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An investigation of Fokas system using two new modifications for the trigonometric and hyperbolic trigonometric function methods

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Abstract
In this work, two new adaptations for the trigonometric and hyperbolic trigonometric function approaches have been presented. These two modifications, entitled modified extended rational sin–cos function technique and modified extended rational sinh–cosh function method, have been applied for the first time to the Fokas system that represents the nonlinear pulse propagation in monomode fiber optics. We intend to produce innovative, explicit traveling waves, solitons, and periodic wave solutions. These achieved outcomes are presented in the form of exponential functions, trigonometric hyperbolic functions, and combination constructions of the exponential functions along with the trigonometric and hyperbolic trigonometric functions. The obtained solutions reveal significant features of the physical phenomenon and are new. The investigated model incorporates the notions of dispersion, transverse diffusion, degree of dispersion, nonlinear pairing, nonlinear immersion, and the force of the nonlinear interaction among the two components of the system. For the most accurate visual evaluation of the physical importance and dynamic properties, we have presented the findings in a variety of plots, which involve two- and three-dimensional representations. One or more elements in our research that are unique, such as newly modified methodologies, is a new observation that leads researchers to invest in new solutions.

Keywords Fokas system · Traveling wave transformation · Modified extended rational sinh–cosh function method · Monomode fiber optics · Modified extended rational sin–cos

Mathematics Subject Classification 35C07 · 35Dxx · 45Kxx · 65Mxx · 45K05

1 Introduction

In the present day of computer networking and communications, the topic of study in the theory of solitons and their utilization in fiber optics is becoming increasingly essential. An optical soliton is a flash of light that travels without distortion owing to dispersion or other causes. Both temporal and spatial solitons will be addressed, combined with the physical

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components that make them feasible. In this situation, the optical pulse could begin to create a stable nonlinear pulse known as an optical soliton. The dispersion of the fiber material restricts the bit rate of transmission. Fiber loss is the sole element that contributes to the decline of the pulse quality through expansion in the pulse width.

The complex nonlinear (2 + 1)-dimensional Fokas system that demonstrates nonlinear pulse propagation in monomode fiber optics has the following form:

$$\begin{aligned} &iu_t + \beta_1 u_{xxx} + \beta_2 uv = 0 \\ &\beta_3 v_y - \beta_4(|u|^2)_x = 0, \end{aligned} \tag{1}$$

that derived in 1994 by Fokas (1994) employing the inverse spectral method, the non-linear pulse propagation in monomode fiber optic is represented by the complex functions $v(x, y, t)$ and $u(x, y, t)$. β_1 symbolizes the dispersion coefficient, which characterizes the degree of dispersion in the system, β_2 denotes the nonlinear pairing parameter, which indicates the intensity of the nonlinear dealing among the two components of the system, u and v, β_3 symbolizes the transverse diffusion parameter, which specifies the amount of dispersion in the transverse direction, β_4 illustrates the nonlinear immersion coefficient, which represents the amount of a saturated state of the nonlinear participation. Differing versions of (1) have been examined using various methodologies, including Riccati expansion and Ansatz methods (Khater 2021), the generic Kudryashov's method, the Sardar sub-equation approach, and Bernoulli sub-equation function method (Ali et al. 2023b), the truncated Painlevé approach (Thilakavathy et al. 2023), generalized Riccati equation mapping and Kudryashov methods (Kumar and Kumar 2023a), using a modified mapping method (Mohammed et al. 2023), the extended rational versions of sinh–cosh and sin–cos methods (Wang et al. 2022), the bilinear transformation method (Chen et al. 2019; Rao et al. 2015), the bilinear Kadomtsev-Petviashvili hierarchy reduction method (Rao et al. 2021), the bilinear forms of Hirota's method (Rao et al. 2019), the exponential function method (Wang 2022), the elliptic function expansion forms of the Jacobian method (Tarla et al. 2022), the singular manifold, and the expansion forms of G'/G^2, Sine-Gordon methods (Alrebdi et al. 2022), the polynomial method that depends on the complete discrimination (Zhang et al. 2023).

Numerous research works examine considerable analytical and semi-analytical techniques for getting the exact solution of NPDEs, including the modified version of the exponential-function method (Muhamad et al. 2023), the extended rational forms of sin–cos and sinh–cosh methods (Mahmud et al. 2023a, b), Bernoulli and its improved version (Baskonus et al. 2022a, b; Mahmud et al. 2023c, d), The transformation of Laplace has been used for solving the fractional system in the form of Caputo fractional derivatives (Tanriverdi et al. 2021), it is worth mentioning that the main source of these modifications are (Mahmud 2023; Muhamad 2023f), the extended auxiliary equation mapping and extended direct algebraic methods (Iqbal et al. 2018a, b, 2019; Seadawy et al. 2019, 2020a, b; Seadawy and Iqbal 2021), the extension of the modified rational expansion method (Seadawy et al. 2021), the modification form of extended auxiliary equation mapping method (Lu et al. 2018; Iqbal and Seadawy 2020; Seadawy and Iqbal 2023), the extended modified rational expansion method (Seadawy et al. 2022), the generalized exponential rational function method (Ghanbari and Gómez-Aguilar 2019a, b; Ghanbari and Baleanu 2020; Ghanbari 2019; Ghanbari et al. 2018; Ghanbari and Kuo 2019; Ghanbari and Baleanu 2019), the five methods mentioned therein (Khater and Ghanbari 2021), the reproducing kernel method (Ghanbari and Akgül 2020), the extended rational sinh-Gordon method and $\exp(-\phi(\eta))$ expansion function method (Shafqat-ur-Rehman and Ahmad 2023; Rehman and Ahmad

2023), the modified generalized exponential rational function method, and the modified rational sinh–cosh and sin–cos methods (Rehman et al. 2022, 2023a, b; Ahmad et al. 2023; Ahmad 2023), the modified Sardar sub-equation method (Ali et al. 2023a). Considering this context, we can notice an array of methodologies used by several academics to express their ideas in exploring the mathematical models that describe situations in real life (Gasmi et al. 2023; Jafari et al. 2023; Srinivasa and Mundewadi 2023; Bilal et al. 2023; Kumar and Kumar 2023b; Nasir et al. 2023). Overall, some shortcomings and adverse characteristics in the prior versions of these methods became the motivation for us to come up with these two additional enhancements.

This scholarly investigation has been laid out as follows: Sect. 1 is specialized for listing the literature relevant to the approaches and the examined model in a short overview. The methodologies of the described approaches are detailed in Sect. 2. The formulation of the recommended techniques for constructing specific semi-analytic solutions to Eq. (1) is presented in Sect. 3. In Sect. 4, the concluding remarks of the study have been provided agreeably. Finally, the last Sect. 5, is dedicated to the analysis and discussion of the results that were collected.

2 Formulation of the modification methods

Always, the configuration of the presented approaches commonly depends on the following step:

Step 1 Let the next NPDE be followed.

$$\mathscr{S}\left(\mathscr{T}, \mathscr{T}_x, \mathscr{T}_t, \mathscr{T}_y, \mathscr{T}_{xt}, \mathscr{T}_{xx}, \mathscr{T}_{yt}, \mathscr{T}_{xyt}, \dots\right) = 0, \tag{2}$$

wherein $\mathscr{T} = \mathscr{T}(x, y, t)$. By setting

$$\mathscr{T}(x, y, t) = \mathscr{R}(\mathscr{P}), \ \ \mathscr{P} = \delta_1 x + \delta_2 y - \delta_3 t, \tag{3}$$

where δ_1, δ_2 and δ_3 are non-zero arbitrary parameters. If (3) is substituted in (2), then the outcome is presented as follows

$$\mathscr{I}(\mathscr{R}, \mathscr{R}', \mathscr{R}'', \dots) = 0, \tag{4}$$

herein

$$\mathscr{R} = \mathscr{R}(\mathscr{P}), \ \ \mathscr{R}' = \frac{d\mathscr{R}}{d\mathscr{P}}, \ \ \mathscr{R}'' = \frac{d^2\mathscr{R}}{d\mathscr{P}^2}, \dots$$

Step 2 Initially, we created these two modified solution forms:

1. For the first modification, let the solution to (4) take the following forms:

$$\mathscr{T}(\mathscr{P}) = \frac{\gamma_0 + \gamma_1 \sinh(\mu\mathscr{P})}{\gamma_2 \sinh(\mu\mathscr{P}) \pm \gamma_3 \cosh(\mu\mathscr{P})}, \ \ \gamma_2 \sinh(\mu\mathscr{P}) \pm \gamma_3 \cosh(\mu\mathscr{P}) \neq 0, \tag{5}$$

or,

$$\mathscr{T}(\mathscr{P}) = \frac{\gamma_0 + \gamma_1 \cosh(\mu\mathscr{P})}{\gamma_2 \sinh(\mu\mathscr{P}) \pm \gamma_3 \cosh(\mu\mathscr{P})}, \ \ \gamma_2 \sinh(\mu\mathscr{P}) \pm \gamma_3 \cosh(\mu\mathscr{P}) \neq 0, \tag{6}$$

2. For the second modification, suppose that the solutions to (4) take the following forms:

$$\mathscr{T}(\mathscr{P}) = \frac{\gamma_0 + \gamma_1 \sin(\mu\mathscr{P})}{\gamma_2 \sin(\mu\mathscr{P}) \pm \gamma_3 \cos(\mu\mathscr{P})},\ \gamma_2 \sin(\mu\mathscr{P}) \pm \gamma_3 \cos(\mu\mathscr{P}) \neq 0, \tag{7}$$

or,

$$\mathscr{T}(\mathscr{P}) = \frac{\gamma_0 + \gamma_1 \cos(\mu\mathscr{P})}{\gamma_2 \sin(\mu\mathscr{P}) \pm \gamma_3 \cos(\mu\mathscr{P})},\ \gamma_2 \sin(\mu\mathscr{P}) \pm \gamma_3 \cos(\mu\mathscr{P}) \neq 0, \tag{8}$$

where in (5–8), the $\mu,\ \gamma_i,$ for $i = 0, 1, 2, 3$ are intended coefficients that will be identified later such that

$$\gamma_0^2 + \gamma_1^2 \neq 0,\ \gamma_2^2 + \gamma_3^2 \neq 0,$$

and a wave number $\mu \neq 0$.

Step 3 Anonymous, also known as parameters, might be found by substituting one of (5–8) into (4), putting together all the terms that have the same powers as and equating to zero all the coefficients for the same power terms, this process produces a set of algebraic equations. Identifying the solutions to the obtained algebraic system using different symbolic computing tools is possible.

Step 4 By re-installing the obtained results of $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ and μ into one of (5–8), the solution to (4) will be derived, and thereafter, the solution to (2) is obtained.

3 Implementations of the recommended methods

Implementing waveform transformation

$$u(x, y, t) = U(\xi)e^{i\kappa\xi}, v(x, y, t) = V(\xi),\ \xi = \delta_1 x + \delta_2 y - \delta_3 t, \tag{9}$$

to (1), then one gets the following:

$$\begin{aligned} &\kappa\delta_3 U - i(\delta_3 - 2\beta_1\kappa\delta_1^2)U' - \beta_1\kappa^2\delta_1^2 U + \beta_1\delta_1^2 U'' + \beta_2 UV = 0 \\ &\beta_3\delta_2 V' - 2\beta_4\delta_1 UU' = 0. \end{aligned} \tag{10}$$

directly from the second part of (10), one obtains:

$$V = \frac{\beta_4\delta_1}{\beta_3\delta_2}U^2. \tag{11}$$

By substituting (11) into the first part of (10), the following is the outcome:

$$\kappa\delta_3 U - i(\delta_3 - 2\beta_1\kappa\delta_1^2)U' - \beta_1\kappa^2\delta_1^2 U + \beta_1\delta_1^2 U'' + \frac{\beta_2\beta_4\delta_1}{\beta_3\delta_2}U^3 = 0. \tag{12}$$

By splitting the real and imagined components of (12), the operators end up with:

$$\begin{aligned} &\kappa\delta_3 U - \beta_1\kappa^2\delta_1^2 U + \beta_1\delta_1^2 U'' + \frac{\beta_2\beta_4\delta_1}{\beta_3\delta_2}U^3 = 0 \\ &- i(\delta_3 - 2\beta_1\kappa\delta_1^2)U' = 0. \end{aligned} \tag{13}$$

From the imaginary part of (13), one immediately obtains:

$$\kappa = \frac{\delta_3}{2\beta_1 \delta_1^2}. \tag{14}$$

By substituting (14) into the real part of (13) after simplifications, the following is the result:

$$\frac{\delta_3^2}{4\beta_1 \delta_1^2} U + \beta_1 \delta_1^2 U'' + \frac{\beta_2 \beta_4 \delta_1}{\beta_3 \delta_2} U^3 = 0. \tag{15}$$

A recommended equation to suppose the trial solution is the ordinary differential equation (15).

3.1 Implementation of MER sinh–cosh M to the examined model

To solve (1) by employing the MER sinh-cosh M, suppose that (15) has a solution with the following form:

$$\frac{\gamma_1 \sinh(\mu\xi) + \gamma_0}{\gamma_2 \sinh(\mu\xi) + \gamma_3 \cosh(\mu\xi)}. \tag{16}$$

In (16), $\mu, \gamma_0, \gamma_1, \gamma_2,$ and γ_3 are unknown purposeful parameters that must be demonstrated later by taking into account that

$$\mu \neq 0,\ \gamma_0^2 + \gamma_1^2 \neq 0,\ \gamma_2^2 + \gamma_3^2 \neq 0,$$

and μ is a wave number. Moreover, the derivatives of (16) with respect to ξ are taking the following forms:

$$U' = \frac{\gamma_1 \gamma_3 \mu - \gamma_0 \mu \big(\gamma_3 \sinh(\mu\xi) + \gamma_2 \cosh(\mu\xi)\big)}{\big(\gamma_2 \sinh(\mu\xi) + \gamma_3 \cosh(\mu\xi)\big)^2}, \tag{17}$$

and

$$\begin{aligned} U'' = &- \frac{2\big(\gamma_3 \mu \sinh(\mu\xi) + \gamma_2 \mu \cosh(\mu\xi)\big)\big(\gamma_1 \gamma_3 \mu - \gamma_0 \mu \big(\gamma_3 \sinh(\mu\xi) + \gamma_2 \cosh(\mu\xi)\big)\big)}{\big(\gamma_2 \sinh(\mu\xi) + \gamma_3 \cosh(\mu\xi)\big)^3} \\ &- \frac{\gamma_0 \mu \big(\gamma_2 \mu \sinh(\mu\xi) + \gamma_3 \mu \cosh(\mu\xi)\big)}{\big(\gamma_2 \sinh(\mu\xi) + \gamma_3 \cosh(\mu\xi)\big)^2}. \end{aligned} \tag{18}$$

Subbing (16)–(18) into (15), one gets the following:

$$\left.\begin{aligned}
&-4\beta_1^2\beta_3\gamma_0\gamma_2^2\delta_2\delta_1^4\mu^2\sinh^2(\mu\xi)-4\beta_1^2\beta_3\gamma_0\gamma_3^2\delta_2\delta_1^4\mu^2\sinh^2(\mu\xi)-4\beta_1\beta_2\beta_4\gamma_0^3\delta_1^3\\
&+8\beta_1^2\beta_3\gamma_1\gamma_3^2\delta_2\delta_1^4\mu^2\sinh(\mu\xi)+8\beta_1^2\beta_3\gamma_1\gamma_2\gamma_3\delta_2\delta_1^4\mu^2\cosh(\mu\xi)-\beta_3\gamma_0\gamma_3^2\delta_2\delta_3^2\\
&+4\beta_1^2\beta_3\gamma_0\gamma_3^2\delta_2\delta_1^4\mu^2-4\beta_1\beta_2\beta_4\gamma_1^3\delta_1^3\sinh^3(\mu\xi)-\beta_3\gamma_1\gamma_2^2\delta_2\delta_3^2\sinh^3(\mu\xi)\\
&-\beta_3\gamma_1\gamma_3^2\delta_2\delta_3^2\sinh^3(\mu\xi)-12\beta_1\beta_2\beta_4\gamma_0\gamma_1^2\delta_1^3\sinh^2(\mu\xi)-\beta_3\gamma_0\gamma_2^2\delta_2\delta_3^2\sinh^2(\mu\xi)\\
&-\beta_3\gamma_0\gamma_3^2\delta_2\delta_3^2\sinh^2(\mu\xi)-12\beta_1\beta_2\beta_4\gamma_0^2\gamma_1\delta_1^3\sinh(\mu\xi)-\beta_3\gamma_1\gamma_3^2\delta_2\delta_3^2\sinh(\mu\xi)\\
&-2\beta_3\gamma_1\gamma_2\gamma_3\delta_2\delta_3^2\sinh^2(\mu\xi)\cosh(\mu\xi)-2\beta_3\gamma_0\gamma_2\gamma_3\delta_2\delta_3^2\sinh(\mu\xi)\cosh(\mu\xi)\\
&-8\beta_1^2\beta_3\gamma_0\gamma_2\gamma_3\delta_2\delta_1^4\mu^2\sinh(\mu\xi)\cosh(\mu\xi)-8\beta_1^2\beta_3\gamma_0\gamma_2^2\delta_2\delta_1^4\mu^2=0.
\end{aligned}\right\} \tag{19}$$

In (19) collecting all the coefficients with the same powers of $\cosh^{\tau_1}(\mu\mathscr{P})\sinh^{\tau_2}(\mu\mathscr{P})$ where $\tau_1,\tau_2=0,1,2,3$ and equating them to zero. From the coefficients of $\cosh^{\tau_1}(\mu\mathscr{P})\sinh^{\tau_2}(\mu\mathscr{P})$, one creates a system as given below:

$$\left.\begin{aligned}
&-8\beta_1^2\beta_3\gamma_0\gamma_2^2\delta_2\delta_1^4\mu^2+4\beta_1^2\beta_3\gamma_0\gamma_3^2\delta_2\delta_1^4\mu^2-4\beta_1\beta_2\beta_4\gamma_0^3\delta_1^3-\beta_3\gamma_0\gamma_3^2\delta_2\delta_3^2=0,\\
&8\beta_1^2\beta_3\gamma_1\gamma_2\gamma_3\delta_1^4\delta_2\mu^2=0,\\
&8\beta_1^2\beta_3\gamma_1\gamma_3^2\delta_2\delta_1^4\mu^2-12\beta_1\beta_2\beta_4\gamma_0^2\gamma_1\delta_1^3-\beta_3\gamma_1\gamma_3^2\delta_2\delta_3^2=0,\\
&-8\beta_1^2\beta_3\gamma_0\gamma_2\gamma_3\delta_2\delta_1^4\mu^2-2\beta_3\gamma_0\gamma_2\gamma_3\delta_2\delta_3^2=0,\\
&-4\beta_1^2\beta_3\gamma_0\gamma_2^2\delta_2\delta_1^4\mu^2-4\beta_1^2\beta_3\gamma_0\gamma_3^2\delta_2\delta_1^4\mu^2-12\beta_1\beta_2\beta_4\gamma_0\gamma_1^2\delta_1^3-\beta_3\gamma_0\gamma_2^2\delta_2\delta_3^2\\
&-\beta_3\gamma_0\gamma_3^2\delta_2\delta_3^2=0,\\
&-2\beta_3\gamma_1\gamma_2\gamma_3\delta_2\delta_3^2=0,\\
&-4\beta_1\beta_2\beta_4\gamma_1^3\delta_1^3-\beta_3\gamma_1\gamma_2^2\delta_2\delta_3^2-\beta_3\gamma_1\gamma_3^2\delta_2\delta_3^2=0.
\end{aligned}\right\} \tag{20}$$

One creates the following cases by solving (20).

Case 1 The following are the parameters that were obtained from solving (20):

$$\begin{aligned}\gamma_2&=-\frac{\sqrt{2\beta_1\beta_3\gamma_3^2\delta_1\delta_2\mu^2-\beta_2\beta_4\gamma_0^2}}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\sqrt{\delta_1}\sqrt{\delta_2}\mu};\\ \gamma_1&=0;\delta_3=-2i\beta_1\delta_1^2\mu.\end{aligned} \tag{21}$$

The following set of solutions to (1) has been identified by replacing (21) gathering with (16) into (15).

$$u_1=\frac{\gamma_0e^{\delta_2\mu y+\delta_1\mu\left(x+2i\beta_1\delta_1\mu t\right)}}{\gamma_3\cosh\left(\delta_2\mu y+\delta_1\mu\left(x+2i\beta_1\delta_1\mu t\right)\right)-\frac{\sqrt{2\beta_1\beta_3\gamma_3^2\delta_1\delta_2\mu^2-\beta_2\beta_4\gamma_0^2}\sinh\left(\delta_2\mu y+\delta_1\mu\left(x+2i\beta_1\delta_1\mu t\right)\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\sqrt{\delta_1}\sqrt{\delta_2}\mu}}, \tag{22}$$

and

$$v_1=\frac{\beta_4\gamma_0^2\delta_1}{\beta_3\delta_2\left(\gamma_3\cosh\left(\mu\left(2i\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)\right)-\frac{\sqrt{2\beta_1\beta_3\gamma_3^2\delta_1\delta_2\mu^2-\beta_2\beta_4\gamma_0^2}\sinh\left(\mu\left(2i\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\sqrt{\delta_1}\sqrt{\delta_2}\mu}\right)^2}. \tag{23}$$

Graphs of (22) and (23) where $\beta_1=-\frac{8}{3};\beta_2=\frac{1}{2};\beta_3=\frac{9}{4};\beta_4=\frac{2}{3};\mu=-\frac{2}{3};\delta_1=\frac{2}{5};\delta_2=-\frac{1}{2};y=-\frac{3}{2};\gamma_0=\frac{5}{2};\gamma_3=\frac{1}{2}$, and $-20\le x\le 20,\ -20\le t\le 20$ are given in the following:

For the values of t that are mentioned below, one reaches:

The values of t are mentioned in the legend below.

Case 2 The following are the parameters that were obtained from solving (20):

$$\begin{aligned}\gamma_0&=-\frac{\sqrt{\beta_1}\sqrt{\beta_3}\gamma_3\sqrt{\delta_1}\sqrt{\delta_2}\mu}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}};\\ \gamma_1&=-\frac{i\sqrt{\beta_1}\sqrt{\beta_3}\gamma_3\sqrt{\delta_1}\sqrt{\delta_2}\mu}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}};\gamma_2=0;\delta_3=-\sqrt{2}\beta_1\delta_1^2\mu.\end{aligned} \tag{24}$$

The following set of solutions to (1) has been determined by re-installing (24) with (16) into (15).

$$\begin{aligned}u_2=&-\frac{\sqrt{\beta_1}\sqrt{\beta_3}\sqrt{\delta_1}\sqrt{\delta_2}\mu\exp\left(-\frac{i\mu\left(\sqrt{2}\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)}{\sqrt{2}}\right)\operatorname{sech}\left(\mu\left(\sqrt{2}\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}\\ &-\frac{\sqrt{\beta_1}\sqrt{\beta_3}\sqrt{\delta_1}\sqrt{\delta_2}\mu\exp\left(-\frac{i\mu\left(\sqrt{2}\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)}{\sqrt{2}}\right)\left(i\tanh\left(\mu\left(\sqrt{2}\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)\right)\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}},\end{aligned} \tag{25}$$

and

$$v_2=\frac{\beta_1\delta_1^2\mu^2\left(\operatorname{sech}\left(\mu\left(\sqrt{2}\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)\right)+i\tanh\left(\mu\left(\sqrt{2}\beta_1\delta_1^2\mu t+\delta_1x+\delta_2y\right)\right)\right)^2}{2\beta_2}. \tag{26}$$

Profile of the solutions in (25) and (26) where $\beta_1=\frac{8}{3};\beta_2=\frac{1}{2};\beta_3=\frac{5}{4};\beta_4=\frac{2}{5};\mu=-\frac{3}{4};\delta_1=\frac{5}{2};\delta_2=\frac{3}{2};y=-\frac{3}{2};\gamma_0=\frac{5}{2};\gamma_3=\frac{1}{2};$ and $-20\le x\le 20$, are given bellow for the different values of t that mentioned in the legend

Case 3 The following are the parameters that were reached from solving (20):

$$\gamma_0=-\frac{i\sqrt{\beta_3}\gamma_3\sqrt{\delta_2}\delta_3}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}\delta_1^{3/2}};\gamma_1=0;\gamma_2=0;\mu=\frac{i\delta_3}{2\beta_1\delta_1^2}. \tag{27}$$

By inserting (27) and (16) into (15), the following set of solutions to (1) have been gained:

$$u_3=-\frac{i\sqrt{\beta_3}\sqrt{\delta_2}\delta_3\exp\left(\frac{i\delta_3\left(-\delta_3t+\delta_1x+\delta_2y\right)}{2\beta_1\delta_1^2}\right)\sec\left(\frac{\delta_3\left(-\delta_3t+\delta_1x+\delta_2y\right)}{2\beta_1\delta_1^2}\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}\delta_1^{3/2}}, \tag{28}$$

and

$$v_3 = -\frac{\delta_3^2 \sec^2\left(\frac{\delta_3(-\delta_3 t+\delta_1 x+\delta_2 y)}{2\beta_1\delta_1^2}\right)}{2\beta_1\beta_2\delta_1^2}. \tag{29}$$

Remark 1 Similarly, by assuming that (6) is the trial solution to (15), some other set solutions to (1) may be obtained using the same prior process.

3.2 Implementation of MER sin–cos M to the examined model

To solve (1) by employing the MER sin–cos M, suppose that (15) has a solution with the following structure:

$$\frac{\lambda_1\cos(\mu\xi)+\lambda_0}{\lambda_3\sin(\mu\xi)+\lambda_2\cos(\mu\xi)}. \tag{30}$$

In (16), $\mu, \lambda_0, \lambda_1, \lambda_2,$ and λ_3 are unknown purposeful parameters that must be demonstrated later by taking into account that

$$\mu \neq 0,\ \lambda_0^2+\lambda_1^2 \neq 0,\ \lambda_2^2+\lambda_3^2 \neq 0,$$

and μ is a wave number. Moreover, the successive derivatives of (16) according to ξ are taking the forms below.

$$U' = \frac{\lambda_0\lambda_2\mu\sin(\mu\xi)-\lambda_3\mu\left(\lambda_0\cos(\mu\xi)+\lambda_1\right)}{\left(\lambda_3\sin(\mu\xi)+\lambda_2\cos(\mu\xi)\right)^2}, \tag{31}$$

and

$$\begin{aligned} U'' = &-\frac{2\left(\lambda_3\mu\cos(\mu\xi)-\lambda_2\mu\sin(\mu\xi)\right)\left(\lambda_0\lambda_2\mu\sin(\mu\xi)-\lambda_3\mu\left(\lambda_0\cos(\mu\xi)+\lambda_1\right)\right)}{\left(\lambda_3\sin(\mu\xi)+\lambda_2\cos(\mu\xi)\right)^3} \\ &+\frac{\lambda_0\lambda_3\mu^2\sin(\mu\xi)+\lambda_0\lambda_2\mu^2\cos(\mu\xi)}{\left(\lambda_3\sin(\mu\xi)+\lambda_2\cos(\mu\xi)\right)^2}. \end{aligned} \tag{32}$$

Subbing (30)–(32) into (15), one gets the following:

$$\left.\begin{aligned} &4\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_2^2\mu^2\sin^2(\mu\xi)-4\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_3^2\mu^2\sin^2(\mu\xi)+4\beta_1\beta_2\beta_4\delta_1^3\lambda_0^3 \\ &-8\beta_1^2\beta_3\delta_2\delta_1^4\lambda_1\lambda_2\lambda_3\mu^2\sin(\mu\xi)+8\beta_1^2\beta_3\delta_2\delta_1^4\lambda_1\lambda_3^2\mu^2\cos(\mu\xi) \\ &-8\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_2\lambda_3\mu^2\sin(\mu\xi)\cos(\mu\xi)+4\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_2^2\mu^2 \\ &+8\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_3^2\mu^2+\beta_3\delta_2\delta_3^2\lambda_0\lambda_3^2\sin^2(\mu\xi)+4\beta_1\beta_2\beta_4\delta_1^3\lambda_1^3\cos^3(\mu\xi) \\ &+\beta_3\delta_2\delta_3^2\lambda_1\lambda_2^2\cos^3(\mu\xi)+12\beta_1\beta_2\beta_4\delta_1^3\lambda_0\lambda_1^2\cos^2(\mu\xi)+\beta_3\delta_2\delta_3^2\lambda_0\lambda_2^2\cos^2(\mu\xi) \\ &+12\beta_1\beta_2\beta_4\delta_1^3\lambda_0^2\lambda_1\cos(\mu\xi)+2\beta_3\delta_2\delta_3^2\lambda_1\lambda_2\lambda_3\sin(\mu\xi)\cos^2(\mu\xi) \\ &+\beta_3\delta_2\delta_3^2\lambda_1\lambda_3^2\sin^2(\mu\xi)\cos(\mu\xi)+2\beta_3\delta_2\delta_3^2\lambda_0\lambda_2\lambda_3\sin(\mu\xi)\cos(\mu\xi)=0. \end{aligned}\right\} \tag{33}$$

In (33), by collecting all the coefficients with the same powers of $\cos^{\tau_1}(\mu\mathscr{P})\sin^{\tau_2}(\mu\mathscr{P})$ where $\tau_1,\tau_2=0,1,2,3$ and equating them to zero. From the coefficients of $\cos^{\tau_1}(\mu\mathscr{P})\sin^{\tau_2}(\mu\mathscr{P})$, one creates a system as given below:

$$\left.\begin{aligned}
&4\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_2^2\mu^2+8\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_3^2\mu^2+4\beta_1\beta_2\beta_4\delta_1^3\lambda_0^3=0,\\
&-8\beta_1^2\beta_3\delta_1^4\delta_2\lambda_1\lambda_2\lambda_3\mu^2=0,\\
&4\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_2^2\mu^2-4\beta_1^2\beta_3\delta_2\delta_1^4\lambda_0\lambda_3^2\mu^2+\beta_3\delta_2\delta_3^2\lambda_0\lambda_3^2=0,\\
&8\beta_1^2\beta_3\delta_2\delta_1^4\lambda_1\lambda_3^2\mu^2+12\beta_1\beta_2\beta_4\delta_1^3\lambda_0^2\lambda_1=0,\\
&2\beta_3\delta_2\delta_3^2\lambda_0\lambda_2\lambda_3-8\beta_1^2\beta_3\delta_1^4\delta_2\lambda_0\lambda_2\lambda_3\mu^2=0,\\
&\beta_3\delta_2\delta_3^2\lambda_1\lambda_3^2=0,\\
&12\beta_1\beta_2\beta_4\delta_1^3\lambda_0\lambda_1^2+\beta_3\delta_2\delta_3^2\lambda_0\lambda_2^2=0,\\
&2\beta_3\delta_2\delta_3^2\lambda_1\lambda_2\lambda_3=0,\\
&4\beta_1\beta_2\beta_4\delta_1^3\lambda_1^3+\beta_3\delta_2\delta_3^2\lambda_2^2\lambda_1=0.
\end{aligned}\right\} \tag{34}$$

By solving (34), the following cases are created:

Case 1 The following are the parameters that were obtained from solving (34):

$$\beta_1=\frac{\delta_3}{2\delta_1^2\mu};\lambda_1=0;\lambda_2=0;\lambda_0=\frac{i\sqrt{\beta_3}\sqrt{\delta_2}\sqrt{\delta_3}\lambda_3\sqrt{\mu}}{\sqrt{\beta_2}\sqrt{\beta_4}\sqrt{\delta_1}}. \tag{35}$$

The following set of solutions to (1) has been identified by replacing (35) gathering with (30) into (15).

$$u_4=\frac{i\sqrt{\beta_3}\sqrt{\delta_2}\sqrt{\delta_3}\sqrt{\mu}\left(\cot\left(\mu\left(-\delta_3t+\delta_1x+\delta_2y\right)\right)+i\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\sqrt{\delta_1}}, \tag{36}$$

and

$$v_4=-\frac{\delta_3\mu}{\beta_2}\csc^2\left(\mu\left(-\delta_3t+\delta_1x+\delta_2y\right)\right). \tag{37}$$

Graphs of (36) and (37) where $\beta_2=\frac{1}{4};\beta_3=\frac{7}{2};\beta_4=\frac{5}{3};\mu=\frac{1}{2};\delta_1=\frac{2}{5};\delta_2=\frac{3}{2};\delta_3=\frac{3}{4};y=\frac{3}{2}$, and $-10\le x\le 10$, $-10\le t\le 10$ are given in the following:

Where the values of t are mentioned in the legend, one gets:

Case 2 The following are the parameters that were obtained from solving (34):

$$\delta_1=\frac{i\sqrt{\delta_3}}{\sqrt{2}\sqrt{\beta_1}\sqrt{\mu}};\lambda_1=0;\lambda_2=0;\lambda_0=\frac{i\sqrt[4]{-2}\sqrt[4]{\beta_1}\sqrt{\beta_3}\sqrt{\delta_2}\sqrt[4]{\delta_3}\lambda_3\mu^{3/4}}{\sqrt{\beta_2}\sqrt{\beta_4}}. \tag{38}$$

The following set of solutions to (1) has been determined by re-installing (38) with (30) into (15).

$$u_5 = \frac{2\sqrt[4]{-2}\sqrt[4]{\beta_1}\sqrt{\beta_3}\sqrt{\delta_2}\sqrt[4]{\delta_3}\mu^{3/4}e^{\frac{\sqrt{2}\sqrt{\delta_3}\sqrt{\mu}x}{\sqrt{\beta_1}}+2i\delta_3\mu t}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(e^{\frac{\sqrt{2}\sqrt{\delta_3}\sqrt{\mu}x}{\sqrt{\beta_1}}+2i\delta_3\mu t}-e^{2i\delta_2\mu y}\right)}, \tag{39}$$

and

$$v_5 = \frac{\delta_3\mu}{\beta_2}\csc^2\left(\mu\left(-\delta_3 t+\frac{i\sqrt{\delta_3}x}{\sqrt{2}\sqrt{\beta_1}\sqrt{\mu}}+\delta_2 y\right)\right). \tag{40}$$

Profile of the solutions in (39) and (40) where $\beta_1=\frac{8}{3};\beta_2=\frac{1}{8};\beta_3=-\frac{1}{4};\beta_4=\frac{2}{5};\mu=\frac{1}{2};\delta_1=\frac{5}{2};\delta_2=-\frac{1}{2};\delta_3=\frac{5}{2};y=-\frac{3}{2}$ and $-20\le x\le 20,\ -20\le t\le 20$ are given below:

For the values of t that are mentioned in the legend, one reaches:

For the values of t that are mentioned in the legend, one obtains:

Remark 2 Similarly, by assuming that (7) is the trial solution to (15), some other set solutions to (1) may be obtained using the same prior process.

4 Conclusion

The present study describes the first implementation of two modified trigonometric analytic methods on a complex nonlinear (2 + 1)-dimensional Fokas system. The studied model is constructed to explain the nonlinear pulsed transmission in monomode fibers with optical features. Our novel modification approaches are the modified extended rational sinh–cosh method and the modified extended rational sin–cos method. The outcomes have been illustrated by numerous innovative and unique solutions that have been stated by traveling waves, oscillating, soliton types, and exponential rational functions blended with trigonometric and hyperbolic trigonometric functions. The updated approaches are trustworthy, influential, and straightforward in discovering semi-analytic solutions to mathematical models in numerous domains, such as mathematics, physics, biology, and engineering. The detected results have been detailed in three dimensions, contour surfaces, and two-dimensional graphs that represent the impact of temporal progression. The two- and three-dimensional displays help us better appreciate the qualities of the acquired outcomes. The obtained outcomes have all been properly validated by putting the created findings back into their linked equations. The functioning and behavior of the graphs mostly rely on the specified numerical values that are supplied for the optional coefficients. For the future scope of the work, we recommend that the authors use these two modifications, which we believe are useful, practical, and effective. It will play a significant role in forthcoming research related to applied science.

5 Results and discussion

The following statements have been added to clarify the distinguishing characteristics of our updating methods: We have acquired a collection of solutions that are difficult to get through the utilization of prior iterations of these techniques. The adjustments we have

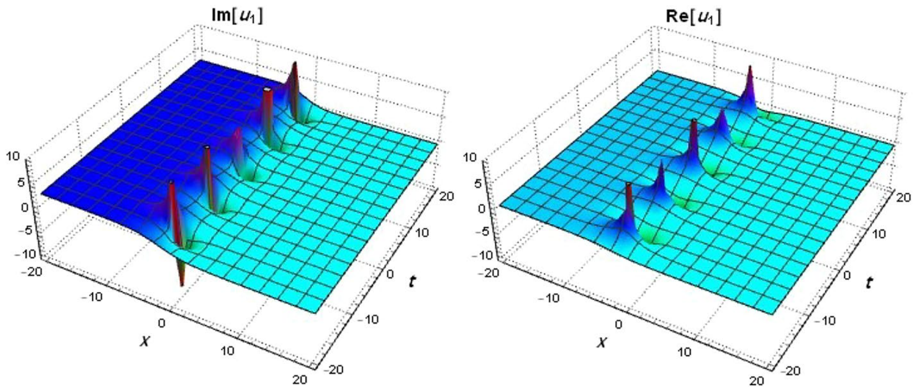

Fig. 1 3D figures of (22)

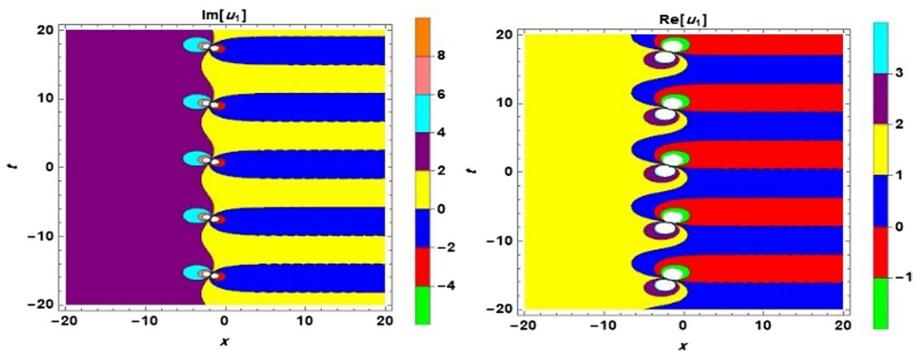

Fig. 2 Contour surfaces of (22)

made are dependable, efficient, and swiftly adaptable to many mathematical models. Some shortcomings and unfavorable variables in the past versions of these procedures supplied the impetus for us to arrive at these two further enhancements. Although no analytical technique is devoid of drawbacks, positively, there are major benefits to our modifications for portraying the formulated solution in (22) and in (23) that are unreachable to acquire by employing the prior old versions. The singular breather solitons in both x and t are shown in Figs. 1, 2, 3, 4 and 5. Figure 6 represents a solitary wave on the left and a bright soliton on the right-hand side. Figures 7, 8, 9, 10 and 11 represent periodic and traveling wave solutions. The two interacting breather solitons are illustrated in Figs. 12, 13 and 14. The dark soliton on the right-hand side and the solitary waveform on the left-hand side can be observed in Figs. 15, 16 and 17.

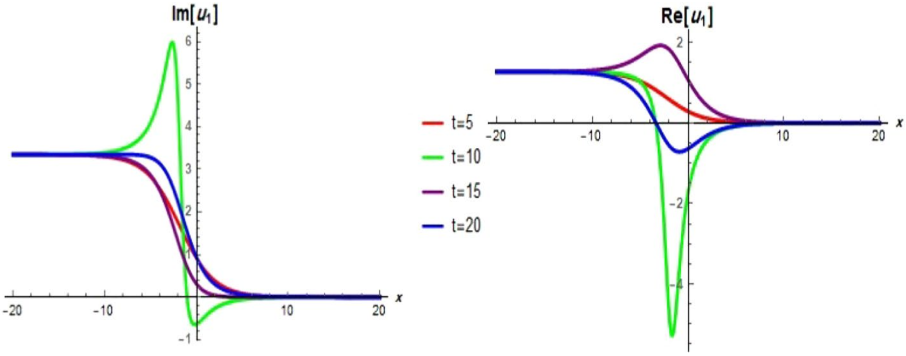

Fig. 3 2D graphs of (22)

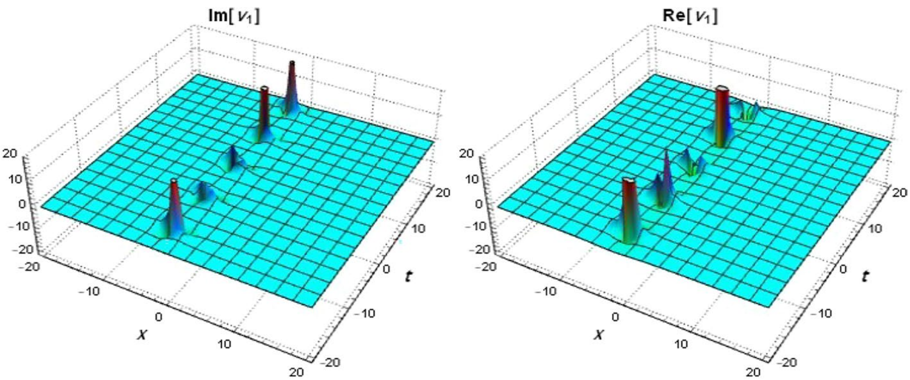

Fig. 4 3D figures of (23)

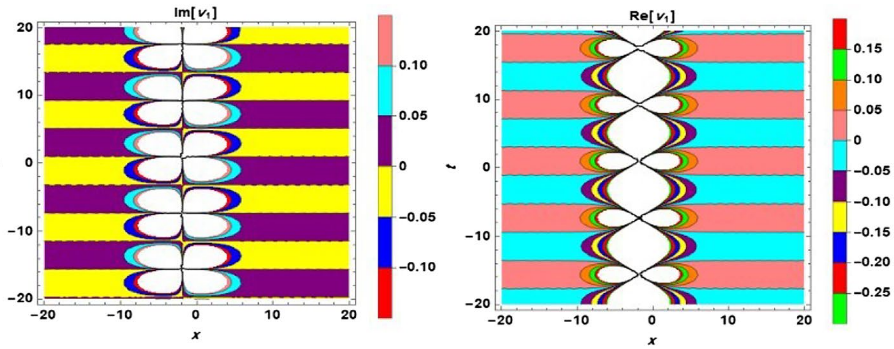

Fig. 5 Contour surfaces of (23)

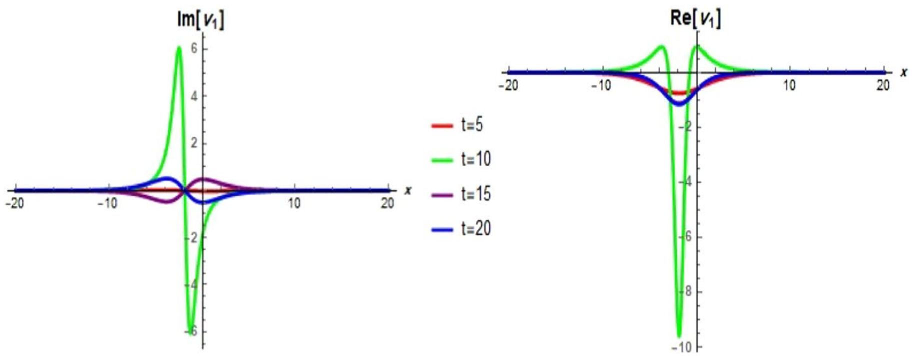

Fig. 6 2D graphs of (23)

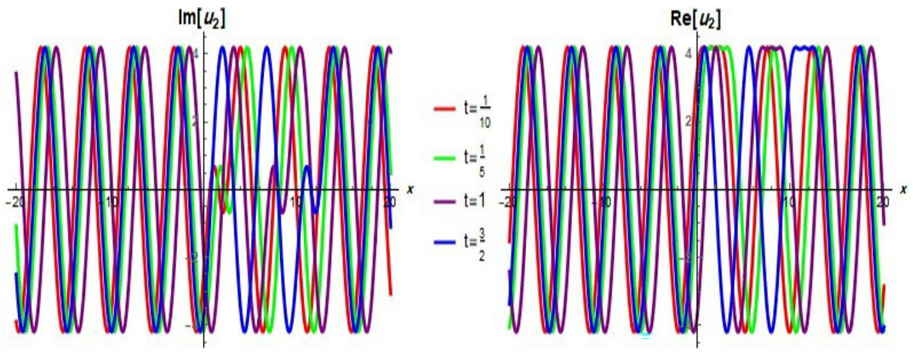

Fig. 7 2D graphs to (25)

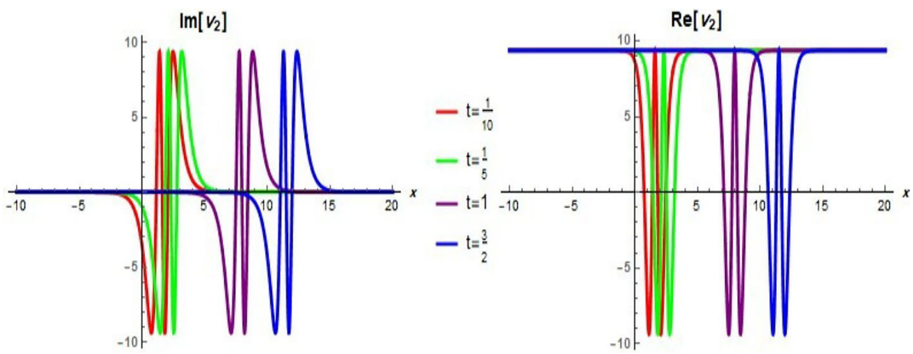

Fig. 8 2D graphs to (26)

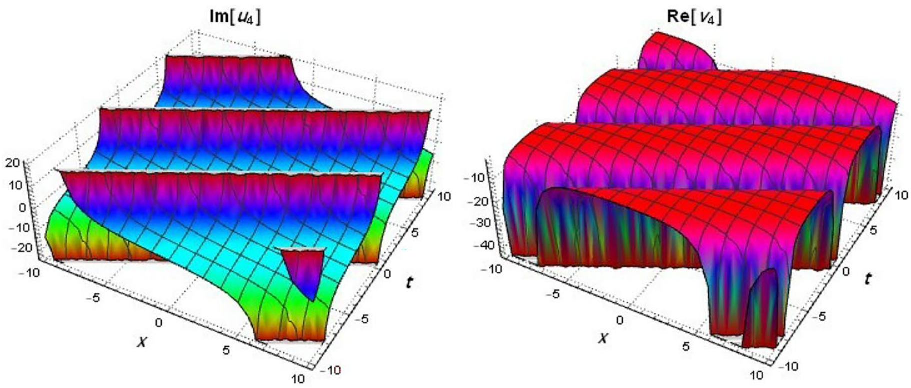

Fig. 9 3D figures represent the imaginary part of (36) and the real part of (37)

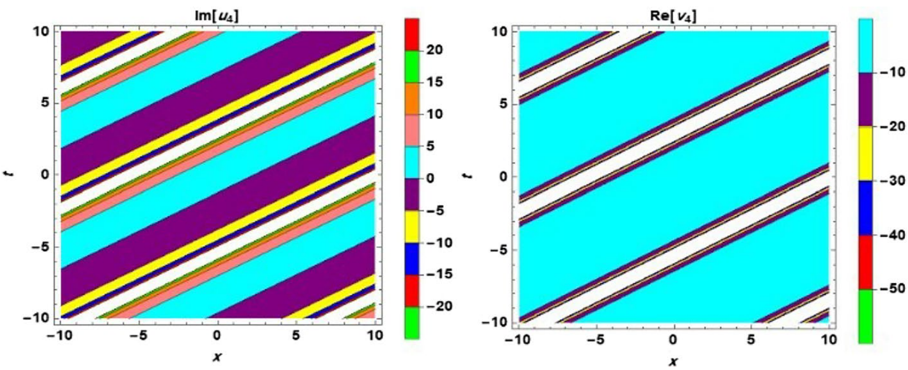

Fig. 10 Contour surfaces represent the imaginary part of (36) and the real part of (37)

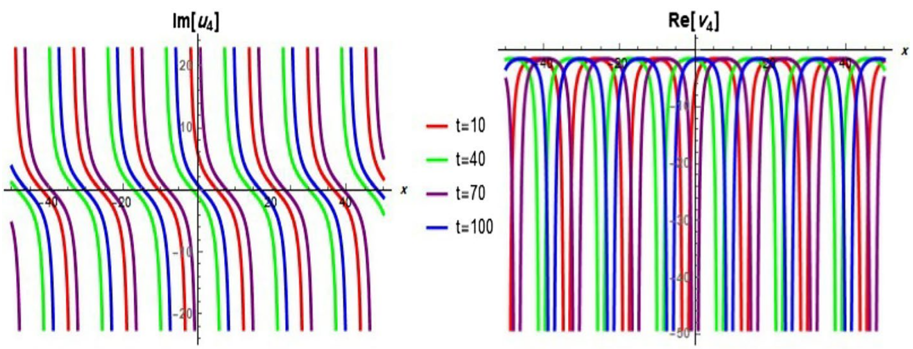

Fig. 11 2D graphs represent the imaginary part of (36) and the real part of (37)

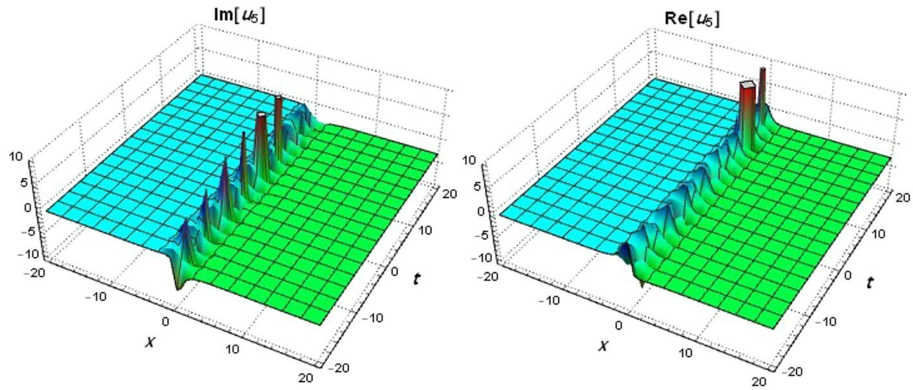

Fig. 12 3D figures of (39)

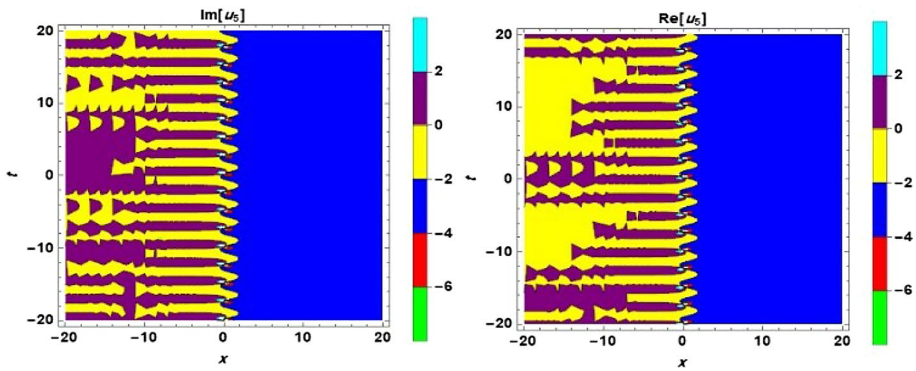

Fig. 13 Contour surfaces of (39)

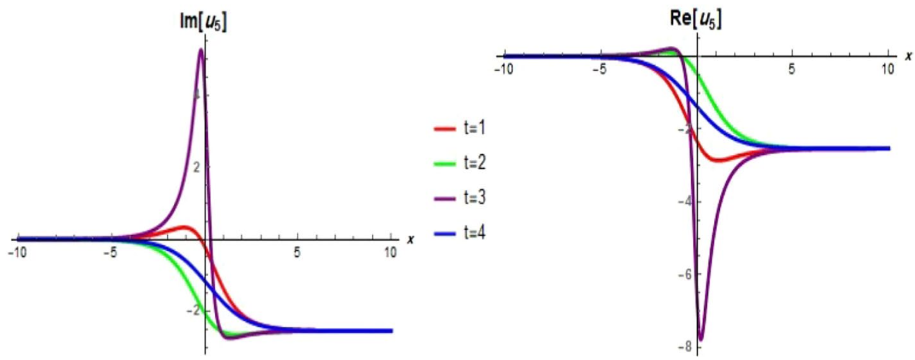

Fig. 14 2D graphs of (39)

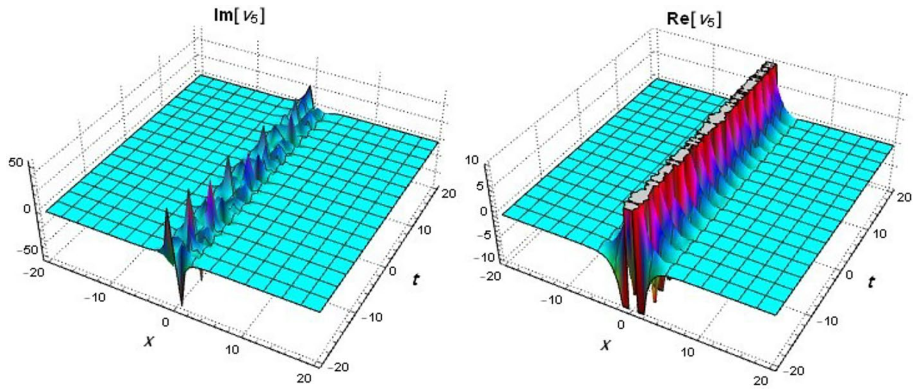

Fig. 15 3D figures of (40)

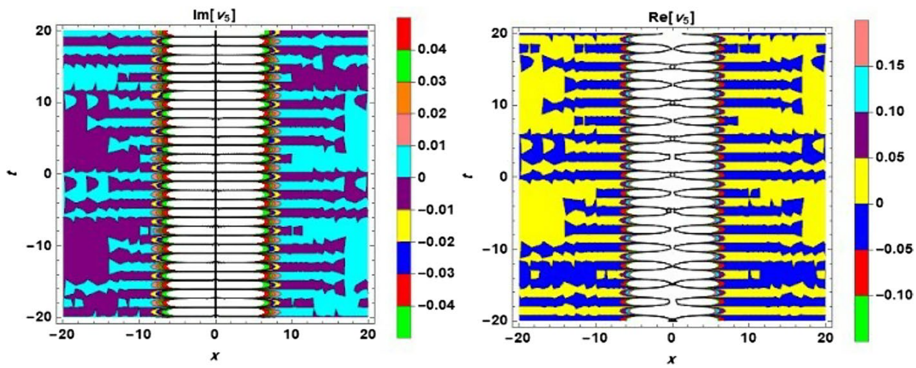

Fig. 16 Contour surfaces of (40)

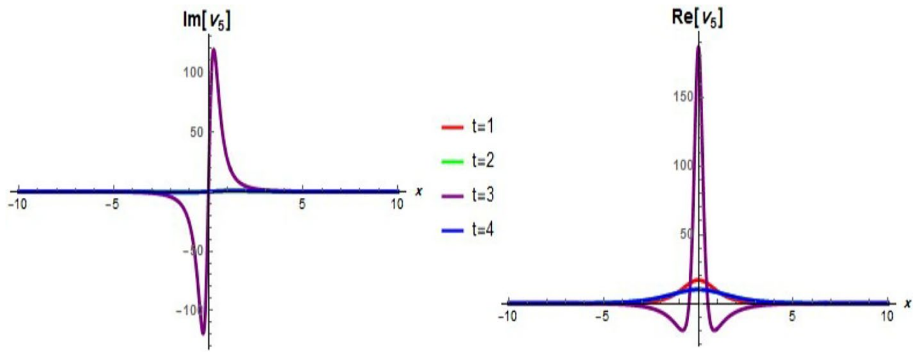

Fig. 17 2D graphs of (40)

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Declarations

Ethical approval The authors confirm their adherence to ethical standards.

Conflict of interest The authors indicate that there is no conflict between their interests in publishing this work.

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