Nonlinear Dyn (2015) 80:1661–1664
DOI 10.1007/s11071-015-2069-2

EDITORIAL

Fractional dynamics and its applications

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Published online: 10 April 2015

The calculus of fractional order started more than three centuries ago. It was firstly mentioned by Leibniz, and represents a generalization of the classical integer-order differential and integral calculus. In the last sixty years, fractional calculus had played a very important role in various fields such as physics, chemistry, mechanics, electricity, biology, economy and control theory. Moreover, it has been found that the dynamical behavior of many complex systems can be properly described by fractional-order models. Such tool has been extensively applied in many fields which has seen an overwhelming growth in the last three decades.

The Special Issue on Fractional Dynamics and Its Applications of the journal Nonlinear Dynamics includes a collection of 19 papers, encompassing the most important areas of current research on fractional dynamics. The papers of the present special issue can be categorized as follows:

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- dynamical analysis of fractional differential equations and systems;
- fractional-order control theory;
- modeling with fractional calculus and applications.

Various aspects of the fractional generalization of the field theory have been actively studied. In [1], Tarasov considers non-relativistic field equations with the Riesz fractional derivatives of non-integer order. A connection of these equations with microscopic (lattice) models is discussed. By using the background field and the mean field methods, the author obtains corrections to linear and equilibrium solutions caused by the weak nonlinearity.

Čermák and Kisela in [2] formulate explicit necessary and sufficient conditions for the local asymptotic stability of equilibrium points of a kind of fractional differential equation involving two Caputo derivatives. Then, using the recent developments on linearization methods in fractional dynamical systems, they extend the results to the original nonlinear equation.

Meerschaert et al. [3] study a fractional wave equation, replacing the second time derivative by a Caputo derivative of order between one and two. The authors show that the fractional wave equation governs a stochastic model for wave propagation, with deterministic time replaced by the inverse of a stable subordinator, whose index is one-half the order of the fractional time derivative.

The discrete dynamic behavior and its applications have received considerable attention in various applied areas, such as synchronization control, secure communication, biomolecular network evolution, and so on. In [4], Wu and Baleanu report the delayed logistic equation discretized by means of the discrete fractional calculus approach and the related discrete chaos. The Lyapunov exponent together with the discrete attractors and the bifurcation diagrams are given.

Piezoelectric materials play a significant role in harvesting ambient vibration energy. Due to their inherent characteristics and electromechanical interaction, the system damping for piezoelectric energy harvesting can be adequately characterized by fractional calculus. Cao et al. [5] introduce a fractional model for magnetically coupling broadband energy harvesters under low-frequency excitation and investigate their nonlinear dynamic characteristics.

Based on the features of digital image encryption and high-dimensional chaotic sequences, Zhao et al. [6] propose a symmetric digital image encryption algorithm by a new improper fractional-order chaotic system.

Control and synchronization of fractional-order chaotic systems have attracted the attention of many scholars, and several techniques have been used in the scope of chaotic systems. Chaos control and synchronization of second-order non-autonomous fractional complex chaotic systems are discussed by Aghababa in [7].

Wang et al. [8] study the compactness and R_δ-structure of the set of trajectories on a closed domain for the fractional evolution inclusion. Moreover, the authors discuss the R_δ-structure of the set of trajectories to the control problem corresponding to the inclusion.

Although a considerable amount of research has been carried out in the field of fractional-order controllers, the majority of the results deal with stable processes. Limited research has been reported regarding unstable processes. Muresan et al. [9] propose a methodology for designing and tuning fractional-order controllers for a class of unstable second-order processes. The design is carried out using the stability analysis of fractional-order systems, by means of Riemann surfaces and a proper mapping in the w-plane. The resulting fractional-order controllers are implemented with a graphical programming on industrial equipment and are validated experimentally using a laboratory scale magnetic levitation unit.

Fractional-order models have been widely used in modeling and identification of thermal systems. A general model in this category is considered as the model of thermal systems, and a fractional-order controller is proposed for controlling such systems by Badri and Tavazoei [10] . The proposed controller is a generalization of the traditional PI algorithms. The parameters of this controller can be obtained by using a recently introduced tuning method that can simultaneously ensure the following three requirements: desired phase margin, desired gain crossover frequency, and phase flatness of the Bode plot at this frequency.

Shahiri et al. [11] investigate robust control of nonlinear proton exchange membrane fuel cells against uncertainty using fractional complex-order control.

In [12], Boroujeni and Momeni introduce an iterative method to design optimal non-fragile H^∞ observer for Lipschitz nonlinear fractional-order systems. It is shown that not only the iterative method is successful in finding the proper boundary condition, but also the performance of the proposed observer satisfies both non-fragility and robustness to external disturbances with an acceptable accuracy.

Almeida and Torres [13] present a discrete method for solving fractional optimal control problems, where the dynamic control system depends on integer-order and Caputo fractional derivatives. Their approach consists in approximating the initial fractional-order problem by a new one that involves integer-order derivatives only. The latter problem is discretized, by application of finite differences, and then solved numerically.

Saidi et al. [14] discuss robust proportional, integral and derivative (PID) controller of fractional-order design via numerical optimization. Three new frequency-domain design methods are proposed. Several numerical examples are presented to show the efficiency of each proposed method and discuss the obtained results. Also, an application to the liquid carbon monoxide level control is presented.

Inspired in dynamic systems theory and Brewer's contributions to applying it to economics, Machado and Mata [15] establish a bond graph model. Two main variables, a set of inter-connectivities based on nodes and links (bonds) and a fractional-order dynamical perspective, prove to be a good macro-economic representation of countries' potential performance in nowadays globalization. Several experiments analyzed the influence of the memory effects and characterized the dynamical behavior in light of the fractional approach.

Fractional tumor development is considered by Iomin [16] in the framework of one-dimensional continuous time random walks (CTRW) in the presence of chemotherapy. The chemotherapy influence on the CTRW is studied by means of observations of both stationary solutions due to proliferation and fractional evolution in time.

In [17], Coffey et al. emphasize the rectifying effect of a strong bias field superimposed on a strong ac field on the electric polarization (or magnetization) of an assembly of noninteracting dipolar particles. Furthermore, they suggest that experiments should be designed so as to detect the frequency-dependent dc nonlinear response introduced by the bias field. The results can explain the anomalous nonlinear relaxation of complex dipolar systems, where the relaxation process is characterized by a broad distribution of relaxation times. The advantage of having kinetic equations incorporating the anomalous relaxation becomes clearly visible as it enables to study the effect of the nonlinear anomalous behavior on fundamental parameters associated with the fractional diffusion.

Multimedia streaming of three-dimensional (3D) stereoscopic videos over last-generation networks subject to bandwidth limitations is an open problem. Nigmatullin et al. [18] propose a fractional exponential reduction moments approach based on the statistics of the so-called fractional moments. Each random sequence of frames in 3D videos can be analyzed and reduced to a finite set of parameters that allow fitting to the sequence by exponential functions, followed by a characterization and classification of the video with a kind of fingerprint. The approach allows comparing real streams and numerical data output from fractional dynamical models by means of the reduced parameters.

In [19], Muthukumar et al. construct a fractional-order dynamical system that exhibits chaotic and reverse chaotic attractors. This is achieved by changing the sign of the one parameter which involves in the existence of the phase reversal function. A new method of fast projective synchronization of fractional-order dynamical systems is introduced. An affine cipher is proposed for secure communication based on the solutions of the synchronized fractional-order chaotic systems with the support of the sender's and receiver's date of birth.

This selection of high-quality works represents the state of the art in the application of fractional calculus in several distinct research areas. The guest editors hope that these findings will support readers in opening new horizons.

Acknowledgments We would like to thank Professor Ali H. Nayfeh for his kind support on the publication of this special issue. We also wish to express my appreciation to the authors of all articles in this special issue for the excellent contributions as well as many reviewers for their high-quality work on reviewing the manuscripts.

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