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ERRATUM

Erratum to: Updated numerical integration method for stability calculation of Mathieu equation with various time delays

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The authors regret there is a negligence in Sects. 2.1 and 3 in the original publication.

The solution of Eq. (2) in Sect. 2.1 can be expressed in the integral form when $\mathbf{A}(t)$ is a constant matrix

$$\mathbf{x}(t) = \mathrm{e}^{\mathbf{A}(t)\cdot(t-t_1)}\mathbf{x}(t_1) + \sum_{j=1}^{s}\int_{t_1}^{t} \mathrm{e}^{\mathbf{A}(t)\cdot(t-\xi)}\mathbf{B}_j(\xi)\cdot\mathbf{x}(\xi-\tau_j)\mathrm{d}\xi, \quad (3)$$

where $\mathbf{x}(t_1)$ denotes the state value at $t = t_1$.

At each small subinterval $[t_{i-1}, t_i]$, Eq. (3) is represented as

$$\mathbf{x}(t) = \mathrm{e}^{\mathbf{A}(t)\cdot(t-t_{i-1})}\mathbf{x}(t_{i-1}) + \sum_{j=1}^{s}\int_{t_{i-1}}^{t} \mathrm{e}^{\mathbf{A}}(t)\cdot(t-\xi)\mathbf{B}_j(\xi) \cdot\mathbf{x}(\xi-\tau_j)\mathrm{d}\xi, \quad t\in[t_{i-1}, t_i]. \quad (4)$$

It is assumed that $\mathbf{A}(t)$ is a constant matrix on the small interval $t \in [t_{i-1}, t_i]$. For convenience, $\mathbf{A}(t)$ is replaced with $\mathbf{A}(t_i)$ as an approximation. For the corresponding discretized time points $t_i = t_1 + (i-1)h$ $(i = 1, \ldots, n+1)$, $\mathbf{x}(t_i)$ is approximated by Newton–Cotes formulas according to Ding et al. [1].

$$\mathbf{x}(t_i) = \mathrm{e}^{\mathbf{A}(t_i)\cdot(t_i-t_{i-1})}\mathbf{x}(t_{i-1}) + \sum_{j=1}^{s}\frac{t_i-t_{i-1}}{2}\Big[\mathrm{e}^{\mathbf{A}(t_i)\cdot(t_i-t_{i-1})}\mathbf{B}_j\mathbf{x}(t_{i-1}-\tau_j) + \mathbf{B}_j\mathbf{x}(t_i-\tau_j)\Big]. \quad (5)$$

Since the Newton–Cotes formula has the local truncation error $\mathcal{O}(h^3)$, the second term of right-hand side of Eq. (5) has the local truncation error $\mathcal{O}(h^3)$. Therefore, the discretization error of the proposed method is $\mathcal{O}(h^3)$, which can also be verified via the convergence of critical eigenvalues.

Equation (39) in Sect. 3 can be expressed in the integral form when $\mathbf{A}(t)$ is a constant matrix

$$\mathbf{x}(t) = \mathrm{e}^{\mathbf{A}(t)\cdot(t-t_1)}\mathbf{x}(t_1) + \int_{t_1}^{t}\mathrm{e}^{\mathbf{A}(t)\cdot(t-\xi)}\left[\int_{-\sigma}^{0}\mathbf{B}(\theta)\mathbf{x}(\xi+\theta)\mathrm{d}\theta\right]\mathrm{d}\xi. \quad (40)$$

The period interval T is also equally discretized with a time step h. At each small subinterval $[t_{i-1}, t_i]$, Eq. (40) is represented as

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$$\mathbf{x}(t) = \mathrm{e}^{\mathbf{A}(t)\cdot(t-t_{i-1})}\mathbf{x}(t_{i-1}) + \int_{t_{i-1}}^{t} \mathrm{e}^{\mathbf{A}(t)\cdot(t-\xi)} \left[\int_{-\sigma}^{0} \mathbf{B}(\theta)\mathbf{x}(\xi+\theta)\mathrm{d}\theta\right]\mathrm{d}\xi, \quad t \in [t_{i-1}, t_i]. \tag{41}$$

It is assumed that $\mathbf{A}(t)$ is a constant matrix on the small interval $t \in [t_{i-1}, t_i]$. For simplicity, $\mathbf{A}(t)$ can be replaced with $\mathbf{A}(t_i)$ as an approximation. At the corresponding discretized time points $t_i = t_1 + (i-1)h$ $(i = 1, \ldots, n+1)$, we approximate $\mathbf{x}(t_i)$ by using Newton–Cotes formulas, leading to

$$\begin{aligned} \mathbf{x}(t_i) &= \mathrm{e}^{\mathbf{A}(t_i)\cdot(t_i-t_{i-1})}\mathbf{x}(t_{i-1}) + \int_{t_{i-1}}^{t_i} \mathrm{e}^{\mathbf{A}(t_i)\cdot(t_i-\xi)} \left[\int_{-\sigma}^{0} \mathbf{B}(\theta)\mathbf{x}(\xi+\theta)\mathrm{d}\theta\right]\mathrm{d}\xi, \quad t \in [t_{i-1}, t_i] \\ &\approx \mathrm{e}^{\mathbf{A}(t_i)\cdot h}\mathbf{x}(t_{i-1}) + \frac{h}{2}\left\{ \left[\int_{-\sigma}^{0} \mathbf{B}(\theta)\mathbf{x}(t+\theta)\mathrm{d}\theta\right]\Bigg|_{t=t_i} + \mathrm{e}^{\mathbf{A}(t_i)\cdot h} \cdot \left[\int_{-\sigma}^{0} \mathbf{B}(\theta)\mathbf{x}(t+\theta)\mathrm{d}\theta\right]\Bigg|_{t=t_{i-1}} \right\}. \end{aligned} \tag{42}$$

However, it does not affect the results and conclusions of the paper. The authors would like to apologize for any inconvenience caused to the readers.

Reference

1. Ding, Y., Zhu, L.M., Zhang, X.J., Ding, H.: Numerical integration method for prediction of milling stability. J. Manuf. Sci. Eng. **133**(3), 031005 (2011)