Event-triggered model predictive control of positive systems with random actuator saturation

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Abstract This paper investigates the event-triggered model predictive control of positive systems with actuator saturation. Interval and polytopic uncertainties are imposed on the systems, respectively. First, a new model with actuator saturation obeying Bernoulli distribution is established, which is more general and powerful for describing the saturation phenomenon than the saturation in a deterministic way. Then, a linear event-triggering condition is constructed based on the state and error signal. Under the event-triggering condition, an interval estimate approach is presented to reach the positivity and stability of the systems. The saturation part in the controller is technically transformed into a non-saturation part. Thus, a linear programming approach is proposed to compute the event-triggered controller gain and the corresponding gain of attraction domain. A predictive algorithm is introduced for the computation of the event-triggered controller parameters. Finally, an example is provided to illustrate the validity of the design.

Keywords Positive systems · Event-triggered model predictive control · Linear programming

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1 Introduction

In real world, there exist a large amounts of quantities taking nonnegative values such as population, the concentration of liquid, and economic index. It may lead to modeling error when describing these nonnegative quantities via general dynamic systems (non-positive). Positive systems can be used to accurately describe the systems consisting of nonnegative quantities. Such kind of systems has drawn many interests in the control field owing to their effectiveness in medical treatment [1, 2], communication [3, 4], water systems [5] and challenges in theory [6–9]. Over past two decades, much effort has been devoted to stability [10, 11], control synthesis [12, 13], observation [14], filter [15], etc. It is shown in the existing literature that linear copositive Lyapunov functions and linear programming are more effective for positive systems than other ones. Despite these interesting results on positive systems, there are still many open issues of positive systems. For instance, existing results ignore the practicability of the presented control approaches, the optimal control of positive systems is rarely investigated, and few results consider the corresponding issues of constrained positive systems. Furthermore, it is necessary to introduce some new approaches to positive systems to solve these open issues [16–18].

In [19], an ℓ_1 -gain model predictive control (MPC) was proposed for positive systems with disturbances. Linear framework-based MPC and distributed MPC

were introduced for constrained positive systems in [20,21] and [22], respectively. Linear copositive Lyapunov functions associated with linear programming were commonly used in [19–22]. MPC is an advanced control approach and can explicitly deal with uncertainties and constraints of systems [23–25]. MPC is also a kind of optimal control strategy, in which the control input is chosen by computing an optimization problem at each time instant. These literature [19–22] have partly solved the optimal control problem of positive systems. It should be noted that the MPC contains untractable issues since the optimization algorithm is continuously implemented at each time instant. This increases computation burden. Moreover, it may be unnecessary to update the control input when the prescribed system performance is satisfied. A so-called event-triggered control strategy was introduced to reduce the cost of control design owing to frequent samples [26,27]. In the event-triggered control framework, the control input is not updated until some prescribed event conditions are satisfied. This class of control strategies is more practical and advanced than the traditional time-sampled control (the control behavior is variable with time). An event-triggering approach was used to reduce the workload of the communication traffic in networks [28,29]. By means of Lyapunov-like functions method and average dwell time technique, the finite-time event-triggered control problem of switched systems was solved in [30]. Under the assumption that the plant is input-to-state stable with respect to measurement errors, the control law was updated once the given event-triggering condition was violated [31]. An event-triggered MPC was proposed in [32] for discrete-time systems subject to exogenous disturbances, where it was shown that the event-triggering implementation can keep the state in an explicitly computable set. An event-triggered MPC algorithm was designed for continuous-time systems based on a dual-mode approach [33]. For nonlinear systems, a nonlinear event-triggered MPC was proposed in [34]. It is verified in [35] that the event-triggered MPC scheme can compensate for the communication delays actively rather than passively. More results on event-triggered MPC can refer to [36–39]. To the best of the authors' knowledge, there are no results on the event-triggered MPC of positive systems. The reasons mainly lie in three aspects. First, the research on positive systems is more complex than general systems. Take the control and observation of positive systems into account.

The controller design requires not only the stability of the systems but also the positivity of the systems. The observer design guarantees not only the stability of the error system derived from the original system and the observer, but also the positivity of the observer. Second, the research approaches of positive systems are different from general systems. As pointed out in the first paragraph, some new approaches should be introduced for positive systems. It was also verified in [4–13] that a linear approach is more effective for positive systems. This implies that the research approaches of general systems cannot be directly developed for positive systems. Third, few results consider both the MPC and the event-triggered control of positive systems. The previous two aspects directly lead to the delay of the research of positive systems to general systems. Up to now, the MPC and the event-triggered control are still open to positive systems. These reasons imply that it is important to investigate the event-triggered MPC of positive systems. This motivates us to carry out the work.

Actuator saturation is a universal phenomenon for a control system owing to limited capacity of actuator elements. Actuator saturation may lead to the performance deterioration of the systems and even cause instability of the systems. Some meaningful achievements have been devoted to the saturation issues of general systems [40–43] and positive systems [44–46], where it is assumed that the saturation occurs in a deterministic way. Generally, the aging of elements arises with time and frequent use. Thus, it induces the saturation of elements. On the other hand, the external factors may cause the saturation of actuator. For example, the irregular environment change will worsen the pumping volume of valve exposed to the environment, most of capacitances are sensitive to the temperature and high temperature will weaken the performance of the capacitances, etc. These imply that it is not very suitable to consider the saturation in a deterministic way. In [47] and [48], random phenomenon was introduced for describing nonlinearities and delays. The literature [49] further considered random sensor saturation of networked systems. In [50], a distributed MPC with random actuator saturation was investigated. The random saturation is more general and practical than the deterministic case [40–46]. However, there are results on neither the event-triggered MPC nor random actuator saturation of positive systems. Considering the differences of research approaches between general systems and positive systems and the complexity of posi-

tive systems, there is still room to focus on the presented topic in this paper.

This paper proposes an event-triggered MPC of positive systems with random actuator saturation. Different from [47–50], linear Lyapunov functions are employed and linear programming-based predictive algorithm is addressed in this paper. Compared with [19–21], the saturation description is more general and practical, and the presented control strategy is more advanced and has less design cost. The contributions of the paper have three aspects: (i) a linear event-triggered MPC framework is constructed for positive systems, (ii) a matrix decomposition-based MPC controller design is proposed, and (iii) a linear programming-based MPC algorithm is presented for positive systems with random actuator saturation. The rest of the paper is organized as follows. Section 2 gives the problem formulation. In Sect. 3, the event-triggered MPC is proposed for interval and polytopic systems, respectively. An illustrative example is provided in Sect. 4. Section 5 concludes the paper.

Notation: Let $\mathbb{R}$ (or $\mathbb{R}_+$), $\mathbb{R}^n$ (or $\mathbb{R}_+^n$), and $\mathbb{R}^{n \times m}$ be the sets of real numbers, n -dimensional vectors (or, nonnegative), and $n \times m$ -dimensional matrices, respectively. For a matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, $A \geq 0$ (> 0) means that $a_{ij} \geq 0$ ($a_{ij} > 0$) $\forall i, j = 1, \dots, n$. $A \leq 0$ (< 0) means that $a_{ij} \leq 0$ ($a_{ij} < 0$) $\forall i, j = 1, \dots, n$. Similarly, $A \geq B$ ($A \leq B$) means that $a_{ij} \geq b_{ij}$ ($a_{ij} \leq b_{ij}$) $\forall i, j = 1, \dots, n$. $\mathbb{N}$ denotes the set of integers. $\mathbb{N}^+$ represents the positive integers. The identity matrix is denoted as I . Define $\mathbf{1}_m = (\underbrace{1, \dots, 1}_m)^T$ and $\mathbf{1}_m^{(i)} = (\underbrace{0, \dots, 0}_i, \underbrace{1, 0, \dots, 0}_{m-i})^T$. For a vector x , $\|x\|_1$ and $\|x\|_2$ represent its 1-norm and Euclidean norm, respectively. $\mathbb{E}\{\cdot\}$ and $\text{Prob}\{\cdot\}$ refer to mathematical expectation and probabilities. If not stated, vectors and matrices possess compatible dimensions.

2 Problem formulation

Consider a class of time-varying systems with random actuator saturation:

$$\begin{aligned} x(k+1) = & A(k)x(k) + B(k) \\ & \times (\alpha(k)\text{sat}(u(k)) + (1 - \alpha(k))\beta(k)u(k)), \end{aligned} \quad (1)$$

where $k \in \mathbb{N}$, $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input, and $\alpha(k)$ and $\beta(k)$ are two independent stochastic variables; $\text{sat}(u(k)) = (\text{sat}(u_1(k)), \dots, \text{sat}(u_m(k)))^T$ represents the saturation function, where $\text{sat}(u_i(k)) = \text{sign}(u_i(k)) \min\{1, |u_i(k)|\}$, $i = 1, \dots, m$ and $\text{sign}(\cdot)$ is the sign function; the system matrices $A(k) \in \mathbb{R}^{n \times n}$ and $B(k) \in \mathbb{R}^{n \times m}$ are time-varying and located in an interval uncertain set:

$$\Omega_1 = \{[A(k) \ B(k)] : \underline{A} \leq A(k) \leq \bar{A}, \underline{B} \leq B(k) \leq \bar{B}\}, \quad (2)$$

where $\underline{A} \geq 0$ and $\underline{B} \geq 0$, or a polytopic uncertain set:

$$\begin{aligned} \Omega_2 = & \{[A(k) \ B(k)] : [A(k) \ B(k)] \\ & = \sum_{p=1}^L \lambda_p [A^{(p)} \ B^{(p)}]\}, \end{aligned} \quad (3)$$

where $A^{(p)} \geq 0$, $B^{(p)} \geq 0$, and $\sum_{p=1}^L \lambda_p = 1$, $\lambda_p \geq 0$. The random actuator saturation is governed by stochastic variables $\alpha(k)$ and $\beta(k)$ with:

$$\begin{aligned} \text{Prob}\{\alpha(k) = 1\} = \bar{\alpha}, \quad \text{Prob}\{\alpha(k) = 0\} = 1 - \bar{\alpha}, \\ \text{Prob}\{\beta(k) = 1\} = \bar{\beta}, \quad \text{Prob}\{\beta(k) = 0\} = 1 - \bar{\beta}, \end{aligned} \quad (4)$$

where $0 \leq \bar{\alpha} \leq 1$ and $0 \leq \bar{\beta} \leq 1$.

The system (1) is called interval uncertain system and polytopic uncertain system if its system matrices satisfy the constraints in (2) and (3), respectively. Polytopic uncertain systems consist of several vertex systems, which employ different operation models at different time instants or different operation status. This class of systems has been extensively applied for describing time-varying systems and nonlinear systems. Many MPC literature also used polytopic uncertain systems to model practical dynamical processes [23, 24, 50]. Interval uncertain systems are another important modeling method for uncertain systems. They only need to provide upper and lower bounds of systems matrices. On the one hand, interval uncertain systems are a special class of polytopic uncertain systems when polytopic uncertain systems only include two vertex systems. On the other hand, polytopic uncertain systems can be rewritten as interval uncertain systems if the maximal and minimal elements of all vertex system matrices in polytopic uncertain systems are assigned to the upper and lower bound matrices in interval uncertain systems, respectively. From

this point, interval uncertain systems are much easier in modeling than polytopic uncertain systems. Some other key points should be noted. First, the control designs of the two classes of uncertain positive systems are different. This will be further stated later. Second, interval uncertain positive systems possess nice properties not only in modeling but also the control design. The positivity of the lower bound system and the stability of the upper bound system can lead to the positivity and stability of the whole interval uncertain positive system. These reasons are why we employ polytopic and interval uncertain systems in the paper. There have also been some positive systems literature that considered the two classes of uncertain systems [7, 8, 16, 17].

Remark 1 In practice, actuator saturation is a universal phenomenon owing to the limited capacity of elements. The actuator saturation issue has been one of important issues in the control field. Some efforts are devoted to the actuator saturation of general systems [40, 41, 43], the systems with MPC approaches [42, 50], positive systems [44–46], etc. The system (1) proposes a more general actuator saturation model, i.e., random actuator saturation. If $\alpha(k) = 1$, the actuator is subject to saturation; if $\alpha(k) = 0$ and $\beta(k) = 1$, the actuator runs normally; if $\alpha(k) = 0$ and $\beta(k) = 0$, it means that the information transmitted via the actuator is lost. It is clear that the system (1) considers not only the actuator saturation but also the information loss of the actuator.

Definition 1 [6, 7] A system is positive if its state and outputs are nonnegative for any nonnegative initial conditions and nonnegative inputs.

Lemma 1 [6, 7] A system $x(k+1) = Ax(k) + Bu(k)$ is positive if and only if $A \geq 0$ and $B \geq 0$.

By Lemma 1, the system (1) with interval uncertainty (2) and polytopic uncertainty (3) are all positive.

Lemma 2 [6, 7] Give a matrix $0 \leq A \in \mathbb{R}^{n \times n}$, then the following statements are equivalent:

- (i) The matrix A is a Schur matrix;
- (ii) There is a vector $v > 0$ in $\mathbb{R}^n$ such that $(A - I)v < 0$.

Remark 2 For a positive system $x(k+1) = Ax(k) + Bu(k)$, it was shown in [20] that there does not exist a nonnegative control law. The control synthesis of a positive system is to find a control law such that the closed-loop system is positive and stable. It does not require the nonnegative property of the control law. This point was also stated in [8].

This paper is to design the following event-triggered MPC controller for the system (1):

$$u(k+i|k) = F(k_s)\hat{x}(k), k \in [k_s, k_{s+1}), \quad (5)$$

where $F(k_s) \in \mathbb{R}^{m \times n}$ is the controller gain, $\hat{x}(k) = x(k_s|k_s)$ is the sample state, and $k_s \in \mathbb{N}$ and $k_{s+1} \in \mathbb{N}$, $s \in \mathbb{N}$ are the s th and $(s+1)$ th event-triggering time instants, respectively. Define the error $\tilde{x}(k) = \hat{x}(k) - x(k)$. The event-triggering condition is given as:

$$\|\tilde{x}(k)\|_1 > x^T(k)\epsilon, \quad (6)$$

where $0 < \epsilon \in \mathbb{R}^n$ is given. The control input will be updated if the condition (6) is satisfied. Otherwise, the control input is fixed, that is, $u(k+i|k) = F(k_s)x(k_s)$, $k \in [k_s, k_{s+1})$. Under the fixed control input, the performances of the system may be deteriorated. This may cause the divergence of the state. Thus, the term $\|\tilde{x}(k)\|_1$ will become larger. At the time instant k_{s+1} , the condition (6) holds. Immediately, the control input is updated by $u(k+i|k) = F(k_{s+1})x(k_{s+1})$ to avoid the further divergence of the system.

Remark 3 For the event-triggered control of non-positive systems [27–31], an Euclidean norm-based inequality: $\|\tilde{x}(k)\|_2 > \rho\|x(k)\|_2$ was usually chosen as the event-triggering condition, where $\rho > 0$. Then, a quadratic Lyapunov function approach is always used for dealing with the synthesis of the systems. Under the quadratic Lyapunov function, the Euclidean norm-based event-triggering condition is tractable for transforming the error correlation term into the state correlation term. Thus, the event-triggered control issue becomes a general time-triggered control issue. In most of existing literature on positive systems [5–11], it has been verified that a linear Lyapunov function is more powerful and suitable for positive systems. This reason mainly lies in the positivity of positive systems. Under the positivity condition, a linear Lyapunov function can be employed for positive systems. Naturally, an event-triggering condition $\|\tilde{x}(k)\|_1 > \rho\|x(k)\|_1$ is used for positive systems in terms of 1-norm inequality. The condition (6) further relaxes the commonly used condition since it replaces the vector $\mathbf{1}_n$ by ϵ .

Denote $\mathbb{D}$ as the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. Assume that each element in $\mathbb{D}$ is labeled as D_ℓ , $\ell = 1, 2, \dots, 2^m$ and set $D_\ell^- = I - D_\ell$. If $D_\ell \in \mathbb{D}$, then $D_\ell^- \in \mathbb{D}$.

Lemma 3 [41] Give matrices $K \in \mathbb{R}^{m \times n}$ and $H \in \mathbb{R}^{m \times n}$, then

$$\text{sat}(Kx(k)) = \sum_{\ell=1}^{2^m} \tilde{h}_{\ell} (D_{\ell} K + D_{\ell}^{-} H)x(k)$$

for $x(k) \in \Psi(H_i) \triangleq \{x(k) \mid |H_i x(k)| \leq 1, i = 1, 2, \dots, m\}$, where $\sum_{\ell=1}^{2^m} \tilde{h}_{\ell} = 1$, $0 \leq \tilde{h}_{\ell} \leq 1$, and H_i is the i th row of the matrix H .

In this paper, the matrix H is a variable to be designed later and $H \prec 0$.

Definition 2 [12] A positive stochastic system is mean-square stable if for given initial state x_0 and initial mode r_0 , $\mathbb{E}\{\|x\|_1 | x_0, r_0\} \rightarrow 0$ as $k \rightarrow \infty$.

Under Definition 2, the set $\Lambda = \{x \mid \mathbb{E}\{\|x\|_1 | x_0, r_0\} \rightarrow 0\}$ is the attraction domain. Define a cone set $\Phi = \{x(k) \in \mathbb{R}_+^n \mid x^T(k)v \leq 1\}$, where $0 \prec v \in \mathbb{R}^n$. Under the controller (5), it should be guaranteed that the cone set Φ is contained in the attraction domain Λ .

3 Main results

This section will propose the event-triggered MPC for the system (1). In the first subsection, we establish a framework about constraints and cost function of the system (1). The second subsection presents the MPC controller design. In the third subsection, the constraints are handled. The forth section constructs a cone invariant set and addresses the robust stability of the system (1).

3.1 MPC framework

The state and input constraints are given as:

$$\begin{aligned} \|x(k)\|_1 &\leq \delta, \\ \|u(k)\|_1 &\leq \eta, \end{aligned} \quad (7)$$

where $\delta > 0$ and $\eta > 0$ are known. Following the method in [20], we assume that $u(k) \prec 0$, that is, $F(k_s | k_s) \prec 0$. This paper is to design the controller (5) to solve the optimization problem:

$$\begin{aligned} \min_{u(k+i|k), i \in \mathbb{N}} \quad & \max_{[A(k+i) \ B(k+i)] \in \Omega_1(\text{Or}, \Omega_2), i \in \mathbb{N}} J_{\infty}(k), \\ \text{s.t.} \quad & (1) \text{ and } (7) \end{aligned} \quad (8)$$

with

$$J_{\infty}(k) = \mathbb{E} \left\{ \sum_{i=0}^{\infty} (x(k+i|k)^T \zeta + \bar{u}(k+i|k)^T \varrho) \right\}, \quad (9)$$

where $x(k+i|k)$ is the future state and $\bar{u}(k+i|k) = \alpha(k+i)\text{sat}(u(k+i|k)) + (1-\alpha(k+i))\beta(k+i)u(k+i|k)$ is the term related to control law predicted at time k , $0 \prec \zeta \in \mathbb{R}^n$, and $0 \succ \varrho \in \mathbb{R}^m$.

Choose a linear copositive Lyapunov function:

$$V(i, k) = x(k+i|k)^T v, \quad (10)$$

where $0 \prec v \in \mathbb{R}^n$. A robust stability condition is introduced:

$$\begin{aligned} \mathbb{E}\{\Delta V(i, k)\} &= \mathbb{E}\{V(i+1, k) - V(i, k)\} \\ &\leq -\mathbb{E}\{x(k+i|k)^T \zeta + \bar{u}(k+i|k)^T \varrho\}. \end{aligned} \quad (11)$$

Summing both sides of (11) from $i = 0$ to ∞ gives

$$J_{\infty}(k) \leq V(0, k) - \mathbb{E}\{V(\infty, k)\}, \quad (12)$$

which implies that $J_{\infty}(k)$ is upper-bounded. Note the fact that $J_{\infty}(k)$ is a nonnegative infinite series, then $\lim_{i \rightarrow \infty} \mathbb{E}\{x(k+i|k)^T \zeta + \bar{u}(k+i|k)^T \varrho\} = 0$. Owing to $x(k+i|k)^T \zeta \geq 0$ and $\bar{u}(k+i|k)^T \varrho \geq 0$, it follows that $\lim_{i \rightarrow \infty} \mathbb{E}\{x(k+i|k)^T \zeta\} = 0$. This in turn gives $\mathbb{E}\{V(\infty, k)\} = 0$. Therefore, we have

$$J_{\infty}(k) \leq V(0, k). \quad (13)$$

Then, the problem (8) can be solved by computing the minimum of $\gamma(k)$ for

$$V(0, k) = x(k|k)^T v \leq \gamma(k) \quad (14)$$

under the condition (11), where $\gamma(k) \geq 0$ is a variable to be minimized.

The MPC framework in (7)–(14) contains five linear elements: linear constraint (7), linear cost function (9), linear Lyapunov function (10), linear robust stability condition (11), and linear computation method (i.e., linear programming). Such a linear MPC framework is proposed based on the positivity property of positive systems. As stated in Remark 3, linear Lyapunov functions have been used in the literature. Accordingly, the cost function (9) and the stability condition (11) are

introduced. The use of 1-norm-based constraint (7) is because it is easy to be transformed into linear computation condition. Considering these points, the linear MPC framework in (7)–(14) is different from existing MPC of general systems [23–25]. A similar linear MPC framework was also successfully developed for positive systems in [19–22].

3.2 Event-triggered MPC

The traditional MPC control law is updated at each time instant by implementing the corresponding predictive algorithm. The event-triggered MPC is to find a trade-off between the optimal performance and the computation cost. The predictive algorithm of the event-triggered MPC is implemented only at the event-triggering time instants. This means that the event-triggering condition has priority over the performance of the systems.

Theorem 1 (a) Interval uncertainty *If there exist constants $\mu_1 > 1, 0 < \mu_2 < 1, \mu_3 > 0, \gamma(k) > 0$ and $\mathfrak{R}^n$ vectors $v > 0; \underline{\xi}(k), \xi^{(i)}(k), \bar{\xi}(k) < 0; \underline{\zeta}(k), \zeta^{(i)}(k), \bar{\zeta}(k) < 0$ such that*

$$\mathbf{1}_m^T (\underline{B}^T v + \varrho) \underline{A} + Q_{1\ell} \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k) (I + \mathbf{1}_n \epsilon^T) + Q_{2\ell} \sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k) (I + \mathbf{1}_n \epsilon^T) \geq 0, \quad (15a)$$

$$\bar{A}^T v + \bar{\alpha} \bar{\zeta}(k) + c \bar{\xi}(k) - Q_3 \bar{\zeta}(k) - Q_4 \bar{\xi}(k) + \varsigma - v < 0, \quad D_\ell = 0, \quad (15b)$$

$$\bar{A}^T v + (\bar{\alpha} + c) \bar{\xi}(k) - (Q_3 + Q_4) \bar{\xi}(k) + \varsigma - v < 0, \quad D_\ell = I, \quad (15c)$$

$$\bar{A}^T v + (\bar{\alpha} \mu_2 + c) \bar{\xi}(k) + \bar{\alpha} \mu_2 \bar{\zeta}(k) - Q_3 \bar{\zeta}(k) - (Q_3 + Q_4) \bar{\xi}(k) + \varsigma - v < 0, \quad D_\ell \neq 0, I, \quad (15d)$$

$$\mathbf{1}_m^T (\underline{B}^T v + \varrho) \geq \mu_3, \quad (15e)$$

$$\mathbf{1}_m^T (\bar{B}^T v + \varrho) \leq \mu_1 \mathbf{1}_m^T (\underline{B}^T v + \varrho), \quad (15f)$$

$$\mathbf{1}_m^T D_\ell^T \underline{B}^T v \geq \mu_2 \mathbf{1}_m^T \underline{B}^T v, \quad D_\ell \neq 0, D_\ell \neq I, \quad (15g)$$

$$\underline{\xi}(k) < \xi^{(i)}(k) < \bar{\xi}(k), \quad \underline{\zeta}(k) < \zeta^{(i)}(k) < \bar{\zeta}(k), \quad (15h)$$

$$\mu_3 v + \zeta^{(i)}(k) > 0, \quad (15i)$$

and (14) hold $\forall \ell = 1, \dots, 2^m$ and $i = 1, \dots, m$, then under the event-triggered MPC control law (5) with

$$F(k) = \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)}, \quad (16)$$

and the gain of attraction domain

$$H(k) = \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)}, \quad (17)$$

the system (1) is positive, the condition (11) holds, the performance index (8) is reached, and the state will kept in the set $\Psi(H_i)$ for any initial state starting from the set Φ , where $Q_{1\ell} = \bar{B} D_\ell$, $Q_{2\ell} = \bar{B} D_\ell^-$, $c = (1 - \bar{\alpha}) \bar{\beta}$, $Q_3 = \mu_1 \bar{\alpha} \epsilon \mathbf{1}_n^T$, and $Q_4 = \mu_1 (1 - \bar{\alpha}) \bar{\beta} \epsilon \mathbf{1}_n^T$.

(b) Polytopic uncertainty *If there exist constants $\mu_1 > 1, 0 < \mu_2 < 1, \mu_3 > 0, \gamma(k) > 0$ and $\mathfrak{R}^n$ vectors $v > 0; \underline{\xi}(k), \xi^{(i)}(k), \bar{\xi}(k) < 0; \underline{\zeta}(k), \zeta^{(i)}(k), \bar{\zeta}(k) < 0$ such that*

$$\mathbf{1}_m^T (\widehat{B}^T v + \varrho) A^{(p)} + Q_{1\ell p} \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k) (I + \mathbf{1}_n \epsilon^T) + Q_{2\ell p} \sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k) (I + \mathbf{1}_n \epsilon^T) \geq 0, \quad (18a)$$

$$A^{(p)T} v + \bar{\alpha} \bar{\zeta}(k) + c \bar{\xi}(k) - Q_3 \bar{\zeta}(k) - Q_4 \bar{\xi}(k) + \varsigma - v < 0, \quad D_\ell = 0, \quad (18b)$$

$$A^{(p)T} v + (\bar{\alpha} + c) \bar{\xi}(k) - (Q_3 + Q_4) \bar{\xi}(k) + \varsigma - v < 0, \quad D_\ell = I, \quad (18c)$$

$$A^{(p)T} v + (\bar{\alpha} \mu_2 + c) \bar{\xi}(k) + \bar{\alpha} \mu_2 \bar{\zeta}(k) - Q_3 \bar{\zeta}(k) - (Q_3 + Q_4) \bar{\xi}(k) + \varsigma - v < 0, \quad D_\ell \neq 0, I, \quad (18d)$$

$$\mathbf{1}_m^T (\widehat{B}^T v + \varrho) \geq \mu_3, \quad (18e)$$

$$\mathbf{1}_m^T (B^{(p)T} v + \varrho) \leq \mu_1 \mathbf{1}_m^T (\widehat{B}^T v + \varrho), \quad (18f)$$

$$\mathbf{1}_m^T D_\ell^T B^{(p)T} v \geq \mu_2 \mathbf{1}_m^T \widehat{B}^T v, \quad D_\ell \neq 0, D_\ell \neq I, \quad (18g)$$

$$\underline{\xi}(k) < \xi^{(i)}(k) < \bar{\xi}(k), \quad \underline{\zeta}(k) < \zeta^{(i)}(k) < \bar{\zeta}(k), \quad (18h)$$

$$\mu_3 v + \zeta^{(i)}(k) > 0, \quad (18i)$$

and (14) hold $\forall \ell = 1, \dots, 2^m$, $i = 1, \dots, m$, and $p = 1, \dots, L$, then under the event-triggered MPC control law (5) with

$$F(k) = \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)}, \quad (19)$$

and the gain of attraction domain

$$H(k) = \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)}, \quad (20)$$

the system (1) is positive, the condition (11) holds, the performance index (8) is reached, and the state will be kept in the set $\Psi(H_i)$ for any initial state starting from the set Φ , where $Q_{1\ell p} = B^{(p)} D_\ell$, $Q_{2\ell p} = B^{(p)} D_\ell^-$, $c = (1 - \bar{\alpha})\bar{\beta}$, $Q_3 = \mu_1 \bar{\alpha} \in \mathbf{1}_n^T$, $Q_4 = \mu_1 (1 - \bar{\alpha})\bar{\beta} \in \mathbf{1}_n^T$, $\widehat{B} = [\hat{b}_{ij}]$ with $\hat{b}_{ij} = \min_{p=1, \dots, L} \{b_{ij}^{(p)}\}$ and $b_{ij}^{(p)}$ is the i th row j th column element of the matrix $B^{(p)}$.

Proof Assume that $x(t) \in \Psi(H_i)$. By Lemma 3 and the control law (5),

$$\begin{aligned} \text{sat}(F(k)\widehat{x}(k)) &= \sum_{\ell=1}^{2^m} \hbar_\ell \left(D_\ell F(k)x(k) + D_\ell F(k)\widetilde{x}(k) \right. \\ &\quad \left. + D_\ell^- H(k)x(k) + D_\ell^- H(k)\widetilde{x}(k) \right). \end{aligned} \quad (21)$$

The closed-loop system (1) in the interval $k \in (k_s, k_{s+1})$ is

$$\begin{aligned} x(k+1|k) &= A(k)x(k|k) + B(k) \\ &\quad \times \left(\alpha(k) \sum_{\ell=1}^{2^m} \hbar_\ell (D_\ell F(k_s) + D_\ell^- H(k)) \right. \\ &\quad \left. + (1 - \alpha(k))\beta(k)F(k_s) \right) x(k|k) + B(k) \\ &\quad \times \left(\alpha(k) \sum_{\ell=1}^{2^m} \hbar_\ell (D_\ell F(k_s) + D_\ell^- H(k)) \right. \\ &\quad \left. + (1 - \alpha(k))\beta(k)F(k_s) \right) \widetilde{x}(k). \end{aligned} \quad (22)$$

Noting the conditions $\xi(k) < 0$, $\zeta(k) < 0$, it is easy to get $\xi^{(i)}(k) < 0$, $\zeta^{(i)}(k) < 0$ by (15h) and (18h). By (15e) and (18e), we have $F(k_s) < 0$ and $H(k) < 0$. Give any initial condition $x(k_0|k_0) \geq 0$. From (6),

$$-\mathbf{1}_n \epsilon^T x(k_0|k_0) \leq \widetilde{x}(k_0|k_0) \leq \mathbf{1}_n \epsilon^T x(k_0|k_0). \quad (23)$$

Then,

$$\begin{aligned} x(k_0+1|k_0) &\geq A(k_0)x(k_0|k_0) + B(k_0) \left(\alpha(k_0) \sum_{\ell=1}^{2^m} \hbar_\ell (D_\ell F(k_0) \right. \\ &\quad \left. + D_\ell^- H(k_0)) \right) x(k_0|k_0) + (1 - \alpha(k_0))\beta(k_0)F(k_0)x(k_0|k_0) \end{aligned}$$

$$\begin{aligned} &+ D_\ell^- H(k_0)) + (1 - \alpha(k_0))\beta(k_0)F(k_0) \Big) x(k_0|k_0) \\ &+ B(k_0) \left(\alpha(k_0) \sum_{\ell=1}^{2^m} \hbar_\ell (D_\ell F(k_0) + D_\ell^- H(k_0)) \right. \\ &\quad \left. + (1 - \alpha(k_0))\beta(k_0)F(k_0) \right) \mathbf{1}_n \epsilon^T x(k_0|k_0) \\ &= \alpha(k_0) \left(A(k_0) + B(k_0) \left(\sum_{\ell=1}^{2^m} \hbar_\ell (D_\ell F(k_0) \right. \right. \\ &\quad \left. \left. + D_\ell^- H(k_0)) (I + \mathbf{1}_n \epsilon^T) \right) \right) x(k_0|k_0) + (1 - \alpha(k_0)) \\ &\quad \times \left(A(k_0) + \beta(k_0)B(k_0)F(k_0) (I + \mathbf{1}_n \epsilon^T) \right) x(k_0|k_0) \\ &= \alpha(k_0) \sum_{\ell=1}^{2^m} \hbar_\ell \left(A(k_0) + B(k_0) (D_\ell F(k_0) \right. \\ &\quad \left. + D_\ell^- H(k_0)) (I + \mathbf{1}_n \epsilon^T) \right) x(k_0|k_0) + (1 - \alpha(k_0)) \\ &\quad \times \left(A(k_0) + \beta(k_0)B(k_0)F(k_0) (I + \mathbf{1}_n \epsilon^T) \right) x(k_0|k_0) \end{aligned} \quad (24)$$

for $k \in (k_s, k_{s+1})$. The closed-loop system (1) in the event-triggered time instant k_0 is

$$\begin{aligned} x(k_0+1|k_0) &= A(k_0)x(k_0|k_0) + B(k_0)\alpha(k_0) \\ &\quad \times \sum_{\ell=1}^{2^m} \hbar_\ell \left(D_\ell F(k_0) + D_\ell^- H(k_0) \right) x(k_0|k_0) \\ &\quad + B(k_0)(1 - \alpha(k_0))\beta(k_0)F(k_0)x(k_0) \\ &= \alpha(k_0) \sum_{\ell=1}^{2^m} \hbar_\ell \left(A(k_0) + B(k_0) (D_\ell F(k_0) \right. \\ &\quad \left. + D_\ell^- H(k_0)) \right) x(k_0|k_0) + (1 - \alpha(k_0)) \\ &\quad \times \left(A(k_0) + \beta(k_0)B(k_0)F(k_0) \right) x(k_0|k_0). \end{aligned} \quad (25)$$

Using $F(k|k) < 0$ and $H(k) < 0$, it is clear that

$$\begin{aligned} A(k_0) + B(k_0) (D_\ell F(k_0) + D_\ell^- H(k_0)) (I + \mathbf{1}_n \epsilon^T) \\ \leq A(k_0) + B(k_0) (D_\ell F(k_0) + D_\ell^- H(k_0)), \\ A(k_0) + \beta(k_0)B(k_0)F(k_0) (I + \mathbf{1}_n \epsilon^T) \\ \leq A(k_0) + \beta(k_0)B(k_0)F(k_0). \end{aligned} \quad (26)$$

This means that the positivity of the system (1) in the event-triggered time instant can be guaranteed if the system (1) in (k_s, k_{s+1}) is positive. Therefore, we only prove that the system (1) is positive in (k_s, k_{s+1}) .

(a) *Interval uncertainty* By (15a) and (15e), it follows that

$$\begin{aligned} & A(k_0) + B(k_0)(D_\ell F(k_0) + D_\ell^- H(k_0))(I + \mathbf{1}_n \epsilon^T) \\ & \geq \underline{A} + \overline{B} \left(D_\ell \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \right. \\ & \quad \left. + D_\ell^- \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k_0)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \right) (I + \mathbf{1}_n \epsilon^T) \\ & = \frac{1}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \left(\mathbf{1}_m^T (\underline{B}^T v + \varrho) \underline{A} \right. \\ & \quad \left. + \overline{B} \left(D_\ell \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0) + D_\ell^- \sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k_0) \right) \right. \\ & \quad \left. \times (I + \mathbf{1}_n \epsilon^T) \right) \geq 0. \end{aligned} \quad (27)$$

Then, the following inequality holds:

$$\begin{aligned} & A(k_0) + \beta(k_0)B(k_0)F(k_0)(I + \mathbf{1}_n \epsilon^T) \\ & \geq \underline{A} + \overline{B} \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} (I + \mathbf{1}_n \epsilon^T) \\ & = \frac{1}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \left(\mathbf{1}_m^T (\underline{B}^T v + \varrho) \underline{A} \right. \\ & \quad \left. + \overline{B} \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0) (I + \mathbf{1}_n \epsilon^T) \right) \\ & \geq \frac{1}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \left(\mathbf{1}_m^T (\underline{B}^T v + \varrho) \underline{A} \right. \\ & \quad \left. + \overline{B} \left(D_\ell \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0) + D_\ell^- \sum_{i=1}^m \mathbf{1}_m^{(i)} \zeta^{(i)T}(k_0) \right) \right) \\ & \quad \times (I + \mathbf{1}_n \epsilon^T) \geq 0. \end{aligned} \quad (28)$$

Together with (24) gives $x(k_0 + 1|k_0) \geq 0$. By recursive deduction, it is easy to obtain $x(k_0 + i|k_0) \geq 0, \forall i \in \mathbb{N}^+$. Therefore, the interval system (1) is positive.

By (22), we have

$$\begin{aligned} & \mathbb{E}\{V(k_s + 1, k) - V(k_s, k)\} = \mathbb{E}\{x^T(k + k_s + 1|k)v\} - x^T(k + k_s|k)v \\ & = \mathbb{E}\left\{x^T(k + k_s|k)A^T(k_s)v + x^T(k + k_s|k) \right. \\ & \quad \times \left(\alpha(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell (F^T(k_s)D_\ell^T + H^T(k_s)D_\ell^{-T}) \right. \\ & \quad \left. \left. + (1 - \alpha(k_s))\beta(k_s)F^T(k_s) \right) B^T(k_s)v - x^T(k + k_s|k)v \right\} \end{aligned}$$

$$\begin{aligned} & + \tilde{x}^T(k_s) \left(\alpha(k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell (F^T(k_s)D_\ell^T + H^T(k_s)D_\ell^{-T}) \right. \\ & \quad \left. + (1 - \alpha(k_s))\beta(k_s)F^T(k_s) \right) B^T(k_s)v \Big) \\ & = x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(A^T(k_s)v + \bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v \right. \\ & \quad \left. + H^T(k_s)D_\ell^{-T} B^T(k_s)v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v - v \right) \\ & \quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(\bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v \right. \\ & \quad \left. + H^T(k_s)D_\ell^{-T} B^T(k_s)v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v \right). \end{aligned} \quad (29)$$

By (4), the condition (11) holds if the following condition is valid:

$$\begin{aligned} & x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(A^T(k_s)v + \bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v \right. \\ & \quad \left. + H^T(k_s)D_\ell^{-T} B^T(k_s)v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v - v \right) \\ & \quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(\bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v \right. \\ & \quad \left. + H^T(k_s)D_\ell^{-T} B^T(k_s)v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v \right) \\ & \leq -\mathbb{E} \left(x(k + k_s|k)^T \varsigma + (\alpha(k + k_s)\text{sat}(u^T(k + k_s|k)) \right. \\ & \quad \left. + (1 - \alpha(k + k_s))\beta(k + k_s)u^T(k + k_s|k))v \right) \\ & = -\sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(x(k + k_s|k)^T \varsigma + \tilde{x}^T(k_s) (\bar{\alpha}(F^T(k_s)D_\ell^T \varrho \right. \\ & \quad \left. + H^T(k_s)D_\ell^{-T} \varrho) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)\varrho) \right), \end{aligned} \quad (30)$$

which is equivalent to

$$\begin{aligned} & x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(A^T(k_s)v + \varsigma + \bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v \right. \\ & \quad \left. + H^T(k_s)D_\ell^{-T} B^T(k_s)v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v - v \right) \\ & \quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(\bar{\alpha}(F^T(k_s)D_\ell^T (B^T(k_s)v + \varrho) + H^T(k_s)D_\ell^{-T} \right. \\ & \quad \left. \times (B^T(k_s)v + \varrho)) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)(B^T(k_s)v + \varrho) \right) \leq 0. \end{aligned} \quad (31)$$

By (23),

$$\begin{aligned}
& x^T(k + k_s | k) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(A^T(k_s) v + \varsigma + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s) v \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} B^T(k_s) v) + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s) v - v \right) \\
& + \tilde{x}^T(k_s) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho)) \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho)) + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s) v + \varrho) \right) \\
& \leq x^T(k + k_s | k) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(A^T(k_s) v + \varsigma + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s) v \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} B^T(k_s) v) + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s) v - v \right) \\
& - x^T(k + k_s | k) \epsilon \mathbf{1}_n^T \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho)) \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho)) + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s) v + \varrho) \right) \\
& = x^T(k + k_s | k) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(A^T(k_s) v + \varsigma \right. \\
& \quad \left. + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s) v + H^T(k_s) D_\ell^{-T} B^T(k_s) v) \right. \\
& \quad \left. + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s) v - v \right) - \epsilon \mathbf{1}_n^T \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho)) \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho)) + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s) v + \varrho) \right). \quad (32)
\end{aligned}$$

To further transform (32) into a tractable condition, we consider D_ℓ via three cases.

Case I: $D_\ell = 0$. From (2), (15f), (15h), (16), (17), we obtain

$$\begin{aligned}
& F^T(k_s) D_\ell^T B^T(k_s) v = 0, \\
& H^T(k_s) D_\ell^{-T} B^T(k_s) v \leq \frac{\bar{\zeta}(k_s) \mathbf{1}_m^T B^T(k_s) v}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \leq \bar{\zeta}(k_s), \\
& F^T(k_s) B^T(k_s) v \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T B^T(k_s) v}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \leq \bar{\xi}(k_s), \\
& F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho) = 0, \\
& H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho) \geq \frac{\zeta(k_s) \mathbf{1}_m^T (B^T(k_s) v + \varrho)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \\
& \quad \geq \frac{\zeta(k_s) \mathbf{1}_m^T (\underline{B}^T v + \varrho)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \geq \mu_1 \underline{\zeta}(k_s), \\
& F^T(k_s) (B^T(k_s) v + \varrho) \geq \frac{\xi(k_s) \mathbf{1}_m^T (B^T(k_s) v + \varrho)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \\
& \quad \geq \frac{\xi(k_s) \mathbf{1}_m^T (\underline{B}^T v + \varrho)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \geq \mu_1 \underline{\xi}(k_s). \quad (33)
\end{aligned}$$

Together with (2), (32) becomes

$$\begin{aligned}
& x^T(k + k_s | k) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(A^T(k_s) v + \varsigma \right. \\
& \quad \left. + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s) v + H^T(k_s) D_\ell^{-T} B^T(k_s) v) \right.
\end{aligned}$$

$$\begin{aligned}
& \quad \left. + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s) v - v \right) \\
& + \tilde{x}^T(k_s) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho)) \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho)) \right. \\
& \quad \left. + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s) v + \varrho) \right) \\
& \leq x^T(k + k_s | k) \left(\bar{A}^T v \right. \\
& \quad \left. + \bar{\alpha} \bar{\zeta}(k_s) + c \bar{\xi}(k_s) - Q_3 \underline{\zeta}(k_s) - Q_4 \underline{\xi}(k_s) + \varsigma - v \right).
\end{aligned}$$

By (15b), $x^T(k + k_s | k) (\bar{A}^T v + \bar{\alpha} \bar{\zeta}(k_s) + c \bar{\xi}(k_s) - Q_3 \underline{\zeta}(k_s) - Q_4 \underline{\xi}(k_s) + \varsigma - v) < 0$. Together with (28), the condition (11) holds.

Case II: $D_\ell = I$. Using a similar method to (33),

$$F^T(k_s) D_\ell^T B^T(k_s) v = F^T(k_s) B^T(k_s) v \leq \bar{\xi}(k_s),$$

$$H^T(k_s) D_\ell^{-T} B^T(k_s) v = 0,$$

$$F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho) = F^T(k_s) (B^T(k_s) v + \varrho) \geq \mu_1 \underline{\xi}(k_s),$$

$$H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho) = 0,$$

$$F^T(k_s) (B^T(k_s) v + \varrho) \geq \mu_1 \underline{\xi}(k_s).$$

Together with (2), (29) is transformed into

$$\begin{aligned}
& x^T(k + k_s | k) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(A^T(k_s) v + \varsigma + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s) v \right. \\
& \quad \left. + H^T(k_s) D_\ell^{-T} B^T(k_s) v) + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s) v - v \right) \\
& + \tilde{x}^T(k_s) \sum_{\ell=1}^{2m} \tilde{h}_\ell \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho)) + H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho) \right. \\
& \quad \left. + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s) v + \varrho) \right) \leq x^T(k + k_s | k) \left(\bar{A}^T v + (\bar{\alpha} + c) \bar{\xi}(k_s) \right. \\
& \quad \left. - (Q_3 + Q_4) \underline{\xi}(k_s) + \varsigma - v \right).
\end{aligned}$$

By (15c), $x^T(k + k_s | k) (\bar{A}^T v + (\bar{\alpha} + c) \bar{\xi}(k_s) - (Q_3 + Q_4) \underline{\xi}(k_s) + \varsigma - v) < 0$. Together with (28), the condition (11) holds.

Case III: $D_\ell \neq 0$ and $D_\ell \neq I$. By (15f)–(15h),

$$F^T(k_s) D_\ell^T B^T(k_s) v \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T D_\ell^T B^T(k_s) v}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \leq \mu_2 \bar{\xi}(k_s),$$

$$H^T(k_s) D_\ell^{-T} B^T(k_s) v \leq \frac{\bar{\zeta}(k_s) \mathbf{1}_m^T D_\ell^{-T} B^T(k_s) v}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \leq \mu_2 \bar{\zeta}(k_s),$$

$$F^T(k_s) B^T(k_s) v \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T B^T(k_s) v}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \leq \bar{\xi}(k_s),$$

$$\begin{aligned}
F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho) & \geq \frac{\xi(k_s) \mathbf{1}_m^T D_\ell^T (B^T(k_s) v + \varrho)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \\
& \geq \mu_1 \underline{\xi}(k_s),
\end{aligned}$$

$$H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho) \geq \frac{\zeta(k_s) \mathbf{1}_m^T D_\ell^{-T} (B^T(k_s) v + \varrho)}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)}$$

$$\begin{aligned}
&\geq \mu_1 \underline{\xi}(k_s), \\
F^T(k_s)(B^T(k_s)v + \varrho) &\geq \frac{\underline{\xi}(k_s) \mathbf{1}_m^T (B^T(k_s)v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\
&\geq \frac{\underline{\xi}(k_s) \mathbf{1}_m^T (\widehat{B}^T v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \geq \mu_1 \underline{\xi}(k_s).
\end{aligned}$$

Together with (2), (29) is transformed into

$$\begin{aligned}
&x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \bar{h}_\ell \left(A^T(k_s)v + \varsigma + \bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v \right. \\
&\quad \left. + H^T(k_s)D_\ell^{-T} B^T(k_s)v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v - v \right) \\
&\quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \bar{h}_\ell \left(\bar{\alpha}(F^T(k_s)D_\ell^T (B^T(k_s)v + \varrho) + H^T(k_s)D_\ell^{-T} (B^T(k_s)v + \varrho)) \right. \\
&\quad \left. + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)(B^T(k_s)v + \varrho) \right) \leq x^T(k + k_s|k) \left(\bar{A}^T v + (\bar{\alpha}\mu_2 + c)\bar{\xi}(k_s) \right. \\
&\quad \left. + \bar{\alpha}\mu_2\bar{\xi}(k_s) - Q_3\bar{\xi}(k_s) - (Q_3 + Q_4)\bar{\xi}(k_s) + \varsigma - v \right).
\end{aligned}$$

By (15d), $x^T(k + k_s|k)(\bar{A}^T v + (\bar{\alpha}\mu_2 + c)\bar{\xi}(k_s) + \bar{\alpha}\mu_2\bar{\xi}(k_s) - Q_3\bar{\xi}(k_s) - (Q_3 + Q_4)\bar{\xi}(k_s) + \varsigma - v) < 0$. Thus, the condition (11) holds. The performance index (8) can be obtained by implementing the optimization:

$$\min \gamma(k) \quad \text{s.t. (15) and (14)}. \quad (34)$$

For the initial state in Φ , we have $x^T(k)v \leq 1$ by (11), that is, $x(k) \in \Phi, \forall k \in \mathbb{N}$. This implies that the set Φ is the domain of attraction of the system (1). By (15e) and (15i), $v \geq -\frac{\xi^{(i)}(k)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)}$. Then, $-x^T(k)H_i^T(k) = -\frac{x^T(k)\xi^{(i)}(k)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \leq x^T(k)v \leq 1$, which implies that $\Phi \subseteq \Psi(H_i)$.

(b) *Polynomial uncertainty* By (18a) and (18e), it follows that

$$\begin{aligned}
&A(k_0) + B(k_0)(D_\ell F(k_0) + D_\ell^{-T} H(k_0))(I + \mathbf{1}_n \epsilon^T) \\
&= \sum_{p=1}^L \lambda_p \left(A^{(p)} + B^{(p)} \left(D_\ell \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \right. \right. \\
&\quad \left. \left. + D_\ell^{-T} \frac{\sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \right) (I + \mathbf{1}_n \epsilon^T) \right) \\
&= \frac{1}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \sum_{p=1}^L \lambda_p \left(\mathbf{1}_m^T (\widehat{B}^T v + \varrho) A^{(p)} \right. \\
&\quad \left. + B^{(p)} \left(D_\ell \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0) \right. \right. \\
&\quad \left. \left. + D_\ell^{-T} \sum_{i=1}^m \mathbf{1}_m^{(i)} \xi^{(i)T}(k_0) \right) (I + \mathbf{1}_n \epsilon^T) \right) \\
&\geq 0.
\end{aligned}$$

Then, the following inequality holds:

$$\begin{aligned}
&A(k_0) + \beta(k_0)B(k_0)F(k_0)(I + \mathbf{1}_n \epsilon^T) \\
&\geq A(k_0) + B(k_0)F(k_0)(I + \mathbf{1}_n \epsilon^T) \\
&\geq A(k_0) + B(k_0)(D_\ell F(k_0) + D_\ell^{-T} H(k_0))(I + \mathbf{1}_n \epsilon^T) \\
&\geq 0.
\end{aligned}$$

Using (24) gives $x(k_0 + 1|k_0) \geq 0$. Therefore, the polytopic system (1) is positive.

Using a similar method to (29)–(32), we have

$$\begin{aligned}
&x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \bar{h}_\ell \left(A^T(k_s)v + \varsigma \right. \\
&\quad \left. + \bar{\alpha}(F^T(k_s)D_\ell^T B^T(k_s)v + H^T(k_s)D_\ell^{-T} B^T(k_s)v) \right. \\
&\quad \left. + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^T(k_s)v - v \right) \\
&\quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \bar{h}_\ell \left(\bar{\alpha}(F^T(k_s)D_\ell^T (B^T(k_s)v + \varrho) \right. \\
&\quad \left. + H^T(k_s)D_\ell^{-T} (B^T(k_s)v + \varrho)) \right. \\
&\quad \left. + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)(B^T(k_s)v + \varrho) \right) \\
&\leq x^T(k + k_s|k) \sum_{p=1}^L \sum_{\ell=1}^{2^m} \bar{h}_\ell \left((A^{(p)})^T v \right. \\
&\quad \left. + \varsigma + \bar{\alpha}(F^T(k_s)D_\ell^T B^{(p)T} v \right. \\
&\quad \left. + H^T(k_s)D_\ell^{-T} B^{(p)T} v) + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)B^{(p)T} v - v \right) \\
&\quad - \epsilon \mathbf{1}_n^T (\bar{\alpha}(F^T(k_s)D_\ell^T (B^{(p)T} v + \varrho) \\
&\quad \left. + H^T(k_s)D_\ell^{-T} (B^{(p)T} v + \varrho)) \right. \\
&\quad \left. + (1 - \bar{\alpha})\bar{\beta}F^T(k_s)(B^{(p)T} v + \varrho)) \right). \quad (35)
\end{aligned}$$

In order to transform (35) into a tractable condition, we consider D_ℓ via three cases.

Case I: $D_\ell = 0$. From (3), (18f), (18h), (19), (20), we obtain

$$\begin{aligned}
&F^T(k_s)D_\ell^T B^{(p)T} v = 0, \\
&H^T(k_s)D_\ell^{-T} B^{(p)T} v \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T B^{(p)T} v}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \leq \bar{\xi}(k_s), \\
&F^T(k_s)B^{(p)T} v \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T B^{(p)T} v}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \leq \bar{\xi}(k_s), \\
&F^T(k_s)D_\ell^T (B^{(p)T} v + \varrho) = 0, \\
&H^T(k_s)D_\ell^{-T} (B^{(p)T} v + \varrho) \geq \frac{\xi(k_s) \mathbf{1}_m^T (B^{(p)T} v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\
&\quad \geq \mu_1 \xi(k_s), \\
&F^T(k_s)(B^{(p)T} v + \varrho) \geq \frac{\xi(k_s) \mathbf{1}_m^T (B^{(p)T} v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)}
\end{aligned}$$

$$\geq \mu_1 \underline{\xi}(k_s). \quad (36)$$

Together with (3), (35) becomes

$$\begin{aligned} & x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(A^T(k_s)v + \varsigma \right. \\ & \quad + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s)v + H^T(k_s) D_\ell^{-T} B^T(k_s)v) \\ & \quad + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s)v - v) \\ & \quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s)v + \varrho)) \right. \\ & \quad + H^T(k_s) D_\ell^{-T} (B^T(k_s)v + \varrho)) \\ & \quad + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s)v + \varrho) \Big) \\ & \leq x^T(k + k_s|k) \sum_{p=1}^L \left(A^{(p)T} v \right. \\ & \quad + \bar{\alpha} \bar{\zeta}(k_s) + c \bar{\xi}(k_s) \\ & \quad \left. - Q_3 \underline{\zeta}(k_s) - Q_4 \underline{\xi}(k_s) + \varsigma - v \right). \end{aligned}$$

By (18b), $x^T(k + k_s|k) \sum_{p=1}^L (A^{(p)T} v + \bar{\alpha} \bar{\zeta}(k_s) + c \bar{\xi}(k_s) - Q_3 \underline{\zeta}(k_s) - Q_4 \underline{\xi}(k_s) + \varsigma - v) < 0$. Together with (28), the condition (11) holds.

Case II: $D_\ell = I$. Using a similar method to (36),

$$\begin{aligned} & F^T(k_s) D_\ell^T B^{(p)T} v = F^T(k_s) B^{(p)T} v \leq \bar{\xi}(k_s), \\ & H^T(k_s) D_\ell^{-T} B^{(p)T} v = 0, \\ & F^T(k_s) D_\ell^T (B^{(p)T} v + \varrho) = F^T(k_s) (B^{(p)T} v + \varrho) \geq \mu_1 \underline{\xi}(k_s), \\ & H^T(k_s) D_\ell^{-T} (B^{(p)T} v + \varrho) = 0, \\ & F^T(k_s) (B^{(p)T} v + \varrho) \geq \mu_1 \underline{\xi}(k_s). \end{aligned}$$

Together with (3), (35) is transformed into

$$\begin{aligned} & x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(A^T(k_s)v + \varsigma \right. \\ & \quad + \bar{\alpha} (F^T(k_s) D_\ell^T B^T(k_s)v + H^T(k_s) D_\ell^{-T} B^T(k_s)v) \\ & \quad + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s)v - v) \\ & \quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s)v + \varrho)) \right. \\ & \quad + H^T(k_s) D_\ell^{-T} (B^T(k_s)v + \varrho)) \\ & \quad + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s)v + \varrho) \Big) \\ & \leq x^T(k + k_s|k) \sum_{p=1}^L \left(A^{(p)T} v \right. \\ & \quad \left. + (\bar{\alpha} + c) \bar{\xi}(k_s) \right) \end{aligned}$$

$$- (Q_3 + Q_4) \underline{\xi}(k_s) + \varsigma - v).$$

By (18c), $x^T(k + k_s|k) \sum_{p=1}^L (A^{(p)T} v + (\bar{\alpha} + c) \bar{\xi}(k_s) - (Q_3 + Q_4) \underline{\xi}(k_s) + \varsigma - v) < 0$. Together with (35), the condition (11) holds.

Case III: $D_\ell \neq 0$ and $D_\ell \neq I$. By (18f)-(18h),

$$\begin{aligned} & F^T(k_s) D_\ell^T B^{(p)T} v \\ & \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T D_\ell^T B^{(p)T} v}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \leq \mu_2 \bar{\xi}(k_s), \\ & H^T(k_s) D_\ell^{-T} B^{(p)T} v \\ & \leq \frac{\bar{\zeta}(k_s) \mathbf{1}_m^T D_\ell^{-T} B^{(p)T} v}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \leq \mu_2 \bar{\zeta}(k_s), \\ & F^T(k_s) B^{(p)T} v \\ & \leq \frac{\bar{\xi}(k_s) \mathbf{1}_m^T B^{(p)T} v}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \leq \bar{\xi}(k_s), \\ & F^T(k_s) D_\ell^T (B^{(p)T} v + \varrho) \\ & \geq \frac{\underline{\xi}(k_s) \mathbf{1}_m^T D_\ell^T (B^{(p)T} v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \geq \mu_1 \underline{\xi}(k_s), \\ & H^T(k_s) D_\ell^{-T} (B^{(p)T} v + \varrho) \\ & \geq \frac{\underline{\zeta}(k_s) \mathbf{1}_m^T D_\ell^{-T} (B^{(p)T} v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \geq \mu_1 \underline{\zeta}(k_s), \\ & F^T(k_s) (B^{(p)T} v + \varrho) \\ & \geq \frac{\underline{\xi}(k_s) \mathbf{1}_m^T (B^{(p)T} v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \geq \frac{\underline{\xi}(k_s) \mathbf{1}_m^T (\widehat{B}^T v + \varrho)}{\mathbf{1}_m^T (\widehat{B}^T v + \varrho)} \\ & \geq \mu_1 \underline{\xi}(k_s). \end{aligned}$$

By (3), (35) is transformed into

$$\begin{aligned} & x^T(k + k_s|k) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \\ & \quad \left(A^T(k_s)v + \varsigma + \bar{\alpha} \right. \\ & \quad \left(F^T(k_s) D_\ell^T B^T(k_s)v + H^T(k_s) D_\ell^{-T} B^T(k_s)v) \right. \\ & \quad \left. + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) B^T(k_s)v - v \right) \\ & \quad + \tilde{x}^T(k_s) \sum_{\ell=1}^{2^m} \tilde{h}_\ell \end{aligned}$$

$$\begin{aligned}
& \left(\bar{\alpha} (F^T(k_s) D_\ell^T (B^T(k_s) v + \varrho) \right. \\
& \quad + H^T(k_s) D_\ell^{-T} (B^T(k_s) v + \varrho)) \\
& \quad \left. + (1 - \bar{\alpha}) \bar{\beta} F^T(k_s) (B^T(k_s) v + \varrho) \right) \\
& \leq x^T(k + k_s | k) \\
& \quad \left(A^{(p)T} v + (\bar{\alpha} \mu_2 + c) \bar{\xi}(k_s) \right. \\
& \quad \left. + \bar{\alpha} \mu_2 \bar{\xi}(k_s) - Q_3 \underline{\xi}(k_s) \right. \\
& \quad \left. - (Q_3 + Q_4) \underline{\xi}(k_s) + \varsigma - v \right).
\end{aligned}$$

By (18d), $x^T(k + k_s | k)(A^{(p)T} v + (\bar{\alpha} \mu_2 + c) \bar{\xi}(k_s) + \bar{\alpha} \mu_2 \bar{\xi}(k_s) - Q_3 \underline{\xi}(k_s) - (Q_3 + Q_4) \underline{\xi}(k_s) + \varsigma - v) < 0$. Together with (35), the condition (11) holds. The rest of the proof can be obtained using a similar method to that in the interval uncertainty case and is omitted. This completes the proof. $\square$

Remark 4 In the traditional MPC design, the optimization algorithm is implemented at each sample instant to compute the control input. Such a kind of computation process requires high computation efficiency of the algorithm. It is known that the algorithm will grow in size and complexity over many iterations with the increase of the dimension of the systems. In some cases, the algorithm is delayed to the step length of the sample. It directly leads to the failure of the MPC algorithm. Meanwhile, the design cost is high. Indeed, the control law is not needed to be updated when the required performances of the systems are achieved or takes some trade-off between the design cost and the performances. In [20] and [21], the MPC of positive systems was first considered. Some similar design cost problems mentioned above exist in [20] and [21]. Theorem 1 introduces the event-triggered MPC strategy to positive systems. On one hand, the event-triggered strategy can reduce the design cost of the MPC controller. On the other hand, the event-triggered MPC can reduce the risk of the control failure owing to the lower efficiency of the computation. The event-triggered MPC provides a trade-off framework between the performances of the systems and the event-triggering condition.

Remark 5 The controller gain (16) and the gain of attraction domain (17) are designed based a matrix decomposition approach. In detail, an $n \times n$ matrix is decomposed into the sum of n sub-matrices in which

the elements in $n - 1$ rows are all zero. Thus, these sub-matrices can be given by the product of the vector $\mathbf{1}_n^{(i)}$ and a variable vector, where $i = 1, 2, \dots, n$. The objective of the matrix decomposition-based design approach is to easily transform the corresponding conditions into linear programming. A similar approach is used for (19) and (20). This approach is different from the traditional linear matrix inequalities-based MPC design [23, 24, 35, 37]. It has also been shown in existing literature [19, 20, 22, 44–46] that the linear programming approach is more effective in coping with the positivity of positive systems than linear matrix inequalities.

Remark 6 For the control design of interval uncertain system (1), the condition (15a) is to guarantee the positivity of the lower bound system matrix of interval uncertain system (1). The conditions (15b)–(15d) are to achieve the stability of the upper bound system matrix of interval uncertain system (1). Thus, the positivity and stability of interval uncertain system (1) are reached. An indirect design approach is utilized in the control design of interval uncertain system (1). For polytopic uncertain system (1), the condition (18a) and the conditions (18b)–(18d) are to achieve the positivity and stability of each vertex system of polytopic uncertain system (1). Furthermore, the gains in (16) and (17) are also different from the gains in (19) and (20).

To compute the conditions (15) and (18) in terms of linear programming, the parameters μ_1 , μ_2 , and μ_3 are required to be known. How to choose these parameters is key to the computation of the MPC controller. We provide some discussions and a suggested algorithm to solve (15). The case in (18) is similar to (15).

Noting the condition (15g), it is clear that μ_2 can be found by computing the following condition:

$$\mathbf{1}_m^T D_\ell^T \underline{B}^T \geq \mu_2 \mathbf{1}_m^T \underline{B}^T, \quad D_\ell \neq 0, D_\ell \neq I. \quad (37)$$

Moreover, we obtain $\mu_2 \in (0, \bar{\mu}_2]$, where $\bar{\mu}_2 = \min_{i=1, \dots, n}$

$n, j = 1, \dots, m \frac{\sum_{j=1}^m b_{ij}}{b_{ij}}$ and b_{ij} is the i th row j th column element of the matrix $\underline{B}$. The introduction of the condition (15g) is only for convenience of derivation. From the proof in Theorem 1, the condition (15i) is used to guarantee the invariant property of the set $\Psi(H_i)$. The parameter μ_2 is only used in (15g). Considering these points, one can first find a value of μ_1 such that the conditions (15a)–(15f) are feasible, where μ_3 is replaced by zero. Then, we have $\mu_3 \in (0, \mathbf{1}_m^T (\underline{B}^T v + \varrho)]$.

Algorithm 1: Searching parameters for MPC

Input : The lower and upper bounds of the system matrices $A(k)$ and $B(k)$;
 $\bar{h}_1, \bar{h}_2, \bar{h}_3, \chi_0 \in \mathbb{N}^+, \bar{\chi} \in \mathbb{N}^+$ and other necessary parameters of the systems;
Output: μ_1, μ_2, μ_3 ;

```

1 Define a set  $\Theta$ ;
2 Initialize the model parameters  $\mu_1 = 1 + \bar{h}_1, \mu_2 = \bar{h}_2, \mu_3 = 0$ , and  $\chi_1 = 1$ ;
3 while  $\mu_3 \leq \chi_1 \bar{h}_3$  or  $\mu_3 \leq \bar{\chi} \bar{h}_3$  do
4   repeat
5     Solve linear programming (15a)-(15h);
6     repeat
7       Solve linear programming (15a)-(15h);
8       if (15a)-(15h) is feasible then
9         Save  $(\mu_1, \mu_2, \mu_3)$  to  $\Theta$ ;
10         $\chi_1 = \max(\chi_1, \lfloor \frac{\mathbf{1}_m^T (B^T v + \varrho)}{\bar{h}_3} \rfloor)$ ;
11      else
12         $\mu_1 \leftarrow \mu_1 + \bar{h}_1$ ;
13      end
14    until  $\mu_1 \geq 1 + \chi_0 \bar{h}$ ;
15    if (15a)-(15h) is unfeasible and  $\mu_1 \geq 1 + \chi_0 \bar{h}$  then
16       $\mu_2 \leftarrow \mu_2 + \bar{h}_2$ ;
17    end
18  until  $\mu_2 \geq \bar{\mu}_2$ ;
19   $\mu_3 \leftarrow \mu_3 + \bar{h}_3$ ;
20 end

```

Remark 7 Algorithm 1 provides a method to choose the corresponding parameters. It should be pointed out that Algorithm 1 may be failed if the set M_1 or M_2 is empty for the step lengths $\bar{h}_1, \bar{h}_2$, and $\bar{h}_3$ as small as possible and the value of χ as big as possible. In this case, the event-triggered MPC control in Theorem 1 is invalid. On the other hand, the optimal solutions based on Algorithm 1 are local since they are dependent on the given parameters.

From Theorem 1, it can be found that the gain of domain of attraction $H(k)$ is key to the positivity of the system (1). The gain $H(k)$ is chosen as variable and may be different at different sample instants. This increases the feasibility of the conditions in Theorem 1. Indeed, if the conditions in Theorem 1 is feasible at the initial sample time instant k_0 , then the positivity of the system (1) is valid for all sample time instants by choosing $H(k) = H(k_0) \forall k \geq k_0$. Theorem 1 presents a reliable approach to the computation of $H(k)$.

3.3 Handling constraints

In this subsection, we will handle the constraints on the systems.

Theorem 2 (Handling constraints)

(a) *Interval uncertainty* If there exist constants $\mu_1 > 1, 0 < \mu_2 < 1, \mu_3 > 0, \gamma(k) > 0, \varpi > 0$ and $\mathfrak{N}^n$ vectors $v \succ 0; \underline{\xi}(k) \prec 0, \bar{\xi}^{(i)}(k), \bar{\xi}(k) \prec 0; \underline{\zeta}(k), \zeta^{(i)}(k), \bar{\zeta}(k) \prec 0$ such that (14), (15), and

$$v \geq \varpi \mathbf{1}_n, \quad (38a)$$

$$\gamma(k) \leq \delta \varpi, \quad (38b)$$

$$\eta \mathbf{1}_m^T (\underline{B}^T v + \varrho) \mathbf{1}_n + \delta \sum_{i=1}^m \bar{\xi}^{(i)}(k) \mathbf{1}_m^{(i)T} \mathbf{1}_m \geq 0, \quad (38c)$$

hold $\forall \ell = 1, \dots, 2^m$ and $i = 1, \dots, m$, then the constraints in (7) are handled under the control law (5) with (16) and the gain of attraction domain (17).

(b) *Polytopic uncertainty* If there exist constants $\mu_1 > 1, 0 < \mu_2 < 1, \mu_3 > 0, \gamma(k) > 0, \varpi > 0$ and $\mathfrak{N}^n$ vectors $v \succ 0; \underline{\xi}(k), \bar{\xi}^{(i)}(k), \bar{\xi}(k) \prec 0; \underline{\zeta}(k), \zeta^{(i)}(k), \bar{\zeta}(k) \prec 0$ such that (18), (14), and

$$v \geq \varpi \mathbf{1}_n, \quad (39a)$$

$$\gamma(k) \leq \delta \varpi, \quad (39b)$$

$$\eta \mathbf{1}_m^T (\widehat{B}^T v + \varrho) \mathbf{1}_n + \delta \sum_{i=1}^m \bar{\xi}^{(i)}(k) \mathbf{1}_m^{(i)T} \mathbf{1}_m \geq 0, \quad (39c)$$

hold $\forall \ell = 1, \dots, 2^m, i = 1, \dots, m$, and $p = 1, \dots, L$, then the constraints in (7) are handled under the control law (5) with (19) and (20).

Proof (a) *Interval uncertainty.* From (11) and (14), we obtain $x^T(k + k_s + 1|k)v \leq x^T(k + k_s|k)v \leq \dots \leq x^T(k|k)v \leq \gamma(k)$. Using (38a) and (38b) gives $\varpi x^T(k + k_s + 1|k)\mathbf{1}_n \leq x^T(k + k_s + 1|k)v \leq \gamma(k) \leq \delta \varpi$, which implies that the constraint (7a) is handled. By (38c),

$$\eta \mathbf{1}_n + \delta \frac{\sum_{i=1}^m \bar{\xi}^{(i)}(k) \mathbf{1}_m^{(i)T}}{\mathbf{1}_m^T (\underline{B}^T v + \varrho)} \mathbf{1}_m \geq 0. \quad (40)$$

Using (16), it is clear that $-\delta F^T(k_s)\mathbf{1}_m \leq \eta \mathbf{1}_n$. Thus, $-\delta x^T(k + k_s|k)F^T(k_s)\mathbf{1}_m \leq \eta x^T(k + k_s|k)\mathbf{1}_n$, that is, $-u^T(k + k_s|k)\mathbf{1}_m \leq \frac{\eta}{\delta} x^T(k + k_s|k)\mathbf{1}_n$. By (36), we have $-u^T(k + k_s|k)\mathbf{1}_m = \|u(k + k_s|k)\|_1 \leq \frac{\eta}{\delta} x^T(k + k_s|k)\mathbf{1}_n \leq \eta$, which implies that the constraint (7b) is satisfied.

- (b) *Polytopic uncertainty* The proof is similar to that in (a) and omitted. $\square$

3.4 Stability analysis

In this subsection, we first present the invariant set of the systems. Then, an optimization algorithm is given to compute the event-triggered MPC control law. Moreover, the feasibility of the optimization design is provided. Finally, the robust stability of the systems is addressed.

Lemma 4 *The set $\Phi = \{x(k) \in \mathbb{R}_+^n | x^T(k)v \leq 1\}$ is the invariant set of the system (1) provided the conditions in Theorem 1 hold.*

Under Theorem 1, the condition (11) is satisfied. Thus, Lemma 4 is valid. For a system without saturation, the invariant set is key to the feasibility of the predictive algorithm. For the system (1) with saturation, the invariant set is first used to achieve the validity of Lemma 3 as well as the feasibility of the algorithm.

Algorithm 2 *Step 1* : Implement Algorithm 1 to find the set Θ . Choose one element in Θ and implement the optimization at the initial sample time instant $k_0 = 0$:

$$\min \gamma(k) \quad \text{s.t. (15), (14) and} \quad (41) \quad \text{(s.t. (18), (14) and (39)).} \quad (41)$$

Step 2 : Judge whether the event-triggering condition (6) is satisfied. If the condition (6) is not satisfied at the sample instants $k = 1, 2, \dots, k_1 - 1$, then the predictive control law $u(k) = u(k_0) = F(k_0)x(k_0|k_0)$. Otherwise, implement (41) at the event-triggering time instant k_1 to compute the predictive control law $u(k_1) = F(k_1)x(k_1|k_0)$.

Step 3 : Repeat Step 2 for all event-triggering time instants $k_2, k_3, \dots, \aleph$, where $\aleph$ is the last event-triggering time instants in the predictive step length.

Step 4 : Implement the control law $u(k) = F(\aleph)x(k|k_0)$ for $k > \aleph$.

Theorem 3 *Suppose that Algorithm 2 is feasible at the initial sample time instant k_0 , then it is also feasible for all $k_s > k_0$, where k_s represents the s th event-triggering time instant. Moreover, the event-triggered MPC control law (16) with the gain of domain of attraction (17) [(19) with the gain of attraction domain (20)] can guarantee the robustly stability of the interval system (1) [the polytopic system (1)].*

Proof Algorithm 2 is feasible at the initial sample time instant k_0 , that is, the conditions (15), (14), and (38) are valid. Denote the optimization solution by

$$\begin{aligned} \prod(k_0) = \{ & \mu_1, \mu_2, \mu_3, \gamma^*(k_0), \varpi, v, \underline{\xi}^*(k_0), \\ & \xi^{(i)*}(k_0), \bar{\xi}^*(k_0), \underline{\zeta}^*(k_0), \\ & \zeta^{(i)*}(k_0), \bar{\zeta}^*(k_0) \}. \end{aligned} \quad (42)$$

At the $k_0 + 1$ th sample time instant, choose

$$\begin{aligned} \prod(k_0 + 1) = \{ & \mu_1, \mu_2, \mu_3, \gamma(k_0 + 1) = \gamma^*(k_0), \varpi, v, \\ & \underline{\xi}(k_0 + 1) = \underline{\xi}^*(k_0), \xi^{(i)}(k_0 + 1) = \xi^{(i)*}(k_0), \\ & \bar{\xi}(k_0 + 1) = \bar{\xi}^*(k_0), \\ & \underline{\zeta}(k_0 + 1) = \underline{\zeta}^*(k_0), \\ & \zeta^{(i)}(k_0 + 1) = \zeta^{(i)*}(k_0), \\ & \bar{\zeta}(k_0 + 1) = \bar{\zeta}^*(k_0) \}. \end{aligned} \quad (43)$$

It is easy to obtain that (15) and (38) are valid under $\prod(k_0 + 1)$. Noting the condition (11), we have $x^T(k_0 + 1|k_0)v \leq x^T(k_0 + 1|k_0)v \leq \gamma^*(k_0) = \gamma(k_0 + 1)$. Thus, the condition (14) holds. This implies that Algorithm 2 is feasible at $k_0 + 1$. Repeat the process, we have that Algorithm 2 is feasible at $k_1 - 1$. Choose a feasible set $\prod(k_1 - 1)$ at $k_1 - 1$. It is not hard to verify the validity of (15), (14), and (38). By a recursive deduction, the feasibility of Algorithm 2 is reached. For the polytopic system (1), the proof of the feasibility is similar and omitted.

By Theorem 1 and the feasibility of Algorithm 2, the Lyapunov function $V(i, k)$ is strictly decreasing with respect to i . Thus, the system (1) is robustly stable. $\square$

From the proof of Theorem 3, the robust stability condition (11) plays a key role for the feasibility of Algorithm 2 and the stability of the system (1). The solutions constructed in (43) are given by virtue of (41). Therefore, it is easy to guarantee the feasibility of the conditions (15) and (38). However, the condition (14) is dependent the system state $x(k)$. Under the condition (11), the condition (14) is valid.

Remark 8 This paper provides a novel event-triggered MPC framework for positive systems with saturation. From the viewpoint of the MPC, the event-triggering strategy is more practical than the time-triggering one [19,20]. From the viewpoint of positive systems, the randomly occurring saturation is more general than the

certain saturation case in [44–46]. The framework can be further developed for the event-triggered distributed MPC of positive systems, the event-triggered MPC of stochastic positive systems, and other similar issues of positive.

This paper attempts to develop the event-triggered control strategy for the MPC of positive systems. There are still open issues though a linear event-triggered MPC framework is constructed in the paper. Algorithm 1 provides a suggested method for choosing the parameters μ_1 , μ_2 , and μ_3 . However, it is not the best way to choose these parameters. A significant improvement is to present a new design to remove the parameters restriction. This issue is interesting in the further work. Meanwhile, all existing results on the MPC of positive systems are concerned with linear systems. There are no results on the MPC of nonlinear positive systems though some attention has been paid to the same topic of nonlinear systems (non-positive) [38, 51, 52]. The nonlinearity of positive systems is essentially difficult to be explored since there are no tractable methods to define the positivity of a nonlinear system [46]. The MPC framework in this paper and the results on nonlinear positive systems in [46] provide a possibility to further investigate the MPC of nonlinear positive systems.

4 Illustrative example

SEIR (susceptible-exposed-infected-removed) is one of the most fundamental models of epidemics [53]. It describes the evolution of four classes of individuals in a certain zone, namely the susceptible S , capable of contracting the disease and becoming infectious; the asymptomatic E and symptomatic I infectious, capable of transmitting the disease to susceptibilities; and the recovered R , permanently immune (after healing or dying). Such a simple model represents well a generic behavior of epidemics, and a related advantage consists in a small number of parameters to identify. This is an important outcome in the case of a virus attack with a limited amount of data available. In [54], a modified SEIR discrete-time model was proposed, where it has been used to model the epidemics trend of COVID-19 in China. The base model was proposed as follows:

$$S(k+1) = S(k) - b \frac{p_c I(k) + r(k) E(k)}{N} S(k), \quad (44a)$$

$$E(k+1) = (1 - \sigma) E(k) + b \frac{p_c I(k) + r(k) E(k)}{N} S(k), \quad (44b)$$

$$I(k+1) = (1 - \chi) I(k) + \sigma E(k), \quad (44c)$$

$$R(k+1) = R(k) + \chi I(k), \quad (44d)$$

where $k \in \mathbb{N}$ (the set of nonnegative integers) is the time counted in days ($k = 0$ corresponds to the beginning of measurements or prediction), N denotes the total population, the parameter $0 < \chi < +\infty$ represents the mortality and recovery rates, the parameter $0 < b < +\infty$ corresponds to the infection rate of the virus transmission from infectious to susceptibilities, $0 < \sigma < +\infty$ is the incubation rate by which the exposed develop symptoms; $0 < p_c < +\infty$ corresponds to the number of contacts for the infectious I , $p_c \leq r(k) < +\infty$ is the number of contacts per person per day for the exposed population E . It is not hard to know that the system (44) is positive since the number of four classes individuals is nonnegative. For the corresponding analysis and synthesis, one must employ a positive system approach.

Noting the fact that $R(k)$ in (44d) is only dependent on that $I(k)$, $R(k)$ is easy to be obtained when $I(k)$ is known. Therefore, we only consider (44a)–(44c). It is clear that $0 < b \frac{p_c I(k) + r(k) E(k)}{N} < 1$. In [53] and [54], some identification methods were used to obtain the values of parameters b , p_c , $r(k)$, σ , χ . Here, we do not employ any identification methods to estimate those parameters and only assume that these parameters satisfy some certain polytopic uncertainty conditions. It is also necessary to point out that the exposed and infectious population will affect the susceptible population, the symptomatic persons will affect the asymptomatic persons, and the susceptible persons can become the symptomatic persons. Based on these points, the SEIR model (44) is further modified as:

$$\begin{pmatrix} S(k+1) \\ E(k+1) \\ I(k+1) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} S(k) \\ E(k) \\ I(k) \end{pmatrix}, \quad (45)$$

where $a_{11} = 1 - b \frac{p_c I(k) + r(k) E(k)}{N}$, $a_{22} = 1 - \sigma$, $a_{32} = \sigma$, $a_{33} = 1 - \chi$ and a_{12} , a_{13} , a_{21} , a_{23} , a_{31} are unknown nonnegative weighted coefficients. In (45), there is no any external control strategy imposed on the system. The system (45) is a predicted model to estimate the population of four classes of individuals. The estimation is only the first step in the SEIR research. A more

important issue is how to contain the spread of epidemic. Therefore, it is necessary to provide some effective control strategies to achieve the expected objective. Thus, a control input $u(k)$ is introduced to the system (45). At present, quarantine is one of available strategies in the absence of specific drugs and vaccines. From the viewpoint of control theory, quarantine is to move the people from the infection zone to a safe zone or restrict their behaviors. In such a case, the control input should be non-positive, that is, $u(k) \leq 0$. Noting the control design in Theorem 1, it can be found that the theoretical design in Theorem 1 meets the desire of quarantine.

Considering the people's daily need and normal life, complete quarantine is not a good idea, especially, when the epidemic situation is under control. A balance between the epidemic and social development is limited quarantine. Limited quarantine means that the quarantine strategy is dependent on the epidemic situation. When the epidemic situation is serious, a corresponding strict quarantine is implemented. Contrarily, the quarantine strategy is unchanged. Limited quarantine is a better strategy for containing the infectious and can promote social development. It is also clear that limited quarantine follows the philosophy of the event-triggered control stated in this paper.

In reality, the quarantine strategy is likely to meet some problems. For instance, some individuals who need to be quarantined are not settled in a sudden epidemic outbreak. Such a sudden event holds the random property. Meanwhile, the lack of people's implement ability is exposed in a sudden event. This may be owing to limited human and material resources allocation in some regions or because people are not well prepared for the epidemic. Anyhow, the limited implement ability is just a class of saturation phenomena. These imply that random actuator saturation can be employed to model the mentioned practical processes.

By these analysis above, it is reasonable to use the system (1) with polytopic uncertainty to re-construct SEIR for epidemics, where

$$A^{(1)} = \begin{pmatrix} 0.42 & 0.37 & 0.29 \\ 0.29 & 0.38 & 0.33 \\ 0.30 & 0.24 & 0.39 \end{pmatrix}, \quad A^{(2)} = \begin{pmatrix} 0.42 & 0.32 & 0.22 \\ 0.30 & 0.39 & 0.37 \\ 0.30 & 0.24 & 0.45 \end{pmatrix},$$

$$B^{(1)} = \begin{pmatrix} 0.25 & 0.29 \\ 0.26 & 0.28 \\ 0.29 & 0.27 \end{pmatrix}, \quad B^{(2)} = \begin{pmatrix} 0.29 & 0.31 \\ 0.28 & 0.28 \\ 0.26 & 0.28 \end{pmatrix},$$

$$\bar{\alpha} = 0.40, \bar{\beta} = 0.85,$$

$$\epsilon = (0.05 \ 0.08 \ 0.11)^T, \delta = 3.8 \times 10^7, \eta = 9 \times 10^7.$$

Give $\varrho = (-0.0010 \ -0.0010 \ -0.0010)^T$, $\zeta = (0.0010 \ 0.0010)^T$, and $x(0) = (6 \times 10^7 \ 9 \times 10^7 \ 2 \times 10^4)^T$. The corresponding parameters are provided only from a numerical aspect and they are not from some existing official data. For example, it is assumed from numerical point of view that there are 6×10^7 susceptible population, 9×10^7 exposed population, and 2×10^4 infectious population at some zone at some time instant. By Algorithm 1, we can obtain a set feasible parameters $\mu_1 = 3$, $\mu_2 = 0.1$, and $\mu_3 = 0.3$. Implement Algorithm 2 via one iteration, we have

$$v = \begin{pmatrix} 6.8000 \\ 6.8000 \\ 8.0508 \end{pmatrix} \times 10^8, \quad \underline{\xi} = \begin{pmatrix} -1.7796 \times 10^8 \\ -1.3453 \times 10^8 \\ -0.0040 \end{pmatrix},$$

$$\xi^{(1)} = \begin{pmatrix} -1.7796 \times 10^8 \\ -1.3453 \times 10^8 \\ -0.0030 \end{pmatrix}, \quad \xi^{(2)} = \begin{pmatrix} -1.7796 \times 10^8 \\ -1.3453 \times 10^8 \\ -0.0020 \end{pmatrix},$$

$$\bar{\xi} = \begin{pmatrix} -1.7796 \times 10^8 \\ -1.3453 \times 10^8 \\ -0.0010 \end{pmatrix}, \quad \underline{\zeta} = \begin{pmatrix} -0.0040 \\ -0.0040 \\ -0.0040 \end{pmatrix},$$

$$\zeta^{(1)} = \begin{pmatrix} -0.0030 \\ -0.0020 \\ -0.0030 \end{pmatrix}, \quad \zeta^{(2)} = \begin{pmatrix} -0.0020 \\ -0.0030 \\ -0.0020 \end{pmatrix},$$

$$\bar{\zeta} = \begin{pmatrix} -0.0010 \\ -0.0010 \\ -0.0010 \end{pmatrix}, \quad \varpi = 6.8000.$$

Then, the controller gain and the attract domain gain are

$$F = \begin{pmatrix} -0.1551 & -0.1172 & -2.6144 \times 10^{-12} \\ -0.1551 & -0.1172 & -1.7429 \times 10^{-12} \end{pmatrix},$$

$$H = \begin{pmatrix} -2.6144 & -1.7429 & -2.6144 \\ -1.7429 & -2.6144 & -1.7429 \end{pmatrix} \times 10^{-12}.$$

Let $x(k) = (x_1(k) \ x_2(k) \ x_3(k))^T$, $x'(k) = (x'_1(k) \ x'_2(k) \ x'_3(k))^T$, and $x''(k) = (x''_1(k) \ x''_2(k) \ x''_3(k))^T$ be the states via one, two, and three MPC algorithm iterations, respectively. For each iteration case, implement Algorithm 2 for 100 times and then compute the average values of the states. The sample period is half a day. Figures 1, 2 and 3 show the simulation results of the states under different iterations, Figs. 4 and 5 are the simulations of the control input under different iterations, Fig. 6 shows the event-triggering time instants and intervals, and Fig. 7 gives the domains of attraction with one iteration under different initial conditions. Table 1 shows the event-triggering times under different parameters ϵ .

Fig. 1 Simulations of the states $x_1(k)$, $x'_1(k)$, and $x''_1(k)$

Fig. 2 Simulations of the states $x_2(k)$, $x'_2(k)$, and $x''_2(k)$

Fig. 3 Simulations of the states $x_3(k)$, $x'_3(k)$, and $x''_3(k)$

Fig. 4 Simulations of the inputs $u_1(k)$, $u'_1(k)$, and $u''_1(k)$

Fig. 5 Simulations of the inputs $u_2(k)$, $u_2'(k)$, and $u_2''(k)$

Fig. 6 Event-triggered time instants and intervals

Fig. 7 Attraction domains with different initial conditions

Table 1 Event-triggering times under different parameters ϵ

issues	1	2	3	4	5	6	7	8	9	10
ϵ_1	0.02	0.02	0.02	0.03	0.03	0.03	0.06	0.09	0.11	0.09
ϵ_2	0.04	0.09	0.13	0.04	0.04	0.04	0.04	0.04	0.04	0.09
ϵ_3	0.06	0.06	0.06	0.06	0.09	0.14	0.03	0.03	0.03	0.09
Triggered times	65	47	45	62	54	48	62	52	48	74

From Figs. 1, 2 and 3, it is shown that the converge effect of the states get better with the increase of the iteration times and the infectious case is contained in an acceptable scope at the thirty-fifth sampling time instant. In Figs. 4 and 5, it is shown that a quarantine strategy is implemented for less than 10^7 persons each day at the most serious situation. For a zone with more than 9×10^7 , such a scope of quarantine is reasonable and realizable. In additional, the event-triggered control is suitable for a sudden epidemic situation. On the one hand, such a kind of control strategy is implemented based on the practical case of disease development. On the other hand, it is not easy to continuously update the quarantine strategy each day. From Fig. 6, the number of event-triggering instants is 57. It means that the changing times of quarantine strategy are 57 in 100 sampling instants. Such a quarantine strategy is better than continuous changing one. In practice, an obstacle is how to obtain accurate data of $I(k)$ since there exist some unmeasured, unknown, and undiscovered infectious persons.

In the first nine sets of data in Table 1, we change the values of ϵ_2 , ϵ_3 , and ϵ_1 , respectively. The event-triggering times are reduced with the increase in the corresponding values. Noting the event-triggering condition (6), it is also clear that the condition may not be easily satisfied when the value of ϵ is big. In such a case, the event-triggering times are reduced. However, if the value of ϵ is enough big, the condition (6) may also be easily satisfied or Algorithm 2 cannot get feasible solutions. Thus, the corresponding event-triggering times are increased or the algorithm is ended. The last set of data verifies the former statement. Furthermore, one chooses the values of ϵ_2 , ϵ_3 , and ϵ_1 that are bigger than 0.09, then Algorithm 2 is unfeasible.

5 Conclusion

This paper has proposed an event-triggered MPC control approach to positive systems with randomly occurring actuator saturation. A 1-norm-based event-triggering condition was introduced for the systems. An interval estimate approach was used to transform the error term into the state term. Then, the original system is transformed into an interval system. By virtue of linear programming, an MPC controller with attraction domain gain is designed for the systems. The positivity and stability of the original systems are achieved

by considering the positivity of the lower bound of the interval system and the stability of the upper bound of the interval system, respectively. The obtained MPC approach is supported with a modified SEIR model.

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Declaration

Conflict of interest The authors declare that they have no conflict of interest.

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