Reminiscences of Peter Wynn

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Peter Wynn passed away in December 2017 from a heart attack. He was living in Zacatecas, Mexico. I would like to recall some memories about him.

I made my military service in a scientific laboratory of the French Army in 1968–1969. The building next to mine was another division of the Army, the *Centre de Calcul Scientifique de l'Armement*, which, among other duties, had to solve numerical analysis problems for the civil and military engineers working for the Defense. When freed from my obligations in June 1969, I went to Dr. Francis Ceschino, the head of the numerical analysis group of this service, to ask him if it was possible to be employed there and, simultaneously, to work for a Doctoral Thesis, the French *Thèse d'État ès Sciences Mathématiques*, on a topic which agreed him and a University professor. I was hired, and Ceschino proposed me to work on convergence acceleration methods. At that time in France, there were essentially two centers for doing research in numerical analysis: the University of Paris where Jacques–Louis Lions was working on PDEs, and the University of Grenoble with Noël Gastinel who was interested by other topics. Gastinel accepted to supervise my Thesis on acceleration methods. My first work was to read the papers on Shanks' transformation and on the ε -algorithm of Wynn. Each paper of Wynn was citing another paper of him, I read the new paper and found another citation of him. It seemed to be an endless procedure. Thus, I decided to write a letter to Wynn (at that time the Internet was not developed) asking him to send me a complete list of his publications. He immediately answered and provided me with many reprints of his papers. This was the beginning of many epistolary exchanges. The formal tone of our first letters changed to more personal messages after some time. Peter gave me many advices; he helped me in the proofs

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of several of my results and discussed with me his own research. Once, he wrote to me that the only mathematicians who do not make errors are those who never publish. Without his help, my career would certainly have been different. I defended my Thesis on April 26, 1971. In 1972, he managed to have me invited to participate in the *International Conference on Padé Approximation and Related Matters*, in Boulder, USA, 19–22 June 1972, organized by William B. Jones and Wolfgang J. Thron. This was the first international conference I attended. There I met George Baker Jr., Michael Barnsley, Daniel Bessis, Roy Chisholm, Albert Edrei, John Gammel, Gene Golub, Bill Gragg, Yudell Luke, John Nuttall, Joe Traub, Luc Wuytack, Jean Zinn–Justin, and many others with whom I stayed in contact over the years. There I also met Peter for the first time. After the congress, he invited me to give a seminar at the *Centre de Recherches Mathématiques* of the *Université de Montréal* where he was working at that time. I stayed there a few days discussing mathematics with him, and having meals in good restaurants. Our regular contacts thus continued for many years. Several of my works are a continuation of his papers and vice versa. He discussed with me the results obtained by some of my students and colleagues. He often gave me lengthy explanations. He also took steps so that I could be invited for a long stay in Canada or the USA (but I could not take advantage of these invitations) and to get funds for my visits. I translated for him three papers he wanted to publish in French in the *Comptes Rendus de l'Académie des Sciences de Paris*. Once, he completely rewrote a paper of mine before its submission to a journal. We met at various occasions in Canada (for the ICM 1974), the USA, and France. In particular, he attended the *Colloque d'Analyse Numérique* at La Grande–Motte, France, 26 May–1st June 1975, and a congress on Padé approximants in Toulon and Marseille. The last time we met was at the congress *Rational Approximation with Emphasis on Padé Approximants*, Tampa, USA, 15–17 December 1976, organized by Ed Saff. In 1981, Peter decided to move to Mexico. He wanted to get away, as he wrote to me, *from the many futilities and stupidities of modern academic life for a year or so*. But, because he was able to have a better life there, he decided to stay in the country. He first lived in Guanajuato where he was related to the *Centro de Investigación en Matemáticas*. The cultural life of the city was very rich, and he often went to the theater and to concerts. Peter was a *bon vivant* enjoying good restaurants, but also a man of culture, *un honnête homme* in the sense of the seventeenth century with, in addition, a very British sense of humor. Although, as he wrote to me, he was working hard at mathematics and have settled many matters which previously troubled him, he did not published anything any longer. He was also planning to write one or two books, and to come to Europe *giving circus performances*. But these projects were not realized. However, he was still writing reviews for *Zentralblatt für Mathematik und ihre Grenzgebiete*. He moved to Zacatecas in 1990 and stopped to have scientific contacts as far as I know. His last paper was published in 1981, and his last letter to me dated June 1992.

Without Peter Wynn, the fields of acceleration (or extrapolation) methods, Padé approximation and continued fractions would certainly not have been so much developed, as well as their applications to fixed point methods and other topics in numerical analysis and applied mathematics I owe him a lot. Thank you Peter.

Peter Wynn with C.B. in 1975

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