Correction to: Polynomials with bounds and numerical approximation

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Abstract The result of the main Theorem 2.8 in Després (2017) remains unchanged. Nevertheless the proof must be corrected due to one missprint and missing low order contributions. This correction has its importance in case one desires to implement the method, which is part of our current research.

We use the notations of [1]. The missprint shows up in formula (24), where D was replaced by C in the expressions of b and c . Formula (24) now becomes

$$\begin{cases} a = A\alpha + wB\beta + C\gamma + wD\delta, \\ b = A\beta - B\alpha - C\delta + D\gamma, \\ c = A\gamma + wB\delta - C\alpha - wD\beta, \\ d = A\delta - B\gamma + C\beta - D\alpha. \end{cases} \quad (24)$$

The main correction comes from missing terms proportional to B_{n-2} and D_{n-2} in formula (25). Formula (25) now becomes

$$\begin{cases} A_n^2 - B_{n-1}^2 + C_n^2 - D_{n-1}^2 = 0, \\ 2A_nA_{n-1} + B_{n-1}^2 + 2C_nC_{n-1} + D_{n-1}^2 = 2B_{n-1}B_{n-2} + 2D_{n-1}D_{n-2}. \end{cases} \quad (25)$$

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The modifications to the discussion due by the terms $2B_{n-1}B_{n-2} + 2D_{n-1}D_{n-2}$ are reproduced below.

Degenerated case. If $A_n^2 + C_n^2 = 0$, then $B_{n-1} = D_{n-1} = 0$ and the second formula holds whatever the value of B_{n-2} and D_{n-2} . So $A, C \in P_{n-1}$ and $B, D \in P_{n-2}$. In this case one takes $(\alpha, \beta, \gamma, \delta) = (1, 0, 0, 0)$ so that $(a, b, c, d) = (A, -B, -C, -D)$ and there is nothing more to prove.

Main case. So we consider below the main case $A_n^2 + C_n^2 > 0$. Define

$$\begin{cases} \alpha(x) = \varepsilon(A_n x + \widehat{A}_{n-1}), \\ \beta = \varepsilon B_{n-1}, \\ \gamma(x) = \varepsilon(C_n x + \widehat{C}_{n-1}), \\ \delta = \varepsilon D_{n-1} \end{cases}$$

where $\varepsilon > 0$ is a scaling parameter to be chosen and $\widehat{A}_{n-1}, \widehat{C}_{n-1}$ are unknown at this level of the discussion.

- Let us determine the dominant coefficients of $a, c \in P_{n+1}$ given by (24). An expansion yields

$$\begin{aligned} a(x) = & \varepsilon \left(A_n^2 - B_{n-1}^2 + C_n^2 - D_{n-1}^2 \right) x^{n+1} \\ & + \varepsilon \left(A_n A_{n-1} + A_n \widehat{A}_{n-1} + B_{n-1}^2 - B_{n-1} B_{n-2} + C_n C_{n-1} \right. \\ & \left. + C_n \widehat{C}_{n-1} + D_{n-1}^2 - D_{n-1} D_{n-2} \right) x^n + \text{low order terms.} \end{aligned}$$

Another expansion yields

$$\begin{aligned} c(x) = & \varepsilon (A_n C_n - B_{n-1} D_{n-1} - C_n A_n + D_{n-1} B_{n-1}) x^{n+1} \\ & + \varepsilon (A_n \widehat{C}_{n-1} + A_{n-1} C_n + B_{n-1} D_{n-1} - B_{n-2} D_{n-1} \\ & - C_n \widehat{A}_{n-1} - C_{n-1} A_n - D_{n-1} B_{n-1} + D_{n-2} B_{n-1}) x^n + \text{low order terms.} \end{aligned}$$

Since the two dominant terms vanish, then one gains one order that is $a, c \in P_n$. In order to gain one more order, let us write the technical condition so that $a, c \in P_{n-1}$

$$\begin{cases} A_n A_{n-1} + A_n \widehat{A}_{n-1} + B_{n-1}^2 - B_{n-1} B_{n-2} \\ + C_n C_{n-1} + C_n \widehat{C}_{n-1} + D_{n-1}^2 - D_{n-1} D_{n-2} = 0, \\ A_n \widehat{C}_{n-1} + A_{n-1} C_n + B_{n-1} D_{n-1} - B_{n-2} D_{n-1} \\ - C_n \widehat{A}_{n-1} - C_{n-1} A_n - D_{n-1} B_{n-1} + D_{n-2} B_{n-1} = 0. \end{cases}$$

It can be written as a linear system with unknowns $\widehat{A}_{n-1}, \widehat{C}_{n-1}$ under the form (note one uses (25) to eliminate $-B_{n-1}^2 - D_{n-1}^2$ the right hand side in the first line)

$$(*) \begin{cases} A_n \widehat{A}_{n-1} + C_n \widehat{C}_{n-1} = A_n A_{n-1} + C_n C_{n-1} - B_{n-1} B_{n-2} - D_{n-1} D_{n-2}, \\ -C_n \widehat{A}_{n-1} + A_n \widehat{C}_{n-1} = -C_n A_{n-1} + A_n C_{n-1} + D_{n-1} B_{n-2} - B_{n-1} D_{n-2}. \end{cases}$$

Writing $\widetilde{A}_{n-1} = \widehat{A}_{n-1} - A_{n-1}$ and $\widetilde{C}_{n-1} = \widehat{C}_{n-1} - C_{n-1}$, one gets the linear system with simplified right hand sides

$$\begin{cases} A_n \widetilde{A}_{n-1} + C_n \widetilde{C}_{n-1} = -B_{n-1} B_{n-2} - D_{n-1} D_{n-2}, \\ -C_n \widetilde{A}_{n-1} + A_n \widetilde{C}_{n-1} = D_{n-1} B_{n-2} - B_{n-1} D_{n-2}. \end{cases}$$

The determinant of the linear system is $A_n^2 + C_n^2 > 0$, so there exists a unique solution $\tilde{A}_{n-1}, \tilde{C}_{n-1}$ and a unique $\hat{A}_{n-1}, \hat{C}_{n-1}$. Up to the scaling factor ε , the quadruplet $(\alpha, \beta, \gamma, \delta) \in P_1 \times P_0 \times P_1 \times P_0$ is well-defined.

A direct expansion yields

$$\alpha(x)^2 + \beta^2 w + \gamma(x)^2 + \delta^2 w = \varepsilon^2 \left[\left(A_n^2 - B_{n-1}^2 + C_n^2 - D_{n-1}^2 \right) x^2 + \left(2A_n \hat{A}_{n-1} + B_{n-1}^2 + 2C_n \hat{C}_{n-1} + D_{n-1}^2 \right) x + \left(\hat{A}_{n-1}^2 + \hat{C}_{n-1}^2 \right) \right].$$

The first relation in (*) yields

$$\begin{aligned} & \alpha(x)^2 + \beta^2 w + \gamma(x)^2 + \delta^2 w \\ &= \varepsilon^2 \left(2 \left(A_n A_{n-1} + C_n C_{n-1} - B_{n-1} B_{n-2} - D_{n-1} D_{n-2} + B_{n-1}^2 + D_{n-1}^2 \right) x \right. \\ & \quad \left. + \varepsilon^2 \left(\hat{A}_{n-1}^2 + \hat{C}_{n-1}^2 \right) \right). \end{aligned}$$

The second relation in (25) yields that

$$\alpha(x)^2 + \beta^2 w + \gamma(x)^2 + \delta^2 w = \varepsilon^2 \left(\hat{A}_{n-1}^2 + \hat{C}_{n-1}^2 \right).$$

Next there are two sub-cases.

First sub-case: either $\hat{A}_{n-1}^2 + \hat{C}_{n-1}^2 > 0$ then one takes $\varepsilon = (A_{n-1}^2 + C_{n-1}^2)^{-\frac{1}{2}} > 0$. It insures that $(\alpha, \beta, \gamma, \delta) \in \mathcal{U}_1$.

Second sub-case: or $\hat{A}_{n-1}^2 + \hat{C}_{n-1}^2 = 0$. Since $\hat{A}_{n-1}$ and $\hat{C}_{n-1}$ vanish, by plugging the first relation of (*) into the second one of (25) one obtains $B_{n-1}^2 + D_{n-1}^2 = 0$. Therefore using the first equation of (25), we obtain $A_n^2 + C_n^2 = 0$. This is a contradiction since $A_n^2 + C_n^2 > 0$. So the second sub-case does not hold. So $a, c \in P_{n-1}$.

If $B_{n-2} = D_{n-2} = 0$, then $\hat{A}_{n-1} = A_{n-1}$ and $\hat{C}_{n-1} = C_{n-1}$ which corresponds to the original proof [1]. The remaining part of the proof is the same.

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