$$
\begin{aligned}
\mathbb{E}(\mathcal{R}\Pi(\mathbf{t})) =&\frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(-v^{1-\sigma}k_1\mathbf{I}^a_\sigma(-k_1v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\\
&+k_2v^{2-2\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_1}))^2\\
&+\epsilon_{\mathbf{n}_2}(-v^{1-\sigma}k_2\mathbf{I}^a_\sigma(-k_1v^{2-2\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})+k_2v^{2-2\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1}))\\
&-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(\gamma\epsilon_{\mathbf{n}_1})\epsilon_{\mathbf{n}_2}))^2),\\
\mathbb{E}(\mathcal{R}\Pi\left(\mathbf{n}_1\right)) =&\frac{1}{2}\ \mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1c_6-v^{1-\sigma}\frac{d}{dv}(\gamma k_2v^{1-\sigma}\epsilon_{\mathbf{n}_1}))^2\\
&+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2c_6+v^{1-\sigma}\frac{d}{dv}(\gamma k_1v^{1-\sigma}\epsilon_{\mathbf{n}_2})^2),\\
\mathbb{E}(\mathcal{R}\Pi\left(\mathbf{n}_2\right)) =&\frac{1}{2}\ \mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(v^{1-\sigma}k_1c_7-v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_2^2\epsilon_{\mathbf{n}_2}\epsilon_{\mathbf{n}_1}))^2\\
&+\epsilon_{\mathbf{n}_2}(v^{1-\sigma}k_2c_7+v^{1-\sigma}\frac{d}{dv}(v^{2-2\sigma}k_2k_1)^2),\\
\mathbb{E}(\mathcal{R}\mathbf{G}) =&\frac{1}{2}\mathbf{I}^a_\sigma(1+\epsilon_{\mathbf{n}_1}(-v^{1-\sigma}k_1\mathbf{I}^a_\sigma(\frac{d\gamma}{dv}v^{2-2\sigma}k_1-v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))\\
&-v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d\gamma}{dv}\epsilon_{\mathbf{n}_1})))^2+\epsilon_{\mathbf{n}_2}(-v^{1-\sigma}k_2\mathbf{I}^a_\sigma(\frac{d\gamma}{dv}v^{2-2\sigma}k_1\\
&-v^{2-2\sigma}k_2\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2}))+v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}\frac{d}{dv}(v^{1-\sigma}k_2\epsilon_{\mathbf{n}_2})\epsilon_{\mathbf{n}_2})^2).
\end{aligned}
$$

The application of recursional $\Pi(\mathbf{t}),\Pi\left(\mathbf{n}_1\right),\Pi\left(\mathbf{n}_2\right)$ enegies of the system can be evaluated with different volume fractions for different conformable permeability. The evaluation of

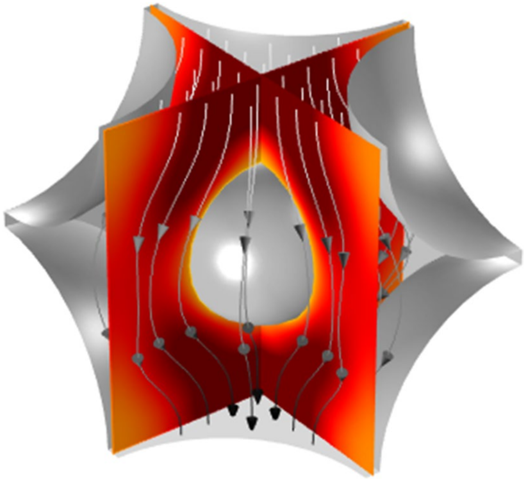

Fig. 1 Recursional$\Pi(\mathbf{t})$ energy with conformable permeability

energy is based in heat conformable transfer and potential of friction with viscosity and thermal conformable $\Pi(\mathbf{t}), \Pi\left(\mathbf{n}_1\right), \Pi\left(\mathbf{n}_2\right)$ conductivity in Figs. 1, 2 and 3.

5 Conclusions

In this paper, we have investigated spacelike magnetic curves according to Bishop frame. Firstly, we have presented conformable derivatives of Lorentz magnetic fields of these magnetic curves. Moreover, we have calculated the conformable derivatives of the normalization and recursional electromagnetic vector fields. Finally, we obtain conformable energy of the normalization and recursional operators for these electromagnetic fields associated with Bishop frame.

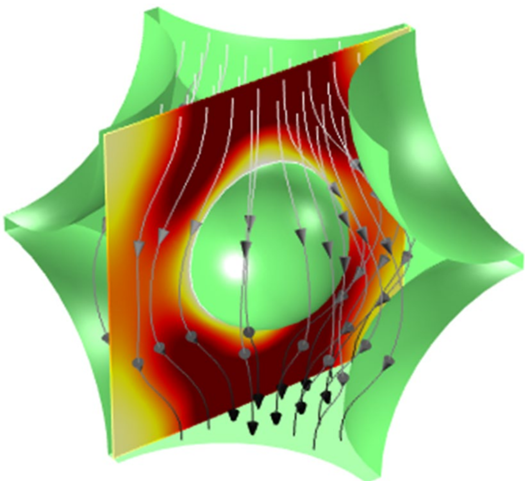

Fig. 2 Recursional$\Pi\left(\mathbf{n}_1\right)$ energy with conformable permeability

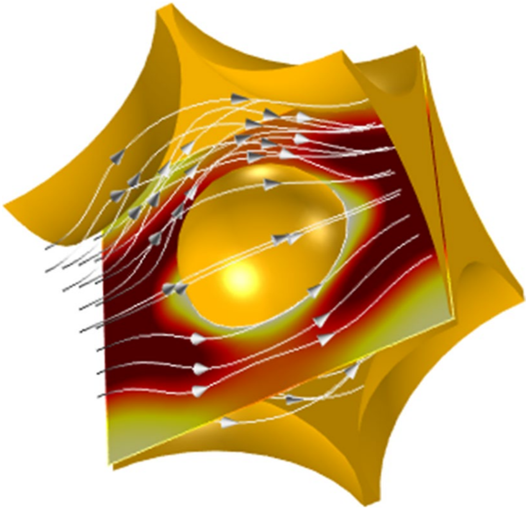

Fig. 3 Recursional$\Pi(\mathbf{n}_2)$ energy with conformable permeability

Author contributions All authors of this research paper have directly participated in the planning, execution, or analysis of this study; All authors of this paper have read and approved the final version submitted.

Funding Open access funding provided by the Scientific and Technological Research Council of Türkiye (TÜBİTAK). No funding was received for the study.

Availability of data and materials No data was used for the research described in the article.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval The contents of this manuscript have not been copyrighted or published previously; The contents of this manuscript are not now under consideration for publication elsewhere.

References

Bishop, R.L.: There is more than one way to frame a curve. Am. Math. Mon. **82**(3), 246–251 (1975)

Bukcu, B., Karacan, M.K.: Special Bishop motion and Bishop Darboux rotation axis of the space curve. J. Dyn. Syst. Geom. Theories **6**(1), 27–34 (2008)

Bükcü, B., Karacan, M.K.: The slant helices according to Bishop frame. Int. J. Comput. Math. Sci. **3**(2), 67–70 (2009a)

Bükcü, B., Karacan, M.K.: Bishop motion and Bishop Darboux rotation axis of the timelike curve in Minkowski 3-space, Kochi J. Math **4**, 109–117 (2009b)

de Andrade, L.G.: Vortex filaments in MHD. Phys. Scr. **73**(5), 484 (2006)

do Carmo, M.P.: Differential Geometry of Curves and Surfaces. Prentice Hall, Englewood Cliffs (1976)

Epp, R.J.: Angular momentum and an invariant quasilocal energy in general relativity. Phys. Rev. D **62**(12), 124018 (2000)

Hayward, G.: Quasilocal gravitational energy. Phys. Rev. D. **47**, 831 (1994)

Karacan, M.K., Bukcu, B.: An alternative moving frame for tubular surfaces around timelike curves in the Minkowski 3-space. Balkan J. Geom. Appl. **12**(2) (2007a)

Karacan, K.M., Bukcu, B.: An alternative moving frame for tubular surface around the spacelike curve with a spacelike binormal in Minkowski 3-space. Math. Morav. **11**, 47–54 (2007b)

Karacan, M.K., Bukcu, B.: An alternative moving frame for a tubular surface around a spacelike curve with a spacelike normal in Minkowski 3-space. Rend. Circ. Mat. Palermo **57**, 193–201 (2008)

Körpinar, T.: New characterizations for minimizing energy of biharmonic particles in Heisenberg spacetime. Int. J. Theor. Phys. **53**, 3208–3218 (2014)

Körpinar, T.: On T-magnetic biharmonic particles with energy and angle in the three dimensional Heisenberg group H. Adv. Appl. Clifford Algebras **28**(1), 9 (2018)

Körpinar, T., Demirkol, R.C.: A new characterization on the energy of elastica with the energy of Bishop vector fields in Minkowski space. J. Adv. Phys. **6**(4), 562–569 (2017)

Körpinar, T., Demirkol, R.C.: A new approach on the energy of elastica and non-elastica in Minkowski space E 2 4. Bull. Braz. Math. Soc. New Ser. **49**(1), 159–177 (2018)

Körpinar, T., Körpinar, Z.: Optical spherical Ss-electric and magnetic phase with fractional q-HATM approach. Optik **243**, 167274 (2021a)

Körpinar, T., Körpinar, Z.: New version of optical spherical electric and magnetic flow phase with some fractional solutions in SH32. Optik **243**, 167378 (2021b)

Körpinar, T., Turhan, E.: Constant energy of time involute particles of biharmonic particles in Bianchi type-I cosmological model spacetime. Int. J. Theor. Phys. **54**, 1654–1660 (2015)

Körpınar, T., Demirkol, R.C., Körpınar, Z.: Magnetic helicity and electromagnetic vortex filament flows under the influence of Lorentz force in MHD. Optik **242**, 167302 (2021a)

Körpınar, T., Demirkol, R.C., Körpınar, Z.: New analytical solutions for the inextensible Heisenberg ferromagnetic flow and solitonic magnetic flux surfaces in the binormal direction. Phys. Scr. **96**(8), 085219 (2021b)

Körpınar, T., Körpınar, Z., Yeneroğlu, M.: Optical energy of spherical velocity with optical magnetic density in Heisenberg sphere space $\mathbb{S}^2_{Heis^3}$. Optik **247**, 167937 (2021c)

Körpinar, T., Demirkol, R.C., Körpınar, Z.: A fractionally magnetized flow of force fields and Fermi–Walker conformable derivative on the unit sphere. Waves Random Complex Media (2022). https://doi.org/10.1080/17455030.2022.2162150

Maluf, J.W., Faria, F.F.: On the construction of Fermi–Walker transported frames. Ann. Phys. **17**(5), 326–335 (2008)

Martinez, E.A.: Quasilocal energy for Kerr black hole. Phys. Rev. D. **50**, 4920 (1994)