Convolution-based time-domain simulation for fluidelastic instability in tube arrays

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Abstract A convolution-based numerical algorithm is presented for the time-domain analysis of fluidelastic instability in tube arrays, emphasizing in detail some key numerical issues involved in the time-domain simulation. The unit-step and unit-impulse response functions, as two elementary building blocks for the time-domain analysis, are interpreted systematically. An amplitude-dependent unit-step or unit-impulse response function is introduced to capture the main features of the nonlinear fluidelastic (FE) forces. Connections of these elementary functions with conventional frequency-domain unsteady FE force coefficients are discussed to facilitate the identification of model parameters. Due to the lack of a reliable method to directly identify the unit-step or unit-impulse response function, the response function is indirectly identified based on the unsteady FE force coefficients. However, the transient feature captured by the indirectly identified response function may not be consistent with the physical fluid-memory effects. A recursive function is derived for FE force simulation to reduce the computational cost of the convolution operation. Numerical examples of two tube arrays,

containing both a single flexible tube and multiple flexible tubes, are provided to validate the fidelity of the time-domain simulation. It is proven that the present time-domain simulation can achieve the same level of accuracy as the frequency-domain simulation based on the unsteady FE force coefficients. The convolution-based time-domain simulation can be used to more accurately evaluate the integrity of tube arrays by considering various nonlinear effects and non-uniform flow conditions. However, the indirectly identified unit-step or unit-impulse response function may fail to capture the underlying discontinuity in the stability curve due to the prespecified expression for fluid-memory effects.

Keywords Tube array · Fluidelastic instability (FEI) · Convolution · Unit-step response function · Unit-impulse response function · Nonlinear fluidelastic (FE) force

Nomenclature

a_j, b_j	Parameters of unit-step response function
A	Vibration amplitude
c_d	Dimensionless fluidelastic (FE) damping coefficient
c_j, d_j	Parameters of unit-impulse response function
c_k	Dimensionless FE stiffness coefficient
C_D	Static drag coefficient

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C_L	Static lift coefficient
D	Tube diameter
f	Tube vibration frequency in flow
f_0	Tube natural frequency in vacuum
F	FE force
F_x	FE force in streamwise direction
F_y	FE force in cross-flow direction
i	Complex unit
I	Unit-impulse response function
$K = 2\pi fD/U$	Reduced frequency
U	
m	Tube mass + fluid added mass
P	Tube pitch
$s = Ut/D$	Dimensionless time
t	Time
U	Pitch flow velocity
U_{cr}	Critical pitch flow velocity
$U_r = U/K$	Reduce pitch flow velocity
fD	
x	Streamwise tube displacement
$X = x/D$	Dimensionless streamwise tube displacement
y	Cross-flow tube displacement
$Y = y/D$	Dimensionless cross-flow tube displacement
β	Nonlinear FE damping coefficient
$\delta(s)$	Dirac delta function
ρ	Fluid mass density
ξ	Tube damping ratio in flow
ξ_0	Tube damping ratio in vacuum
ϕ	Unit-step response function

1 Introduction

Long, slender tube arrays in heat exchangers are susceptible to various flow-induced vibrations, among which fluidelastic instability (FEI) has the greatest potential to induce short-term structural damage [1]. Due to the extremely destructive nature of this instability, considerable efforts have been made in this area in the last few decades [2–8]. It is now well-known that there exist two distinct instability mechanisms, i.e. fluidelastic (FE) damping-controlled and FE stiffness-controlled mechanisms. Excellent reviews on this topic have been provided by several authors [9–12].

There is an increasing interest in advancing the time-domain analysis of FEI to more effectively account for any nonlinear effects (e.g. due to loosened supports, initial axial load, and FE force) and non-uniform flow conditions [13–16]. Both quasi-static and quasi-steady models have been widely used in time-domain analysis [17, 18]. The basic assumption of a quasi-static model is that the FE force on a vibrating cylinder is equal to that on a stationary cylinder whose configuration is identical to the actual instantaneous configuration of the vibrating cylinder [19]. In contrast, a quasi-steady model assumes that the FE force on a vibrating cylinder is equal to that on a stationary cylinder whose configuration is identical to the actual instantaneous configuration modified by the instantaneous velocity [19–21]. Therefore, the FE forces of both models are determined by the instantaneous flow status, while any potential fluid-memory (unsteady) effects are neglected. The fluid-memory effects may significantly affect the energy transformation between a tube and its surrounding flow, and hence, both quasi-static and quasi-steady models are unable to predict FEI with satisfactory accuracy [12]. Although the quasi-steady model is often modified with a time delay, it is well-accepted that the time delay cannot fully capture the fluid-memory effects [22–24]. The unsteady model, which assumes that the FE forces on a vibrating tube are functions of its displacement, velocity, and acceleration, can satisfactorily capture the fluid-memory effects, while the model is expressed in a mixed time–frequency domain [25]. Detailed introduction and comparison of these theories can be found in, e.g. [12, 19, 26].

To effectively capture the fluid-memory effects in the time domain, the complex relationships between tube displacement/velocity and FE force can be simulated by a superposition of scaled and time-shifted fundamental responses, i.e. convolution. The unit-step response function and unit-impulse response function, as two fundamental functions in convolution-based time-domain analysis, have been widely used in the aerodynamics of plate-like structures, e.g. airfoils and bridge decks. Analytical solutions of the unit-step and unit-impulse response functions were made available for a thin airfoil [27]. However, for a bridge deck, a theoretical treatment of the unit-step or unit-impulse response function remains intractable due to complicated flow separation. As a result, the unit-step or unit-impulse response function

of a bridge deck is often identified either experimentally through wind tunnel tests or numerically through computational fluid dynamics-based simulations. Both the direct method, in which the bridge deck is excited with a smoothed unit-step or unit-impulse displacement/velocity, and the indirect method, in which the function is numerically fitted according to the inverse Fourier transform of the frequency-dependent aerodynamic transfer functions, have been advanced [28–31].

The application of a convolution-based time-domain simulation in flow-induced vibration analysis of tube arrays is very limited. Convolution was first introduced to simulate the FE forces of tube arrays by Granger and Paidoussis [22]. They modified the time-delayed term in a quasi-steady model with a unit-step response function-based convolution. The modified model (referred to as a quasi-unsteady model) showed a clearly improved predictive capability compared with the quasi-steady model. Using a vortex wake model, Meskell [23] showed that the unit-step response function can be approximated numerically without using experimental data. However, Granger and Paidoussis [22] and Meskell [23] both focused on the FEI of linear tube arrays in uniform flow, and hence, they converted the model into the frequency domain to determine the critical flow velocity. To the best of the authors' knowledge, the only convolution-based time-domain analysis was conducted recently by Piteau [32], in which the linear FE force on a single flexible tube in a rigid tube array was simulated by a unit-step response function-based approach. Granger and Paidoussis [22] and Piteau et al. [32] identified the unit-step response function based on the vibration frequencies and damping ratios obtained in a free vibration test. Meskell [23] deduced the response function using a vortex wake model. These identification methods should be limited to a tube array with a single flexible tube.

This paper presents a systematic interpretation of the unit-step response function and unit-impulse response function with a specific reference to the simulation of FE forces of tube arrays. Different from previous applications [22, 23, 32], the unit-step or unit-impulse response functions in this paper are determined based on the full unsteady FE force coefficients obtained from the forced vibration tests of Tanaka and Takahara [25]. It is assumed that the effects of each tube in an array on the FE forces of a

tube can be summed linearly. Hence, the response functions determined based on FE force coefficients are applicable for an array with multiple flexible tubes. The connections of these elementary functions with frequency-domain unsteady FE force coefficients are introduced to facilitate the identification of model parameters. Some key numerical issues involved in the time-domain simulation, e.g. the integral of fluid-memory terms and the transient behaviour, are discussed. Amplitude-dependent unit-step and unit-impulse response functions are introduced to capture the main features of the nonlinear FE forces. Numerical examples of two tube arrays, containing both a single flexible tube and multiple flexible tubes, are provided to validate the fidelity of the time-domain simulation.

The subsequent parts of this paper are organized as follows: theoretical backgrounds of the unit-step and unit-impulse response functions, as well as their connections with frequency-domain unsteady FE force coefficients, are introduced in Sect. 2; the amplitude-dependent unit-step and unit-impulse response functions are introduced in Sect. 3; the indirect identification of unit-step and unit-impulse response functions are discussed in Sect. 4; a recursive function is deduced for FE force simulation in Sect. 5 to reduce the computational cost of the convolution operation; results of the numerical examples are provided in Sect. 6; the main conclusions are summarized in Sect. 7.

2 Unit-step and unit-impulse response functions for FE force

2.1 Unit-step response function

Considering an array of static tubes immersed in a two-dimensional flow, the flow-induced lift force acting on a tube can be expressed as

$$F = \frac{1}{2} \rho U^2 D C_L \quad (1)$$

where ρ is the fluid mass density; U is the pitch flow velocity; D is the diameter of the tube; and C_L is the dimensionless lift coefficient of the static tube.

If one of the tubes is disturbed by a unit-step motion in the cross-flow direction, the resulting lift force on

the disturbed tube will exhibit a transient change described by

$$F(s) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \phi(s) \quad (2)$$

where $\partial C_L / \partial Y$ is the derivative of the static lift coefficient with respect to the dimensionless displacement $Y = y/D$ in the cross-flow direction; y is the cross-flow tube displacement; ϕ is the unit-step response function; and s is the dimensionless time.

Inspired by the Jones approximation [33] of the analytical unit-step response function (i.e. the Wagner function [27]) of a thin airfoil, the following general expression is often employed in a case without an analytical solution:

$$\phi(s) = 1 - \sum_{j=1}^n a_j e^{-b_j s} \quad (3)$$

where a_j and b_j ($j = 1 \sim n$) are constants to be identified and $b_j > 0$; n is the order of the unit-step response function.

Equations (2) and (3) indicate that in the case of the step motion, the lift force would, instead of entering a new steady state immediately, undergo a process of transient growth due to the fluid-memory effects.

The extension of Eq. (2) to a general tube motion, starting from rest at $s = 0$, can be obtained in terms of the well-known Duhamel's integral

$$F(s) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \int_0^s \phi(s - \sigma) Y'(\sigma) d\sigma \quad (4)$$

or equivalently, according to an integration by parts and a change of variables

$$F(s) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \left[\phi(0) Y(s) + \int_0^s \phi'(\tau) Y(s - \tau) d\tau \right] \quad (5)$$

where prime represents the derivative with respect to dimensionless time s .

Once the unit-step response function for a tube is available, the FE force induced by an arbitrary motion can be determined according to Eqs. (4) or (5).

Attempts have been made to directly measure the unit-step response function of a separated bluff body (e.g. a rectangular cylinder or a bridge deck) in wind tunnels or based on computational fluid dynamics simulations [28–30]. However, the direct identification method requires further improvement before it

can become a routine identification technique. On the other hand, it is more practical to measure the unsteady FE force coefficients in the frequency domain, as pioneered by Tanaka et al. [25, 34] and Chen et al. [26, 35]. Therefore, a promising indirect method is to fit an equivalent unit-step response function based on the inverse Fourier transform of the measured unsteady FE force coefficients. To this end, Eq. (5) is transformed into the frequency domain as

$$\bar{F}(K) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \left[\phi(0) + \bar{\phi}'(K) \right] \bar{Y} \quad (6)$$

where the overbar represents the Fourier transform operator; $K = 2\pi f D / U$ is the reduced frequency; and f is the tube vibration frequency in flow.

Noting that the Fourier transform of the exponential term $e^{-b_j s}$ in $\phi(s)$ is $1/(b_j + iK)$, the bracketed terms in Eq. (6) can be expressed as

$$\begin{aligned} \phi(0) + \bar{\phi}'(K) &= 1 - \sum_{j=1}^n a_j + \sum_{j=1}^n \frac{a_j b_j^2}{b_j^2 + K^2} \\ &\quad - iK \sum_{j=1}^n \frac{a_j b_j}{b_j^2 + K^2} \end{aligned} \quad (7)$$

On the other hand, the unsteady FE force on a tube is conventionally expressed in a mixed time–frequency domain as [25, 34]

$$F(t) = \frac{1}{2} \rho U^2 D c_k(K) Y(t) + \frac{1}{2} \rho U^2 D c_d(K) \dot{Y}(t) \quad (8)$$

or in a dimensionless form

$$F(s) = \frac{1}{2} \rho U^2 D [c_k(K) Y(s) + c_d(K) Y'(s)] \quad (9)$$

where c_k and c_d are the dimensionless FE stiffness and damping coefficients, respectively; the overdot represents the derivative with respect to time t .

The equivalent expression of Eq. (9) in the frequency domain can be expressed as

$$\bar{F}(K) = \frac{1}{2} \rho U^2 D [c_k(K) + iK c_d(K)] \bar{Y} \quad (10)$$

Comparing Eqs. (6) and (10), the relationships between the Fourier transform of the unit-step response function and the unsteady FE force coefficients may be given by

$$c_k(K) = \frac{\partial C_L}{\partial Y} \left(1 - \sum_{j=1}^n \frac{a_j K^2}{b_j^2 + K^2} \right) \quad (11a)$$

$$c_d(K) = \frac{\partial C_L}{\partial Y} \left(- \sum_{j=1}^n \frac{a_j b_j}{b_j^2 + K^2} \right) \quad (11b)$$

It is then possible to identify the values of a_j and b_j based on c_k and c_d through least squares fit. Details of the identification procedure will be discussed later in Sect. 4.

For the in-line square tube array shown in Fig. 1, the FE forces on tube O are influenced by the vibrations of tube O and its surrounding tubes. To reduce the complexity of the FE force model, it is often assumed that only the effects of tubes R, L, U, and D need to be considered, and the effect of each tube can be summed linearly [25, 26, 34, 35]. Following the variables defined by Tanaka et al. [25, 34], the FE forces on tube O can be expressed as

$$F_y(s) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y_0} \sum_{j=O,R,L,D,U} \left[\int_0^s \phi_{YjY}(s-\sigma) Y_j'(\sigma) d\sigma + \int_0^s \phi_{YjX}(s-\sigma) X_j'(\sigma) d\sigma \right] \quad (12a)$$

$$F_x(s) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y_0} \sum_{j=O,R,L,D,U} \left[\int_0^s \phi_{XjX}(s-\sigma) X_j'(\sigma) d\sigma + \int_0^s \phi_{XjY}(s-\sigma) Y_j'(\sigma) d\sigma \right] \quad (12b)$$

where $j = O, R, L, D$, and U denote the five tubes shown in Fig. 1; X is the dimensionless tube displacement in the streamwise direction; ϕ_{YjY} , ϕ_{YjX} , ϕ_{XjX} , and ϕ_{XjY} are unit-step response functions, in which the

Fig. 1 Configuration of an in-line square tube array

first, second, and third suffixes represent the direction of the force, the position of the vibrating tube, and the direction of the tube motion, respectively. As an example, ϕ_{YRX} represents the unit-step response function related to the y -direction force induced by the x -direction vibration of the right tube (R).

2.2 Unit-impulse response function

Considering again a static tube array immersed in a two-dimensional flow, if one of the tubes is disturbed by a unit-impulse motion in the cross-flow direction, the resulting lift will exhibit a transient change described by

$$F(s) = \frac{1}{2} \rho U^2 D I(s) \quad (13)$$

where I is the unit-impulse response function.

The FE force after a unit-impulse motion will exhibit a transient change from the force determined by the original steady position of the tube and finally fade out to the same value. Therefore, the unit-impulse response function can be divided into a steady part and an unsteady part

$$F(s) = \frac{1}{2} \rho U^2 D \{ [I(s)]_s + [I(s)]_{us} \} \quad (14)$$

where $[I(s)]_s = \frac{\partial C_L}{\partial Y} \delta(s)$ is the steady part, with $\delta(s)$ representing the Dirac delta function; the unsteady part, $[I(s)]_{us}$, converges to zero with increasing time.

The FE force on a tube due to an arbitrary motion can be expressed in terms of the unit-impulse response function as

$$F(s) = \frac{1}{2} \rho U^2 D \int_0^s I(s-\sigma) Y(\sigma) d\sigma \quad (15)$$

To identify the unit-impulse response function based on the unsteady FE force coefficients, Eq. (15) can be transformed into a frequency-domain expression given by

$$\bar{F}(K) = \frac{1}{2} \rho U^2 D \bar{I}(K) \bar{Y} \quad (16)$$

The steady part can be expressed in the frequency domain as

$$[\bar{I}(K)]_s = \frac{\partial C_L}{\partial Y} \quad (17)$$

The frequency-domain expression of the unsteady part can be approximated by the well-known Roger's rational functions [36]

$$[\bar{I}(K)]_{us} = \sum_{j=1}^n c_j \frac{iK}{d_j + iK} \quad (18)$$

Combining Eqs. (17) and (18), the frequency-domain expression of the unit-impulse response function is obtained as

$$\begin{aligned} \bar{I}(K) &= \frac{\partial C_L}{\partial Y} + \sum_{j=1}^n c_j \frac{iK}{d_j + iK} \\ &= \frac{\partial C_L}{\partial Y} + \sum_{j=1}^n \frac{c_j K^2}{d_j^2 + K^2} + iK \sum_{j=1}^n \frac{c_j d_j}{d_j^2 + K^2} \end{aligned} \quad (19)$$

The equivalent time-domain expression of Eq. (19) is

$$I(s) = \frac{\partial C_L}{\partial Y} \delta(s) + \sum_{j=1}^n c_j [\delta(s) - d_j e^{-d_j s}] \quad (20)$$

Comparing Eqs. (10) and (19), the relationships between the Fourier transform of the unit-impulse response function and the unsteady FE force coefficients are given by

$$c_k(K) = \frac{\partial C_L}{\partial Y} + \sum_{j=1}^n \frac{c_j K^2}{d_j^2 + K^2} \quad (21a)$$

$$c_d(K) = \sum_{j=1}^n \frac{c_j d_j}{d_j^2 + K^2} \quad (21b)$$

It is obvious from Eqs. (8), (11), and (21) that $c_j = -a_j \frac{\partial C_L}{\partial Y}$ and $d_j = b_j$. Furthermore, Eqs. (5) and (15) indicate that the unit-impulse response function $I(s)$ is the derivative of the unit-step response function $\phi(s)$ with respect to dimensionless time s , with the exception of $s = 0$. Hence, the FE forces expressed in terms of unit-step and unit-impulse response functions are actually equivalent. This is exactly true as long as these functions are indirectly derived based on the frequency-domain unsteady force coefficients. However, when a direct identification method is utilized, a unit-impulse response function may be preferred over a unit-step response function [37]. If the surrounding tubes are positioned asymmetrically, the FE force due to a sudden upward unit-step motion may differ from that due to a sudden downward unit-step motion. In

contrast, the unit-impulse response function should be able to capture the effects of this asymmetry since the tube is first disturbed in the upward direction and then downward to its original position.

It is well-known that the FE force can be well-approximated by the quasi-steady theory if the reduced flow velocity $U_r = U/fD$ is sufficiently high (or equivalently, if the reduced frequency K is sufficiently low). According to the quasi-steady theory [38], the FE force can be given as

$$F(s) = \frac{1}{2} \rho U^2 D \left[\frac{\partial C_L}{\partial Y} Y(s) - C_D Y'(s) \right] \quad (22)$$

where C_D is the static drag coefficient.

According to Eqs. (9) and (22), it is obvious that as K approaches zero, c_k and c_d should approach $\frac{\partial C_L}{\partial Y}$ and $-C_D$, respectively. To ensure that the simulation of the unit-step or unit-impulse response function is identical to the quasi-steady theory as K approaches zero, it is necessary to constrain $\sum_{j=1}^n \frac{a_j}{b_j} = \frac{C_D}{\partial C_L / \partial Y}$ or $\sum_{j=1}^n \frac{c_j}{d_j} = -C_D$, respectively.

For the flexible tube array shown in Fig. 1, the FE forces on tube O can be expressed as

$$F_y(s) = \frac{1}{2} \rho U^2 D \sum_{j=O,R,L,D,U} \left[\int_0^s I_{yY}(s-\sigma) Y_j(\sigma) d\sigma + \int_0^s I_{yX}(s-\sigma) X_j(\sigma) d\sigma \right] \quad (23a)$$

$$F_x(s) = \frac{1}{2} \rho U^2 D \sum_{j=O,R,L,D,U} \left[\int_0^s I_{xX}(s-\sigma) X_j(\sigma) d\sigma + \int_0^s I_{xY}(s-\sigma) Y_j(\sigma) d\sigma \right] \quad (23b)$$

where I_{yY} , I_{yX} , I_{xX} , and I_{xY} are unit-step response functions. The suffixes of I are the same as those of the unit-step response function ϕ .

3 Consideration of nonlinear FE force

The FE forces are essentially nonlinear functions of tube responses due to the complicated interactions between the fluid and the tubes. The nonlinear FE forces may result in limit cycle oscillations after the critical flow velocity for FEI [39] and/or lead to undesired large amplitude vibrations before the critical velocity (in a hysteresis region) [40]. Furthermore, the effects of nonlinear FE forces may be nonnegligible in evaluating the fretting wear or fatigue life of a tube array [41]. To achieve a more reliable structural

dynamic analysis, there is a rising demand to consider nonlinear FE forces. Some attempts have been made to consider the nonlinear FE effects utilizing the quasi-steady model [39] and the semi-analytical flow channel model [40, 41]. However, these models should be used with caution since they include assumptions that might be unrealistic.

The unsteady model can also be modified with higher-order nonlinear damping and/or stiffness terms to consider the nonlinear FE forces in the frequency domain [42, 43]. As an example, Meskell and Eret [42] introduced a cubic damping term to simulate the postcritical limit cycle oscillations of a flexible tube surrounded by a rigid normal triangular tube array:

$$F(t) = \frac{1}{2}\rho U^2 D c_k(K) Y(t) + \frac{1}{2}\rho U D^2 [c_d(K) \dot{Y}(t) + \beta(K) \dot{Y}^3(t)] \quad (24)$$

where β is a nonlinear FE force coefficient.

To consider the nonlinear effects in the time domain, the Volterra theory [44] provides a promising choice as a direct extension of the linear convolution. However, the identification of higher-order kernels in a Volterra-based model is challenging, and the computational cost increases exponentially with increasing model order. On the other hand, it is known that both the postcritical limit cycle oscillation and hysteresis behaviour of a FE system are essentially related to the amplitude dependency of FE forces [45, 46]. Inada et al. [47] highlighted the amplitude dependency of frequency-domain FE force coefficients for an in-line square tube array and further proposed predicting postcritical vibrations with amplitude-dependent FE force coefficients. Accordingly, it might be beneficial to develop an amplitude-dependent unit-step or unit-impulse response function to capture the main features of the nonlinear FE forces in the time domain. For example, the relationship between the amplitude-dependent unit-step response function $\phi(A, s)$ and frequency-domain FE force coefficients are given by

$$c_k(K, A) = \frac{\partial C_L}{\partial Y} \left[1 - \sum_{j=1}^n \frac{a_j(A) K^2}{b_j^2(A) + K^2} \right] \quad (25a)$$

$$c_d(K, A) = \frac{\partial C_L}{\partial Y} \left[- \sum_{j=1}^n \frac{a_j(A) b_j(A)}{b_j^2(A) + K^2} \right] \quad (25b)$$

where A is the vibration amplitude. The amplitude-dependent FE force coefficients, $c_k(K, A)$ and $c_d(K, A)$, can be identified based either on the displacement signals of free vibration experiments [42], or on harmonic forced vibration experiments with various vibration amplitudes [43].

For a flexible tube array, the nonlinear FE forces on a tube, e.g. tube O in Fig. 1, are then approximated as

$$F_y(s) = \frac{1}{2}\rho U^2 D \frac{\partial C_L}{\partial Y_0} \sum_{j=0,R,L,D,U} \left[\int_0^s \phi_{yy}(A_0, s-\sigma) Y_j'(\sigma) d\sigma + \int_0^s \phi_{yx}(A_0, s-\sigma) X_j'(\sigma) d\sigma \right] \quad (26a)$$

$$F_x(s) = \frac{1}{2}\rho U^2 D \frac{\partial C_L}{\partial Y_0} \sum_{j=0,R,L,D,U} \left[\int_0^s \phi_{xx}(A_0, s-\sigma) X_j'(\sigma) d\sigma + \int_0^s \phi_{xy}(A_0, s-\sigma) Y_j'(\sigma) d\sigma \right] \quad (26b)$$

A practical implementation of Eq. (26) is shown in Sect. 6. The FE force can also be expressed in terms of amplitude-dependent unit-impulse response functions, which is not repeated for brevity. It is noted that the physical meanings of the nonlinear amplitude-dependent unit step/impulse response functions are uncertain. These functions are merely time-domain equivalents of the amplitude-dependent, unsteady FE force coefficients in the considered range of reduced velocities. Despite the lack of a sound physical meaning, it will be shown that the amplitude-dependent unit step/impulse response function is capable of capturing the main nonlinear feature (i.e. amplitude-dependency) of the unsteady FE forces. Since the amplitude-dependent unit step/impulse response function is identified based on the unsteady FE force coefficients (obtained from forced vibration data), the present nonlinear model is believed to be more accurate than the quasi-steady model in calculating the large-amplitude vibrations in the post-instability regime.

4 Identification of unit-step and unit-impulse response functions

The values of a_j and b_j (or c_j and d_j) can be obtained through nonlinear least squares fit based on Eq. (11) (or Eq. (21)). As an example, according to Eq. (11), the values of a_j and b_j can be obtained by minimizing the following error function

$$\Gamma = \sum_{i=1}^Z \left\{ w_1 \left[c_k - \frac{\partial C_L}{\partial Y} \left(1 - \sum_{j=1}^n \frac{a_j K_z^2}{b_j^2 + K_z^2} \right) \right]^2 + w_2 \left[c_d - \frac{\partial C_L}{\partial Y} \left(- \sum_{j=1}^n \frac{a_j b_j}{b_j^2 + K_z^2} \right) \right]^2 \right\} \quad (27)$$

where Z represents the sampling number of the unsteady FE force coefficients involved in the identification; w_1 and w_2 are the weight coefficients of the FE stiffness and damping parts, respectively.

On the other hand, it might be more reasonable to calculate these values to minimize the fitting errors between the forces simulated by the unsteady force coefficients (Eq. (9)) and the forces simulated by the convolutions (Eqs. (4) or (15)). As noted by Piteau [32], the apparently large errors of the fitted c_d and c_k at a low reduced flow velocity do not introduce significant errors in the calculated FE responses. Indeed, according to Eq. (9), the vibration frequency and damping ratio of the tube at any reduced frequency can be deduced as

$$f(K) = f_0 \sqrt{\frac{m}{\rho D^3 c_k(K)/2K^2 + m}} \quad (28a)$$

$$\xi(K) = \xi_0 - \frac{\rho f(K) D^3 c_d(K)}{4m f_0 K} \quad (28b)$$

where m is the tube mass plus the fluid added mass per unit tube length; f_0 is the tube natural frequency in a vacuum; ξ_0 is the tube damping ratio in a vacuum; and ξ is the tube damping ratio in a flow.

It is then obvious that the FE stiffness and FE damping forces are proportional to c_k/K^2 and c_d/K , respectively. Hence, the parameters of the unit-step response function can be obtained by minimizing the following error function

$$\Gamma = \sum_{i=1}^Z \left\{ w_1 \left[\frac{c_k}{K_z^2} - \frac{\partial C_L}{\partial Y} \left(\frac{1}{K_z^2} - \sum_{j=1}^n \frac{a_j}{b_j^2 + K_z^2} \right) \right]^2 + w_2 \left[\frac{c_d}{K_z} - \frac{\partial C_L}{\partial Y} \left(- \sum_{j=1}^n \frac{1}{K_z} \frac{a_j b_j}{b_j^2 + K_z^2} \right) \right]^2 \right\} \quad (29a)$$

Similarly, the parameters of the unit-impulse response function can be obtained by minimizing

$$\Gamma = \sum_{i=1}^Z \left\{ w_1 \left[\frac{c_k}{K_z^2} - \frac{\partial C_L}{\partial Y} \left(\frac{1}{K_z^2} - \sum_{j=1}^n \frac{c_j}{d_j^2 + K_z^2} \right) \right]^2 + w_2 \left[\frac{c_d}{K_z} - \sum_{j=1}^n \frac{1}{K_z} \frac{c_j d_j}{d_j^2 + K_z^2} \right]^2 \right\} \quad (29b)$$

As presented in Eq. (29a) (or Eq. (29b)), both a_j (and c_j), located in the numerators, and b_j (or d_j), located in the denominators, are unknown parameters to be identified. Conventionally, this nonlinear fitting

problem involves selecting a group of initial values and an iterative numerical procedure to find a local optimal solution. The results of such methods are sensitive to the initial values, and sometimes it might be necessary to repeat the whole process by starting with different groups of initial values.

To this end, the nonlinear fitting problem can be degenerated to a linear fitting problem by searching b_j (or d_j) within a predefined range. The identification procedure of the unit-step response function is illustrated in Fig. 2 as an example. For specific values of b_j , the optimal values of a_j can be conveniently identified through a linear fitting. Hence, the nonlinear fitting is linearized during each searching step, and the procedure leads to the optimal solution in the searching range ($b_{\min}$, $b_{\max}$) rather than a local optimal solution that is often obtained in a nonlinear fitting. The unit-impulse response function can be identified with a similar procedure, which is not repeated for brevity.

It is important to note that the unit-step or unit-impulse response function identified based on the unsteady FE force coefficients may not physically represent the true properties of the fluid–structure interaction. Hence, the function is not expected to be applicable outside the range of reduced velocities corresponding to the original experimental data.

5 Integral of fluid-memory terms

A time-domain analysis of FEI entails a step-by-step process in which Eqs. (4) or (15) is employed to simulate the FE force. In both expressions, there are fluid-memory terms related to the whole time-history of the tube motion, which may be extremely time-consuming in terms of the convolution integration. The significant computational cost might prevent the practical application of the time-domain simulation in cases of three-dimensional FE analyses of flexible tube arrays. Fortunately, the computational cost can be largely reduced if the fluid-memory terms are transformed into recursive functions. Taking the unit-step response function as an example, Eq. (4) can be reformulated as

Fig. 2 Flow diagram of unit-step response function identification

$$\begin{aligned}
 F(s) &= \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \int_0^s \phi(s-\sigma) Y'(\sigma) d\sigma \\
 &= \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \int_0^s \left(1 - \sum_{j=1}^n a_j e^{-b_j(s-\sigma)} \right) Y'(\sigma) d\sigma \\
 &= \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} Y(s) - \sum_{j=1}^n A_j(s)
 \end{aligned} \quad (30)$$

where

$$A_j(s) = \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \int_0^s a_j e^{-b_j(s-\sigma)} Y'(\sigma) d\sigma \quad (31)$$

The first term on the right-hand side of the last equal sign in Eq. (30), i.e. $\frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} Y(s)$, is only dependent on the displacement of the current instance.

However, the second term, i.e. $\sum_{j=1}^n A_j(s)$, depends on the entire time history of the tube motion. A recursive function can be generated if there exists an explicit relationship between A_j at dimensionless time step $s + \Delta s$, and its value at dimensionless time step s . A_j at dimensionless time step $s + \Delta s$ can be expressed as

$$\begin{aligned}
 A_j(s + \Delta s) &= \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \int_0^{s+\Delta s} a_j e^{-b_j(s+\Delta s-\sigma)} Y'(\sigma) d\sigma \\
 &= \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} \left[\int_0^s a_j e^{-b_j(s+\Delta s-\sigma)} Y'(\sigma) d\sigma + \int_s^{s+\Delta s} a_j e^{-b_j(s+\Delta s-\sigma)} Y'(\sigma) d\sigma \right]
 \end{aligned} \quad (32)$$

Assuming the integrand of the second term on the right-hand side of the last equal sign in Eq. (32) changes linearly between each dimensionless time step, it might be obtained as

$$A_j(s + \Delta s) = e^{-b_j \Delta s} A(s) + \frac{1}{2} \rho U^2 D \frac{\partial C_L}{\partial Y} a_j e^{-b_j \Delta s} Y'(s) \Delta s \quad (33)$$

It is then possible to calculate the FE force $F(s + \Delta s)$ recursively based on $F(s)$ according to Eqs. (30) and (33). A similar recursive function can be deduced for the unit-impulse response function-based convolution, which is not repeated for brevity. It should be highlighted that to consider the amplitude dependency of FE forces, a_j and b_j should be functions of the vibration amplitude, as presented in Eq. (25). Hence, in a time-domain simulation considering nonlinear FE effects, it is necessary to update a_j and b_j at each step according to the instantaneous vibration amplitude.

6 Numerical examples

6.1 Verification of time-domain simulation

6.1.1 A linear case

A numerical example of an in-line square tube array with pitch ratio $P/D = 1.33$ is provided to verify the fidelity of convolution-based time-domain simulation. The frequency-domain FE force coefficients were measured by Tanaka et al. [25], as given in Fig. 3. The unit-step and unit-impulse response functions are fitted based on these coefficients. Second-order functions (i.e. $n = 2$) are adopted for ϕ_{yOx} , ϕ_{yLy} , ϕ_{yLx} , and ϕ_{xLy} (or I_{yOx} , I_{yLy} , I_{yLx} , and I_{xLy}), while $n = 3$ is adopted for ϕ_{xOx} and ϕ_{xLx} (or I_{xOx} and I_{xLx}) to increase the fitting accuracy. Further increasing the model order does not significantly increase the fitting accuracy. The limiting values of the searching range are $b_{\min} = 0.01$ and $b_{\max} = 4.0$. Parameters a_j and b_j of the resulting unit-step response functions are listed in Table 1. Parameters c_j and d_j of the resulting unit-impulse response functions are $c_j = -a_j \frac{\partial C_L}{\partial Y_0}$ and $d_j = b_j$. Examples of the unit-step and unit-impulse response functions are given in Figs. 4 and 5, respectively.

It is noted that both unit-step and unit-impulse response functions show slight overshooting features. The overshooting feature has also been observed for some separated bluff bodies (e.g. a rectangular cylinder or a bridge deck) [28, 30] and for a flexible tube in

a rigid triangular tube array by Li and Mureithi [24]. Generally, the overshooting phenomenon is believed to be induced by (or at least accompanied by) the flow separation from the structure. Due to the complicated interaction between fluid and structure, it is possible that the transient FE force exceeds the steady-state force, which is reflected as the overshooting phenomenon in the response function. However, it will be shown later that the overshooting of an indirectly identified response function is quite arbitrary and might simply be a result of numerical fitting.

ϕ_{yOx} and ϕ_{xLy} (or I_{yOx} and I_{xLy}) approach to the unit (or zero) after 70 and 200 dimensionless time steps, respectively. The fading time indicates that after a unit disturbance of tube O or tube L, the resulting FE forces on tube O do not become stable until the flow transports $70D$ or $200D$ downstream. The tremendously long fading time is absolutely not physical and is a result of the numerical fitting. The fading time can be reduced by searching b_j in a $(b_{\min}, b_{\max})$ range with a larger $b_{\min}$. Indeed, if a function with shorter fading time is used, the fitted c_d and c_k will approach their quasi-steady values (hence, $c_d \cdot U_r$ or $c_k \cdot U_r^2$ versus U_r will approach a linear or quadratic curve) at a lower reduced wind speed U_r . However, most of the experimental data (especially $c_{d, yLx}$) in Fig. 3 do not show such a trend, which might be due to a systematic experimental error, as discussed by Paidoussis et al. (2010).

Once the unit-step or unit-impulse response function is available, the frequency-domain FE force coefficients can be calculated based on Eqs. (11) or (21) to verify the fidelity of the indirect identification procedure discussed in Sect. 4. The recalculated FE force coefficients are presented in Fig. 3 together with the experimental measurements. The comparisons demonstrate that numerical approximations provide satisfactory results. Further increasing the function order n can produce slightly improved fittings, while the computational costs will also be increased. Hence, unit-step or unit-impulse response functions with $n = 2$ or 3 will be employed in the subsequent time-domain simulations.

The critical pitch flow velocity U_{cr} can be calculated in the frequency domain according to the FE force coefficients (through a complex eigenvalue analysis) or in the time domain according to the unit-step or unit-impulse response functions. To

Fig. 3 Experimental and fitted FE force coefficients for an in-line square tube array

Fig. 3 continued

Table 1 Unit-step response functions for an in-line square tube array

Function	a_1	a_2	a_3	b_1	b_2	b_3
ϕ_{yOy}	-4.74	5.54	0	0.09	0.10	0
ϕ_{xOx}	-3.64	-1.13	2.43	1.35	0.02	0.01
ϕ_{yLy}	0.91	0.49	0	0.01	0.06	0
ϕ_{yLx}	1.89	-0.89	0	0.02	0.08	0
ϕ_{xLy}	1.32	1.90	0	0.01	0.92	0
ϕ_{xLx}	-2.88	4.72	-3.45	2.00	0.01	0.02

Fig. 4 Unit-step response functions for an in-line square tube array **Fig. 5** Unit-impulse response functions for an in-line square tube array

compare the results of time-domain and frequency-domain simulations, stability boundaries are calculated for two configurations: a single flexible tube surrounded by rigid cylinders and a tube array with five flexible tubes in the same row. The in-vacuum

damping ratio is $\xi_0 = 0.2\%$, and the mass-damping parameter $2\pi m \xi_0 / \rho D^2$ is varied by changing the mass ratio $m / \rho D^2$. The resulting $U_{cr} / f_0 D$ are plotted versus the mass-damping parameter $2\pi m \xi_0 / \rho D^2$ in Fig. 6. The stability boundaries obtained by the unit-step and unit-response functions are identical, and hence, only one curve is shown for the time-domain simulations of each configuration. The comparisons prove that the time-domain simulations can satisfactorily calculate the critical state of FEI. It is stated that the nonlinearities (from structural and FE aspects) and nonuniform flow conditions can be implemented straightforwardly in the time-domain simulations. On the other hand, to consider these nonlinear and/or non-uniform effects in the frequency domain, an iterative process is often required, and the computational cost of iteration increases significantly as the nonlinear and/or nonuniform effects become strong [19, 32].

As noted in Fig. 6, there is a discontinuity in the stability curve of the frequency-domain simulation between $1 < 2\pi m \xi_0 / \rho D^2 < 10$. A discontinuity in the stability curve of a tube array has also been noticed by Chen [3] and Price [12]. Hence, it is believed that the discontinuity is an actual feature of the FE system. However, this discontinuity does not exist in the curve of the time-domain simulation. The inconsistency is a result of numerical fitting in the indirect identification of unit-step or unit-impulse response functions. Due to the prescribed forms of these response functions, the variations of the captured FE stiffness and damping effects versus reduced flow velocity follows the rule specified by the response functions. As a result, the captured FE stiffness and damping effects (and hence the predicted stability boundaries) are actually smoothed compared with those of the original FE force coefficients. This is a shortcoming of the time-domain simulation, which is difficult to be improved by simply increasing the order of the unit-step or unit-impulse response function. This is because the frequency-domain equivalent of a unit-step or unit-impulse function should be a continuous function of the reduced frequency (see Eqs. (11) or (21)) regardless of the function order n , which makes the response function unable to capture the discontinuity in the FE effects. For the considered tube configuration, only one stability region exists. Multiple stability regions may exist for other configurations, e.g. a tube row [48], while the capability of the convolution-based time-

Fig. 6 Stability boundaries of FEI for an in-line square tube array

Fig. 7 Pre-instability and post-instability behaviours of an in-line square tube array with $2\pi m \xi_0 / \rho D^2 = 20$: (a) vibration frequency; (b) damping ratio

domain model in capturing the multiple stability regions requires further investigations.

The fidelity of the time-domain simulation is further verified by comparing the simulated vibration frequency and damping ratio in the pre-instability and post-instability ranges. This is done for the first configuration (i.e. a single flexible tube in a rigid tube array) with a mass-damping parameter $2\pi m \xi_0 / \rho D^2 = 20$, as presented in Fig. 7. It is noted that the assumption of linear superposition no longer holds in the post-instability range due to the significantly increased vibration amplitude. The vibration frequency and damping ratio at a specific flow velocity can vary remarkably with increasing vibration amplitude [49], which cannot be captured by a linear FE force model. The comparison in Fig. 7 proves that the time-domain simulation can predict not only the

Fig. 8 Experimental and fitted FE force coefficients for a normal triangular tube array: (a) $c_{d,yOy}$; (b) $c_{k,yOy}$

critical flow velocity but also the pre-instability behaviour of the tube. The vibration frequency increases with increasing U_r due to the positive FE stiffness effects (reflected by the negative $c_{k, yOy}$ values in Fig. 3b), while the damping ratio decreases due to the negative FE damping effects (reflected by the positive $c_{d, yOy}$ values in Fig. 3a). Furthermore, the simulation can be extended into a nonlinear convolution scheme to consider the nonlinear FE force in the post-instability range, which will be discussed in the next example.

6.1.2 A nonlinear case

A numerical example of a single flexible tube in a normal triangular tube array with pitch ratio $P/D = 1.33$ is provided to prove the fidelity of time-domain simulation based on amplitude-dependent unit-step or unit-impulse response functions. Postcritical limit cycle oscillations of this configuration were experimentally measured by Meskell and Fitzpatrick [16]. Meskell and Eret [42] further reported the FE force coefficients (i.e. c_k , c_d , and β , as shown in Eq. (24)) identified from the experimental displacement responses. The amplitude-dependent FE force coefficients $c_{d, yOy}$ can be reconstructed from c_d and β according to an averaging method [50, 51], and the results at vibration amplitudes $A = 0, 0.002$, and 0.004 mm are given in Fig. 8a. Figure 8b presents the results of $c_{k, yOy}$, which are almost independent of vibration amplitude according to [16, 42].

Second-order unit-step response functions are fitted based on these coefficients. Parameters a_j and b_j of the resulting unit-step response functions are listed in Table 2. FE force coefficients fitted by the amplitude-dependent unit-step response functions are presented in Fig. 8 together with the experimental measurements. The comparisons demonstrate that amplitude-

Table 2 Unit-step response functions for a normal triangular tube array at various vibration amplitudes

A (mm)	a_1	a_2	b_1	b_2
0	152.795	− 20.565	4.000	0.021
0.001	147.306	− 20.515	4.000	0.019
0.002	132.269	− 20.391	4.000	0.014
0.003	113.355	− 20.260	4.000	0.010
0.004	15.875	− 22.413	0.540	0.010

Fig. 9 Post-critical responses of a flexible tube in a rigid normal triangular tube array

dependent unit-step response functions can satisfactorily capture the amplitude dependency of FE force coefficients.

The postcritical vibration amplitude can be calculated in the frequency domain according to the amplitude-dependent FE force coefficients (through an iterative process) or in the time domain according to the amplitude-dependent unit-step response functions. To compare the results of time-domain and frequency-domain simulations, the postcritical vibration amplitudes are calculated for this specific configuration with the following mechanical properties: $\xi_0 = 1.5\%$, $f_0 = 6.5$ Hz, and $2\pi m \xi_0 / \rho D^2 = 177.4$. In the frequency-domain simulations, the vibration amplitudes are calculated by a regular iterative procedure (Zhang et al., 2020). The resulting postcritical vibration amplitudes are plotted versus pitch flow velocity in Fig. 9. The comparisons prove that the time-domain simulation can satisfactorily calculate the postcritical response.

6.2 Transient behaviour of time-domain simulation

Since the unit-step or unit-impulse response function is identified indirectly based on the frequency-domain FE force coefficients, it should be remembered that they carry no additional information than that contained in these FE force coefficients. As a result, the shortcomings of conventional frequency-domain analysis, such as the limited applicable reduced frequency range, are remained in the time-domain analysis. In

addition, extra uncertainties can be introduced by numerical fitting. Hence, the indirectly identified unit-step or unit-impulse response function may not correctly reflect the physical fluid-memory effects in terms of both magnitude and duration. It is natural to expect that the distortion of true fluid-memory effects can be reduced if FE force coefficients in a wider reduced frequency range are employed in the identification. However, these coefficients are often only available in a limited reduced frequency range, especially if they are identified in a free vibration test based on the variations of the tube vibration frequency and damping ratio versus flow velocity [32, 52]. In such a case, significant distortion of the fluid-memory effects can be introduced by inappropriate choices of b_j or d_j , for the unit-step or unit-impulse response function, respectively.

To demonstrate the possible distortion of the fluid-memory effects captured by an indirectly identified function, FE force coefficients $c_{d, yOy}$ and $c_{k, yOy}$ of the in-line square tube array within the reduced flow velocity range $3.5 \leq U_r \leq 25$ are utilized to identify the unit-step response function. Three independent runs of the identification procedure were carried out, in which b_j values were searched within (0.08, 4), (0.001, 4), and (0, 4), respectively. The resulting unit-step response functions, denoted as ϕ_1 , ϕ_2 , and ϕ_3 , respectively, are given in Fig. 10.

Table 3 lists the a_j and b_j values of the three unit-step response functions. The FE force coefficients fitted by three unit-step response functions are shown in Fig. 11 together with the original experimental measurements. The unit-step response functions all produce reasonable fittings of the FE force

Fig. 10 Unit-step response functions identified with different searching ranges of b_j

Table 3 Unit-step response functions identified with different searching ranges of b_j

$(b_{\min}, b_{\max})$	a_1	a_2	b_1	b_2
(0.08, 4)	2.15	-1.88	0.1300	0.0900
(0.001, 4)	-18.55	18.63	0.0038	0.0078
(0, 4)	-186.67	186.75	0.0002	0.0006

Fig. 11 FE force coefficients fitted by different unit-step response functions: (a) $c_{d, yOy}$; (b) $c_{k, yOy}$

coefficients. However, the identified unit-step function is largely dependent on the selection of the search range. The fading time of the fluid-memory effects and the transient value (and hence overshooting phenomenon) are quite arbitrary and belong to the uncertainties introduced by the numerical fitting. Small values of b_j may result in a very long fading time as well as an excessively large transient value of

Fig. 12 FE forces induced by harmonic motion for $U_r = 20$

the unit-step response function, as shown by ϕ_2 and ϕ_3 in Fig. 10.

The FE force induced by a harmonic motion for $U_r = 20$ is simulated by the unit-step response functions, as presented in Fig. 12, together with that simulated by the frequency-domain FE force coefficients. It is noted that, due to the long fading time of the unit-step response functions, the FE forces simulated by ϕ_2 and ϕ_3 depart remarkably from those simulated by FE force coefficients. After a short evolution, the force simulated by ϕ_2 collapses to the frequency-domain result. The force simulated by ϕ_3 will eventually match the frequency-domain result after sufficient evolution. However, this cannot be seen in Fig. 12 since the evolution time is very long (approximately 6500 dimensionless time steps). Therefore, it is necessary to assign a reasonable range for b_j or d_j in the indirect identification of the unit-step or unit-impulse response function.

Since all three unit-step response functions in Fig. 10 are fitted based on the unsteady FE force coefficients, it is not possible to tell which of the three functions is closest to the true physical problem. Indeed, none of the three response functions are believed to be physically true. To obtain a response function that reflects the true physical problem, transient information (e.g. forced vibration data with unit-step input or amplitude-varying input) is required for the identification, which is not within the scope of this paper. In addition, the values of $b_{\min}$ and $b_{\max}$ are determined empirically in this paper. Fortunately, the accuracy of the response function (in terms of simulating the FE force) is insignificantly affected by the searching range as long as $b_{\min}$ is not too small.

7 Conclusions

This paper presents a detailed interpretation of a convolution-based numerical algorithm for the time-domain analysis of the fluidelastic instability (FEI) of tube arrays. The unit-step and unit-impulse response functions, as two elementary building blocks for the convolution-based time-domain simulation, are interpreted systematically. An amplitude-dependent unit-step or unit-impulse response function is introduced to capture the main features of the nonlinear fluidelastic (FE) forces in the time domain. Theoretically, there is a close relationship between these elementary functions and the conventional frequency-domain unsteady FE force coefficients. Due to the lack of a reliable method to directly identify the unit-step or unit-impulse response function, it is more realistic to identify the function indirectly based on the FE force coefficients. The indirectly identified unit-step or unit-impulse response function carries no additional information than that contained in the FE force coefficients, which are often only available in a limited reduced frequency range. Hence, the indirectly identified function is not physically true. However, both functions may play an effective role in time-domain FE analysis since they correctly capture the frequency spectrum of FE forces.

Numerical examples of two tube arrays, containing both a single flexible tube and multiple flexible tubes, are provided to validate the fidelity of the time-domain simulation. Numerical results prove that the time-domain simulation can achieve the same level of accuracy as the frequency-domain simulation based on the unsteady FE force coefficients. Time-domain analysis has the superiority of conveniently implementing any nonlinearities (originating from structural and FE aspects) and nonuniform flow conditions. However, the indirectly identified unit-step or unit-impulse response function can fail to capture the discontinuity in the stability curve since the variations of FE stiffness and damping versus reduced flow velocity are smoothed by numerical fitting.

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Declaration

Conflict of interest The authors declare that they have no conflict of interest.

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