

Chains of coupled square dielectric optical microcavities

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Abstract Chains of coupled square dielectric cavities are investigated in a 2-D setting, by means of a quasi-analytical eigenmode expansion method. Resonant transfer of optical power can be achieved along quite arbitrary, moderately long rectangular paths (up to 9 coupled cavities are considered), even with individual standing-wave resonators of limited quality. We introduce an ab-initio coupled mode model, based on a simple superposition of slab mode profiles as a template for the field of individual cavities. Although no loss mechanisms are built in, the model can still help to interpret the results of the former numerical experiments.

Keywords Integrated optics · Numerical modeling · Coupled mode theory · Variational modeling · Dielectric optical microcavities · Resonator chains

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1 Introduction

Coupled-resonator optical waveguides (CROWs) have been discussed already for some years as a means to realize waveguiding along paths with small-size bends, or for interest in their time-delay properties. A variety of concepts exist; among these are series of microring or -disk resonators (Yariv et al. 1999; Little et al. 2004; Morichetti et al. 2007), sequences of defects in photonic crystals or photonic crystal slabs (Bayindir et al. 2000; Reynolds et al. 2001; Scheuer et al. 2005), and coupled dielectric spheres (Astratov et al. 2004; Chen et al. 2006). Some configurations can also be viewed as specific extended cases of photonic molecules (Boriskina 2005, 2006; Boriskina et al. 2007).

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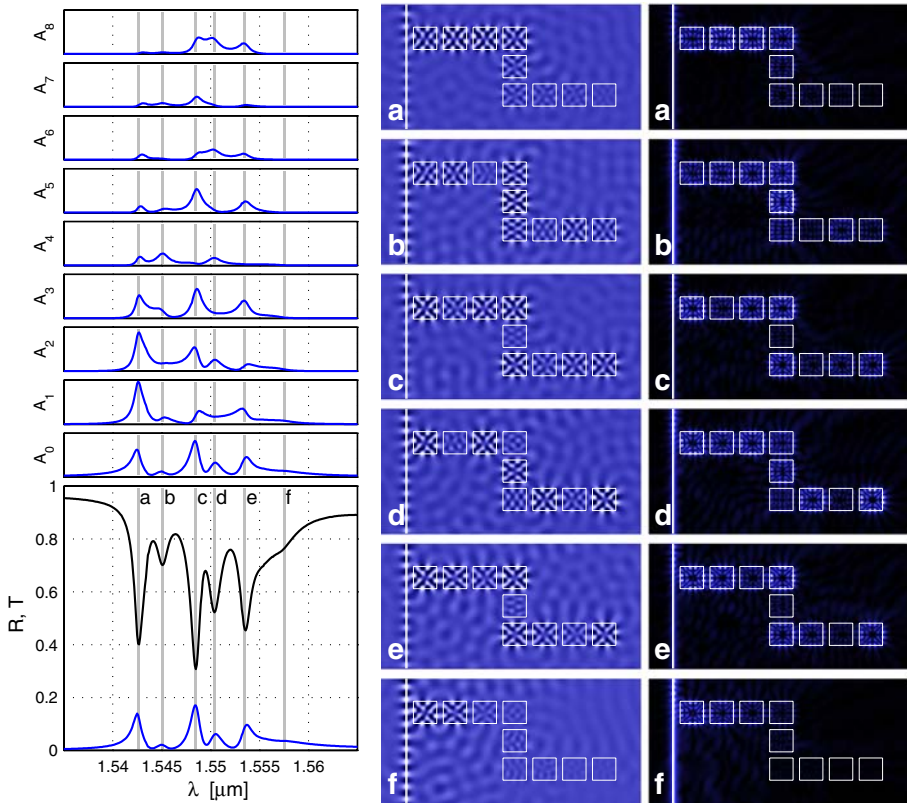


Fig. 2 A double-bend chain with nine square cavities. Parameters, as introduced in Fig. 1: $n_b = 1.45$, $n_g = 3.40$, $t = 0.073 \mu\text{m}$, $w = 1.451 \mu\text{m}$, $g_w = g_r = 0.4 \mu\text{m}$. Left column: reflected power R and transmitted power T in the bus waveguide (bottom), and local field intensities $A_0 - A_8$ in the individual cavities, counting from the input end. Center and right column: field snapshots (center) and optical intensity profiles (right) for the wavelengths (letters a–f) indicated by the gray vertical lines in the spectral plots

properties exhibits a resonance of specific type (cf. Fig. 3(1), Sect. 2, and Ref. Hammer 2002), while no further resonances¹ are supported in the wavelength range of interest (the spectral window in the figures below) by the single cavity.

All following numerical experiments rely on quasi-analytical simulations by means of the QUEP algorithm as described in Ref. Hammer (2004). The computational setting comprises a rectangular computational window with fully transparent boundaries (exception: the corner regions). These are placed, for each of the structures, by $3 \mu\text{m}$ away from the outermost dielectric interfaces, i.e. outside the regions that are shown in the field plots. Per slice and layer, the optical electromagnetic field is expanded into about 10 modes per μm width of the computational window, uniformly for all configurations in this paper. (Occasional) Checks

¹ Due to mechanisms as outlined e.g. in Ref. Hammer and van Groesen (2002); Hammer (2002), the present standing-wave resonances do not occur as regularly as e.g. the almost periodic spectral features found for the traveling-wave whispering gallery modes of not too small dielectric rings or disks. Ref. Lohmeyer (2002) includes a plot of the spectral power transfer for a two-bus configuration with the parameters of Fig. 7 over the larger window $\lambda \in [1.3, 1.7] \mu\text{m}$: the resonance in question is the only significant feature. Definition of a free spectral range seems not appropriate in that case.

with respect to these simulation parameters indicate convergence of the results on the scale of the figures as given. Only 2-D configurations and TE polarized light are considered, the single nonvanishing electric component of the optical field is oriented perpendicular to the x - z -plane (cf. Fig. 1). Figure 2 summarizes results for the S-bend chain.

We look at a spectral range around the design wavelength of $1.55\ \mu\text{m}$. The chain splits the single resonance of one individual cavity into a series of more or less well distinguished resonances, which show up as dips in the relative transmitted guided power T and simultaneous peaks in the relative reflected guided power R . These two quantities do not add up to one, hence a substantial amount of optical power is lost to nonguided waves (lossless dielectric materials are considered), that leave the computational window through the transparent boundaries.

One observes that for a major part the optical fields in each of the chains cavities are very similar to the shape of the resonance supported by a single dielectric square. We therefore evaluated “local amplitudes” for each cavity, the quantities $A_0 - A_8$, as a function of the wavelength, where the index indicates the position of the cavity in the chain, counting from the bus end. A_0 thus represents the intensity in the cavity that is connected to the bus waveguide. More specifically, A_j is the average of the squared local optical field at the eight spots close to the borders of the cavity core where the resonant field of the isolated single cavity would be at maximum. Even for the present configurations with rather pronounced losses, the relatively large intensities A_8 in the last cavity for the pattern c, d, and e indicate that an efficient resonant power transfer along the entire path of the chain is indeed feasible.

The field and intensity plots of Fig. 2 illustrate the shape of the resonances. The field profiles represent time-snapshots of the principal y -component of the real, physical time harmonic electric fields, evaluated at a time within one period, when the maximum of the partly standing waves becomes visible (Alternatively: The overall phase of the complex fields has been chosen such that the spatial maximum of the real part is exhibited. Note that even with that adjustment the overall sign of profiles is still arbitrary). The intensity plots show the absolute square of the complex electric field. Gray levels are scaled to exhibit the local extrema of the individual pattern i.e. the scales are not comparable between different plots (cf. the curves of $A_0 - A_8$). These remarks apply also to all further images in this paper.

Necessarily, the static plots of Fig. 2 can give only a rough impression of the rather complex behaviour of the optical electromagnetic field. It must be emphasized that, unlike the traveling waves observed in circular resonators, these are (almost) purely standing wave resonances: The time-domain view of the stationary, harmonically varying fields of the present chains shows as the major feature an in-place “ \pm -blinking” of the field in each cavity, where the horizontal, vertical, and diagonal nodal lines stay in place. Extremal field strengths in different cavities appear at different moments within the time period. Traveling waves, or partly traveling waves, are observed in the bus waveguide, and outside the cavity and bus cores. The latter contribute to the radiation losses of the structure, if they cross the windows of the computational boundaries, but they could also be a cause of a “large distance”, non-next-neighbor interaction between the cavities.

3 Chain properties

With length and shape we restrict to the two most obvious “parameters” that specify the chain path. Certainly also the influence of other quantities can be of interest. These would be the variables that influence the resonances of individual cavities (refractive index contrast, width/aspect ratio; other shapes or distortions), quantities that determine the interaction

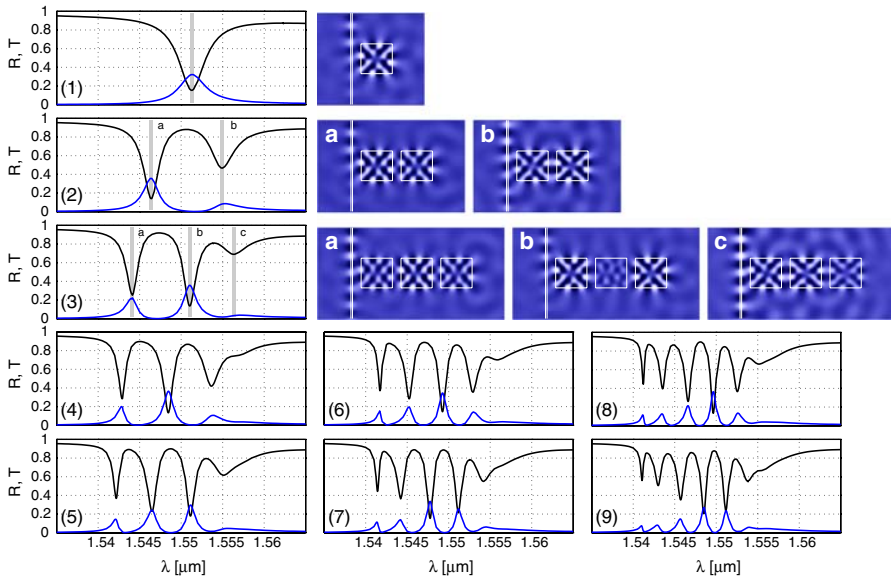


Fig. 3 Straight chains of lengths one (1) up to nine (9) with the parameters of Fig. 2. The plots show the guided wave reflection R and transmission T versus the excitation wavelength λ , and, for chains of length (1), (2), and (3), snapshots of the resonant field pattern at the wavelengths indicated by the gray vertical lines in the spectra

between the cavities (distance, positioning, not necessarily on a rectangular grid), and the parameters that specify the excitation conditions (gap, bus waveguide parameters). If limited to rectangular 2-D configurations, the corresponding studies could be carried out with the methods as used in this paper. One would not necessarily have to restrict to uniform changes of the variables as introduced in Fig. 1: There are indications (Astratov et al. 2004; Boriskina 2007) that more irregular changes of e.g. sizes or distances of individual cavities could be promising just as well. However, given the properties of an individual (optimized) square and its resonance wavelength, we experienced that, beyond choosing plausible values for the gaps between the cavities, for the gap between bus core and the input cavity, and for the width of the bus waveguide² no particular further design strategy was necessary to realize the chain resonances discussed in this section.

3.1 Length of the chain

Figure 3 summarizes the spectral properties of straight chains of lengths up to nine, and examples for resonant field profiles for short chains. With growing length, the pattern found for the larger structures appear to evolve from the single feature in the spectrum of the one-cavity “chain”, and the related extremal field. There are no specific differences between chains with even or odd numbers of squares.³

² Here a phase matching argument involving the access mode and one of the modes that contribute to the cavity resonance (cf. Sect. 5) can be helpful (Hammer 2002).

³ In case of a filter setting with two parallel chains between two bus waveguides, symmetry properties (i.e. also the chain length) might well be significant (Fan et al. 1999).

One observes an increase in the number of spectral peaks with the number of cavities. The interaction between the squares in the composite structures splits the single resonance of the one-cavity chain into several features close to the original wavelength. Examination of the resonant fields reveals a common “supermode” phenomenon (cf. e.g. Ref. Pishko et al. 2007 for a discussion of CROW supermodes), where the extremal profiles supported by the composite, longer chains are made up from properly placed copies of the single-cavity field, superimposed systematically with specific relative amplitudes. Denoting the single cavity mode, with a phase / sign as shown, by a symbol [+], the supermodes of the two-cavity chain could be classified as [++] (a) and [+−] (b), respectively. Correspondingly, one would write the symbols [+++] (a), [+0−] (b), and [+ − +] (c) for the resonances of the three-cavity chain. Note that here the fundamental excitations, the ones that show up at the largest wavelength or the lowest frequency (“energy”) are the ones with the *least* symmetric vectors of amplitudes, i.e. resonances (b) for the two-cavity and (c) for the three cavity chain: Due to the antisymmetry of the single-cavity profile, these composite fields show less “nodes”, vertical nodal lines in the gaps between cavities, than the other supermodes.

While this discussion should apply more or less analogously also for the longer chains, losses come into play, such that the individual spectral peaks start to overlap, such that already for the four cavity chain one can hardly identify all four expected resonances.

3.2 Chain shape

Due to the specific symmetry and strong confinement of the field in the individual cavities one might expect that the particular shape of the chain does not matter as long as it can still be viewed as one sequence with one loose end. Figure 4 compares spectra and resonance pattern of chain configurations with length nine along different paths with one, two, or three bends. The field plots show the resonances at the wavelength of the peak in the transmission pattern that is closest to the most pronounced resonance (c) of Fig. 2. Note that also the straight chain (9) of Fig. 3 could be grouped into this series.

Obviously the shape of the chain path *has* some influence on the spectral response of the present configurations. Effects like corner coupling and radiative or evanescent “long distance” interaction are apparently relevant, not only the next neighbor interaction.

4 Two bus waveguides

The observation of quite high field levels at the loose end of the former structures raises immediately the question, whether the chain can be linked to two bus waveguides, as a means to realize a resonant transfer of optical power between them. Figure 5 shows the spectral power transmission of a straight configuration with nine cavities, Fig. 6 illustrates the fields that establish at some of the resonances.

The spectral curves of Fig. 5 adhere to the scheme stated for standing wave resonators in Manolatu et al. (1999): Off resonance, the input power merely travels down the input bus waveguide, here to output port B. At resonance, a certain part of this input excites the field in the cavity, which in turn distributes this power equally among the four output ports, where in ports A and B superpositions with the incoming/the directly transmitted waves are present. Although the extremum of 25% transmission to all ports is not reached, the present configurations are not too far off this limit. A structure with only a single “connection” between the bus waveguides is hence only partly useful as a wavelength filter device, since at

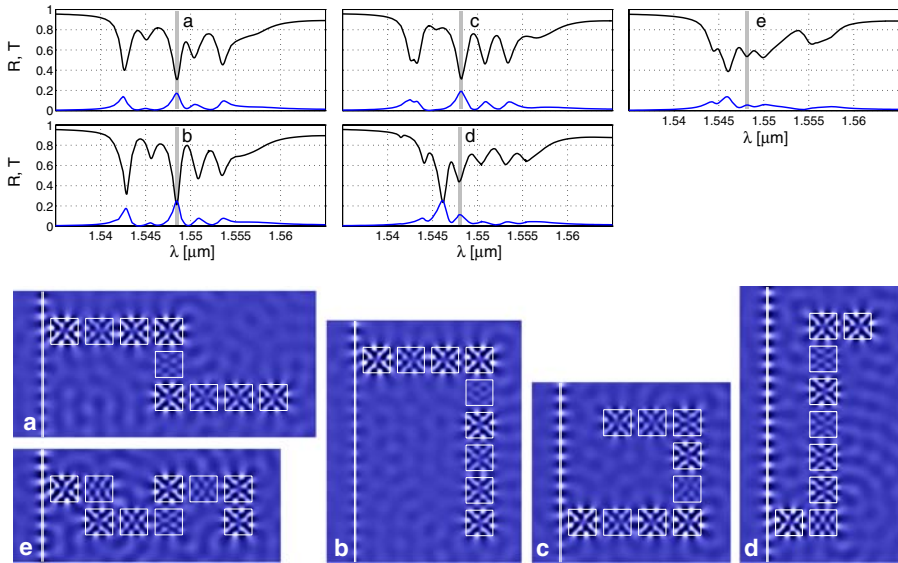


Fig. 4 Spectral guided wave reflection R and transmission T for chains of length nine with different sequential placement of the squares. Parameters are as introduced in Fig. 2. The plots (a–d) of specific resonant fields indicate the respective shape of the chains

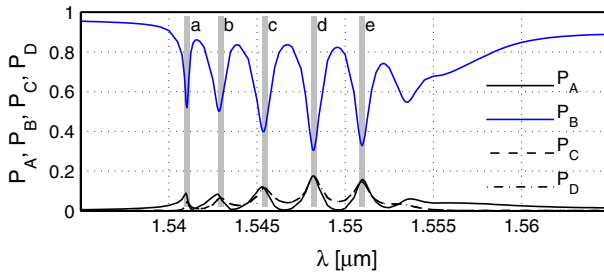


Fig. 5 Spectral power transmission of a straight nine-cavity chain between two parallel bus waveguides, with parameters as given for Fig. 1 (symmetric, straight arrangement) and Fig. 2. The quantities $P_A - P_D$ are the guided relative output powers received by ports A–D upon excitation in port A, as introduced in the plots of Fig. 6

most a quarter of the input arrives at the drop ports C or D. Two coupled parallel connections (Manolatu et al. 1999; Lohmeyer 2002) could offer a way out; corresponding (abstract) concepts have been discussed in (Khan et al. 1999; Fan et al. 1999).

With the introduction of the second bus waveguide the structure is now fully symmetric with respect to the center of the middle cavity. This symmetry shows up nicely in the properties of the series of supermodes as shown in Fig. 6, where the systematic mentioned in Sect. 3.1 applies as well.

5 HCMT model

Although somewhat obscured at some instances by quite pronounced losses, the phenomena observed in the former series of examples are more or less as expected for a sequential

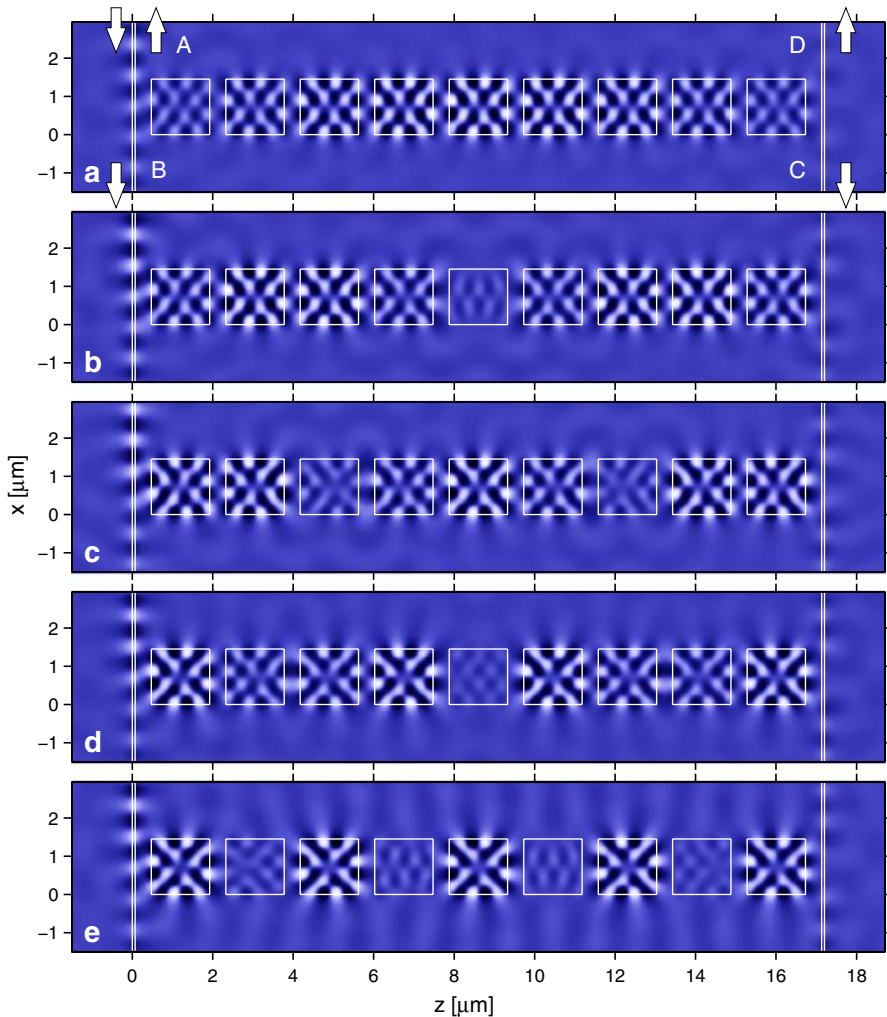


Fig. 6 Shapes of resonant fields supported by the straight nine-cavity chain coupled to two bus waveguides. Plots (a–e) correspond to the wavelengths indicated by the gray vertical lines in Fig. 5

arrangement of coupled oscillators. Still, while “modes of individual cavities” and their “interaction” play a role in the previous discussion, merely the numbers from the QUEP simulations can hardly justify the use of these notions. Therefore, this section tries to found the interpretation on an approximate coupled mode model, which allows to quantify that viewpoint: The applicability range of the model below is defined by how close the ansatz (2) comes to physical reality. A complete failure of the model would imply that the former interpretation is entirely inappropriate. Shortcomings in certain aspects indicate that these aspects are not well represented in the template (2) or in its ingredients. Note that, although the almost analytical HCMT model requires a by far lower computational effort than the QUEP reference calculations, it is here meant to complement, not to replace the latter (cf. the remarks at the end of Sect. 5.3 for comments on possible improvements).

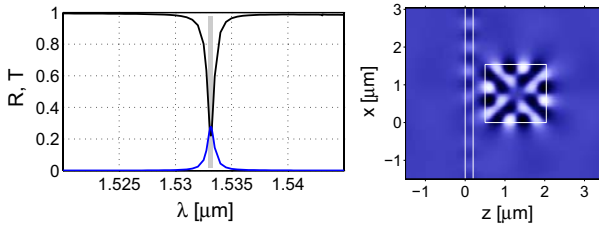


Fig. 7 Spectral power transfer (left) and field profile at resonance $\lambda = 1.5331 \mu\text{m}$ (right) of a single dielectric square coupled to one bus waveguide. Parameters (cf. Fig. 1): $n_b = 1.0, n_g = 3.2, w = 1.54 \mu\text{m}, g_r = 0.39 \mu\text{m}, g_w = 0.3 \mu\text{m}, t = 0.2 \mu\text{m}$; design wavelength $\lambda_0 = 1.532 \mu\text{m}$

To allow for some better chance of an also quantitative comparison we choose a set of parameters similar to Refs. Manolatos et al. (1999); Lohmeyer (2002) with even higher refractive index contrast, such that losses can be expected to have a less significant influence. We shall see that the observations from the HCMT/QUEP comparison lead to an interpretation that applies also to the spectral features found in Sect. 2. Application of the formalism below directly to the structure of Sect. 2 yields results that resemble qualitatively the HCMT data in Fig. 9; the relation to the reference of Fig. 2 would be even less obvious, however.

5.1 Field template

The description in terms of coupled mode theory (CMT) starts with identifying a suitable, if necessary approximate, expression for the resonant optical field supported by an individual square cavity. Figure 7 shows that field for the new parameter set. Note that only that single resonance appears within the wavelength range of interest.

The reasoning of Refs. Hammer and van Groesen (2002); Hammer (2002) suggests that a superposition of suitable slab mode profiles could be useful at this point. One considers the symmetric multimode slab that corresponds to either the vertical or horizontal refractive index cross section of our square, and computes, at the design wavelength close to the former resonance, its guided modes of third order η_0 and fifth order η_1 . Then indeed, according to Fig. 8, the antisymmetric 2-D superposition

$$\psi(x, z) = \eta_0(x) \eta_1(z) - \eta_1(x) \eta_0(z) \tag{1}$$

of these functions results in a profile ψ that appears to be very similar to the shape of the resonance in Fig. 7. ψ can thus serve as the prototype for the mode of a single cavity.

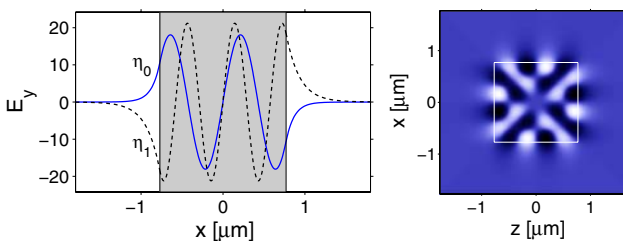


Fig. 8 Left: Third and fifth order guided TE mode profiles of a symmetric three-layer dielectric slab with refractive indices n_b, n_g , and thickness w at wavelength λ_0 , principal electric component E_y . Right: A specific totally antisymmetric 2-D superposition ψ of the two former profiles η_0 and η_1 (cf. the text)

We now consider again the S-bend chain configuration with nine cavities. For what follows it is helpful to extend ψ to an approximation Ψ for the full optical electromagnetic field of a single cavity, by applying the superposition formula also to the magnetic slab mode components. Further, one copy ψ_j of this field is introduced for each cavity $j = 0, \dots, 8$, shifted to the respective cavity center position. Then we assume that the optical electromagnetic field in the chain is well represented by the template

$$\begin{pmatrix} E \\ H \end{pmatrix}(x, z) = u(x) \phi_u(x, z) + d(x) \phi_d(x, z) + \sum_{j=0}^8 r_j \psi_j(x, z). \quad (2)$$

Here ϕ_u and ϕ_d are the—known—upwards and downwards traveling variants of the guided bus modes, the z -dependent mode profiles multiplied by the respective exponential dependence on their propagation coordinate x . Due to the interaction with the chain the contribution of these modes to the overall field will change along the vertical coordinate, therefore x -dependent amplitude functions u and d are introduced. Together with the coefficients r_j of the fields ψ_j associated with the individual squares, these are the unknowns in Eq. 2.

5.2 Solution procedure

In order to apply the procedures of the hybrid analytical/numerical CMT variant (HCMT) of Ref. Hammer (2007), the unknown functions u and d are discretized by 1-D linear finite elements. Discrete coefficients u_l, d_l replace the functions, such that the template (2) assumes the abstract form

$$\begin{pmatrix} E \\ H \end{pmatrix}(x, z) = \sum_k a_k \chi_k(x, z). \quad (3)$$

Here χ_k are either directly the functions ψ_j , or modal elements, i.e. products of the mode field (profile times exponential propagation term) and the hat function of the respective element. $a_k \in \{u_l, d_l, r_j\}$ are the corresponding unknown coefficients, with an abstract index k that covers the bus mode propagation directions, the element numbers, and the indices of the individual cavities.

Basis for the following procedure is then the variational representation of the Helmholtz problem on a suitable 2-D domain that covers the interesting region around the structure, with transparent-influx-boundary conditions that account for the outgoing half-infinite pieces of the bus waveguides in the exterior (Hammer 2007). By restricting the corresponding functional to the discretized field template (3) and looking for vectors of coefficients a_k at which the restricted functional becomes stationary, a small to moderate sized system of linear equations for the a_k can be obtained (only 1-D FEM discretizations are involved). The numerical solution gives a set of coefficients that allows to assemble an approximation to the full optical field. Besides the guided power transmission, the local amplitudes of the directional modes in the bus waveguide, and the coefficients assigned to the fields of the individual cavities can be directly inspected. Full details of the approach are given in Ref. Hammer (2007).

So far the frequency ω has been treated as a fixed parameter. Spectral properties are computed as scans over ω , while keeping the functions η_0, η_1 and hence also ψ_j fixed at the mode profiles that were determined for the design wavelength. This implements the notion that the properties of the chain are determined as interactions of specific resonant modes supported by the individual cavities, each with a fixed associated resonance frequency.

5.3 HCMT results

According to Fig. 9, the present parameter set leads to less pronounced dips in the transmission, but at the same time to sharper, better separated resonances, when compared with Fig. 2. Still the quantitative agreement of the rigorous QUEP simulations with the approximate model is only quite moderate. At least the HCMT predicts a series of resonances in the appropriate region, and the resonant field profiles, as far as identifiable, share common major features such as e.g. the occurrence of small field strengths at certain positions in the chain.

The features in the HCMT spectrum are sharp and pronounced, with a spectral width that is purely determined by the interaction with the bus waveguide. The field patterns corresponding to these well separated resonances could thus be viewed as approximations to the supermodes supported by the entire chain, the coupled resonant modes of the individual cavities, superimposed with specific wavelength dependent relative amplitudes. The HCMT results show a systematic in the supermode forms, i.e. in the distribution of relative amplitudes r_j over cavity positions j throughout the series (a–i) of resonances, as observed in Sect. 3.1.

The time domain view on the stationary, time-harmonic fields reveals that the rather irregular “blinking” pattern mentioned in Sect. 2 is for the present case replaced by a simultaneous change of field amplitudes in all cavities, where extremal levels appear at more or less the same moment in all cavities, in line with the almost real supermode amplitudes as predicted by the HCMT model.

A (slightly speculative) comparison of the HCMT and QUEP results of the present high-contrast configuration with the initial observations of Fig. 2 could lead to the following interpretation of the former, more irregular data: A sequence of nine supermodes are also supported by the former structure with lower contrast; they suffer, however, from much stronger losses, such that their contributions to the spectral transmission partly overlap. The feature (a) in Fig. 2 would thus include contributions from two supermodes of shapes according to resonances (a) and (b) in Fig. 9. Note that an interference of the sequences (a) and (b) of coefficients r_j in Fig. 9 with suitable relative amplitudes can be constructive in the first half of the chain, and destructive in the second half. Then, with some good will, the sequence (b), (c), (d), and (e) of shapes in Fig. 2 corresponds to the profiles (c), (d), (e), and (f) of Fig. 9. Lossy resonances with shapes similar to (g), (h), and (i) of Fig. 9 might contribute to the shoulder (f) of Fig. 2.

The adequate incorporation of optical radiation losses is probably crucial for a quantitative improvement of the CMT model. This would require to provide fields for the actual lossy eigenmodes (leaky modes, quasinormal modes) of the individual cavities. These would be the complex frequency eigensolutions of the open, homogeneous Helmholtz problem. An example for coupled quasi-normal modes supported by defects in 1-D multilayer stacks is given in Refs. Maksimovic et al. (2008a; 2008b). 2- or 3-D configurations require numerical approximations, where solvers for the field shapes and complex eigenfrequencies of open dielectric cavities are less frequently found. Among the few examples are the FEM modules in the commercial package (JCMwave GmbH), and academic programs of more numerical (Sopaheluwakan 2006) (FEM) or analytical character (Prkna et al. 2004; Prkna 2004; Boriskina 2006) (circular cavities). Probably also the near field solutions at resonances (scattered field part) generated by a computational treatment of specific scattering problems (Boriskina et al. 2004, 2005; Pishko et al. 2007), for individual, suitably shaped cavities could be used for this purpose. A third alternative would be to employ real-frequency eigensolutions for the open, leaky individual cavities with artificial gain (“lasing eigenvalue problem” Smotrova and Nosich 2004; Smotrova et al. 2006).

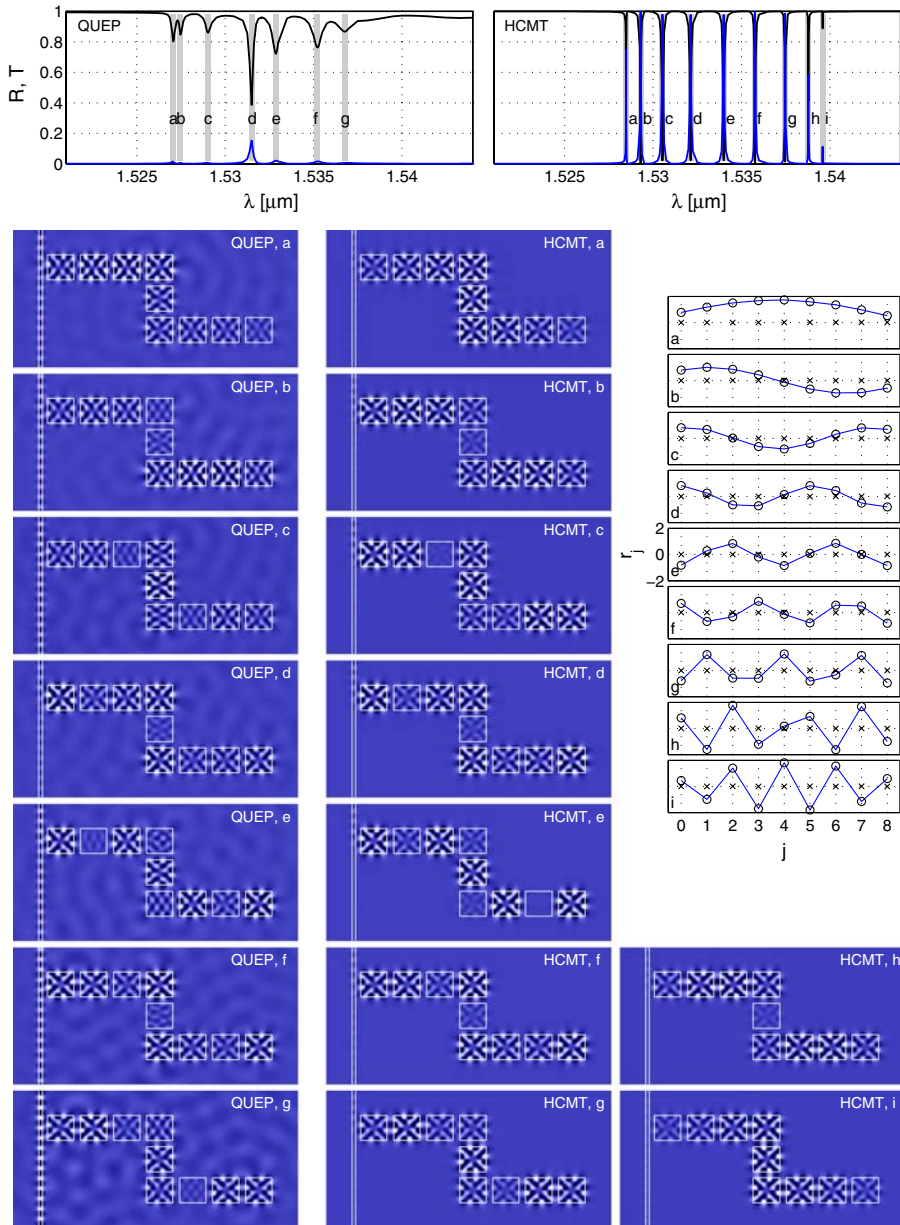


Fig. 9 For an S-bend nine-cavity chain with parameters as given for Fig. 7: Spectral guided wave reflection and transmission, and resonant field pattern, as predicted by the rigorous numerical simulations (QUEP, left) and by the approximate CMT model (HCMT, right). The middle right inset shows the complex coefficients r_j (circles / crosses: real and imaginary parts) with which the individual cavities (index j ; $j = 0$: the bus end of the chain) contribute to the supermodes (a–i)

6 Concluding remarks

Sequential arrangements of square 2-D dielectric optical microcavities have been considered, with one or two ends of the chains coupled evanescently to straight dielectric waveguides that function as input/output ports. Rigorous numerical simulations show that a resonant transfer of optical power along chains of quite arbitrary shape, including rectangular bends, is feasible, even with individual cavities of moderate quality. By varying the shape of the chain path, it was found that a next-neighbor interaction is not sufficient to explain the changes in the power transfer spectrum. Apparently an evanescent or radiative long distance interaction plays a role for the present, not too unrealistic set of design parameters. When looking at chains of growing length we could observe a systematic characteristic of supermode excitation, i.e. collective oscillations of all cavities, and the corresponding splitting of the resonance frequency associated with a single dielectric square.

Our hybrid coupled mode model permits to explain at least some major features of these—at a first glance rather irregular—spectral properties. No free parameters are introduced. The model, however, disregards any radiative losses (so far), and thus cannot be more than an approximation of the resonator chain in a kind of high-Q limit. Note that an extension of the chain, with just one additional unknown per cavity, does hardly imply any additional computational burden for the HCMT model. With a more realistic template for the leaky cavity modes, this could open a way for an adequate treatment of really long chains.

This is admittedly a somewhat academic example, in so far as only the simplest case of 2-D configurations are considered. Still, just as for most concepts discussed nowadays for photonic crystal slabs, it remains to be seen whether these results are relevant for realistic, necessarily high-contrast slab-like 3-D devices.

So far we didn't pay any attention to time-domain properties of the light propagation along the chains. Corresponding studies could concern the slow transfer of light pulses along the chain path (Melloni et al. 2003; Poon et al. 2006), or the time delay properties (Morichetti et al. 2007; Morichetti et al. 2008) of a chain when attached to one branch in an interferometer setting.

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References

- Astratov, V.N., Franchak, J.P., Ashili, S.P.: Optical coupling and transport phenomena in chains of spherical dielectric microresonators with size disorder. *Appl. Phys. Lett.* **85**(23), 5508–5510 (2004)
- Bayindir, M., Temelkuran, B., Ozbay, E.: Propagation of photons by hopping: a waveguiding mechanism through localized coupled cavities in three-dimensional photonic crystals. *Phys. Rev. B* **61**(18), R11 855–R11 858 (2000)
- Boriskina, S.V.: Efficient simulation and design of coupled optical resonator clusters and waveguides. In: *Proceedings of the 7th International Conference on Transparent Optical Networks (ICTON)*, vol. 2, pp. 330–332 (2005)
- Boriskina, S.V.: Spectrally engineered photonic molecules as optical sensors with enhanced sensitivity: proposal and numerical analysis. *J. Opt. Soc. Am.* **23**(8), 1565–1573 (2006)

- Boriskina, S.V.: Coupling of whispering-gallery modes in size-mismatched microdisk photonic molecules. *Opt. Lett.* **32**(11), 1557–1559 (2007)
- Boriskina, S.V., Benson, T.M., Sewell, P.: Photonic molecules made of matched and mismatched microcavities: new functionalities of microlasers and optoelectronic components. *Proc. SPIE* **6452**, 64520X (2007)
- Boriskina, S.V., Sewell, P., Benson, T.M., Nosich, A.I.: Accurate simulation of two-dimensional optical microcavities with uniquely solvable boundary integral equations and trigonometric galerkin discretization. *J. Opt. Soc. Am. A* **21**(03), 393–402 (2004)
- Boriskina, S.V., Benson, T.M., Sewell, P., Nosich, A.I.: Optical modes in 2-d imperfect square and triangular microcavities. *IEEE J. Quantum Electron.* **41**(6), 857–862 (2005)
- Chen, Z., Taflove, A., Backman, V.: Highly efficient coupling and transport phenomena in chains of dielectric microspheres. *Opt. Lett.* **31**(03), 389–391 (2006)
- Fan, S., Villeneuve, P.R., Joannopoulos, J.D., Khan, M.J., Manolatu, C., Haus, H.A.: Theoretical analysis of channel drop tunneling processes. *Phys. Rev. B* **59**(24), 15882–15892 (1999)
- Hammer, M.: Resonant coupling of dielectric optical waveguides via rectangular microcavities: the coupled guided mode perspective. *Opt. Commun.* **214**(1–6), 155–170 (2002)
- Hammer, M.: Quadridirectional eigenmode expansion scheme for 2-D modeling of wave propagation in integrated optics. *Opt. Commun.* **235**(4–6), 285–303 (2004)
- Hammer, M.: Hybrid analytical/numerical coupled-mode modeling of guided wave devices. *J. Lightwave Technol.* **25**(9), 2287–2298 (2007)
- Hammer, M. METRIC—Mode expansion tools for 2D rectangular integrated optical circuits. <http://www.math.utwente.nl/~hammer/Metric/>
- Hammer, M., van Groesen, E.: Total multimode reflection at facets of planar high contrast optical waveguides. *J. Lightwave Technol.* **20**(8), 1549–1555 (2002)
- JCMwave GmbH, Haarer Str. 14a, 85640 Putzbrunn/Munich, Germany. <http://www.jcmwave.com/>
- Khan, M.J., Manolatu, C., Fan, S., Villeneuve, P.R., Haus, H.A., Joannopoulos, J.D.: Mode-coupling analysis of multipole symmetric resonant add/drop filters. *IEEE J. Quantum Electron.* **35**(10), 1451–1460 (1999)
- Little, B.E., Chu, S.T., Absil, P.P., Hryniewicz, J.V., Johnson, F.G., Seiferth, F., Gill, D., Van, V., King, O., Trakalo, M.: Very high-order microring resonator filters for WDM applications. *IEEE Photonics Technol. Lett.* **16**(10), 2263–2265 (2004)
- Lohmeyer, M.: Mode expansion modeling of rectangular integrated optical microresonators. *Opt. Quantum Electron.* **34**(5), 541–557 (2002)
- Maksimovic, M., Hammer, M., van Groesen, E.: Field representation for optical defect microcavities in multilayer structures using quasi-normal modes. *Opt. Commun.* **281**(6), 1401–1411 (2008a)
- Maksimovic, M., Hammer, M., van Groesen, E.: Coupled optical defect microcavities in 1d photonic crystals and quasi-normal modes. In Greiner, C.M., Waechter, C.A. (eds.), *Integrated Optics: Devices, Materials, and Technologies XII*, volume 6896 of *SPIE Proceedings*, pp. 689605–1–689605–11 (2008b)
- Manolatu, C., Khan, M.J., Fan, S., Villeneuve, P.R., Haus, H.A., Joannopoulos, J.D.: Coupling of modes analysis of resonant channel add-drop filters. *IEEE J. Quantum Electron.* **35**(9), 1322–1331 (1999)
- Melloni, F., Morichetti, A., Martinelli, M.: Linear and nonlinear pulse propagation in coupled resonator slow-wave optical structures. *Opt. Quantum Electron.* **35**, 365–379 (2003)
- Morichetti, F., Melloni, A., Breda, A., Canciamilla, A., Ferrari, C., Martinelli, M.: A reconfigurable architecture for continuously variable optical slow-wave delay lines. *Opt. Express* **15**(25), 17273–17281 (2007)
- Morichetti, F., Melloni, A., Ferrari, C., Martinelli, M.: Error-free continuously-tunable delay at 10 Gbit/s in a reconfigurable on-chip delay line. *Opt. Express* **16**(12), 8395–8399 (2008)
- Pishko, S.V., Sewell, P.D., Benson, T.M., Boriskina, S.V.: Efficient analysis and design of low-loss whispering-gallery-mode coupled resonator optical waveguide bends. *J. Lightwave Technol.* **25**(9), 2487–2494 (2007)
- Poon, J.K.S., Zhu, L., DeRose, G.A., Yariv, A.: Transmission and group delay of microring coupled-resonator optical waveguides. *Opt. Lett.* **31**(4), 456–458 (2006)
- Prkna, L.: Rotationally symmetric resonant devices in integrated optics. Ph.D. Thesis, Faculty of Mathematics and Physics, Charles University, Prague, and Institute of Radio Engineering and Electronics, Academy of Sciences of the Czech Republic, Prague, Czech Republic (2004)
- Prkna, L., Čtyroký, J., Hubálek, M.: Ring microresonator as a photonic structure with complex eigenfrequency. *Opt. Quantum Electron.* **36**(1/3), 259–269 (2004)
- Reynolds, A.L., Peschel, U., Lederer, F., Roberts, P.J., Krauss, T.F., de Maagt, P.J.I.: Coupled defects in photonic crystals. *IEEE Trans. Microwav. Theo. Tech.* **49**(10), 1860–1867 (2001)
- Scheuer, J., Palocz, G.T., Poon, J.K.S., Yariv, A.: Coupled resonator optical waveguides—toward the slowing & storage of light. *Opt. Photonics News* **16**(2), 36–40 (2005)
- Smotrova, E.I., Nosich, A.I.: Mathematical study of the two-dimensional lasing problem for the whispering-gallery modes in a circular dielectric microcavity. *Opt. Quantum Electron.* **36**, 213–221 (2004)

- Smotrova, E.I., Nosich, A.I., Benson, T.M., Sewell, P.: Optical coupling of whispering-gallery modes of two identical microdisks and its effect on photonic molecule lasing. *IEEE J. Sel. Top. Quantum Electron.* **12**(1), 78–85 (2006)
- Sopaheluwakan, A.: *Characterization and Simulation of Localized States in Optical Structures*. University of Twente, Enschede, The Netherlands (2006)
- van Groesen, E.W.C., Molenaar, J.: *Continuum Modeling in the Physical Sciences*. SIAM publishers, Philadelphia, USA (2007)
- Vassallo, C.: *Optical Waveguide Concepts*. Elsevier, Amsterdam (1991)
- Yariv, A., Xu, Y., Lee, R.K., Scherer, A.: Coupled-resonator optical waveguide: a proposal and analysis. *Opt. Lett.* **24**(11), 711–713 (1999)